Handout
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A Rational Expectations Equilibrium Model of the Cyclical Behavior of Inventories and Employment

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Notation

$S(t)$ = Sales of the representative firm at time t.

n = Number of firms in the industry, assumed constant over time.

$(1-\sigma)$ = The rate of depreciation per unit of time of inventories of finished goods.

$L(t)$ = The amount of labor employed by the representative firm during time t.

$Q(t)$ = Output of the representative firm at time t.

$I(t)$ = Stock of inventories of finished goods of the representative firm at the beginning of period t.

$P(t)$ = Price of a finished good sold during period t.

$\omega(t)$ = Rental rate of labor at time t.

$Z(t)$ = A (px1) vector whose first element is $\omega(t)$ and whose second element is $P(t)$; the remaining elements of $Z(t)$ are variables that help to predict $\omega(t)$'s and/or future $P(t)$'s.

$\rho(t)$ = A random shock to costs.

$\psi(t)$ = A random shock to costs.

ℓ_1 = A (1xp) row vector with one in the first place and zeroes elsewhere.

ℓ_2 = A (1xp) row vector with one in the second place and zeroes elsewhere.

Ω_t = The information set of the representative firm at time t.

B = A discount factor, $0 < B < 1$.

a, d, e, f, and g are positive scalar constants, $\frac{f}{g} > (1-\sigma)$.

$$\zeta(L) = I - \sum_{j=1}^{q} \zeta_j L^j$$

where ζ_j is a (pxp) matrix, j = 1, ..., q, and I is the (pxp) identity matrix.

$$\delta_\psi(L) = 1 - \sum_{j=1}^{r_\psi} \delta_{\psi_j} L^j,$$

where δ_{ψ_j}, j = 1, ..., r_ψ, is a scalar

$$\delta_\rho(L) = 1 - \sum_{j=1}^{r_\rho} \delta_{\rho_j} L^j,$$

where δ_{ρ_j}, j = 1, ..., r_ρ, is a scalar.

<u>The Problem of the Representative Firm</u>

Maximize

$$\lim_{N\to\infty} \sum_{t=0}^{N} B^t\Big\{P(t)[aL(t)-I(t+1)+\sigma I(t)] - \sigma(t)L(t) - \frac{d}{2}[L(t)+\rho(t)]^2 - \frac{e}{2}[L(t+1)-L(t)]^2 - \frac{f}{2}[I(t)+\psi(t)]^2 - \frac{g}{2}[I(t+1)-\sigma I(t)]^2\Big\} \qquad (4)$$

subject to L_0, I_0 given and

$$\delta_\psi(L)\psi(t) = U_t^\psi \qquad (1)$$

$$\delta_\rho(L)\rho(t) = U_t^\rho \qquad (2)$$

$$\zeta(L)Z(t) = U_t^Z \qquad (3)$$

$\rho(t)$, $\psi(t)$ and $Z(t)$ are of mean exponential order less than $1/\sqrt{B}$, and Ω_t includes at least $\{I(t), I(t-1), \ldots L(t), L(t-1), \ldots \psi(t), \psi(t-1), \ldots \rho(t), \rho(t-1), \ldots \omega(t), \omega(t-1), \ldots P(t), P(t-1), \ldots\}$, as well as the parameters of (1), (2) and (3).

The unique solution of the above problem that satisfies the Euler equations and the transversality condition is:

$$L(t+1) = \lambda_1 L(t) - \frac{\lambda_1 B}{e} \sum_{j=0}^{\infty} (\lambda_1 B)^j [dE_t\rho(t+j+1) + E_t\omega(t+j+1)$$

$$- aE_t P(t+j+1)] \tag{9}$$

$$I(t+1) = \lambda_3 I(t) - \frac{\lambda_3 Bf}{\sigma g} \sum_{j=0}^{\infty} (\lambda_3 B)^j E_t\psi(t+j+1) - \frac{\lambda_3 P(t)}{\sigma g}$$

$$+ (1-\frac{\lambda_3}{\sigma})\frac{\lambda_3 B}{g} \sum_{j=0}^{\infty} (\lambda_3 B)^j E_t P(t+j+1) \tag{14'}$$

$0 < \lambda_1 < 1/\sqrt{B}$, $0 < \lambda_3 < 1/\sqrt{B}$.

