A Modigliani-Miller Theorem for
Open-Market Operations

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Monetary policy is most often conceived of as determining the composition of government net indebtedness; for example, the amount of such indebtedness that takes the form of currency and the amount that takes the form of bonds. Fiscal policy, and, in particular, the size of the deficit on current account, determines the path of government net indebtedness. In this paper I will show that government asset exchanges consistent with an unchanged path of fiscal policy are irrelevant in precisely the sense in which the Modigliani-Miller theorem shows that alternative corporate liability structures are irrelevant. Irrelevance here means that both the equilibrium consumption allocation and the price level are independent of such government asset exchanges. The irrelevance proposition I prove has the following form: if there is an equilibrium with certain properties for one path of portfolios for the government, then that equilibrium is also an equilibrium for some other paths of portfolios for the government.

I prove the irrelevance result for a limited class of environments: models of two-period lived, overlapping generations with a single consumption good that is storable via a constant-returns-to-scale, stochastic storage technology. This class is broad enough to include examples that establish the nonvacuousness of the "if" clause of the proposition. Nonvacuousness requires that there be equilibria in which the private sector voluntarily holds real capital and unbacked government liabilities, liabilities that I call fiat money. Non-vacuousness aside, little attempt has been made to strive for generality.

The class of physical environments is described in Section 1. Section 2 describes the conditions for a perfect-foresight competitive equilibrium for these environments. Markets in contingent claims play a prominent role. The irrelevance proposition is presented, remarked upon, and proved in Section 3. It establishes conditions under which the amount of the consumption good purchased by the government in the open market for fiat money and stored by the government is irrelevant.

As was true of the original application of the Modigliani-Miller theorem to corporate liability structures, the irrelevance proposition is useful for suggesting departures from its assumptions. In Section 4, I describe a departure that arises when the nonnegativity restriction on private gross investment is binding, while, in Section 5, I describe a departure that arises when a legal restriction on minimum money holdings is binding. Open-market operations in government bonds, including the role of government private-sector transaction cost asymmetries, are discussed in Section 6. Two common features of departures from the irrelevance proposition emerge: if two different paths of the government's portfolio necessarily imply different equilibria, then they also imply different paths of fiscal policy. Moreover, at least one of the paths can be viewed as running into barriers of one sort or another on private intermediation.

1. The Physical Environment

Time is discrete and there is a single good. At each date t, a new generation of $N(t)$ 2-period-lived individuals (generation t) appears. Each member h of generation t maximizes the expected value of $u^h(\ ,\ )$, where the first (second) argument is consumption of the good by h in the first (second) period of life and where u^h is strictly increasing, strictly concave and twice differentiable.

At each date t, there is a new aggregate endowment of $Y(t) > 0$ units of the consumption good. This good may be consumed or stored. If $K(t) \geq 0$ is the aggregate amount placed into storage at t, then $Y(t+1) + K(t)x(t+1)$ is the total amount available at $t+1$, where $x(t+1)$ is a random variable drawn independently from period to period from a discrete probability distribution: $x(t+1) = x_i > 0$ with probability f_i; $i=1,2,\ldots,I$. The I-element vector $(x_1,x_2,\ldots,x_I)$ will be denoted x. The value of $x(t+1)$ is observed after time t storage is determined and before generation $t+1$ appears. Note that $K(t)$ is the sum of nonnegative private storage $K^p(t)$ and nonnegative government storage $K^g(t)$.

The supply of fiat money is determined by the government. Changes in it do not require the expenditure of resources by the government and private storage of fiat money neither affects its physical properties nor requires the expenditure of resources.

2. The Market Scheme

I will describe the conditions for a perfect foresight competitive equilibrium in terms of time t markets for claims on time $t+1$ consumption in "state" $x(t+1) = x_i$. The members of generation t in their role as consumers demand such claims. Firms, owned by members of generation t in their role as producers, supply such claims by storing the consumption good and by storing fiat money. In general, the government announces a policy, including a lump-sum tax-transfer scheme, in terms of such claims.

The Consumer's Lifetime Choice Problem

The consumer choice problem of the young of generation t is described in terms of the following notation:

$(c_1^h(t), c_2^h(t))$ - The $(I+1)$ element consumption vector of member h of generation t where $c_1^h(t)$ is first-period consumption and $c_2^h(t) = (c_{21}^h(t), c_{22}^h(t), \ldots, c_{2I}^h(t))$, $c_{2i}^h(t)$ being second-period consumption in "state" $x(t+1) = x_i$.

