

Staff Report WP-18

THE ROLE OF STOCK PORTFOLIO SHIFTS
UNDER ALTERNATIVE EXCHANGE RATE REGIMES

by

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The model set out in this paper is in the tradition of highly aggregative, essentially one-good models of open economies. Such models have been used to analyze (i) possible achievement of internal and external balance by use of monetary and fiscal policy, (ii) the relative potency of monetary and fiscal policy under alternative exchange rate regimes, and, to some extent, (iii) the effects of exchange rate changes on the balance of payments.^{1/} The existing models, however, are generally recognized as deficient in their treatment of capital movements, a treatment which assumes that there accompanies a ceteris paribus once-for-all change in an interest differential only a once-for-all change in the flow of capital between countries. The alternative assumption -- the one I work with -- is that to each constellation of interest rates there exist desired stocks of assets; or more generally, that whether the economy is closed or open, to each constellation of interest rates, wealth and income, there corresponds a desired allocation of wealth among existing assets. The anomalous nature of models invoking the "flow" assumption is brought out by noting that those models confine purchases of foreign assets to take place at a rate per unit time, although they follow the traditional analysis in allowing for once-for-all portfolio shifts between money and domestic earning assets. In addition to imposing this asymmetric view of adjustment to a desired portfolio, such

^{1/}This literature includes Krueger [4], Mundell [5, 6] and Sohmen [8].

models, via their use of the "flow" assumption, almost assume away what has come to be regarded as a major problem, namely, the "volatility" of short-term capital movements.

All this is fairly well accepted. What is missing is an integration of the alternative assumption -- that which allows in a symmetric way for portfolio shifts among assets, including foreign assets -- into a complete model within which the questions mentioned above and others can be analyzed.^{2/} The purpose of this paper is to provide one such model.

The model is of a single country facing the rest of the world which is aggregated into a single foreign country and which in almost all respects is unaffected by what happens in the home country. In addition, there is no mobility of labor or physical capital between countries. The model of the home country is a simple extension of the standard Keynesian closed economy macro-economic model.^{3/} In that spirit, the home country is assumed to produce a single good. The good is different from those produced in the rest of the world, and the home country cannot, by assumption, sell unlimited amounts of it to the rest of the world at a given price. The home country can, however, buy unlimited amounts of foreign goods at a given price.^{4/}

In order to present an analysis that carefully maintains the distinction between stock and flow portfolio shifts, it is convenient to treat time as a continuous variable. Other variables may be discontinuous

^{2/}Floyd [2, 3] attempts such an analysis but seems to fall short of presenting a workable model.

^{3/}For example, see that presented by Bailey [1].

^{4/}One could easily set out a model along the lines described below but in which the home country produces two goods, an export good and a nontraded good, and faces fixed terms of trade.

functions of time. Moreover, although the model determines the time paths of the endogenous variables given the time paths of the exogenous variables, this paper is limited to an examination of alternative equilibria at an instant in time.^{5/}

I. The Model

A. GNP and Aggregate Demand

Write the GNP identity by purchaser as ^{6/}

$$Y = \dot{K} + qK + C + ex + G \quad (1)$$

where all quantities are flows of the home good per unit time:

Y = total output.

\dot{K} = net accumulation of the home good (all by firms).

qK = depreciation of the home country stock of capital, K, assumed identical to the stock of capital held by firms. q is a pure number per unit time and is fixed.

C = purchases of the home good by home residents used up or eaten at a rate equal to the expenditure rate.

ex = purchases of the home good by world residents, i.e., exports.

G = government purchases of the home good assumed used up at a rate equal to the expenditure rate.

^{5/}For a detailed discussion, see Wallace [9].

^{6/}This is identical to

$$Y = \text{domestic absorption} + ex - im/X$$

where im is the quantity of world good(s) purchased per unit time, X is a relative price to be defined below, and domestic absorption = $\dot{K} + qK + C + G + im/X$.

For the most part, I make very simple behavioral assumptions.^{7/}

Thus,

$$\dot{K} = g[r - E(\dot{P}/P) + q - h_K]; \quad g' < 0 \quad (2)$$

$$C = c(y, X); \quad c_y > 0, \quad c_X < 0 \quad (3)$$

$$ex = ex(Y^*, X); \quad ex_{Y^*} > 0, \quad ex_X < 0 \quad (4)$$

where r is the home country interest rate (and nominal yield on equity, see below), $E(\dot{P}/P)$ is the perceived (expected) percentage rate of change of the home price level assumed common to everyone in the economy, h_K is the marginal product of capital (h is the aggregate home production function described below), y is a home disposable income concept to be defined below, Y^* is world GNP which is exogenous (world or foreign variables have "*"s affixed), P is the price of home goods in terms of home currency, P^* is the exogenous price of foreign goods in terms of foreign currency, R is the exchange rate (the price of foreign currency in terms of home currency), and $X \equiv P/RP^*$ is the terms of trade (measured in units of the foreign good per unit of the home good). The import demand function is assumed to be

$$im = im(y, X); \quad im_y > 0, \quad im_X > 0 \quad (5)$$

where im is measured in units of the foreign good per unit time. It follows that the trade balance in units of foreign currency per unit time is $(P/R)ex - P^*im$. It is important to remember that this is a flow.

^{7/} Readers familiar with Sargent and Wallace [7] will recognize what follows below -- especially equation (2) -- as a Keynesian model in contrast to a perfect-market-in-physical-capital model. The latter, which implies equality between the cost of capital and its marginal product, is, in a way, more consistent with the usual trade models in which physical capital is mobile between home industries. It is also more consistent with the interpretation of the one-good assumption as a constant cost production frontier.

B. The Supply Side

The supply side is described by a production function,

$$Y = h(K,L); \quad (6)$$

a demand function for labor written in inverse form,

$$h_L(K,L) = W/P; \quad (7)$$

and an exogenous labor force, L , which is always equal to employment. Thus, the model we shall be analyzing is a "full employment" model, in which output, employment, and the real wage are independent of aggregate demand and are determined by (6), (7), and the exogenous labor force.