Neither (9) nor (14') are decision rules because terms like $E_t\rho(t+j+1)$, $E_t\rho(t+j+1)$, $E_t\psi(t+j+1)$, for all j greater than zero, are unknown to firms at time t. Such terms must be expressed as functions of the elements of agents' information set, Ω_t.

By imposing rational expectations and substituting in the optimal predictions, we find that the decision rules are:

$$L(t+1) = \lambda_1 L(t) - \frac{\lambda_1 B}{e}\{(\ell_1 - a\ell_2)[\frac{L^{-1}-L^{-1}\zeta^{-1}(\lambda_1 B)\zeta(L)}{1-\lambda_1 BL^{-1}}]\}Z(t)$$

$$- \frac{\lambda_1 Bd}{e} \frac{[L^{-1}-L^{-1}\delta_\rho^{-1}(\lambda_1 B)\delta_\rho(L)]}{1-\lambda_1 BL^{-1}}\rho(t) \tag{11}$$

$$I(t+1) = \lambda_3 I(t) + - \frac{\lambda_3 P(t)}{\sigma g} + (1-\frac{\lambda_3}{\sigma})\frac{\lambda_3 B}{g}\ell_2$$

$$[\frac{L^{-1}-L^{-1}\zeta^{-1}(\lambda_3 B)\zeta(L)}{1-\lambda_3 BL^{-1}}]Z(t) - \frac{\lambda_3 Bf}{\sigma g}[\frac{L^{-1}-L^{-1}\delta_\psi^{-1}(\lambda_3 B)\delta_\psi(L)}{1-\lambda_3 BL^{-1}}]\psi(t). \tag{15}$$

If private agents observe $\rho(t)$ and $\psi(t)$ but the econometrician does not, the econometric model becomes

$$L(t+1) = \lambda_1 L(t) - \frac{\lambda_1 B}{e}\{(\ell_1 - a\ell_2)[\frac{L^{-1}-L^{-1}\zeta^{-1}(\lambda_1 B)\zeta(L)}{1-\lambda_1 BL^{-1}}]\}Z(t) + e_1(t) \tag{a}$$

$$I(t+1) = \lambda_3 I(t) + - \frac{\lambda_3 P(t)}{\sigma g} + (1-\frac{\lambda_3}{\sigma})\frac{\lambda_3 B}{g}\ell_2$$

$$[\frac{L^{-1}-L^{-1}\zeta^{-1}(\lambda_3 B)\zeta(L)}{1-\lambda_3 BL^{-1}}]Z(t) + \varepsilon_2(t) \qquad (15')$$

$$\zeta(L)Z(t) = U^Z(t) \qquad (c)$$

or

$$L(t+1) = \lambda_1 L(t) = b(L)Z(t) + e_1(t) \qquad (a')$$

$$I(t+1) = \lambda_3 I(t) + C(L)Z(t) - \frac{\lambda_3}{\sigma g}P(t) + \varepsilon_2(t) \qquad (b')$$

$$\zeta(L)Z(t) = U^Z(t) \qquad (c')$$

where

$$\varepsilon_1(t) = - \frac{\lambda_1 Bd}{e}[\frac{L^{-1}-L^{-1}\delta_\rho^{-1}(\lambda_1 B)\delta_\rho(L)}{1-\lambda_1 BL^{-1}}]\delta_\rho^{-1}(L)U^\rho(t)$$

$$\varepsilon_2(t) = - \frac{\lambda_3 Bf}{\sigma g}[\frac{L^{-1}-L^{-1}\delta_\psi^{-1}(\lambda_3 B)\delta_\psi(L)}{1-\lambda_3 BL^{-1}}]\delta_\psi^{-1}(L)U^\psi(t)$$

$$b(L) = \sum_{j=0}^{q-1} b_j L^j$$

and

$$c(L) = \sum_{j=0}^{q-1} c_j L^j.$$

Notice that because the $b_j(L)$ and $c_j(L)$, $j = 1, \ldots, q - 1$, are nonlinear functions of production parameters, as well as the parameters of $\zeta(L)$, estimation of the system $\{a', b', c'\}$ is subject to cross-equation restrictions.