$(w_1^h(t), w_2^h(t))$ - the corresponding $(I+1)$ element endowment vector of member h of generation t, where $w_2^h(t) = (w_{21}^h(t), w_{22}^h(t), \ldots, w_{2I}^h(t))$.

$s(t)$ - the I-element vector $(s_1(t), s_2(t), \ldots, s_I(t))$ where $s_i(t)$ is the price at time t of one unit of $t+1$ consumption in "state" $x(t+1) = x_i$ in units of time t consumption.

Later, it will be convenient to have a notation for the consumption allocation and endowment allocation of generation t: let $c(t)$ $(w(t))$ be the $N(t)(I+1)$ - element vector consisting of one $(c_1^h(t), c_2^h(t))$ $((w_1^h(t), w_2^h(t)))$ vector for each member h of generation t.

All of this notation is meant to allow for possible dependence of, say, $s(t)$ on $x(t)$, $x(t-1)$ and so on. For any variable $\cdot(t)$, dependence on t is used to denote possible dependence on $x(t), x(t-1),\ldots.$ This is a convenient notation because the young of generation t make choices having observed $x(t), x(t-1),\ldots$.

Member h of generation t is assumed to choose a nonnegative vector $(c_1^h(t), c_2^h(t))$ to maximize $\Sigma_i f_i u^h[c_1^h(t), c_{2i}^h(t)]$ subject to

$$c_1^h(t) + s(t)c_2^h(t) \leq w_1^h(t) + s(t)w_2^h(t) \tag{1}$$

where the vector multiplication is inner-product multiplication. For $s(t)$ and $(w_1^h(t), w_2^h(t))$ that imply a non-empty, bounded budget set, there is a unique maximizing vector $(c_1^h(t), c_2^h(t))$ given by the unique solution to (1) at equality and

$$f_i u_2^h[c_1^h(t), c_{2i}^h(t)] = s_i(t) \sum_{j=1}^{I} f_j u_1^h[c_1^h(t), c_{2j}^h(t)]; \qquad i=1,2,\ldots,I \tag{2}$$

This is all that need be said about consumer demand.

The Choice Problem of Firms

In their role as producers, members of generation t may enter one or both of two lines of business at time t: storing the consumption good or storing money. In each line, any producer maximizes profit as a price-taken with regard to $s(t)$ and the time t and time $t+1$ prices of money.

Profit in terms of time t consumption from storing $k \geq 0$ units of the consumption good is $s(t)xk - k$. Since this is linear in k, the condition that storage be finite in any equilibrium implies as an equilibrium condition

$$(3) \qquad s(t)x \leq 1$$

a condition that must hold with equality if total private storage, $K^P(t)$, is positive.

If $p(t)$ is the price of a unit of money at time t in units of time t consumption and $p(t+1)$ is the price of a unit of money at time $t+1$ in terms of time $t+1$ consumption (a function of $x(t+1)$ as of time t), then profit in terms of time t consumption from storing $m \geq 0$ units of fiat money is $s(t)p(t+1)m - m$. Since this is linear in m, finiteness of the supply of money implies that prices in any competitive equilibrium satisfy

$$(4) \qquad s(t)p(t+1) = p(t)$$

We may write equality here, because if firms store no money, then demand falls short of supply and $p(t) = 0$.

Government Policy Rules

Government policy is a specification at time $t=1$ after $x(1)$ has been observed of paths, possibly contingent, for government consumption at t, $G(t) \geq 0$; the endowment vector for generation t, $w(t)$; government storage at t, $K^g(t)$; and the money supply at t, $M(t) \geq 0$. For $t \geq 1$ and each $x(t)$ in x, these are chosen subject to

$$K^g(t) + G(t) = T(t) + K^g(t-1)x(t) + p(t)[M(t)-M(t-1)] \tag{5}$$

Here $T(t)$, total lump-sum taxes minus transfers at t, is defined by

$$T(t) = Y(t) - \Sigma_h w_1{}^h(t) - \Sigma_h w_{2i}{}^h(t-1)$$

and $M(0)$, $w(0)$ (the endowment of the old at $t=1$), and $K^g(0)$ are assumed given as initial conditions. (The summations over h are over the members of generation t and $t-1$, respectively, a convention that will be used throughout.)