One way to alter the model to allow for a dependence of output on aggregate demand is to replace the assumption of an exogenous labor force equal to employment by the assumption that the money wage, W , is exogenous or is, at least, somewhat inflexible. We stick to the full employment version here because it is simpler to present and because one can infer the effects of disturbances arising in the aggregate demand and portfolio sectors on output and employment in the inflexible money wage model from their effects in the full employment (flexible wage) models; disturbances that increase the domestic price level in the full employment model are expansionary in terms of output and employment in the inflexible money wage model, and vice versa.

C. Assets and Portfolio Equilibrium

Here I make only a slight extension of the simplest closed economy model. I assume that home residents allocate their wealth among three assets: a home earning asset, a foreign earning asset, and domestic money. I further

assume that foreign residents hold no domestic assets although they can, of course, be net creditors or debtors to home residents, but only in the form of their own earning asset.

The home earning asset consists of two separate assets, equities and call loans. The equities represent titles to the earnings of firms. Call loans may be issued by individuals or the government but are assumed not to be distinguishable by issuer. Moreover, call loans and equities are assumed viewed by individuals as perfect substitutes when their real yields, defined to take account of the perceived rate of inflation, are equal. That assumption, in effect, gives us one home earning asset.^{8/} Home money is an asset with a fixed (zero) nominal yield for which there is assumed to be a transactions demand. Home residents are assumed to borrow and lend abroad by issuing or holding call loans denominated in foreign currency with yield equal to the foreign interest rate, r^* . They are, like domestic call loans, zero maturity assets, so that changes in interest rates impose no capital gains or losses on holders.

We now list three equalities between the supplies and demands of each of the three assets, even though only two of the conditions are independent. (Whenever any two hold, the third is implied.)

^{8/} Readers who are unhappy with the perfect substitutes assumption have a number of options. They can alter the model -- make it bigger -- to allow for imperfect substitution between home call loans and equities. Or they can rule out home call loans and allow the home government to engage in open market operations in equities. Or they can alter the nature of the government debt instrument by assuming it to be a perpetuity, which might more reasonably be assumed to be a perfect substitute for equities. Adoption of either of the last two options would make almost no difference as regards results. The assumption in the text is consistent with a constant real yield differential between equities and call loans.

$$(V+B)/\bar{P} = b[r, r^*+E(\dot{R}/R), Y, z] \quad (8)$$

$$M/\bar{P} = m[\dots] \quad (9)$$

$$RF/\bar{P} = f[\dots] \quad (10)$$

Here \bar{P} is an index of home and foreign prices; $\bar{P} = kP + (1-k)RP^*$.

The weight k , which is equal to $C/(C+im/X)$ for some base date, is treated as a constant. V is the nominal value of outstanding equities which I take to be the discounted value of the current earnings stream of all firms,

$$V = \frac{P[Y-(W/P)L-qK]}{r - E(\dot{P}/P)} \quad (11)$$

B is the net debt of the home government (in the form of call loans);

$E(\dot{R}/R)$ is the perceived percentage rate of change of the exchange rate;

F , which is denominated in foreign currency, is the quantity of foreign call loans held by home residents; and z is real wealth;

$$z = (V+B+M+RF)/\bar{P} \quad (12)$$

In (8)-(10) yields are measured as nominal yields in terms of domestic currency, which is one consistent way to measure them. As regards partial derivatives for the functions m , b , and f , we assume (i) that partial derivatives with respect to own interest rates are positive and that those with respect to other rates are negative or zero, (ii) that $m_Y > 0$, and (iii) that for each asset average and marginal holdings with respect to wealth are equal; that is, that wealth elasticities are unity wherever they are defined.

In addition to (8)-(10), there is a condition implied by stock equilibrium in the market in which home currency and foreign currency are traded. To describe the condition, we require a description of the behavior of foreign governments. I shall assume that foreign governments do not intervene in the foreign exchange market. This implies that if the home country is to have a fixed exchange rate vis-à-vis the world, it is the home government's responsibility to peg the rate, and that if it does not, then the exchange rate does, indeed, float. That assumption and the assumption that foreign residents do not hold home assets imply a relationship between changes in home residents' holdings of foreign earning assets, dF , and changes in the home country's stock of reserves, denoted dA , where A consists of holdings of foreign earning assets. The relationship is^{9/}

$$dA + dF = 0. \quad (13)$$

This relationship is crucial to our analysis and is what distinguishes it from analyses which invoke the "flow" assumption described in the introduction. The most important assumption behind (13) is that excess demands and supplies for foreign currency that arise from trade surpluses or deficits, being flows, cannot offset stock excess demands or supplies of foreign currency that result from desired portfolio shifts.^{10/}

It may help to exposit (13) by way of an example. Suppose that home residents wish to add to their holdings of foreign earning assets and hence place orders to buy foreign currency. Under a fixed exchange

^{9/}The symbol $d(.)$ stands for the total differential of $(.)$.

^{10/}This could be remedied by allowing for trade in stocks of goods; for example, transfers of accumulated inventories. But that, in turn, would give us a model in which there was mobility of real capital between countries.

rate regime, the home government must supply the foreign currency which it gets by selling some of its holdings of foreign earning assets. It is easily seen, then, that the net effect is simply an exchange between home residents and the home government as described by (13). It is appropriate to point out at this time that the assumption that foreign residents do not hold home assets is of no importance. At a time when home residents desire to increase their holdings of foreign earning assets, foreign residents, if they did deal in home earning assets, would, in general, want to decrease their holdings of home earning assets. And that would reinforce rather than offset the excess demand for foreign currency created by the actions of home residents.

The last portfolio condition we need is a balance sheet constraint on the portfolio operations of the home government:

$$RdA = dB + dM \quad (14)$$

D. Some Loose Ends

There is a flow constraint for the government which we here write in the form: the real value of the difference between expenditures and receipts from all sources equals the rate at which the government issues liabilities,

$$(G+rB/P) - (T+r*RA/P) = (\dot{M}+\dot{B}-\dot{RA})/P. \quad (15)$$

Here T is the real value of autonomous tax revenues minus transfers exclusive of interest payments. Note that the government earns interest on its reserves if they are positive and pays interest on them if they are negative.