Example

If

$$P(t) = \theta^P P(t-1) + U^P(t)$$

$$\psi(t) = \theta^\psi \psi(t-1) + U^\psi(t)$$

$$\omega(t) = \theta^\omega \omega(t-1) + U^\omega(t)$$

$$\rho(t) = \theta^{\rho}\rho(t-1) + U^{\rho}(t)$$

$$L(t+1) = \lambda_1 L(t) - \frac{\lambda_1 B}{e}\left[\frac{d\theta^{\rho}\rho(t)}{1-\lambda_1 B\theta^{\rho}} + \frac{\theta^{\omega}\omega(t)}{1-\lambda_1 B\theta^{\omega}} - \frac{a\theta^{\rho}\rho(t)}{1-\lambda_1 B\theta^{\rho}}\right]$$

$$I(t+1) = \lambda_3 I(t) - \frac{\lambda_3 Bf}{\sigma g}\,\frac{\theta^{\psi}\psi(t)}{(1-\lambda_3 B\theta^{\psi})} - \frac{\lambda_3 \sigma B\theta^{P} P(t)}{\sigma q(1-\lambda_1 B\theta^{P})}.$$

Competitive Equilibrium

The industry demand curve for final consumption of the good is given by

$$P(t) = A_0 - A_1\overline{S}(t) + U_s(t),\ A_0,\ A_1 > 0$$

where "-" denotes an industry-wide variable.

$$\alpha(L)U_s(t) = V^s(t),\ \alpha(L) = \sum_{j=0}^{r_s} \alpha_j L^j.$$

Let M(t) be a (px1) vector random process that obeys

$$\zeta(L)M(t) = V^M(t). \tag{25}$$

We now let $\omega(t)$ be the first element of M(t).

The representative firm's problem is to choose linear contingency plans for L(t+1) and I(t+1) as functions of the information available at time t, of the form

$$L(t+1) = f_0 + c_L L(t) + c_I I(t) + f_L(L)\overline{L}(t) + f_I(L)\overline{I}(t) + f_M(L)M(t)$$

$$+ f_{u_s}(L)U_s(t) + f_{\psi}(L)\psi(t) + f_{\rho}(L)\rho(t) \tag{28}$$

$$I(t+1) = g_0 + d_L L(t) + d_I I(t) + g_L\overline{L}(t) + g_I\overline{I}(t) + g_M(L)M(t)$$

$$+ g_{u_s}(L)U_s(t) + g_{\psi}(L)\psi(t) + g_{\rho}(L)\rho(t) \tag{29}$$

to maximize

$$\lim_{N\to\infty} E_0 \sum_{t=0}^{N} B^t \{[A_0 - A_1[a\bar{L}(t) - \bar{I}(t+1) + \sigma\bar{I}(t)]][aL(t) - I(t+1)$$

$$+ \sigma I(t) - \omega(t)L(t) - \frac{d}{2}[L(t)+\rho(t)]^2$$

$$- \frac{e}{2}[L(t+1)-L(t)]^2 - \frac{f}{2}[I(t)-\psi(t)]^2$$

$$- \frac{g}{2}[I(t+1)-\sigma I(t)]^2\}$$

subject to L_0, I_0 given and

$$\alpha(L)U_s(t) = V^s(t) \tag{22}$$

$$\delta_\psi(L)\psi(t) = U^\psi(t) \tag{23}$$

$$\delta_\rho(L)\rho(t) = U^\rho(t) \tag{24}$$

$$\zeta(L)M(t) = V^M(t), \tag{25}$$

and firms view the aggregate stock of inventories and labor as evolving according to

$$\bar{L}(t+1) = F_0 + F_L\bar{L}(t) + F_I\bar{I}(t) + F_M(L)M(t) + F_{u_s}U_s(t)$$
$$+ F_\psi(L)\psi(t) + F_\rho(L)\rho(t) \tag{26}$$

$$\bar{I}(t+1) = G_0 + G_L\bar{L}(t) + G_I\bar{I}(t) + G_M(L)M(t) + G_{u_s}U_s(t)$$
$$+ G_\psi(L)\psi(t) + G_\rho(L)\rho(t). \tag{27}$$

Definition: A rational expectations equilibrium is four linear functions (26), (27), (28), and (29) such that

(i) given the aggregate laws of motion (26) and (27), the contingency plans (28) and (29) solve the firm's problem; and