Perfect Foresight Competitive Equilibrium

The question of foresight arises with regard to $p(t+1)$ in (4) and with regard to $w_2{}^h(t)$ in (1). Perfect foresight requires that the i-th element of $p(t+1)$ in (4) equal the equilibrium price of money at $t+1$ in "state" $x(t+1) = x_i$ and that the $w_2{}^h(t)$ vector on the basis of which h chooses at t be realized at $t+1$. Put formally, then, for specified government policy consisting of a possibly contingent sequence $(G(t), w(t), K^g(t))$ defined for

$t \geq 1$, a perfect foresight competitive equilibrium consists of nonnegative sequences $c(t-1)$, $s(t)$, $K(t) \geq K^g(t)$, $p(t)$ and $M(t)$ that for all $t \geq 1$ satisfy (1) at equality and (2) for each h, (3) - (5) and

$$(6) \qquad \Sigma_h(c_{2i}^h(t) - w_{2i}^h(t)) = K^p(t)x(t+1) + p(t+1)M(t)$$

for each $x(t+1)$ in x. The LHS of (6) is the aggregate excess demand of consumers for consumption at $t+1$ in state $x(t+1) = x_i$, while the RHS is the supply of such consumption by firms.

3. The Irrelevance Proposition.

The proposition to be proved is as follows:

If $\{\bar{c}(t-1), \bar{s}(t), \bar{K}(t), \bar{p}(t), \bar{M}(t)\}$ is an equilibrium with $\bar{p}(t) > 0$ for all $t \geq 0$ for the policy $\{G(t), w(t), K^g(t)\} = \{\bar{G}(t), \bar{w}(t), 0\}$, then $\{\bar{c}(t-1), \bar{s}(t), \bar{K}(t), \bar{p}(t), \hat{M}(t)\}$ is an equilibrium for the policy $\{\bar{G}(t), \hat{w}(t), \hat{K}^g(t)\}$, where $\{\hat{K}^g(t)\}$ is any nonnegative sequence bounded by $\{\bar{K}(t)\}$ and $\{\hat{w}(t)\}$ is any $w(t)$ sequence that for all $t \geq 1$ satisfies

(a) $\Sigma_h \hat{w}_1{}^h(t) = \Sigma_h \bar{w}_1{}^h(t)$

(b) $\Sigma_h[\hat{w}_{2i}^h(t) - \bar{w}_{2i}^h(t)] = \hat{K}^g(t)[x(t+1) - \bar{p}(t+1)/\bar{p}(t)]$

for all $x(t+1)$ in x.

(c) $\hat{w}_1{}^h(t) + \bar{s}(t)\hat{w}_2{}^h(t) = \bar{w}_1{}^h(t) + \bar{s}(t)\bar{w}_2{}^h(t)$

for all h.

(The notation "$\{\cdot(t)\}$" means a sequence defined for all $t \geq 1$.)

Before giving a proof, it is worth noting that the proposition is not vacuous. Nonvacuousness is established by showing (i) that there exist economies having equilibria with $p(t) > 0$ for all t and $\bar{K}(t) > 0$ for at least some t when $K^g = 0$; and (ii) showing that there always exists a $\{\hat{w}(t)\}$ that satisfies (a) - (c).

(i) An example. Physical environment: For all t, $N(t) = N$, $Y(t) = yN > 0$ and $u^h(z_1, z_2) = \ln z_1 + \ln z_2$ for all h; $x = (x_1, x_2) = (0.5, 2.0)$ and $f_1 = f_2 = 0.5$.

Policy: For all $t \geq 1$, $G(t) = K^g(t) = 0$ and $w_1^h(t) = y$,

$w_{2i}^h(t) = 0$ for all i and h.

Equilibrium: For all $t \geq 1$, $(s_1(t), s_2(t)) = (2/3, 1/3)$,

$K(t)/N = y/4$, $M(t) = M(1)$, $p(t)M(1)/N = y/4$ and

$(c_1^h(t), c_{21}^h(t), c_{22}^h(t)) = (y/2, 3y/8, 3y/4)$ for all h.