I shall parameterize fiscal policy in terms of G and the sum of the remaining terms on the left-hand side of (15) so that holding fiscal policy constant shall mean holding G and that sum constant. In other words, unless otherwise noted, T is assumed altered to offset the effects of changes in endogenous variables on the flow of interest payments to the public, rB/P , and on the flow of interest receipts on reserves, r^*RA/P .

The balance of payments constraint or the requirement for equality between flow excess demand and supply in the market for currencies is

$$\dot{A} + \dot{F} + P^*im = r^*(A+F) + (P/R)ex. \quad (16)$$

It may be helpful to derive (16) from the "saving equals investment" identity and the government flow constraint, (15). To do that we first write the GNP identity by "use,"

$$PY = PqK + PC + RP^*im + PT - r^*FR - rB + S. \quad (17)$$

Here S is nominal saving which is equal to the rate of change of nominal wealth, $\partial(\bar{P}z)/\partial t$, minus the rate of asset appreciation, $\dot{R}F + (V/v)\dot{v}$ where v is the price per unit of equity. Using the definition of z and the fact that firm financing of net investment implies $\dot{P}K = \dot{V} - (V/v)\dot{v}$, we may write S as

$$S = \dot{P}K + \dot{M} + \dot{B} + \dot{R}F. \quad (18)$$

If we subtract (17) from the GNP identity by "purchaser" [see (1)], we have the "saving equals investment" identity

$$P(\dot{K} + ex + G) = RP^*im + PT - r^*FR - rB + S. \quad (19)$$

Then (16) is obtained by substituting into (19) S from (18) and P(G-T) from (15).

It remains only to describe y, the income concept that appears in the expenditure functions (3) and (5). We assume that

$$y = \frac{P(Y-qK-T)+rB+r*FR}{\bar{P}} = \frac{Y-qK-T+rB/P+r*FR/P}{k+(1-k)/X} \quad (20)$$

Defined this way y differs only slightly from disposable income defined as that level of expenditures that could be undertaken while leaving wealth, z, intact. The latter concept is $y - (\dot{\bar{P}}/\bar{P})z + [\dot{R}F + (V/v)\dot{v}]/\bar{P}$. Moreover, with this definition of y, the restrictions $c_X < 0$ and $im_X > 0$ are consistent with revealed preference theory.

II. Solutions for Exogenous Perceived Quantities

In this section, I solve for instantaneous changes in certain of the endogenous variables as functions of instantaneous changes in exogenous variables by, in effect, approximating the system by a set of linear equations about an initial position. Throughout, I assume that the two perceived quantities, $E(\dot{P}/P)$ and $E(\dot{R}/R)$, are exogenous, independent of current changes in variables. Indeed, except for some discussion of how the results would be affected by departing from this assumption, the perceived quantities are for all practical purposes dropped from the model.

As noted above, the supply side of the model, given K, determines Y, W/P, and L. That means that in manipulating the rest of the model at a moment in time, we may take those variables as predetermined and hence fixed. Given the values of those variables, we can find four relationships

that, depending on the regime specified, allow us to solve for four variables: the domestic price level and interest rate, either the exchange rate or the stock of reserves, and either the money stock or the outstanding stock of government interest-bearing debt.

One of the relationships is an aggregate demand-equals-output condition, which is found by substituting equations (2)-(4) into (1). If we totally differentiate the result, setting the differentials of exogenous and predetermined variables equal to zero and without loss of generality choosing units so that initial values satisfy $P = P^* = R = X = 1$, we find

$$0 = g'dr + c_y dy + (c_X + ex_X)dX. \quad (21)$$

Following the same procedure for initial values, we have from (20)

$$dy = y(1-k)dX - dT + d(rB/P) + d(r^*RF/P).$$

If we make the further assumption that initially $A + F = 0$, then by (13) and our way of parameterizing fiscal policy

$$dy = y(1-k)dX.$$

Hence, in place of (21), we have

$$0 = g'dr + (c_y(1-k)y + c_X + ex_X)dX. \quad (22)$$

There is ambiguity about the sign of dr/dX along this relationship because dy/dX is positive. Thus, while an increase in the terms of trade decreases aggregate demand for home goods by the amount $(c_X + ex_X)dX$, there tends to be an offset because real income increases. I shall assume that the former effect dominates on the ground that a change in X affects income only because I have chosen to treat real income in the rest of the world as exogenous;

X does not affect world income defined in a way comparable to (20). With that assumption, dr/dX is negative for (22), which means that changes in the home interest rate must be accompanied by changes in the terms of trade in the opposite direction in order to keep aggregate demand equal to the predetermined full employment level of home output.

The other three relationships are portfolio conditions. From (9), given Y,

$$dM/\bar{P} - (M/\bar{P})d\bar{P}/\bar{P} = m_r dr + m_{r^*} dr^* + m_z dz \quad (23)$$

while from (10)

$$(R/\bar{P})dF + (F/\bar{P})dR - (RF/\bar{P})d\bar{P}/\bar{P} = f_r dr + f_{r^*} dr^* + f_z dz. \quad (24)$$

From the definition of wealth, (12), and the constraints (13) and (14), it follows -- given Y, W/P , and L -- that

$$dz = (V/\bar{P})(dP/P - dr/r) - z d\bar{P}/\bar{P} + (F/\bar{P})dR \quad (25)$$

where, for convenience, I have assumed the initial value of the exogenous variable $E(\bar{P}/P)$ to be zero. (Equation (25) gives the determinants of instantaneous changes in the public's real wealth.) Substituting (25) into (23) and (24) using the unit wealth elasticity assumptions -- $m_z z = M/\bar{P}$ and $f_z z = RF/\bar{P}$ -- we have in place of (23)

$$\begin{aligned} dM/\bar{P} &= [m_r - (1/r)m_z(V/\bar{P})]dr + m_{r^*}dr^* \\ &+ m_z(V/\bar{P})dP/P + m_z FdR/\bar{P} \end{aligned} \quad (26)$$

and in place of (24)

$$\begin{aligned} (R/\bar{P})dF &= [f_r - (1/r)f_z(V/\bar{P})]dr + f_{r^*}dr^* \\ &+ f_z(V/\bar{P})dP/P - (1-f_z)FdR/\bar{P}. \end{aligned} \quad (27)$$