(ii) the contingency plans of the representative firm (28) and (29) imply the aggregate laws of motion (27) and (28) so that

$$F(\cdot) = nf(\cdot)$$

and

$$G(\cdot) = ng(\cdot).$$

The solution to the above problem reduces to solving the following Euler equation.

$$G_1 Y(t+1) + G_0 Y(t) + G_{-1} Y(t-1) = \mathcal{L}(t)$$

where

$$Y(t) = \begin{bmatrix} L(t) \\ I(t) \end{bmatrix}$$

and

$$\underset{2\times2}{G_0} = \begin{bmatrix} -[Bd+e(1+B)+nBa^2A_1] & -B\sigma nA_1 a \\ -B\sigma nA_1 a & -[B\sigma^2 g+g+Bf+BnA_1+B^2\sigma^2 nA_1] \end{bmatrix}$$

$$\underset{2\times2}{G_{-1}} = \begin{bmatrix} e & 0 \\ anA_1 & g + BnA_1\sigma \end{bmatrix}$$

$$G_1 = \begin{bmatrix} eB & BanA_1 \\ 0 & B\sigma g + B^2 nA_1\sigma \end{bmatrix}$$

and

$$\underset{2\times 2}{L(t)} = \left[\frac{Bd\rho(t) + B\omega(t) - BaU_s(t) - BaA_0}{Bf\psi(t) + U_s(t-1) - B\sigma U_s(t) + A_0 - B\sigma A_0}\right].$$

It turns out that solving the above equations for the equilibrium paths of $\overline{L}(t+1)$ and $\overline{I}(t+1)$ is equivalent to solving the following <u>social planning problem</u>, which consists of maximizing the expected discounted consumer surplus from sales of the good minus the total social costs of production, or

Maximize

$$\lim_{N\to\infty} E_0 \sum_{t=0}^{N} B^t \{n[A_0+Us(t)][aL(t)-I(t+1)+\sigma I(t)]$$

$$- \frac{A_1 n^2}{2}[aL(t)-I(t+1)+\sigma I(t)]^2$$

$$- \omega(t)nL(t) - \frac{dn}{2}[L(t)+\rho(t)]^2$$

$$- \frac{en}{2}[L(t+1)-L(t)]^2 - \frac{fn}{2}[I(t)+\psi(t)]^2$$

$$- \frac{gn}{2}[I(t+1)-\sigma I(t)]^2\} \tag{33}$$

subject to L_0 and I_0 given and

$$\zeta(L)M(t) = V^M(t) \tag{25}$$

$$\alpha(L)U_s(t) = V^s(t) \tag{22}$$

$$\delta_\psi(L)\psi(t) = U^\psi(t) \tag{23}$$

$$\delta_\rho(L)\rho(t) = U^\rho(t). \tag{24}$$

The information set $\overline{\Omega}(t)$ consists of at least $\{L(t), I(t), \overline{\psi}(t), \overline{U}_s(t), \overline{\rho}(t)\}$, and the parameters of the stochastic processes (22), (23), (24), and (25) are known with certainty by the social planner. The maximization is over linear contingency plans setting $\overline{L}(t+1) = nL(t+1)$ and $\overline{I}(t+1) = nI(t)$ as functions of the social planner's information set $\overline{\Omega}(t)$.

The equilibrium laws of motion for $\overline{L}(t+1)$ and $\overline{I}(t+1)$ will then be of the form

$$\overline{L}(t+1) = F_0 + F_L\overline{L}(t) + F_I\overline{I}(t) + F_M(L)M(t) + F_{u_s}(L)U_s(t) + F_\psi(L)\psi(t) + F_\rho(L)\rho(t) \tag{39}$$

$$\overline{I}(t+1) = G_0 + G_L\overline{L}(t) + G_I\overline{I}(t) + G_M(L)M(t) + G_{u_s}U_s(t) + G_\psi(L)\psi(t) + G_\rho(L)\rho(t). \tag{40}$$

Proposition

For the linear quadratic model under consideration, in which production and inventory costs are additively separable, the rational expectations equilibrium laws of motion for labor and inventories decompose, $F_I = G_L = 0$, if and only if the elasticity of consumer demand for industry output is

$$\frac{\overline{S}(t)}{P(t)} \cdot \frac{\partial\overline{S}(t)}{\partial P(t)} = -\infty.$$