(ii) <u>Existence of $\{\hat{w}(t)\}$</u>. One such sequence is given by

$$w_1^h(t) = \bar{w}_1^h(t); \quad w_{2i}^h(t) - \bar{w}_{2i}^h(t) = \hat{K}^g(t)[x(t+1) - \bar{p}(t+1)/\bar{p}(t)]/N(t)$$

for all h, i and $t \geq 1$. This obviously satisfies (a) and (b). To show that it satisfies (c), consider for this scheme

$$\bar{s}_i(t)[w_{2i}(t) - \bar{w}_{2i}(t)] = \hat{K}^g(t)[\bar{s}_i(t)x(t+1) - \bar{s}_i(t)\bar{p}(t+1)/\bar{p}(t)]/N(t)$$

Upon summing both sides over i and using (4), we get

$$\bar{s}(t)[w_2^h(t) - \bar{w}_2^h(t)] = \hat{K}^g(t)[\bar{s}(t)x - 1]/N(t)$$

But this is zero because $\hat{K}^g(t) > 0$ implies $\bar{K}(t) > 0$ and, hence, (3) at equality. This, in turn, implies that the specified $w(t)$ satisfies condition (c).

<u>Proof</u> By condition (c), if $\bar{c}(t)$, $\bar{s}(t)$ and $\bar{w}(t)$ satisfy (1) at equality and (2), then so do $\bar{c}(t)$, $\bar{s}(t)$ and $\hat{w}(t)$. Moreover (3) and (4) hold at equality at the prices $\bar{s}(t)$, $\bar{p}(t)$ with $\hat{K}^p(t) = \bar{K}(t) - \hat{K}^g(t)$. All that remains, then, is to show that (6) is satisfied by the $\hat{M}(t)$ implied by (5) with $p(t) = \bar{p}(t)$.

To find $\hat{M}(t)$, subtract (5) for the $K^g(t) = 0$ policy from (5) for the $K^g(t) = \hat{K}^g(t)$ policy to get

(7) $\hat{K}^g(t) = \hat{T}(t) - \bar{T}(t) + \hat{K}^g(t-1)x(t) + \bar{p}(t)[\hat{M}(t)-\hat{M}(t-1)-\bar{M}(t)+\bar{M}(t-1)]$

Since $M(0)$, $K^g(0)$ and $w_{2i}(0)$ are fixed by initial conditions, condition (a) implies $\hat{T}(1) = \bar{T}(1)$. Thus, for $t=1$, (7) becomes simply

(8) $\hat{K}^g(t) = \bar{p}(t)[\hat{M}(t) - \bar{M}(t)]$

We now show by induction that (8) holds for all $t \geq 1$. If (8) holds for some $\bar{t} \geq 1$, then (7) for $t = \bar{t}+1$ is

(9) $$\hat{K}^g(\bar{t}+1) = \hat{T}(t+1) - \bar{T}(t+1) + \hat{K}^g(\bar{t})x(\bar{t}+1) + \bar{p}(\bar{t}+1)[\hat{M}(\bar{t}+1) - \bar{M}(\bar{t}+1)] - \hat{K}^g(\bar{t})\bar{p}(\bar{t}+1)/\bar{p}(\bar{t})$$

But, for all t

$$\hat{T}(t+1) - \bar{T}(t+1) = \Sigma_h[\bar{w}_{2i}^h(t) - \hat{w}_{2i}^h(t)] = \hat{K}^g(t)[x(t+1) - \bar{p}(t+1)/\bar{p}(t)]$$

where the first equality follows from condition (a) and the second from condition (b). Upon substituting this into (9), we get (8) for $t = \bar{t}+1$ as required.

Now, by the hypothesis of the proposition (see (6)),

$$\Sigma_h\bar{c}_{2i}^h(t) - \Sigma_h\bar{w}_{2i}^h(t) = \bar{K}(t)x(t+1) + \bar{p}(t+1)\bar{M}(t)$$

Upon substituting for $\Sigma_h\bar{w}_{2i}^h(t)$ from condition (b), we have

$$\Sigma_h\bar{c}_{2i}^h(t) - \Sigma_h\hat{w}_{2i}(t) = [\bar{K}(t) - \hat{K}^g(t)]x(t+1) + \bar{p}(t+1)[\bar{M}(t) + \hat{K}^g(t)/\bar{p}(t)]$$

Finally, using (8), we get

$$\Sigma_h\bar{c}_{2i}^h(t) - \Sigma_h\hat{w}_{2i}(t) = [\bar{K}(t) - \hat{K}^g(t)]x(t+1) + \bar{p}(t+1)\hat{M}(t)$$

which is (6) for the asserted equilibrium under the $\hat{K}^g(t)$ policy. This completes the proof.

