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I. Introduction

In this paper we study properties of the ex post forecast distributions of the reduced forms of two quarterly models of the U.S. economy: the "old" FRB-MIT model and the Michigan model.[1/] As part of the study of these distributions, we are able to test for structural change by, in effect, comparing the magnitudes of ex post forecast errors -- differences between mean forecasts and actual outcomes -- to a measure of the dispersion of the distribution of forecasts consistent with the results of estimation.

Our approach is novel only because we know of no other attempt to apply such a test to a simultaneous equations model, let alone to large nonlinear models. There have been studies in which differences between actual outcomes and what we call ex post nonstochastic forecasts (forecasts generated from the point estimates of all parameters) are compared across models including a variety of "naive" models, but those comparisons cannot offer statistical grounds for acceptance or rejection of a model. In contrast, the test we perform determines in a probabilistic sense whether the magnitudes of ex post forecast errors can be attributed entirely to randomness in the economy and to uncertainty stemming from the size of the data set, or, must in part be attributed to structural deficiencies of the model (structure here includes a stochastic specification consistent with the particular estimation procedure employed).

We recognize that there probably have been equation-by-equation tests of the structural equations of certain models, but such tests and those we apply to the reduced form are not perfect substitutes for one another. This is especially true when the forecast period is too short to allow for separate estimation of the complete parameter set. But whether that is the case or not, for many purposes -- in particular, for prediction and control -- the properties of the forecast distributions implied by the reduced forms are of primary interest.

The remainder of the paper is organized as follows. In section II, after a brief description of the models, we describe the statistic we employ and the way it was computed. Our grounds for employing that statistic are presented in section III. The last three sections are devoted to a presentation of results: section IV to basic test results; section V to aspects of the confidence regions and to tests of linear functions of the variables; and section VI to other aspects of the forecast distributions -- comparisons between mean forecasts and nonstochastic forecasts, comparisons between forecast variances from multiperiod endogenous simulations and those from one-period simulations, and comparisons between forecast variances and residual variances.

II. The Specification of the Models and the Computation of the Test Statistics

As noted in the introduction, we test two models in this paper. The first, the Michigan model, is a relatively small model with 24 estimated equations. It has almost no financial sector and operates with the interest rate on 4-6 month commercial paper as its exogenous monetary instrument. The second model, an old version of the FRB-MIT model, has 75 estimated equations and a fairly elaborate financial sector which gives us a choice among possible monetary instruments.[2/] We chose the money stock, because the model has most often been used that way, and, because that is consistent with the estimation procedure; the demand for demand deposits in the FRB-MIT model was estimated with an interest rate as dependent variable and demand deposits as an independent variable.

Both models are estimated on quarterly data, the Michigan model on data for the period 1954-1 through 1967-4, the version of the FRB-MIT model we test on post-Korean War data up through 1968-3. The Michigan model was estimated by two-stage least squares with a special adjustment for serial correlation in two of the equations. Many of the equations are in first-difference form. The FRB-MIT model was estimated by ordinary least squares. In a majority of the estimated equations first-order serial correlation coefficients were estimated, and partial first differences taken.

In both cases, we accept the chosen estimation procedure. In addition, we make the estimation and forecast period breakdown for test purposes according to the above reported estimation periods; that is, we identify the estimation period for test purposes with the reported estimation period for the base model. This gives us a "forecast" period outside the period used to specify the model which seemed to us to yield stronger tests than would another breakdown, since, in general, the specification (functional forms, variables included, etc.) of each model was not determined before viewing the base-period data. Given the data available when we performed the computation, that decision gives us a 12-quarter forecast period for the Michigan model, 1968-1 through 1970-4, and a nine-quarter period for the FRB-MIT model, 1968-4 through 1970-4.

A disadvantage of such a breakdown is that a wider class of tests could be performed if the "estimation" period was shortened and the "forecast" period lengthened enough to allow all parameters to be estimated from data for the "forecast" period alone. In particular, a test of the hypothesis that all parameters changed -- as opposed to tests that certain functions of the parameters change -- might then be possible. However, even then, tests of the class we perform would still be of interest and the calculation of the statistics for them not any simpler.

The models are noncomparable not only with regard to estimation period, but also and perhaps more importantly, with regard to what is taken as exogenous. In all cases we set the forecast-period values of the exogenous variables at their actual values. To do otherwise would mean specifying equations for those variables and, in so doing, venturing far from the reported base models. On balance, the FRB-MIT model takes fewer variables as given than does the Michigan model, which one might expect given their relative sizes. The differences are summarized in a rough way in Table 1. Note that the set of exogenous variables for FRB-MIT is not simply a subset of that for the Michigan model. In particular, we should emphasize that we shall be examining reduced forms as functions of two quite different monetary instruments; the money stock in FRB-MIT, the commercial paper rate in Michigan.

