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Shocks, Learning, and Persistence

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Shocks, Learning, and Persistence  
by  
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In rough outline, the "business cycle" seems to be occasional precipitous drops in output and employment followed by a smooth convergence back to a "full employment" path. There are two common explanations for the business cycle.

One explanation is that there are occasional permanently nonneutral shocks to the economy. Most macroeconomic models assume this. A problem with this explanation is the apparent return to an unchanged growth path. One solution to this problem is that the short-run adjustment to a shock dwarfs the lasting effect upon the growth path.

A second explanation for the business cycle is occasional transitorily nonneutral shocks to the economy. The business cycle model of Lucas (5) belongs to this class. A problem with this explanation is the convergence back to the growth path. Why is the return not immediate? A solution to this problem, too, is the short-run adjustment to a shock.

There are three classes of short-run adjustment mechanisms for these two explanations for the business cycle. First, stock adjustments are often assumed. A decrease in output caused by a shock reduces capital stock or increases inventories. In subsequent periods, output is effected negatively until the stock returns to its growth path. The models of Kydland and Prescott (3) and of Bryant (1) are of this form, for example. A problem with this adjustment mechanism is that, empirically, stock adjustments over the business cycle seem to be small.

A second mechanism to produce persistence is costs of adjustment which decrease over time. After reducing, say, employment, it is costly to restore

it quickly to its previous level. The model of Lucas (4) is of this form, for example. A problem with this mechanism is that the precipitous employment declines characterizing business cycles indicate that these adjustment costs are either small or asymmetric.

The last mechanism for adjustment is the learning process. Typically, it is assumed that economic agents face a signal extraction problem. It takes the agent time to learn the nature of a shock, and until she does, she assumes there is some chance that a permanent real adjustment is called for. Naturally, this explanation only works for shocks that are frequently transitory. The Lucas (5) model is of this type. A problem with this mechanism is that in most models if agents shared information the adjustment period would disappear. Therefore, one needs a reason why individuals cannot share information. Brute force restrictions on communication are unconvincing. Therefore, one needs a model showing that individuals' solution strategies involve successfully concealing their information. However, such models are complicated.

This paper presents a different model of the learning process. A "business cycle" is generated by the process of learning following a shock, even with complete sharing of information. Indeed, a key difference in the model is that all agents have the same information. This implies that one cannot attenuate the business cycle by improving information flow, counter to many models of output and employment fluctuation.

There is a problem common to both permanent and transitory shock explanations for the business cycle. Neither can explain the most obvious asymmetry in the stylized business cycle: we do not observe precipitous rises from the growth path in output and employment followed by convergence back to that path. The proposed new model does generate this asymmetry, however.

The Model

Now let us turn to our model. Our model assumes real exogenous shocks to the economy. It seems more realistic to assume shocks to supplies of inputs than the other possibilities: shocks to technologies or preferences. However, observed shocks to input supplies start a learning process only if the implied changed ratios of inputs have unknown effects on outputs. To capture this effect while retaining a simple model, we assume a nonstochastic labor supply and treat shocks to the output technology as a function of this single input alone.

The model is as follows. Time is discrete and without beginning or end. There are an uncountably infinite number of individuals indexed by  $s \in [0,1]$  who live and have lived forever. The individuals are equally endowed with an equal amount of labor each period, the total amount of labor being  $L$ . There is a single nonstorable but transferable consumption good. This consumption good alone enters individuals' utility functions, it enters them positively without satiation. The utility functions are strictly concave.

Now let us turn to the technologies for converting labor into the consumption good. Each individual is endowed with four linear technologies for producing the consumption good,  $f_{i,s}(L_{i,s}) = a_{i,s}L_{i,s}$ ,  $L_{i,s} \geq 0$ ,  $i = 0, 1, 2, 3$ . An individual can only use one of the technologies 1, 2, and 3 in a period, and a technology can only be used by one individual.

$$(a_{0,s}, a_{1,s}, a_{2,s}, a_{3,s}) \in [0,1] \times U\{(1,0,0), (0,1,0), (0,0,1)\}$$
$$\equiv [0,1] \times \bigcup_{i=1}^3 E_i.$$

The  $a_{i,s}$  are random variables distributed as follows. The  $a_{i,s}$  for all  $i,s$  are determined in a drawing. An (independent) drawing occurs in the beginning of any period with probability  $p > 0$  if a drawing has not occurred in the past two periods, and with probability zero otherwise. If a drawing does not occur in a period,  $a_{i,s}$  equal their previous period values. In a drawing,  $a_{0,s}$  are all equal

to  $a_0$ , which is distributed uniformly on  $[0,1]$ .  $(a_{1,s}, a_{2,s}, a_{3,s})$  are i.i.d. and equal  $E_1$ ,  $E_2$ , or  $E_3$  with probability  $1/3$  each. The outcome of the drawing on  $a_0$  is known after the drawing occurs, while the outcome on  $\{(a_{1,s}, a_{2,s}, a_{3,s})\}$  is unknown. The fact that a drawing has occurred is known.

Now we turn to our solution of the model. Individuals can perfectly diversify against the risk of technologies 1, 2, and 3 by forming firms of positive mass, or risk sharing in some other way. As the individuals are risk averse, they do so. We examine the behavior of such firms.