Before going on to discuss various departures from the assumptions of the irrelevance proposition, I want to comment on the sense in which the $\hat{w}(t)$ scheme holds fiscal policy fixed.

In general, different paths of the government's portfolio imply correspondingly different paths of interest earnings for the government. Consistent with the standard national-income-accounts practice of treating net interest received by the government as a component of taxes minus transfers, unchanged fiscal policy in the face of alternative paths for the government's portfolio calls for adjustments in other components of taxes minus transfers. The $\hat{w}(t)$ sequence constitutes such an adjustment.

As shown in the proof, conditions (a) and (b) on $\{\hat{w}(t)\}$ imply equation (8) for all $t \geq 1$: the path of net government wealth at $\{\bar{p}(t)\}$ is unaffected by the path of the government's portfolio -- $\bar{p}(t)\bar{M}(t)$ is net government indebtedness under the $K^g(t) = 0$ policy while $\bar{p}(t)\hat{M}(t) - \hat{K}^g(t)$ is net government indebtedness under the alternative policy. But to get irrelevance, in addition to holding the path of "aggregate" fiscal policy fixed, the distributional aspects of fiscal policy must also be held fixed. This is accomplished by condition (c). Indeed, if one adopts (8) for all $t \geq 1$ and condition (c) as a definition of holding fiscal policy constant, then the irrelevance proposition can be viewed as (i) giving conditions under which fiscal policy can be held fixed; and (ii) showing that *if* fiscal policy *is* *in* fact held *fixed*, then irrelevance holds. In this connection, note that an easy induction argument shows that condition (c) and equation (8) for all $t \geq 1$ imply conditions (a) and (b). Moreover, equation (8) can be taken as supplying a definition of "earnings" on alternative portfolios.

4. Binding Nonnegativity of Private Storage

If $K^g(t) > \bar{K}(t)$ for some t, then there is no value of $K^p(t)$ consistent with unchanged total accumulation at t and irrelevance cannot hold. Moreover, in simple examples at least, $K^g(t) > \bar{K}(t)$ amounts to a subsidy on storage financed by lump-sum taxes with the subsidy being greater and the price of money lower the greater is K^g.

We will illustrate this for the economy of the example given in the last section except that we now assume $K^g(t)/Y(t) = \theta$, $w_{2i}^h(t) = \theta y[x(t+1)-1]$ for all h and $t \geq 1$ and $\Sigma_h w_{21}^h(0) = K^g(0)x(1)$. For each θ in [0, 1/2], there is a stationary equilibrium with $p(t) = p_\theta > 0$ for all $t \geq 1$.[1/] For $\theta \leq 1/4$, the irrelevance proposition holds. For $\theta > 1/4$, the stationary solution is found by first solving the relevant versions of (1), (2), (4), and (6) with $K^p(t) = 0$ for c_1^h, c_{21}^h, c_{22}^h, s_1, s_2 and $(p_\theta M_\theta)$.[2/] Then p_θ may be found using the relevant version of (5); namely,

$$(10) \qquad \theta y = p_\theta M_\theta/N - (\bar{p}\bar{M}/N)(p_\theta/\bar{p}) = p_\theta M_\theta/N - (y/4)(p_\theta/\bar{p})$$

where $\bar{p}$ and $\bar{M}$ are the equilibrium values for $K^g = 0$. Without displaying the numerical solutions, we can show that $p_\theta/\bar{p} < 1$ for some θ.

In a stationary equilibrium for this economy

$$(11) \qquad y = c_1(t) + K^g/N + (\bar{p}\bar{M}/N)(p_\theta/\bar{p}) = c_1(t) + \theta y + (y/4)(p_\theta/\bar{p})$$

which simply describes the disposition of the per capita endowment of the young at $t = 1$, $p_\theta\bar{M}/N$ being the amount that goes to the current old. For this example, $c_1(t)$ is equal to half of wealth, which by (1) and (4) implies

(12) $$c_1(t) = y - \theta y(1-sx) \geq y - \theta y/2$$

The inequality follows from noting that sx is a minimum at $s_1 = 1$ for s satisfying (4). This inequality and (11) imply $p_\theta/\bar{p} \leq (2-3/\theta)$ or $p_\theta/\bar{p} < 1$ for $\theta > 1/3$.