Changes in the exchange rate appear in (26) and (27) multiplied by the initial value of home residents' holdings of foreign earning assets, F , which can be positive, negative, or zero. We shall assume that initial holdings are zero. That, by way of $f_z = RF/\bar{P}$, implies that $f_z = 0$, so that (26) becomes

$$dM/\bar{P} = [m_r - (1/r)m_z(V/\bar{P})]dr + m_{r^*}dr^* + m_z(V/\bar{P})dP/P \quad (28)$$

while (27), again using (13), becomes

$$-(R/\bar{P})dA = f_r dr + f_{r^*} dr^*. \quad (29)$$

These are two of the portfolio relationships we shall use. The third is simply (14), which, for convenience, is repeated here:

$$RdA = dB + dM. \quad (30)$$

Equations (28)-(30) along with (22), which we rewrite here as (31)

$$0 = g'dr + (c_y(1-k)y + c_x + ex_x)(dP-dR), \quad (31)$$

constitute the system we shall work with.

Equations (28)-(31) contain six potentially endogenous variables: dM , dr , dP , dA , dR , and dB . We study three exchange regimes in detail. In each regime two of those six variables are exogenous. We designate as regime (i) a floating exchange rate system in which dA is exogenous and either dM or dB is exogenous. We designate as regime (ii) a gold standard fixed rate system in which dR and dB are exogenous and as regime (iii) a fixed rate

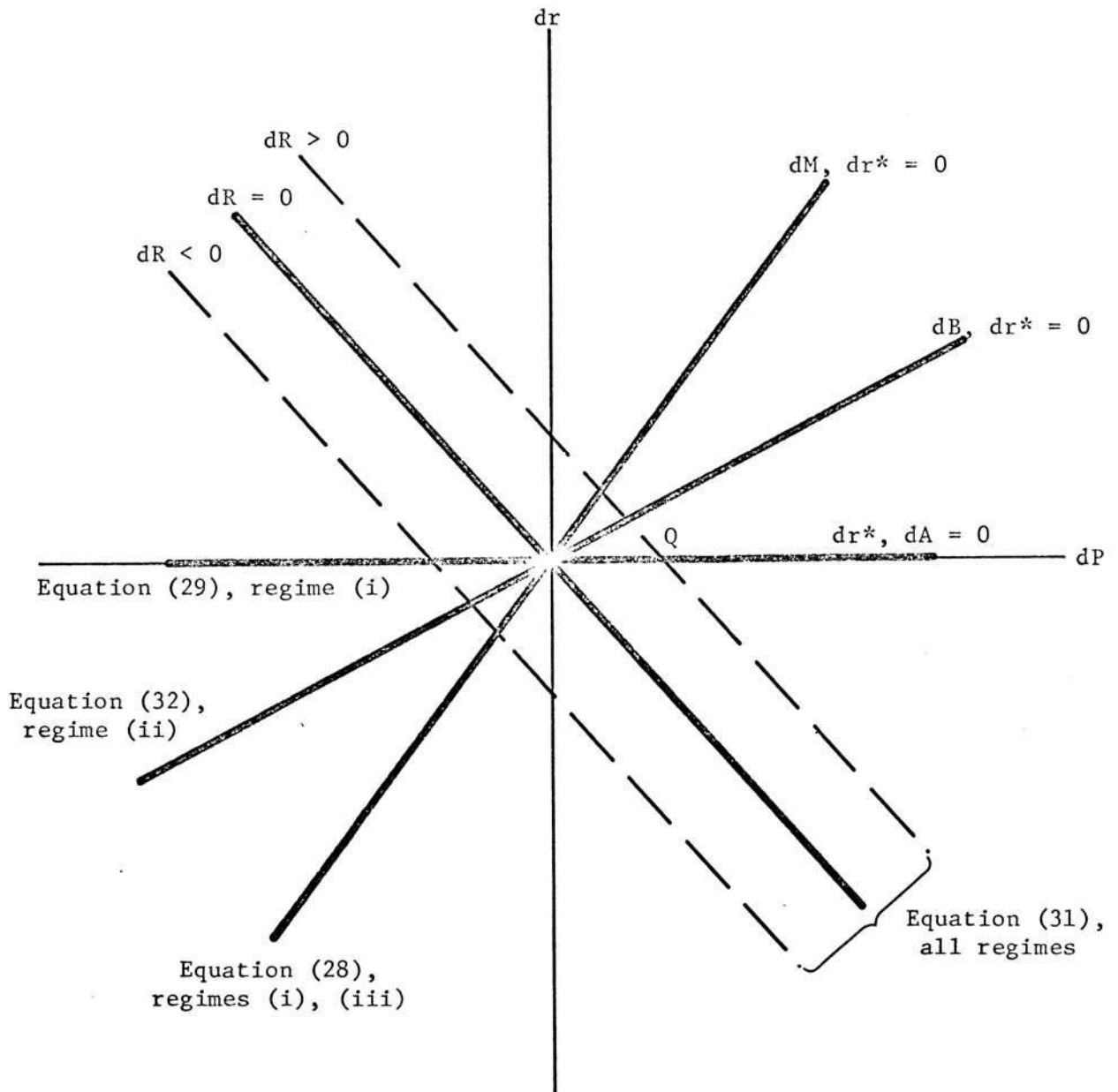
system with sterilization of reserve movements in which dR and dM are exogenous.^{11/} We shall present the relevant relationships for each regime, all obtainable from (28)-(31), graphically.

In Figure 1, the deviation of the home interest rate from its initial equilibrium value is measured along the vertical axis and that for the home price level along the horizontal axis. The set of downward sloping lines represents relationship (31). It applies in all three regimes, although the exchange rate, R , is a dependent variable only in the floating rate system, regime (i). The set is drawn holding fiscal policy, as described above, constant; expansionary (contractionary) fiscal policy shifts the set to the right (left). It is also drawn for a given value of the foreign price level in terms of foreign currency; an increase (decrease) in P^* shifts the set to the right (left).