In order to make a test whose statistical properties can (in principle) be determined, the models must be specified in stochastic terms. This means that for the types of tests we wish to make, more must be specified or assumed about the models than has been reported. It follows that the model tested is, in effect, a composite between a base model reported by its originators and our addendum, which will be described in detail below. One point, however, deserves mention here. We assume that the structural equation residuals are independent across equations. This is consistent with both the reported estimation procedures and the lack of reported covariances. We admit, though, that abandoning that assumption could have far-reaching effects on test results. It should therefore be stressed that <u>our test results imply something about our addendum as well as about the base models</u>.

We have chosen to use a test statistic of the general form

(1) $D=(C\hat{y})'(C\hat{\sum}C')^{-1}(C\hat{y})/r$

Here $\hat{y}'=(\hat{y}_{11}\hat{y}_{12},\ldots\hat{y}_{1M},\hat{y}_{21}\ldots\hat{y}_{2M},\ldots\hat{y}_{n1},\ldots\hat{y}_{nM})$ is an nM-element vector of deviations between mean forecasts of endogenous variables, $\hat{Y}$, and actual outcomes, Y, n being the number of endogenous variables for which we compute statistics (12 for the Michigan model, 16 for the FRB-MIT model) and M the number of quarters in the forecast period (12 for the Michigan model, nine for FRB). $\sum$ is the (MnxMn) covariance matrix of $\hat{y}$, and C is an (rxMn) matrix of constants, $r\leq Mn$. $\hat{Y}$ and $\hat{\sum}$ are computed conditional on the values of the endogenous variables during the estimation period. They depend in no way on forecast-period values of the endogenous variables.

D is a quadratic form in the deviations of the actual values of the endogenous variables from their expected values given the observations in the estimation period. Rejection regions and confidence intervals are set up using an F-distribution with r numerator degrees of freedom and q denominator degrees for D, where we take for q a rough average of the degrees of freedom (in estimating the residual) for the structural equations of the model. (In fact, for both models we set q=48 and always use 5 percent rejection regions and 95 percent confidence regions.) The implied confidence intervals or acceptance regions are ellipsoids in the space of deviations.[3/]

Because the models consist of sets of nonlinear structural equations, we estimate $\hat{Y}$ and $\hat{\Sigma}$ by way of Monte Carlo experiments. That is done by repeatedly drawing values of the structural parameters consistent with the estimation period mean and covariance estimates, and values of the residuals consistent with the estimation period residual variance estimates, and for each drawing generating an Mn element observation on y. For each model we take 300 random drawings and take as $\hat{Y}$ the (Mn-element) vector of averages of those observations and as $\hat{\Sigma}$ the sample (MnxMn) covariance matrix.

The random parameters are generated one structural equation at a time.4/ Letting $\hat{\hat{\alpha}}_i$ stand for the column vector of random parameters of the ith estimated equation, a priori sample values of $\hat{\alpha}_i$ are generated by the matrix equation.

$$(2) \qquad \hat{\hat{\alpha}}_i = \hat{\alpha}_i + R_i' v$$

where $\hat{\alpha}_i$ is the estimation period vector of point estimates, v is a column vector of independent, mean zero, variance one, random variables generated by a random number generator5/ (drawn independently for different equations), and R_i is a matrix such that $R_i'R_i$ equals the estimation period estimated covariance matrix of the point estimator. It follows, then, that $\hat{\alpha}_i$ generated by equation (2) has mean $\hat{\alpha}_i$ and covariance matrix $R_i'R_i$, the estimated covariance matrix of the point estimator.

The additive disturbance for each estimated equation is random both among runs and among periods in each run. It is chosen independently across time and equations according to

$$(3) \qquad w_i(j) = \sigma_i v$$

where $w_i(j)$ is the residual for the ith equation at time j, σ_i is the estimation period residual standard error of the ith estimated equation, and v is a random variable with the same properties as the v in (2). (Note that the v's referred to in (2) and (3) are drawn independently.)