An alternative way to generate stationary equilibria with $\theta > 1/4$ is to treat $(p_\theta/\bar{p})$ as a policy instrument; the interpretation is that the government announces a price of money, p_θ, satisfying $0 < p_\theta/\bar{p} < 1$ at which it is willing to sell (or buy) money in exchange for the consumption good at any time. The equilibrium is found by solving (10) and the relevant versions of (1), (2), (4), (6) and $K^p(t) = 0$ for θ, c_1^h, c_{21}^h, c_{22}^h, s_1, s_2 and $p_\theta M_\theta$.

There is, of course, nothing "neutral" about alternative values of $p_\theta/\bar{p}$ accomplished in either of these equivalent ways, a point I will comment on at the end of the next section.

5. Globally Binding Legal Minimum Money Holdings

The model described in section 2 is one of voluntarily-held money, equation (4) being a consequence. In fact, equation (4) is a consequence if <u>some</u> money is held voluntarily. But it is easy to construct a model and there may be historical instances in which money is held <u>only</u> to meet prescribed legal restrictions. In such situations, money can have value in an equilibrium with the LHS of (4) less than the RHS and irrelevance need not hold.

I will illustrate the nonirrelevance possibility by way of an example with a "reserve requirement": storage of k units of the consumption good from t to $t+1$ must be accompanied by storage of money from t to $t+1$ whose value at t is at least equal to ρk for $\rho \geq 0$. The physical environment of the economy I use is $N(t) = 1$ and $Y(t) = y > 0$ for all t, $u^h(c_1, c_2) = u(c_1, c_2)$ with c_1 and c_2 being normal goods, and $x = (x_1) = \bar{x} > 1$. For policy I assume $G(t) = 0$, $K^g(t) = K^g$, $w_1^{\,h}(t) = y$ and $w_2^{\,h}(t) = K^g(\bar{x}-1)$ for all $t \geq 1$, and $\Sigma_h w_{21}^h(0) = K^g(0)x(1)$. I will describe the dependence of the stationary equilibrium on the parameter K^g.

At any price s, profit from storing k units of the consumption good from t to $t+1$ consists of the profit from storing the good and the profit from storing the required money; namely, $(s\bar{x} - 1)k + [sp(t+1)/p(t) - 1]\rho k$. It follows that at $p(t+1) = p(t) = p > 0$, $s \leq (1+\rho)/(\bar{x}+\rho) < 1$ in any competitive equilibrium. It also follows that no additional money is stored at any $p(t+1) = p(t) = p > 0$. That being so, the relevant version of (6) implies $c_2 = (\bar{x}-\rho)K^p + K^g(\bar{x}-1)$. This and (1) imply $c_1 = y - (1+\rho)K^p$. Then, letting

$v(c_1, c_2)$ denote the function $u_1(c_1, c_2)/u_2(c_1, c_2)$, we may summarize (1) - (4) and (6) by condition

$$(13) \qquad v[y - (1+\rho)K^p, (\bar{x}+\rho)K^p + K^g(\bar{x}-1)] = (\bar{x}+\rho)/(1+\rho) \quad \text{if} \quad K^p > 0.$$

A second condition on K^p and K^g (and p) is the relevant version of (5),

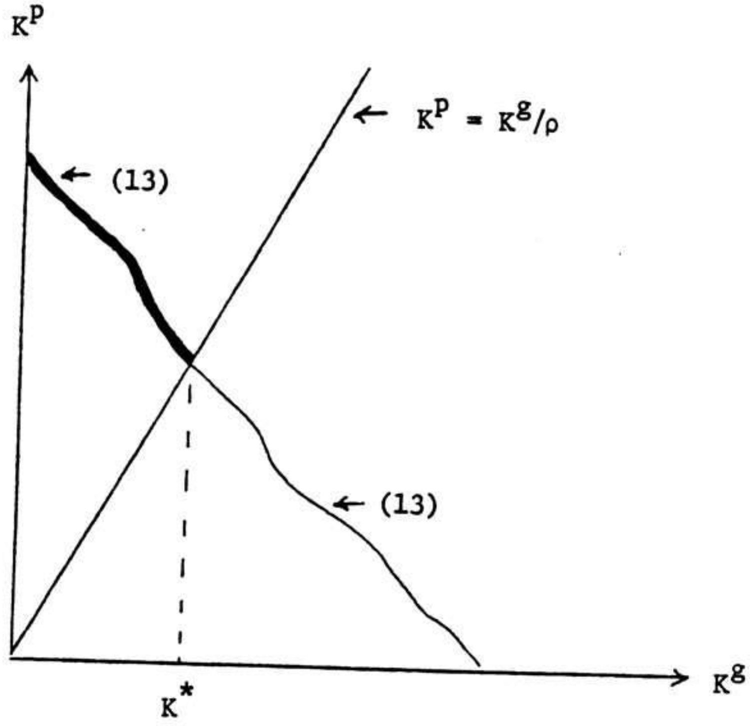
$$(14) \qquad K^g = \rho K^p - p\bar{M}$$