For the floating rate system, regime (i), equations (28) and (29) apply with dM exogenous in (28) and dA exogenous in (29). In Figure 1, the former is shown as the steeply upward sloped line, while the latter is shown coincident with the horizontal axis. It is easily verified that increases (decreases) in the money stock, M , or in the foreign interest rate, r^* , shift relationship (28) to the right (left). The horizontal relationship, (29), shifts upward (downward) for increases (decreases) in r^* or A . Notice that the floating rate equation system is entirely recursive; equation (29) determines dr , (28) determines dP , (31) determines dR , and (30) determines dB .

^{11/}The reasoning behind calling regime (ii) a gold standard system is as follows. With $dB = 0$, we have from (30) $RdA = dM$. But since $dA = -dF$ [see (13)], we have $RdF + dM = 0$. This is the portfolio constraint that applies to residents of a country in a world using a single commodity as money. In such a world, the exchange rate is fixed without government intervention in the foreign exchange market.

Figure 1. The Determination of the Home Price Level and Interest Rate



The two fixed rate systems introduced above differ only in that regime (ii) is one in which capital movements affect the money stock while in regime (iii) the money stock is exogenous. In both systems, the stock of reserves, A, is endogenous. Under regime (iii), dr and dP are determined jointly by equations (28) and (31). Then dA can be found from (29) and dB from (30).^{12/} In regime (ii), dB is exogenous. The portfolio condition shown in Figure 1 for regime (ii) is found by substituting the value of dM from (30) into (28); and by substituting the value of dA from (29) into that, the resulting relationship is

$$\begin{aligned}
 -dB = & [m_r - (1/r)m_z(V/\bar{P}) + f_r]dr + (m_{r^*} + f_{r^*})dr^* \\
 & + m_z(V/\bar{P})dP/P.
 \end{aligned}
 \tag{32}$$

It follows that dr/dP from (32), although positive, is less than dr/dP from (28). Given dP and dr determined jointly by (31) and (32) and given dB which is exogenous under regime (ii), dA can be found from (29) and dM from (30).

A. The Workings of the Floating Rate Regime Without Intervention, dA = 0

1. An Open Market Operation: dM = -dB ≠ 0.

Given dr = 0 from (29), it follows from (28) that

$$dP/P = (dM/M)(M/\bar{P})/m_z(V/\bar{P}) = (dM/M)z/(V/\bar{P})$$

^{12/}The regime (iii) model can also be used to analyze a passive payments policy, under which the actions of the home government are the same as under the floating rate regime, but under which the rest of the world intervenes and pegs the exchange rate. In such a system, equations (28) and (31) determine the home price level and interest rate so that except for the fact that the rest of the world suffers any implied change in reserves, the model is indistinguishable from the regime (iii) model.

so that the price level moves in the same direction as the money stock. It also follows from (22) that $dX = 0$ so that

$$dR/R = dP/P.$$

That, in turn, implies that the allocation of income and output is unaffected, a kind of neutrality result. In terms of Figure 1, relationship (28) shifts, giving rise to a new equilibrium at an altered P and R along the horizontal axis.

Although the system of equations for regime (i) is entirely recursive, the workings of the model can perhaps be better understood in terms of a different kind of description of the passage from the initial equilibrium to the new equilibrium. The description I shall give depends implicitly on the following dynamic scheme:

<u>Market</u>	<u>Adjusting Price</u>
Labor	Money wage, W
Home good	Price level, P
Home earning assets	Home interest rate, r
Foreign currency	Exchange rate, R

To fix ideas, suppose $dM > 0$. That means the government tries to grant additional loans or pays off debt it had outstanding. The implied excess stock supply of money and excess stock demand for home earning assets tend to push r down. That, in turn, gives rise to

- (i) An excess stock demand for foreign earning assets and an accompanying excess stock demand for foreign money in the foreign exchange market which push up the exchange rate, R .

- (ii) An increase in desired net investment [see (2)] the stimulating effect of which on aggregate demand is reinforced by the effect of the increase in the exchange rate on exports and home consumption of the home good. The increase in aggregate demand pushes up the price level, P .

The increases in R and P serve to reduce the initial excess supply of money since \bar{P} increases. That allows r to move back toward its initial position which works to wipe out the excess demand for foreign earning assets and the matching excess demand for foreign currency. The movement of r back toward its initial position and the rise in P through its effect on X work to wipe out the stimulus to aggregate demand, thus getting us to the new equilibrium described above.

In the story just given, it is suggested that orders are placed in the foreign exchange market, orders for foreign currency. Those orders never get executed. There is no source for an offsetting stock excess supply of foreign currency. The presence of those orders does push the exchange rate up, but that, by itself, does not cause a reduction in the excess demand for foreign currency. It could if the model were altered to allow, for example, for an inverse dependence of $E(R/R)$ on $R - R^0$, the difference between the ruling exchange and the initial exchange rate. Such an assumption could be interpreted to mean that people view the initial exchange rate, R^0 , as the "long-run" equilibrium rate and, therefore, expect departures from it to be wiped out over time. Here, with $E(R/R)$ assumed exogenous, excess demand in the market for foreign currency is reduced and wiped out only because r moves back toward its initial position, which it attains when equilibrium is restored.

The fact that r does not change unless r^* changes is, perhaps, surprising. There seem to be three important assumptions responsible for that result: The first is the absence of speculation; that is, the absence of any relationship between $E(\dot{R}/R)$ and the value of the exchange rate. The second is the absence of intervention by the monetary authority; that is, $dA = 0$. The third is the distinction between stock excess demands and supplies arising from desired portfolio shifts and flow excess demands and supplies arising from trade in goods. We treat the two as incommensurate and hence do not allow one to offset the other. We have made other assumptions, but these seem not to be critical. The assumption that foreigners do not deal in domestic assets has no bearing on the result. The assumptions about wealth elasticities and initial conditions are in the spirit of assumptions that rule out "distribution effects."

It is important to note that home and foreign earning assets are not assumed to be perfect substitutes. That is evidenced by the fact that a finite amount of intervention, $dA \neq 0$, allows r to change relative to r^* . It is true, however, that without intervention, the properties of the floating rate system we are examining are independent of the absolute magnitudes of f_r and f_{r^*} .