Given (2) and (3), a single M-period simulation run may be thought of as generated as follows. First a random set of parameters is drawn for each estimated equation. Those drawings constitute the parameter values for the run. Then, residuals are drawn, one for each estimated equation. These are embedded in the equations, and a solution, $y^{(1)}= y_{11}, y_{21}, \ldots y_{n1}$, obtained via the Gauss-Seidel iterative procedure. That solution is dependent on estimation-period values of all variables and on forecast-period values of exogenous variables. Then a new set of residuals is drawn, again according to (3), and a solution, an observation on $y^{(2)}$, obtained. That observation is again dependent on estimation-period values of all variables and on forecast-period values of exogenous variables, and, in addition, is dependent on the previously solved for value of $y^{(1)}$. Proceeding in this way, observations on $y^{(3)}, y^{(4)}, \ldots y^{(M)}$ are obtained. As noted above, for the principle tests, we performed 300 such M-period endogenous simulation runs for each model.[6/]

III. Considerations in Selecting Statistics and Distributions Employed

The statistic we have chosen to use, described in the preceding section, is a slightly modified version of the statistic:

(4) $$D=\hat{g}' \hat{\Sigma}^{-1} \hat{g}/r$$

where:

(a) g is an r-vector of estimable parameters of the joint distribution of the endogenous variables y and the null hypothesis can be stated as

(5) H_0: $g=0$.

(b) $\hat{g}$ is the estimate of g made without regard to the restriction imposed by the null hypothesis.

(c) $\hat{\Sigma}$ is an estimate of the covariance of $\hat{g}$, also made without the null restriction.

(d) We have attempted to use "best" estimates in all cases.

With our particular models and small sample sizes, there are no available tests with known optimality properties (in terms of power). Therefore, in choosing both the general form of the test statistic (4), and the particular estimates and modifications used, along with the distribution used to define the critical region, we have been guided by known results for more simple models and asymptotic results for a general class of models which include ours.

We are painfully aware, however, that our model is too distant from the simple models and our sample too small to provide any rigorous justification at this time. Nevertheless in our judgment there are enough favorable indications to justify its use relative to a nonstatistical test.

The simple models to which we refer are the normal linear model (NLM) where the covariance matrix is known up to a scalar multiple and certain reduced-form models.

Let $Y \sim N(X\beta, \sigma^2 V)$ where Y is Tx1, X is a fixed TxK matrix, β is a Kx1 vector of unknown coefficients, V a known TxT matrix, and σ^2 an unknown scalar. The classical F-statistic for the hypothesis $g=C\beta-d=0$, where C is a fixed rxK matrix and d a fixed rx1 vector, is given by

(6) $$\hat{g}'\hat{\Sigma}^{-1}\hat{g}/r,$$

where $\hat{g}=C\hat{\beta}-d$, $\hat{\Sigma}=\hat{\sigma}^2C[X'V^{-1}X]^{-1}C'$, $\hat{\sigma}^2=\hat{u}'\hat{u}/T-K$, $\hat{u}=Y-X\hat{\beta}$, and $\hat{\beta}=(X'V^{-1}X)^{-1}X'V^{-1}Y$. This statistic has an F(r,T-K) distribution under the null hypothesis, and the critical region given by requiring that (6) be less than some constant is known to yield a UMP invariant test, i.e., UMP among those tests whose equal-power surfaces are certain "natural" ellipsoids.

Now consider a reduced-form model given by $Y\sim N(XA,\Sigma\otimes I_T)$, where Y is Txn, X is TxK, A is Kxn, Σ is an (nxn), unknown matrix, I_T is TxT, and the covariance matrix is partitioned according to columns of Y. For the hypothesis HA-D=0, where H is pxK, an invariant set of statistics is given by the roots of the determinantal equation in s

(7) $$|(H\hat{A}-D)'[H(X'X)^{-1}H']^{-1}(H\hat{A}-D)-(np)s\hat{V}|=0$$

where $\hat{A}=(X'X)^{-1}X'Y$, $\hat{U}=Y-X\hat{A}$, and $\hat{V}=\hat{U}'\hat{U}/T-K$.
The sum of these roots is given by the trace

(8) $$\mathrm{tr}\hat{V}^{-1}(H\hat{A}-D)'[H(X'X)^{-1}H'](H\hat{A}-D)/np$$

which can be put in the form

(9) $$g'\hat{\Sigma}^{-1}g/r$$

with $g'=[(h_1A-d_1),\ldots(h_pA-d_p)]$, r=np and h_i, d_i the ith rows of H and D. If p=1, (9) is a constant multiple of a Hotelling T^2 statistic, i.e., it has the distribution of (T-K)F(r,T-K-n+1)/T(T-K-n+1). Here it should be noted that the inversion of $\hat{V}$ loses n-1 more degrees of freedom. Except in the cases n=1 or p=1, there is more than one nonzero root of (7) and no UMP test exists. It should also be noted, though, that the joint distribution of the roots under the null hypothesis is independent of V. The reader is referred to Lehmann [7], and Rao [8] for a thorough treatment of these linear finite-sample-size models.