Here $\bar{M}$ is what the money supply would be if $K^g = 0$. Since $p\bar{M} \geq 0$, this implies $K^g \leq \rho K^p$.

Our first task is to find the pairs (K^g, K^p) that satisfy this inequality and (13). The bold faced curve in Figure 1 constitutes this set.[3/] It follows that for any K^g in $[0, K^*)$, there exists a stationary equilibrium; find K^p from (13) and, then, p from (14). It is immediate that p is decreasing K^g.

In this example and that of the last section, open market operations have the usually asserted qualitative effects on the price of money. This implies that the welfare of the current old (at t=1) is affected in a similar way by such operations. But there the similarity ends. In section 4, $K^g > \bar{K}$ implies net taxes on the young and, in simple examples, makes everyone worse off than they are with $K_g \leq \bar{K}$. In this section, $K_g > 0$ implies a net subsidy to the young and makes them better off than with $K^g = 0$. Note that in both cases, unchanged fiscal policy is not consistent with different government portfolios. Note also that open market operations seem to be consistent with "neutrality" in the sense of an unchanged real equilibrium only when the irrelevance proposition holds. When it holds, "neutrality" is accompanied by an unchanged price of money.

Figure 1



6. Government Bonds and Private Versus Government Intermediation Costs

So far I have described open-market operations in (titles to) real capital. In the United States, open-market operations are largely conducted in government bonds, which for this discussion, I assume take the form of default-free, zero coupon titles to fiat money in the future (discount bonds).

In order for open-market operations in such bonds to matter, it is necessary that these bonds not always sell at face value. But getting coexistence of voluntarily-held fiat money and interest-bearing bonds is not easy.[4/] Consider a bond which at time t is a title to one dollar at time t+k. At time t+k-1, it and one dollar are both titles to one dollar at t+k. Hence, if both are held, then the bond must sell for one dollar at t+k-1. By induction, then, the bond must sell for one dollar at time t. It seems evident that to avoid this one must somehow place barriers in the way of trading in bonds.

The necessity to restrict bond trading is, of course, one of the messages of the inventory models of money demand (Baumol (1952), Tobin (1956), Miller-Orr (1966)). In those models, individuals and firms require money in order to make purchases. Why, though, cannot government bonds be "spent"? One answer is that bonds are not "spent" because they are available only in large, inconveniently sized denominations.

To convince yourself that this indivisibility is the only thing that makes bonds different from currency, consider the following hypothetical situations. Suppose the Federal Reserve stood ready to convert on demand large denomination Treasury Bills into small denomination bills that are equivalent in terms of face value and maturity date. Would Treasury Bills sell at a discount in such circumstances? Alternatively, imagine that the Federal Reserve ceased issuing currency in anything but thousand-dollar denominations. Absent a prohibition, there would presumably appear private sector, one-hundred-percent-reserve intermediaries who, on the model of mutual funds, would issue smaller denominations. In such circumstances, the thousand-dollar bill would sell at a discount in terms of smaller denomination intermediary liabilities.[5/] Indeed, this situation would be approximated if the Federal Reserve were to charge for new currency in a way that reflects its costs, a proposal now under consideration.

If government bonds sell at a discount only because they are issued in large denominations which on the margin at least have to be intermediated by the private sector using a costly technology, then one is immediately led to ask whether the government should ever issue such things. If government resource costs do not depend on the composition of its liabilities, then, price discrimination considerations aside, it should not. Indeed, this answer is implied by the inventory models of money demand. An increase in bonds and a decrease in money in those models is accompanied by an increase in the yield on bonds sufficient to induce additional trips-to-the-bank, additional phone calls to the broker, and so on. The higher interest on bonds must be financed by higher taxes, the effect being a higher subsidy on trips-to-the-bank financed by higher taxes. Explicit general equilibrium expositions of this sort of distortion are given in Bryant and Wallace (1979 a, b).