2. A Change in r^*

There is nothing neutral about this. To fix ideas, suppose $dr^* > 0$. Then by (29), $dr > 0$ (the horizontal relationship in Figure 1 shifts upward). The increases in r and r^* decrease desired money holdings so that P must increase [see (28)]. Also, given $dr > 0$, it follows from (22) that $dX < 0$ and hence that

$$dR/R > dP/P.$$

Pursuing this result, from (16) with $\dot{dA} = 0$ (an extension of the no intervention assumption), we have

$$\dot{dF} = (ex/X + ex_X)XdX - P^*(im_Y(dy/dX) + im_X)dX.$$

Since $dy/dX > 0$, under quite weak conditions requiring that export and import functions be sufficiently elastic, $\dot{dF} > 0$; that is, there is an improvement in the trade balance.^{13/} Assuming $\dot{dF} > 0$, we have an interesting result.

The rise in r^* creates a stock excess demand for foreign earning assets, a desired portfolio shift out of home earning assets into foreign earning assets. The attempt to carry out that transfer leads to changes in variables, particularly in r , that wipe out that stock excess demand. But when equilibrium is achieved, \dot{F} has increased and \dot{K} has decreased. Thus, although the home economy is unable to reallocate wealth toward foreign assets, it does reallocate saving away from accumulation of home assets to accumulation of foreign assets.

3. Fiscal Policy

Suppose there is an increase in government expenditures financed either by an increased rate of borrowing from the public or by an increased rate of money creation.^{14/} Working within Figure 1, the set of downward

^{13/} One sufficient condition is an elasticity of export demand with respect to X greater than unity in absolute value. Below we shall see that $\dot{dF} > 0$ helps imply a certain kind of stability.

^{14/} Under present assumptions -- namely, that $E(\dot{P}/P)$ and $E(\dot{R}/R)$ are exogenous -- r , P , R , and the composition of home output are all independent of the financing decision. The proof is that equations (28)-(31) do not contain M or B .

sloping lines shifts to the right. Neither the home interest rate nor the price level changes. The exchange rate falls by enough to generate the required release of output from the sum of exports and consumption of home goods.

Under weak conditions on export and import elasticities (the same as required above), the trade balance worsens in this case; namely, $d\dot{F} < 0$. This implies that for the economy as a whole, the increase in government expenditures is in some degree financed by borrowing from the rest of the world.

B. The Workings of the Fixed Exchange Rate Systems

1. An Open Market Operation

Comparable open market operations for regimes (ii) and (iii) can be defined as follows. In regime (ii) define the operation in terms of a given change in B. In regime (iii) define it as a change in M of equal magnitude to the regime (ii) change in B, but of opposite sign. (In each case the monetary authority is exchanging call loans for money with the public.) It follows, then, from equations (28) and (32) that such an operation gives rise to equal horizontal shifts of the upward sloping curves in Figure 1: $dM = -dB > 0$ produces rightward shifts; $dM = -dB < 0$ produces leftward shifts. Since the aggregate demand-equals-output relationship is unaffected by the open market operation, the effects of the operation are larger under regime (iii) than under regime (ii).

From the effects on r and P that can be determined from Figure 1, there follow effects on the composition of output by use and on the stock of reserves. Suppose the operation is such that $dM = -dB < 0$. Then r rises and P falls. The increase in r is accompanied by an increase in the home country stock of reserves as home residents are induced to borrow abroad. Net

investment, \dot{K} , declines, while exports rise. The effect on home consumption of home goods is uncertain since the relative price effect works to increase C , while the income effect [see (20)] works to decrease C . Imports decrease and, thus, the trade balance in units of foreign currency per unit time improves given sufficiently elastic export and import demand functions.

2. A Change in r^*

To fix ideas, suppose r^* increases. In Figure 1, relationship (28) shifts to the right, while relationship (32) shifts to the left, although the former shift may be considered relatively insignificant since its magnitude horizontally is proportional to the magnitude of m_{r^*} . Under regime (ii), the price level falls and the home interest rate rises.

In both regimes, there is a shift on the part of the public from home assets to foreign assets. More foreign assets are acquired under regime (iii) and, correspondingly, there is a larger loss of reserves under that regime.

3. A Change in P^* or R

The analysis of a rise in P^* is identical to the analysis of a rise in R , a devaluation. So to be specific, I shall deal with a devaluation. Under our assumptions, neither of the portfolio equilibrium curves in Figure 1 is affected by changes in P^* or R . A devaluation has the effect of shifting the aggregate demand-equals-output relationship in Figure 1 to the right, thus implying increases in r and P under both regimes.

The rise in r implies a portfolio shift out of foreign earning assets that is accompanied by an increase in the home stock of reserves, A . The rise in r also produces a decline in \dot{K} , which implies that $C + ex$ must increase to keep aggregate demand equal to output. But that can only

happen by way of a decline in X ; namely, $dP/P < dR/R$. It follows that exports increase and that imports decrease. A favorable effect on the trade balance occurs if certain minimal conditions on the elasticities of the export and import demand functions are satisfied. If these are the effects of a devaluation, what then do we make of the assertion that in order for a devaluation to be successful, it must be accompanied by contractionary monetary and/or fiscal policy?

Suppose that in Figure 1 the post devaluation aggregate demand-equals-output relationship passes through point Q . If somehow the new equilibrium is at Q , then the devaluation is ineffective. The key lies in the unchanged interest rate at Q . If r does not change, then neither does the home country stock of reserves nor X . It follows then that all other real variables are left unchanged. Thus, if one defines contractionary monetary policy as synonymous with a rise in r , the assertion I set out to question is true. Note, moreover, that nothing prevents the government from engaging in a monetary policy that keeps r fixed.