In the absence of such restrictions, the econometric model becomes

$$\overline{L}(t+1) = F_0 + F_L\overline{L}(t) + F_I\overline{I}(t) + F_M(L)M(t) + F_{u_s}(L)U_s(t) + \textstyle\sum_L(t) \tag{41}$$

$$\overline{I}(t+1) = G_0 + G_L\overline{L}(t) + G_I\overline{I}(t) + G_M(L)M(t) + G_{u_s}(L)U_s(t) + \textstyle\sum_I(r) \tag{42}$$

$$\alpha(L)U_s(t) = V^s(t) \tag{22}$$

and

$$\zeta(L)M(t) = V^M(t) \tag{25}$$

where

$$\textstyle\sum_L = F_\psi(L)\psi(t) + F_\rho(L)\rho(t)$$

and

$$\textstyle\sum_I = G_\psi(L)\psi(t) + G_\rho(L)\rho(t).$$

<u>Estimation</u>

Define

$$y(t) = \begin{bmatrix}\bar{L}(t)\\ \bar{I}(t)\end{bmatrix} \quad \phi(t) = \begin{bmatrix}\psi(t)\\ \zeta(t)\end{bmatrix}, \ \textstyle\sum(t) = \begin{bmatrix}\sum_L(t)\\ \sum_I(t)\end{bmatrix}$$

$$\pi_1(L) = \begin{bmatrix}F_L(L) & F_I(L)\\ G_L(L) & G_I(L)\end{bmatrix}, \ \pi_2(L) = \begin{bmatrix}F_M(L) & F_{u_s}(L)\\ G_M(L) & G_{u_s}(L)\end{bmatrix}$$

$$2(t)d = \begin{bmatrix}M(t)\\ U_s(t)\end{bmatrix}, \ \pi_3(L) = \begin{bmatrix}F_\psi(L) & F_\rho(L)\\ G_\psi(L) & G_\rho(L)\end{bmatrix}, \ \delta(L) =$$

$$\begin{bmatrix}\delta_\psi^{-1}(L) & 0\\ 0 & \delta_\rho^{-1}(L)\end{bmatrix}$$

$$U(t) = \begin{bmatrix}U_\psi(t)\\ U_\rho(t)\end{bmatrix}, \text{ and } V(t) = \begin{bmatrix}V^M(t)\\ V^s(t)\end{bmatrix}.$$

Hence, our system may be written as

$$y(t+1) = \pi_1(L)y(t) + \pi_2(L)2(t) + \textstyle\sum(t)$$

$$\alpha(t) = \pi_3(L)\phi(t)$$

$$\alpha(L)U_s(t) = V^s(t)$$

$$\zeta(L)M(t) = V^M(t).$$

Introduce the new process

$$C(t) = U(t) - \lambda V(t)$$

where $C(t)$ is a (2x1) column vector and λ is a (2x(P+1)) matrix where $E[C(t)V(t)] = 0$. Note that if $U(t)$ and $V(t)$ are uncorrelated $\lambda = [0]$ and $C(t) = U(t)$.

Then the system to be estimated is

$$y(t+1) = [I-\pi_1(L)]^{-1}[\pi_2(L)\Phi(L)+\pi_3(L)\delta(L)\lambda]V(t)$$

$$+ [I-\pi_1(L)]^{-1}[\pi_3(L)\delta(L)]C(t) \tag{a}$$

$$\zeta(L)M(t) = V^M(t) \tag{b}$$

$$EC_tC_{t-j} = 0, \; EV_t^M V_{t-j}^{M'} = 0$$

for $j \neq 0$ and

$$EC_t V^M_{t-j=0}$$

for all j. Note that $M(t)$ is strictly exogenous in the equation of the above system.

Define

$$\bar{y}_t = \begin{bmatrix} y_1 \\ \vdots \\ y_t \end{bmatrix}, \; \bar{M}_T = \begin{bmatrix} M_1 \\ \vdots \\ M_T \end{bmatrix}.$$

Then the normal log likelihood function for $(\bar{y}_t,\bar{M}_t)$ is

$$\mathcal{L}_T = -\frac{1}{2}(T+TP)\log 2\pi - \frac{1}{2}\log\det\Gamma_T - \frac{1}{2}[\bar{y}_T' M_T']\Gamma_T^{-1}\begin{bmatrix}\bar{y}_T\\ M_T\end{bmatrix}$$

where

$$\mathcal{L}_T = E\begin{bmatrix}\bar{y}_T\\ \bar{M}_T\end{bmatrix}\begin{bmatrix}\bar{y}_T\\ M_T\end{bmatrix}',$$

where the mean $(\bar{y}_T',\bar{M}_T')$ is zero because we are dealing in deviations from the mean.