As the last example shows, multivariate UMP tests do not often exist for finite sample sizes. However, Wald [9] has shown that in an asymptotic sense the results for the NLM hold for a wide variety of models and hypotheses (including the above reduced-form model). The main requirements are that the estimates of g be maximum likelihood (or asymptotically equivalent to m.l.) and that $\hat{\Sigma}$ be the estimated asymptotic covariance matrix. Normality as such is not required. In the limit, the statistic (4) has a $F(r,\infty)$ or $\chi^2(r)/r$ distribution under the null hypothesis which is used to set up the critical region.

The rest of this section is devoted to comments on the major choices that were made in adapting the general statistic (4) to cur model.

As noted above the two models which we tested were estimated by ordinary least squares (OLS) or two-stage least squares (2SLS) by their originators. We have continued to use their point estimates of the model parameters, along with the covariance matrix for the "estimation" period. We are therefore treating their estimates as the best available. This means that to some extent a statistical test of the structure is confused with a test of the hypotheses implicit in the type of estimates the originator was willing to accept.

Given the form of the model, the general hypothesis which we would like to test is that the coefficient values and the distribution of the structural disturbances are the same in the "estimation" and "forecast" periods. The alternate hypotheses are that the structural coefficient values are different in the estimation and forecast periods, but, and we emphasize this, that the distribution of the structural disturbances remains the same.

However, given that our choice of estimation and forecast periods involves a short forecast period, it is not possible to test the complete hypothesis stated above; i.e., there is no appreciable power against a subset of the above alternate hypotheses. This means that there exists a set of functions of our parameters such that our test has relatively high power against the alternative that they change any given amount, and has relatively low power against the alternative that the complementary set of function changes.

We do not see any practical method to determine and estimate precisely those (estimable) functions which we can test with high power. Therefore we have chosen to test the conditional means of the endogenous variables in the forecast period -- given the actual values of the exogenous variables in both periods and of the endogenous variables in the "estimation" period. (Note that this does not imply we are testing all reduced-form parameters. In fact, we are only testing certain functions of the reduced-form parameters.) We could have tested the parameters in several equations, equation by equation, by either an OLS or 2SLS method. Given the assumption of the independence of disturbances across structural equations, this would be appropriate asymptotically. We thought, however, that our method, which is equivalent to using restricted predictors of endogenous variables, might give some added power for small samples.

To construct our statistic we assumed that the best estimate of the conditional means from the "forecast" period data were the actual values in that period. But, since our alternate hypothesis stated that the <u>structural</u> and not the <u>reduced-form</u> residual distribution remained the same, no estimate of the "forecast" period variance could be made without using the null hypothesis. Given this complication, we decided to compute the statistic as a prediction interval test, which is exactly equivalent to the test (4) for the NLM, see Chow [1], when g is of the form

$$(10) \qquad g=C(m_1-m_2),$$

where C is a given rectangular matrix and m_1-m_2 is the vector of differences between the predicted conditional means using "estimation" and "forecast" data respectively.

Our choice of a distribution to compute the critical region was based on the behavior of the simple and asymptotic cases mentioned above. An F distribution for finite samples is consistent with a χ^2 asymptotically. In addition the simple models indicate that an F might be an appropriate way to take account of the fact that the covariance matrix must be estimated.

As an approximation to the "denominator" degrees of freedom we use a rough average number of degrees of freedom for our equations in the "estimation" period. We did not attempt to subtract the additional degrees of freedom suggested by the "reduced form" simple model because of our assumption of independence of residuals across equations.

IV. Basic Test Results

Before turning to test results, it may be helpful to focus on some of the raw data. Figure 1 shows a number of single-quarter forecast distributions for real GNP from the Michigan model; while Figure 2 shows such distributions for the GNP deflator. Figures 3 and 4 show corresponding distributions from the FRB-MIT model. There is a clear-cut relationship between the forecast span and the variances of those distributions: the greater the forecast span, the greater the variance. We shall argue below that this arises mainly from the presence in the models of lagged endogenous variables and the fact that the greater the forecast span, the greater the number of those variables generated randomly within the simulations. Notice that in Figure 4, at each date the actual value of the deflator lies outside the estimated distribution of possible outcomes forecast by the FRB-MIT model.

We limit all our testing to a subset of the endogenous variables of the models: for Michigan, the 12 variables listed in Table 2, for FRB-MIT, the 16 variables listed in Table 3. For Michigan, the list includes an exhaustive breakdown of the endogenous components of nominal GNP -- variables 3, 5, 9, and 12 -- while for FRB-MIT it includes a similar breakdown except that imports, which are endogenous, are excluded. Tables 2 and 3 contain a variable-by-variable view of the output; for each variable and each date, we list the actual value, the actual minus the mean value (the means of distributions like those in Figures 1-4), and the standard error of forecast (standard deviations of distributions like those in Figures 1-4).