But may not the market impose a fixed r on the economy, thus making a devaluation ineffective? To answer yes is to assume that the partial derivatives of f with respect to r and r^* are indefinitely large in absolute value. In that case, devaluation is a futile gesture, except as it affects the home price level, unless it is accompanied by other policies. And if that is the situation, a deficient stock of reserves cannot be a problem. The government can get as large a stock of reserves as it wishes by selling call loans (bonds) -- that is, by increasing B -- because those conditions on the partial derivatives of f mean that residents view home and foreign earning assets as perfect substitutes and are willing to hold

unlimited amounts of B, borrowing abroad to finance their holdings. Those borrowings are accompanied by matching increases in the home country stock of reserves.^{15/}

As regards the traditional motives for devaluation, our conclusion is as follows. A devaluation unaccompanied by changes in fiscal or commercial policies will not increase the home stock of reserves or improve the trade balance (no matter how elastic the export and import demand functions) unless the devaluation is accompanied by a rise in the home interest rate.

4. The Mix of Monetary and Fiscal Policy

Under either fixed rate regime, the use of monetary and fiscal policy makes possible the achievement of (dr, dP) combinations in any quadrant of Figure 1, and, in particular, makes possible the combination $dr > 0, dP = 0$. We shall take $dP = 0$ as defining "domestic balance." "International balance" has usually been defined to mean the attainment of some desired payments balance, \dot{A} , perhaps zero. The fact that $dr > 0$ can be achieved while maintaining "domestic balance" was treated by Mundell [5] as implying control of \dot{A} by way of the presumed effect of r on \dot{F} . In this model, control of r implies control of F , and hence of the stock of reserves, A , not the balance of payments, \dot{A} .

Indeed, the equations we have been working with do not determine \dot{F} and hence do not determine \dot{A} . Under the fixed rate regimes, \dot{F} is not determined simply by the level of government expenditures and taxes and the level of the money stock. It depends on how those government policy variables and the other exogenous variables vary over time.

^{15/}This also shows that a country faced with such f and b functions can insulate its own price level from that in the rest of the world only by changing its exchange rate, appreciating in response to increases in P^* , depreciating in response to decreases in P^* .

III. Time Paths of the Variables and the Role of Expectations

So far we have examined the implied behavior of certain variables at a moment in time given that a disturbance impinges at that time. Thus, for example, in the fixed rate regimes, a devaluation at time t gives rise to an increase in the stock of reserves at time t . But, clearly, we are interested in more than such instantaneous responses. Indeed, since observations are always averages or integrals over finite time intervals, even if our interest is mainly in the "short-run," in order to say anything about observations it is necessary to describe responses at least in an interval about the point at which a disturbance occurs. Happily, something can be said. If the exogenous variables are continuous and differentiable functions of time in an interval, then so are the endogenous variables. Thus, for example, if a devaluation at time t is the only discontinuous disturbance that occurs in an interval about time t , then there exists a $\delta > 0$, such that an average of the stock of reserves over the interval $[t, t+\delta)$ is greater than an average over the interval $(t-\delta, t)$. In that sense, at least, the model has testable implications.

Although the model is capable of determining the time paths of the endogenous variables as functions of the time paths of the exogenous variables, a priori not much can be said about those time paths. Without assuming the structure to be linear, which would be highly implausible, we cannot display the reduced form explicitly. Moreover, given only qualitative assumptions about the parameters of the structure and about the time paths of the exogenous variables, even if the nonlinear reduced form could be found, it is doubtful that much could be deduced about it.

To see this, suppose

$$r(t) = r[x(t)]$$

is the reduced form equation for the home interest rate under one of the regimes, where $x(t) = [x_1(t), x_2(t), \dots, x_N(t)]$ is the vector of exogenous variables under that regime.^{16/} Given the function r , $\dot{\partial r} / \partial x_j$ can be found by first differentiating r with respect to time:

$$\dot{r}(t) = \sum_{i=1}^N r_i[x(t)] \dot{x}_i(t)$$

where $r_i(x) = \partial r / \partial x_i$. Then

$$\dot{\partial r}(t) / \partial x_j(t) = \sum_i \dot{x}_i(t) \partial r_i[x(t)] / \partial x_j(t) = \sum_i \dot{x}_i(t) [\partial^2 r / \partial x_i(t) \partial x_j(t)].$$

Obviously, assumptions that imply a sign restriction for $\partial r / \partial x_j$ do not by themselves imply a sign restriction for $\dot{\partial r} / \partial x_j$. The latter requires more assumptions about the time paths of the exogenous variables and more assumptions about the shape of the function r , which is equivalent to more assumptions about the shapes of the functions in the structure. To illustrate this concretely, we shall pursue the analysis of the effects of a change in the foreign interest rate under regime (i), the floating rate system.

From the supply side equations, (6) and (7), we have

$$\dot{Y} = h_K \dot{K} + h_L \dot{L}$$

$$h_{LK} \dot{K} + h_{LL} \dot{L} = (\dot{W}/P).$$

^{16/} Above, we did not solve explicitly for the function r , but we did find the partial derivatives of r -- $r_i(x) \equiv \partial r / \partial x_i$, $i = 1, 2, \dots, N$ -- by way of the implicit function theorem.

Differentiating these with respect to r^* , treating \dot{L} as exogenous, and using the prior results, $dK/dr^* = dY/dr^* = d(W/P)/dr^* = 0$, we have

$$\dot{dY}/dr^* = h_K \dot{dK}/dr^*$$

$$h_{LK} \dot{dK}/dr^* = d(W/P)/dr^*.$$

For regime (i), we found that under weak conditions on export and import elasticities, $dK/dr^* < 0$ and $dF/dr^* > 0$. It follows from the first of these and the supply side equations that $dY/dr^* < 0$ and, therefore, that something must happen to reduce the rate of growth of total demand for home output. It follows from the second inequality that something must happen to make home residents increase the rate at which they desire to add to holdings of foreign earning assets.