Hannan (1970) has suggested an approximation to the above log likelihood function $\mathcal{L}_T$,

$$\begin{aligned} {}^{*}_{T} = &-\frac{1}{2}(T+TP)\log 2\pi - \frac{1}{2}\sum_{t=1}^{T}\log\left\{\det[S(\omega_J)]\right\} \\ &-\frac{1}{2}\sum_{t=1}^{T}\operatorname{trace}\,[S(\omega_J)^{-1}I(\omega_J)] \end{aligned}$$

where $I(\omega_j)$ is the periodogram for the $(\bar{Y}_t,\bar{M}_T)$ process at frequency $\omega_j = \frac{2\pi j}{T}$ and $S(\omega)$ is the theoretical spectral density matrix of $(\bar{Y}_T,\bar{M}_T)$ process. One then maximizes $\mathcal{L}_T^*$ by one of several acceptable iterative methods, beginning from an initial consistent estimate of the free parameters.

Appendix B - Numerical Examples

We now consider various numerical examples of the competitive equilibrium emerging from the model of Section II.2.

As we indicated in Section II, in order to compute the equilibrium laws of motion for $[\bar{L}(t+1),\bar{I}(t+1)] = [nL(t+1),nI(t+1)]$, we solve the following social planning problem; maximize

$$E_0 \sum_{t=0}^{\infty} B^t [n[A_0+U_s(t)][aL(t)-I(t+1)+\sigma I(t)] \tag{33}$$

$$- \frac{A_1 n^2}{2}[aL(t)-I(t+1)+\sigma I(t)]^2$$

$$- w(t)nL(t) - \frac{dn}{2}[L(t)+\rho(t)]^2 - \frac{en}{2}[L(t+1)-L(t)]^2$$

$$- \frac{fn}{2}[I(t)+\Psi(t)]^2 - \frac{gn}{2}[I(t+1)-\sigma I(t)]^2$$

subject to

$$\alpha(L)U_s(t) = V^s(t) \tag{22}$$

$$\delta_\Psi(L)\Psi(t) = U^s(t) \tag{23}$$

$$\delta_\rho(L)\rho(L) = U^\rho(t) \tag{24}$$

$$\zeta(L)M(t) = V^m(t) \tag{25}$$

where $w(t)$ is the first element of the (Px1) vector process $M(t)$. At time t the social planner knows $\{L(t),I(t),\bar{L}(t),\bar{I}(t)\}$ and $\{M(t),M(t-1),\ldots,U_s(t),U_s(t-1),\ldots,\rho(t),\rho(t-1),\ldots,\Psi(t),\Psi(t-1),\ldots\}$, as well as the parameters of (22), (23), (24), (25), and those of the demand schedule, A_0 and A_1.

The maximization is over linear contingency plans for setting $[L(t+1), I(t+1)]$ as functions of the elements of the planner's information set at time t. Given the optimal decision rule for $[L(t+1),I(t+1)]$, the equilibrium laws of motion for $[\bar{L}(t+1),\bar{I}(t+1)]$ is obtained by using $[\bar{L}(t+1),\bar{K}(t+1)] = n[L(t+1), I(t+1)]$. For all the examples, solutions are arrived at by iterating on the matrix Riccati difference equation until the convergence criterion is fulfilled. In particular, successive iterations were performed on the feedback law

$$F_t = \beta[Q+\beta B^1 P_t B]^{-1} B^1 P_t A$$

where iterations on the matrix Riccati difference equation

$$P_{t+1} = \beta A^1 P_t A + R - \beta^2 A^1 P_t B[Q+\beta B^1 P_t B]^{-1} B^1 P_t A$$

were started from $P_0 = 0$. Convergence was claimed when the norm, defined as the maximum absolute value over the elements of $(F_{t+1}-F_t)$, was less than 10^{-5}.