To the extent that the structure embodied in each estimated model applies over the forecast period, the standard errors of forecast in Tables 2 and 3 measure the precision of single-date, single-variable forecasts made conditional on values of the variables assumed to be exogenous. For some variables, those standard errors seem quite large. For real GNP for the Michigan model, they range from almost 1 percent of the level for the first quarter of the forecast period to about 5 percent for the twelfth quarter; for the FRB-MIT model they range from about three-fourths of 1 percent in the first quarter to almost 4 percent in the ninth quarter.

For any variable at any date, the ratio of the forecast error -- the second entry -- to the standard error of forecast -- the third entry -- is a single-variable version of the D of sections II and III and can be treated as a t statistic with 48 degrees of freedom, $t_{.05}(48)=2.01$. The F statistics in the last column are for each variable over all quarters of the forecast period.[7/] The relevant 5 percent critical values are $F_{.05}(12,48)=1.96$ for the Michigan model, and $F_{.05}(9,48)=2.08$ for the FRB-MIT model. For the Michigan model, F statistics for the GNP deflator, business fixed investment and the corporate AAA bond interest rate exceed the critical value; for the FRB-MIT model, F's for the GNP deflator, the two interest rates, nonresidential structures, and state and local purchases exceed the critical value. It is interesting that despite differences between forecast periods and exogenous variable sets, the models fail on roughly similar sets of variables: sets which include the GNP deflator, business fixed investment, and the long-term interest rate.[8/]

In interpreting the F statistics in Tables 2 and 3, it should be noted that if the model predicted zero correlations among outcomes for the same variable in different quarters, the F statistic for each variable would equal the average of the squared t statistics for the variable. Some examples of the correlations among variables are given in Tables 4 and 5 which, in each case, contain submatrices from the matrix of simple correlations between all pairs of the nM variables. The simple correlations between real GNP at different dates are given in the upper left-hand block; those between the GNP deflator at different dates in the lower right-hand block; and those between the two variables in the upper right-hand block. The submatrices for the two models are remarkably similar.

In each case, the correlations between forecasts of a variable at one date and at another date are positive. Moreover, the correlations decline as the time span between the dates increases: namely, looking from the diagonal either across a row or up a column. More interestingly, holding the span between dates fixed, the correlations tend to increase with time: namely, looking down from upper left to lower right on other than the main diagonals. This occurs despite the fact that the variances in Tables 2 and 3 increase with time and implies that the within-path covariance increases even faster. In a sense, it suggests that individual forecast paths become increasingly smooth as the fixed initial set of lagged endogenous variables gets less and less important.

The similarity between correlation matrices for the two models extends to the off-diagonal block. The pattern of asymmetry is common to both models. Real GNP is negatively correlated with past prices and positively correlated with future prices, although the former gets weaker and the latter stronger the further one gets from the beginning of the forecast period.

The positive correlations between real GNP at t and at t+j in Table 4 help explain, for example, why the F statistic for the Michigan model for the vector of GNP outcomes is lower than the average of the squared t's, which is 1.32. The actual forecast errors for real GNP for that model are all of the same sign; the model underpredicts real GNP in every quarter. But given the positive correlations in the upper left block of Table 4, those errors cast less doubt on the model than would a sequence of errors of similar absolute magnitude but with randomly varying signs. An average of the squared t's takes account only of the absolute magnitudes. In contrast, the F statistic credits the model for predicting correctly that forecast errors for different dates will be positively correlated.

Table 6 contains joint test results across variables and time. For the Michigan model, tests are performed for variables 2-12 in Table 2 for the first quarter, the first four quarters, the first eight quarters, and all 12 quarters. Real GNP is omitted, because an identity connects it to the deflator and the endogenous components of GNP. (The test statistics are virtually unaffected by including real GNP and omitting one of the other variables entering the identity. They would be completely unaffected if the identity were linear.) For the FRB-MIT model, tests are performed on all 16 variables in Table 3 for the first quarter, the first four, the first eight, and all nine. Given the variable-by-variable tests in Tables 2 and 3 and the seemingly large standard errors of forecast exhibited there, these results are somewhat surprising. They suggest that neither model's structure is adequate during the forecast period, although that result comes through less strongly for Michigan than for FRB-MIT. Loosely speaking, if these results are put along side Table 2 and 3 results, they suggest that although the models predict fairly well the correlations over time between forecast errors for single variables, they do not correctly predict the correlations among forecast errors for different variables.