The objective function of the firm is to maximize output. The output is then divided equally between individuals. An individual can learn about his technologies 1, 2, and 3 by using an arbitrarily small amount of labor in one of them a period. We assume that they can learn in every period even if  $L_{1,s} = L_{2,s} = L_{3,s} = 0$ , with the understanding that this produces the supremum of outcomes with policies for which  $(L_{1,s}, L_{2,s}, L_{3,s}) \neq 0$ . Therefore,  $1/3$  of the individuals find a good technology in the first period after a shock (drawing),  $1/3$  find a good technology in the second period after a shock, and the rest in the third period by elimination.

Now we are ready to describe the firm's behavior. The firm treats identical individuals identically. Let  $L_0^1$  be the first period after shock per person input of labor in the  $0^{\text{th}}$  technology. Let  $L_0^{2,1}$  be the second period per person input of labor in the  $0^{\text{th}}$  technology of individuals who do not find a good technology in the first period. Let  $L_0^{2,2}$  be the second period per person input of labor in the  $0^{\text{th}}$  technology of individuals who do find a good technology in the first period. Let  $L_0^3$  be the third period per person input of labor in the  $0^{\text{th}}$  technology. Then, in the first period, the firm's output per person is

$$a_0 L_0^1 + 1/3(L - L_0^1).$$

In the second period it is:

$$1/3[a_0L_0^{2,2} + L - L_0^{2,2}] + 2/3[a_0L_0^{2,1} + 1/2(L - L_0^{2,1})]$$

and in the third period it is:

$$a_0L_0^3 + L - L_0^3.$$

This is maximized for

$$a_0 \in \left\{ \begin{array}{l} [0, 1/3] \\ (1/3, 1/2) \\ (1/2, 1) \end{array} \right\}$$

by

$$\left\{ \begin{array}{l} L_0^1 = L_0^{2,1} = L_0^{2,2} = L_0^3 = 0 \\ L_0^1 = 1, L_0^{2,1} = L_0^{2,2} = L_0^3 = 0 \\ L_0^1 = L_0^{2,1} = 1, L_0^{2,2} = L_0^3 = 0 \end{array} \right\}.$$

We can ignore the set of measure zero where  $a_0 \in \{1/3 \cup 1/2 \cup 1\}$ .

Now we turn to one interpretation of this result. Define  $W^i$  as the sum of labor in technologies 1, 2, and 3 per person, and  $Q^i$  as the sum of output in technologies 1, 2, and 3 per person, in the  $i^{\text{th}}$  period following a shock. One way to view this is that technology 0 is "leisure" or working at home, out of the economy. Then our solution can be tabled as in Table 1.

Table 1

Employment and Output Post Shock

$a_0$	$w^1$	$w^2$	$w^3$	$Q^1$	$Q^2$	$Q^3$
$[0, 1/3)$	L	L	L	1/3L	2/3L	L
$(1/3, 1/2)$	0	L	L	0	2/3L	L
$(1/2, 1)$	0	1/3L	L	0	1/3L	L

We have, then, our stylized business cycle. Note that increasing the value of leisure decreases employment and output, while it makes individuals better off.

Embellishments

Now we briefly suggest some possible embellishments to the model that might make it more "realistic."

First, instead of assuming individual technologies, one can assume a multi-good world where each set of technologies produces a separate single good. Individuals do not, then, all jump to good technologies, and one does not have to impose "immobility." One still has to impose some "lumpiness" to guarantee that the learning process takes more than one period. For example, one can assume that only the total output of a good is observable, and have  $(a_{1,s}, a_{2,s}, a_{3,s})$  be uniformly distributed on or within the unit cube.

Second, we assumed that shocks are neutral in the sense of not altering the production possibility curve. One could also assume that shocks alter the production possibility curve, say  $(a_{1,s}, a_{2,s}, a_{3,s}) \in U \alpha E_i$ ,  $\alpha \in [0, 2]$ . If one does so, a positive shock, an increase in  $\alpha$ , can easily produce a temporary decrease in output and employment.

Lastly, note that the model does not yield productivity rising in a downturn. This is easily "fixed" by having different skill levels of workers, say  $a_{1,s_j}, a_{2,s_j}, a_{3,s_j} \in U z_j E_i$ ,  $z_j \in [0, 2]$ . If one does so, the relatively unskilled (low  $z$ ) workers are unemployed more frequently, tending to raise productivity in a downturn.

## References

- (1) Bryant, J. 1978, "Relative Prices and Inventory Investment," Journal of Monetary Economics 4, 85-102.
- (2) \_\_\_\_\_. 1979, "Demand Management: An Illustrative Example," Staff Report #46, Federal Reserve Bank of Minneapolis.
- (3) Kydland, F., and E. Prescott. 1978, "On the Possibility and Desirability of Stabilization Policy," Working Paper, Carnegie-Mellon University.
- (4) Lucas, R. 1967, "Adjustment Costs and the Theory of Supply," Journal of Political Economy 75, 321-334.
- (5) \_\_\_\_\_. 1977, "Understanding Business Cycles," Journal of Monetary Economics 5, 7-30.