This particular asymmetry -- namely, bonds impose resource costs on the public but do not allow the government any cost savings -- is far-fetched.[6/] Klein (1973) argued that any issuer is induced to issue large denomination securities only because issuing smaller denominations would involve additional resource costs. Certainly, Federal Reserve Bank money departments (currency handling departments) use up resources. Indeed, sufficient symmetry would give rise to the irrelevance result. It is possible that more government bonds and less currency outstanding implies no more than a shift of intermediation activities from the government to the private sector with the additional interest cost to the government being matched by reduced government consumption in the form of reduced resource expenditures on processing currency.

Whether or not such symmetry holds, the analysis of open-market operations in government bonds in models that are consistent with the coexistence of voluntarily held valued money and interest bearing bonds is very different from the analysis of open-market operations in typical macroeconomic models. While the latter pay lip service to the inventory models of money demand, their results seem suspiciously like those that would come from a model with a globally binding legal restriction on minimum money holdings. For example, a careful drawing out of the implications of the inventory models of money demand has to recognize that accompanying alterations in the interest rate on safe assets must be alterations in the amount of resources expended on trips-to-the-bank. This, in turn, alters the amount of output available for consumption and investment.

Concluding Remarks

Most economists are aware of considerable evidence showing that the price level and the amount of money are closely related. That evidence, though, does not imply that the irrelevance proposition is inapplicable to actual economies. The irrelevance proposition applies to asset exchanges under some conditions. Most of the historical variation in money supplies has not come about by way of asset exchanges; gold discoveries, banking panics, and government deficits and surpluses account for much of it. Nothing in the models for which the irrelevance proposition holds denies that such occurrences alter the price level in the usual way. The applicability of the irrelevance proposition can, perhaps, be judged by examining periods of exogenous asset exchanges. Two episodes that come to mind are the 1920's Federal Reserve gold sterilization program and the large purchases of government bonds by the Federal Reserve in the post World War II pre-accord period.

Perhaps the main plea to be made for the irrelevance proposition is that it, and the environments in which it holds, should serve as the starting point for analyses of government asset exchanges. This is the same plea that is made for the Modigliani-Miller theory as a theory of corporate liability structures. The applicability of complete competitive markets to open market operations seems no more far-fetched than its applicability to corporate liability structures. After all, economies of complete competitive markets are ones in which a prohibition on the institution of limited liability does not matter. This last implication seems, if anything, more far-fetched than the notion that it matters little whether or not the government stands ready to convert its large denomination liabilities into a wide range of equivalent smaller denomination liabilities.

Footnotes

1/ It is not true that 1/2 is an upper bound on θ. An (unattainable) upper bound is 2/3.

2/ The solution for s_1 is $[5\theta/2 - 2 + 2(1 + \theta/2 + 73\theta^2)^{1/2}]/6\theta$. One must, of course, verify that the (s_1, s_2) solution satisfies (3).

3/ That (13) is as pictured follows from c_1 and c_2 being normal goods, which implies $v_1 < 0$, $v_2 > 0$.

4/ The word voluntary is crucial. In the presence of a globally binding restriction on minimum money holdings, it is easy to get this coexistence. For example, in the model of Section 5, a small amount of one-period government bonds that do not qualify as reserves sells at $(1+\rho)/(x+\rho)$ per dollar of face value.

5/ Intermediary liabilities have the following form. Upon demand, the intermediary pays out a \$1,000 Federal Reserve note in exchange for its notes with a total face value equal to \$1,000. In general, though, when presented with a \$1,000 Federal Reserve note, it pays out its own notes with a total face value less than \$1,000, or, equivalently, charges a fee for its own notes.

6/ I am indebted to Robert E. Lucas, Jr. of the University of Chicago and to my colleague, Christopher A. Sims for emphasizing the arbitrariness of this asymmetry.

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Corrections for "A Modigliani-Miller Theorem for Open Market Operations"

page 13 -- line 9: change " p   0" to "$p_\theta > 0$"

1st term in equation (11): change "$(p_\theta(\bar{p})$ to $(p_\theta/\bar{p})$

page 15 --- 2nd line from bottom: change "x" to "$\bar{x}$".

page 16 -- lines 7 and 8 from bottom: change "K_g" to "K^g".