V. Aspects of the Confidence Ellipsoids and Tests on Linear Functions of the Variables

As indicated above, the tests which we perform correspond to examining ellipsoids. Geometrically, for an r-vector of "deviations" y, the examination involves first setting up a standard ellipsoid, a region in r-dimensional space consisting of all r-vectors of y satisfying

$$y'\Sigma^{-1}y \leq 1$$

where Σ is the covariance matrix of y, which we estimate by $\hat{\Sigma}$. The ellipsoid has r axes, one corresponding to each of the characteristic roots,

$\lambda_1 \geq \lambda_2 \geq \ldots \geq \lambda_r$ and corresponding vectors, $v_1 v_2, \ldots v_r$, of $\sum$. The ith axis lies in the one dimensional subspace spanned by v_i, is centered on 0, and has radius length $\lambda_i^{1/2}$. The sum of the λ's equals the sum of the diagonal elements of $\sum$.

The ellipsoid to make a joint test of a set of $s \leq r$ linear functions of y, $c_i'y$, i=1,2,...,s, at the significance level δ, is found in two steps. First, the radius length of each axis of the standard ellipsoid is multiplied by $[sF_\delta(s,q)]^{1/2}$. Second, the resulting ellipsoid is projected perpendicularly onto the s-dimensional subspace spanned by the vectors $c_1, c_2, \ldots, c_s$. If we define the (sxr) matrix C by $C' = (c_1, c_2, \ldots, c_s)$, the projection, which is the desired ellipsoid is given by the set of vectors y satisfying

$$(Cy)'(C \sum C')^{-1} Cy \leq sF_\delta(s,q)$$

In the last section, we examined $\hat{\sum}$ to some extent and performed tests which involved choosing for C those matrices consisting of different sets of rows of the identity matrix of order nM. In this section we shall examine the shapes of the ellipsoids for certain subvectors of y, and shall perform tests on linear functions of them: first, tests suggested by the shapes of the ellipsoids; and then a test of interest, a priori.

We are interested in the shape of the ellipsoid as a means of summarizing the forecast distributions. Thus, v_1 is the length-one vector such that the variance of $v_1'y$ is a maximum equal to λ_1. In a sense, then $v_1'y$ is the linear combination about which the model has least to say. Similar interpretations can be given to $v_2'y, v_3'y, \ldots, v_r'y$, where $v_r'y$ is the linear combination with minimum variance. We are also interested in how well the model actually predicts these linear combinations.

We begin with results for the M-vector of deviations of each variable for the different dates of the forecast period. Although we are interested in the shape of each of the M-dimensional ellipsoids, space does not permit us to list all roots and their corresponding vectors. We can, however, give an example, and since an almost completely uniform pattern emerged for every variable in both models, an example chosen at random is very informative.

As illustrated by the vectors in Table 7, the general pattern of characteristic vectors is that those associated with lower variance exhibit higher frequency oscillations. In each case, v_1, the vector associated with the highest variance component, exhibits cycles with a period much greater than the forecast period, (i.e., frequency near 0), while v_2 and v_3 exhibit periods with frequency close to the length of the forecast period. The vector associated with the lowest variance typically has a period of two quarters. A second feature of the canonical form is that the first one or two components account for a very large percentage of the variance.

While we cannot present all characteristic vectors, we can present each root and the test statistic for each corresponding linear combination. In Tables 8 and 9 we give for each variable the M roots (ranked from largest to smallest and expressed as a fraction of the sum) and the corresponding test statistic, $(v_i y)^2/\lambda_i$, which can be evaluated using an F(1,q) distribution.[9/] Note that the F statistics in Tables 2 and 3 are simply averages of these. Although we do not discern any clear pattern from these tables directly, by splitting the characteristic vectors into high and low variance groups, certain features can be noticed.

For each variable, we have divided the M-dimensional space into a space of high variance linear combinations (in a sense, those about which the model has little to say) and a space of low variance linear combinations (those about which the model has a lot to say). The test results for each subspace are given in Tables 10 and 11. The parameter k, which is the dimension of the high variance space was determined as follows. Given that the roots are ranked from largest (λ_1) to smallest -- k=4 if $\lambda_4/\lambda_1>.05$, k=3 if $\lambda_4/\lambda_1<.05$ and $\lambda_3/\lambda_1>.05$, k=2 if $\lambda_3/\lambda_1<.05$ and $\lambda_2/\lambda_1>.05$, while k=1 if $\lambda_2/\lambda_1<.05$. Given the value of k for each variable, the high variance test statistic for that variable is the average of the corresponding first k test statistics in Tables 8 and 9, while the low variance test statistic is the average of the remaining M-k. The former can be treated as F(k,q) and the latter as F(M-k,q). Since the results for the FRB-MIT model (Table 11) are clearer than those for Michigan, we discuss them first.

The variables which did not pass the nine-period test for the FRB-MIT model are y_2, y_5, y_6, y_{11}, y_{13}. None of these variables pass the joint test of the high variance linear combinations, but all of them except y_6 pass the joint test of the low variance linear combination. Thus the actual data seem to exhibit a low frequency component with higher variance than the model itself. This can be interpreted to mean that the real world differs from the model in the direction of a naive model. Another way of stating this result is that the model tends to compensate sufficiently for high frequency autocorrelation but not for low frequency autocorrelation.