For all of the examples we assumed $\beta = .7$, $n = 1{,}000$, $a = .8$, $d = 1.5$, $e = 1.4$, $f = 1.2$. We also set $A_0 = 0$, which is equivalent to setting constant terms in the equilibrium $[\bar{L}(t+1),\bar{I}(t+1)]$ equal to zero. As such, the equilibrium describes variables measured in deviations from the mean.

(I) $A_1 = .010$, $g = 1.3$, $\sigma = 0.0$ or a depreciation rate of 100 percent.

$$\begin{bmatrix} L(t+1) \\ I(t+1) \end{bmatrix} = \begin{bmatrix} .33750 & 0 \\ .20678 & 0 \end{bmatrix} \begin{bmatrix} L(t) \\ I(t) \end{bmatrix}$$

$$+ \begin{bmatrix} -.09568 & -.17696 & -.05481 & .01021 \\ -.05862 & -.10842 & -.07034 & -.01672 \end{bmatrix} \begin{bmatrix} w(t) \\ \rho(t) \\ \Psi(t) \\ U_s(t) \end{bmatrix}$$

(II) $A_1 = .010$, $g = 1.3$, $\sigma = .2$

$$\begin{bmatrix} L(t+1) \\ I(t+1) \end{bmatrix} = \begin{bmatrix} .33550 & -.02521 \\ .20511 & .16661 \end{bmatrix} \begin{bmatrix} L(t) \\ I(t) \end{bmatrix}$$

$$+ \begin{bmatrix} -.09442 & -.17432 & -.05696 & .01031 \\ -.05732 & -.10565 & -.07345 & -.01666 \end{bmatrix} \begin{bmatrix} w(t) \\ \rho(t) \\ \Psi(t) \\ U_s(t) \end{bmatrix}$$

(III) $A_1 = .010,\ g = 1.3,\ \sigma = .5$

$$\begin{bmatrix} L(t+1) \\ I(t+1) \end{bmatrix} = \begin{bmatrix} .33603 & -.05845 \\ .20700 & +.42522 \end{bmatrix} \begin{bmatrix} L(t) \\ I(t) \end{bmatrix}$$

$$+ \begin{bmatrix} -.09361 & -.17230 & -.06218 & .01019 \\ -.05661 & -.10371 & -.08064 & -.01692 \end{bmatrix} \begin{bmatrix} w(t) \\ \rho(t) \\ \Psi(t) \\ U_s(t) \end{bmatrix}$$

(IV) $A_1 = .010,\ g = 1.3,\ \sigma = .9$

$$\begin{bmatrix} L(t+1) \\ I(t+1) \end{bmatrix} = \begin{bmatrix} .36381 & -.02064 \\ .24159 & .87532 \end{bmatrix} \begin{bmatrix} L(t) \\ I(t) \end{bmatrix}$$

$$+ \begin{bmatrix} -.10340 & -.19117 & -.08547 & .00811 \\ -.06817 & -.12573 & -.10970 & -.01955 \end{bmatrix} \begin{bmatrix} w(t) \\ \rho(t) \\ \Psi(t) \\ U_s(t) \end{bmatrix}$$

(V) $A_1 = .001,\ g = 1.3,\ \sigma = 0.0$

$$\begin{bmatrix} L(t+1) \\ I(t+1) \end{bmatrix} = \begin{bmatrix} .41677 & 0 \\ .08218 & 0 \end{bmatrix} \begin{bmatrix} L(t) \\ I(t) \end{bmatrix}$$

$$+ \begin{bmatrix} -.12199 & -.22734 & -.02233 & .04130 \\ -.02405 & -.04483 & -.12271 & -.06580 \end{bmatrix} \begin{bmatrix} w(t) \\ \rho(t) \\ \Psi(t) \\ U_s(t) \end{bmatrix}$$

(VI) $A_1 = .001,\ g = 1.3,\ \sigma = .2$

$$\begin{bmatrix} L(t+1) \\ I(t+1) \end{bmatrix} = \begin{bmatrix} .41615 & -.01037 \\ .08021 & .13823 \end{bmatrix} \begin{bmatrix} L(t) \\ I(t) \end{bmatrix}$$

$$+ \begin{bmatrix} -.12167 & -.22668 & -.02267 & .04169 \\ -.02290 & -.04246 & -.12689 & -.06474 \end{bmatrix} \begin{bmatrix} w(t) \\ \rho(t) \\ \Psi(t) \\ U_s(t) \end{bmatrix}$$