From (10),

$$\dot{F} = \partial[(\bar{P}/R)f]/\partial t = (\bar{P}/R)\dot{f} + (\dot{\bar{P}}/R)f$$

so that

$$\dot{dF} = (\dot{\bar{P}}/R)df + f d(\dot{\bar{P}}/R) + \dot{f} d(\bar{P}/R) + (\bar{P}/R)\dot{d\dot{f}}.$$

Since f is zero initially and df is zero under the floating rate regime, we have

$$\dot{dF} = \dot{f} d(\bar{P}/R) + (\bar{P}/R)\dot{d\dot{f}}.$$

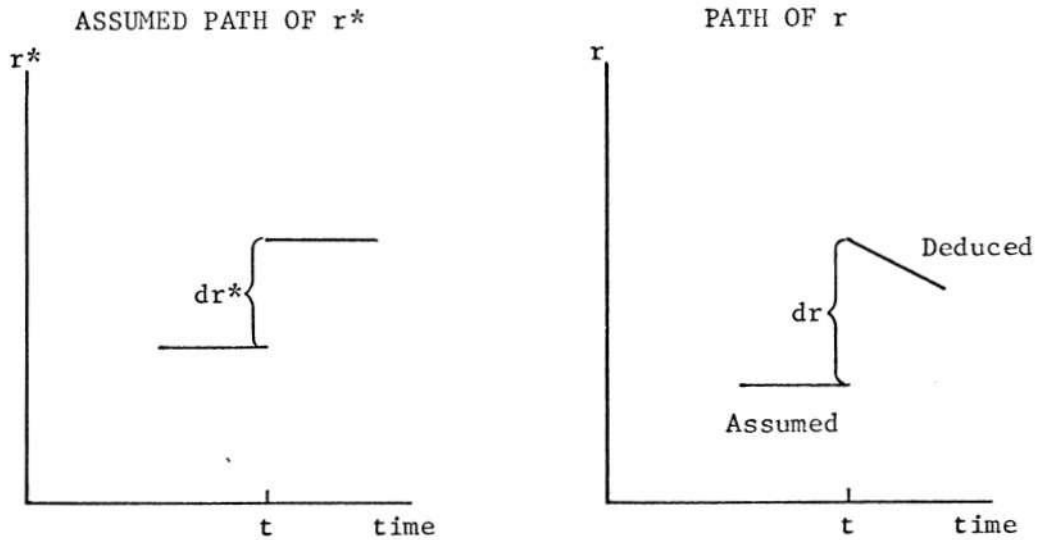
Since $d(\bar{P}/R)/dr^* \neq 0$, we must make some assumption about the initial value of \dot{f} . That comes down to assumptions about initial values of \dot{r} and \dot{r}^* .

Assuming that both are zero, the unit wealth elasticity assumption implies that \dot{f} is zero initially. Then

$$\dot{dF} = (\bar{P}/R)\dot{df} = (\bar{P}/R)(f_r \dot{dr} + f_{r^*} \dot{dr}^* + z \dot{df}_z).$$

But, $f_z = f/z$ implies that $df_z = 0$. So, if $\dot{dr}^* = 0$ and $dF/dr^* > 0$, it follows that $\dot{dr}/dr^* < 0$. Since we previously found that $dr/dr^* > 0$ and since we are assuming that $\dot{r} = 0$ initially, we may summarize this part of our result diagrammatically as in Figure 2.

Figure 2. Floating Exchange Rate Regime:
Effects on Path of r of Change in r^* at t



Under the further assumption that $\dot{h}_K = 0$ initially, it follows that

$$d(\partial K/\partial t)/dr^* = g' [dr/dr^* - h_{KK} \dot{dK}/dr^*] > 0.$$

Thus, while investment, \dot{K} , declines in response to an increase in r^* , the decline immediately begins to be reversed. That result and $dY/dr^* < 0$ imply that $\dot{dX}/dr^* > 0$, the latter being required to slow down the rate of growth of aggregate demand. Thus, the movement of the terms of trade over time is also such as to wipe out the initial jump.

This is an appropriate time to recall that the recursiveness we have used to determine some of the properties of the model involved the assumption that the expectations, $E(\dot{R}/R)$ and $E(\dot{P}/P)$, are not determined "rationally." Put differently, our results depend on the assumption that those variables are not equal to the model's forecast of \dot{R}/R and \dot{P}/P , but are predetermined, given by the past. As has been pointed out elsewhere [9], the assumption that expectations are equal to the model's forecasts makes solutions at any moment in time dependent on a specification of the time paths of the exogenous variables into the indefinite future and on the implied paths of the endogenous variables. While such considerations are outside the scope of this paper, it is not out of place to suggest that the imposition of some degree of rationality would provide an interesting way to study the role of speculation. For example, one could ask for what sets of parameter values and expected time paths of the exogenous variables is "partly rational" speculation stabilizing when contrasted with no speculation? By no speculation I mean the case of static expectations examined in this paper; namely, people expecting the current value of the price level and exchange rate to be maintained indefinitely. By "partly rational" speculation, I mean the case in which $E(\dot{R}/R)$ is equated to the model's solution value for \dot{R}/R (and similarly for $E(\dot{P}/P)$), while expectations of higher order time derivatives of R and P are assumed exogenous. Stability might be defined in terms of the magnitudes of the responses of the endogenous variables to exogenous shocks, for example, the magnitudes of dP/dr^* , dR/dr^* , and dr/dr^* .

IV. Concluding Remarks

The main task of this paper was to present a model in which the menu of assets among which wealth must be allocated includes a foreign earning asset. It should be emphasized, though, that while keeping to that basic portfolio framework, many kinds of models could be constructed.

The model I have presented does come to grips with the role of capital movements. It fully rationalizes why changes in the distribution of reserves in fixed rate systems and changes in exchange rates in floating rate systems may seem to be dominated by capital movements. The reason is that excess demands and supplies generated by desired portfolio shifts, being stock excess demands and supplies, cannot be offset by trade flows. They have to be accommodated -- in fixed rate systems, by changes in the distribution of reserves; in the no intervention floating rate system, by changes in interest and exchange rates that make private asset holders willing on net to hold their initial portfolio.

As it stands, the model I've set out cannot be used to judge the relative desirability of the different regimes. That question might, however, be considered within a stochastic version of the model in which current disturbances could not be offset immediately by current policy actions and in which policy, particularly in the fixed rate regimes, is more fully described. Then, given a criterion -- a plausible one in the context of the model described above is minimization of the variance of \bar{P} -- one could determine conditions under which one regime is better than another.

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