For the Michigan model where variables y_2, y_5 and y_7 did not pass the 12-period test, variables y_2 and y_7 fail the joint test of the high variance (low frequency) linear combination and pass the joint test of the low variance (high frequency) linear combination.

We also examined the ellipsoid generated by several variables jointly. In particular, we examined the characteristic vectors and values for the covariance matrix for real GNP, the GNP deflator, and the unemployment rate.[10/] It would have conveniently fit with our interpretation of the eigen vectors of single variables as frequency components if the joint eigen vectors could have been described as the components of the (3x3) correlation matrix for each frequency, with (approximately) distinct frequencies uncorrelated. This, alas, was not the case. The components of the single variables are obviously correlated across components. For example, the highest variance (joint) component had (roughly) the same form as in the single-variable analysis for the GNP and unemployment partitions, but the price partition behaved in a manner similar to the second and third single-variable components. Indeed, we were not able to find any useful general interpretation of these joint components.

This completes our examination of linear combinations suggested by the forecast distributions themselves. We now examine annual averages, a set of linear combinations which might be considered of interest, a priori.

We present joint test results for all the variables for which quarterly forecasts were tested in Tables 2 and 3. For the Michigan model, we test annual forecasts for the first year, the first two years jointly, and all three years jointly. For the FRB-MIT model we omit the first quarter of the forecast period and test annual averages for 1969, and for 1969 and 1970 jointly. In each case, the test statistic (1) is computed using the relevant matrix C. The results are given in Table 12.

As a forecaster of annual averages, the Michigan model fails the test for the whole forecast period, but passes it for one- and two-year horizons. While the relative standing of the model for different horizons is the same as in Table 6, the model is more consistent as a forecaster of annual averages. The same kind of comparison cannot be made for the FRB-MIT model, because all joint tests on quarterly forecasts were inclusive of

1968-4. Nevertheless, the poor showing of FRB-MIT as an annual forecaster over 1969 and 1970 is not entirely surprising. In the quarterly tests, the model did better forecasting only 1968-4 than it did forecasting for any longer period.

VI. Other Properties of the Forecast Distributions

1. Nonstochastic Point Forecasts and Their Relationship to Mean Forecasts

Tables 13 and 14 contain nonstochastic point forecasts, those minus mean forecasts, and the standard errors of the mean forecasts, which we take to be the standard errors of forecast in Tables 2 and 3 divided by the square root of 299 -- 299 is the number of Monte Carlo replications minus one. The nonstochastic point forecasts for each model are obtained from a single endogenous simulation over the forecast period with parameters and residuals set at their means: the parameters at their point estimates, the residuals at zero.

For both models, there are some large discrepancies between points and means. A single joint test for each model -- to determine whether all the discrepancies could arise from sampling error attributable to the Monte Carlo experiment -- yields an F statistic equal to 4.85 for the Michigan model, and one equal to 5.55 for the FRB-MIT model, in each case exceeding the relevant 5 percent critical value. In a statistical sense, at least, points do not adequately represent means, which is what one expects to find for any model other than one consisting of estimated linear reduced-form equations. Of course, despite the high values of the test statistics, one might still want to use the nonstochastic estimates because they can be obtained more cheaply. The important point, though, is that such a judgment would be hard to make before appraising the kind of discrepancies that result for each model.

2. A Sequence of One-Quarter Forecast Distributions

The variation over time of the standard errors of forecast in Tables 2 and 3 could, in principle, be traced to two different sources. One involves the presence in both models of lagged endogenous variables: the greater the forecast span, the greater the number of lagged random disturbances affecting forecasts by way of their effects on the values of lagged endogenous variables. The other involves changes in average initial conditions: each standard error of forecast is a function of the fixed values of the predetermined variables conditional on which the forecast is being made. By analogy with linear models, we expect standard errors of forecast to be larger the more distant are the values of the predetermined variables from their sample period means. And since most variables in these models are stated in terms of levels, deviations of predetermined variables from their means can be expected to increase with time during the forecast period.

The results in Tables 15 and 16 allow us to draw some inferences about the importance of each source of variation. The statistics in these tables are derived from sequences of one quarter simulations in which lagged endogenous variables are each quarter set equal to actual values.[11/] Hence the standard errors of forecasts in Tables 15 and 16 vary only because average initial conditions change.

Our suggestion that most of the increase in variance in Tables 2 and 3 is attributable to the presence of lagged endogenous variables seems largely correct. In Table 15, there is, if anything, only a slight tendency for variances to increase. There is a clearer pattern in the results for the FRB-MIT model, but the rate of increase is very small relative to that in Table 3.