(VII) $A_1 = .001,\ g = 1.3,\ \sigma = .5$

$$\begin{bmatrix} L(t+1) \\ I(t+1) \end{bmatrix} = \begin{bmatrix} .41483 & -.02911 \\ .07540 & .33669 \end{bmatrix} \begin{bmatrix} L(t) \\ I(t) \end{bmatrix}$$

$$+ \begin{bmatrix} -.12102 & -.22535 & -.02660 & .04253 \\ -.02038 & -.03725 & -.13095 & -.06184 \end{bmatrix} \begin{bmatrix} w(t) \\ \rho(t) \\ \Psi(t) \\ U_s(t) \end{bmatrix}$$

(VIII) $A_1 = .001,\ g = 1.3,\ \sigma = .9$

$$\begin{bmatrix} L(t+1) \\ I(t+1) \end{bmatrix} = \begin{bmatrix} .41219 & -.06546 \\ .06450 & .55782 \end{bmatrix} \begin{bmatrix} L(t) \\ I(t) \end{bmatrix}$$

$$+ \begin{bmatrix} -.11980 & -.22280 & -.02073 & .04428 \\ -.01507 & -.02624 & -.12885 & -.05484 \end{bmatrix} \begin{bmatrix} w(t) \\ \rho(t) \\ \Psi(t) \\ U_s(t) \end{bmatrix}$$

(IX) $A_1 = .010,\ g = .3,\ \sigma = .9$

$$\begin{bmatrix} L(t+1) \\ I(t+1) \end{bmatrix} = \begin{bmatrix} .42445 & -.02823 \\ .31959 & .86650 \end{bmatrix} \begin{bmatrix} L(t) \\ I(t) \end{bmatrix}$$

$$+ \begin{bmatrix} -.12307 & -.22848 & -.11481 & .00343 \\ -.09188 & -.17011 & -.14633 & -.02571 \end{bmatrix} \begin{bmatrix} w(t) \\ \rho(t) \\ \Psi(t) \\ U_s(t) \end{bmatrix}$$

(X) $A_1 = .010,\ g = .1,\ \sigma = .9$

$$\begin{bmatrix} L(t+1) \\ I(t+1) \end{bmatrix} = \begin{bmatrix} .44195 & -.03050 \\ .34190 & .86389 \end{bmatrix} \begin{bmatrix} L(t) \\ I(t) \end{bmatrix}$$

$$+ \begin{bmatrix} -.12888 & -.23995 & -.12337 & .00210 \\ -.09882 & -.18317 & -.15692 & -.02746 \end{bmatrix} \begin{bmatrix} w(t) \\ \rho(t) \\ \Psi(t) \\ U_s(t) \end{bmatrix}$$

(XI) $A_1 = .010,\ g = .00001,\ \sigma = .9$

$$\begin{bmatrix} L(t+1) \\ I(t+1) \end{bmatrix} = \begin{bmatrix} .45171 & -.03178 \\ .35431 & .86242 \end{bmatrix} \begin{bmatrix} L(t) \\ I(t) \end{bmatrix}$$

$$+ \begin{bmatrix} -.13215 & -.24580 & -.12817 & .00135 \\ -.10272 & -.19051 & -.16284 & -.02843 \end{bmatrix} \begin{bmatrix} w(t) \\ \rho(t) \\ \Psi(t) \\ U_s(t) \end{bmatrix}$$

While we report the following regularities observed in the specific examples calculated, no claims are made for their robustness in the face of alternative specifications for equations (22), (23), (24), (25), and the other parameters of the model.

$$\frac{\partial L(t+1)}{\partial L(t)} > 0,\ \frac{\partial L(t+1)}{\partial I(t)} < 0,\ \frac{\partial I(t+1)}{\partial L(t)} > 0,\ \frac{\partial I(t+1)}{\partial L(t)} > 0,\ \frac{\partial I(t+1)}{\partial I(t)} > 0$$

$$\frac{\partial I(t+1)}{\partial \Psi(t)} < 0,\ \frac{\partial I(t+1)}{\partial U_s(t)} < 0,\ \frac{\partial L(t+1)}{\partial \rho(t)} < 0,\ \frac{\partial L(t+1)}{\partial U_s(t)} > 0$$