Note by the way that the corresponding 1968-1 mean forecasts and variances in Tables 2 and 15 and the 1968-4 values in Tables 3 and 16 differ only because they were generated from different samples of random variables. The differences give some indication of sampling error that arises from the Monte Carlo procedure given a sample size of 300.

3. Residual Standard Errors

For a single linear equation, the forecast variance is the sum of two parts -- the residual variance and the variance of the mean forecast -- where the latter is attributable entirely to parameter estimate variance which approaches zero as the sample size increases. The forecast variances we have computed cannot be split up in this way because structural parameters and residuals enter the reduced form nonlinearly. Thus, if we had computed the variances of mean forecasts from a set of simulation experiments in which only parameters were drawn randomly and added them to the corresponding residual variances computed from experiments in which only residuals were drawn randomly, we would not expect the sum to equal the forecast variance. Nevertheless, it is of interest to examine the residual variance, because it provides an estimate of the part of the forecast variance that, in principle, is independent of the amount of data available and that can be reduced only by altering the specification of the model.

In Tables 17 and 18 we report the ratio of each residual standard error to the corresponding standard error of forecast from Tables 2 and 3.[12/] For both models, the ratios tend to decline with time although the pattern is more consistent and far more pronounced for the Michigan model. For example, consider the results for real GNP in the ninth quarter of the forecast period for both models. While the standard error of forecast is about 25 billion for both models (see Tables 2 and 3), for the Michigan model only about 50 percent is directly attributable to the structure of the model and would remain no matter how large a data set had been available; for the FRB-MIT model about 75 percent is attributable to the structure of the model. The models differ more in this respect than in almost any other we've examined.

Appendix 1

Check for Strange Runs

The models we deal with are nonlinear. Hence, in general, there will not be a unique solution. The solution procedure, the Gauss-Seidel iterative routine, finds a solution. But, as illustrated by Friedman [4], there is no guarantee that quarter by quarter the solution is not switching, say, between alternative roots of a quadratic equation. The procedure outlined below is designed to discover such anomalies from an examination of the within-run behavior of each variable. The procedure is designed to discover runs in which the path over time of any variable exhibits unusually large jumps or oscillations.

Let $y_i(t)$ be the solved-for value of the ith variable at date t in a particular simulation run where $i=1,\ldots n$, and $t=1,\ldots M$. Let $x_i(t) = y_i(t)-\bar{y}_i(t)-[y_i(t-1)-\bar{y}_i(t-1)]$ where $y_i(0)=\bar{y}_i(0)$ -- the actual value of the ith variable in the last quarter of the estimation period -- and where for $t>0$, $\bar{y}_i(t)$ is the mean forecast of the ith variable at the tth quarter. The variance of $x_i(t)$ is $V_i(t)=S_i(t,t)+S_i(t-1,t-1)-2S_i(t,t-1)$, where $S_i(a,b)$ is the covariance of the ith variable between quarters a and b. We compute the ratio

$$R(i,t)=|x_i(t)|/[V_i(t)]^{1/2}$$

which we expect to be large for runs for which the solution routine is oscillating quarter by quarter between different multiple solutions.

Since for Michigan, M=12 and n=12, and for FRB-MIT, M=9 and n=16, and since for each model we performed 300 simulations, we end up with 43,200 observations on R for each model. The distribution of R for each model is summarized below.

Interval	Frequency	
	Michigan	FRB-MIT
0 - 2.0	41,272	41,237
2.0 - 3.0	1,811	1,814
3.0 - 4.0	103	140
4.0 - 5.0	8	9
5.0 - 6.0	6	0
6.0 -	0	0

Since the results are strongly in accord with what we would expect from a normal distribution for x, we concluded that there were no "strange runs" among our simulations. For example, if the distribution of R is normal, approximately 4.5 percent of the sample, 1,965 observations, should be greater than 2.0. For Michigan 1,928 fell in that range while for FRB-MIT, 1,963 did.

Appendix 2

Since the computer programs that were written to solve the Michigan and FRB-MIT models were not designed for our computations, it was necessary to add a significant amount of new coding. Our computations required two major programming additions: the first was to include a stochastic residual in each structural equation which was consistent with the form of the estimated equation; the second was a subprogram that generated random coefficients and residuals consistent with the distributions implied by estimation.

To check our residual coding and the randomization procedure, a program was written to generate for the estimation period 100 sets of stochastic predictions of the dependent variables and a nonstochastic set. For each equation we generated predictions using actual values of right-hand side endogenous variables and then calculated two statistics: a residual variance

$$\hat{\sigma}^2 = (\hat{y}-y)'(\hat{y}-y)/(N-k)$$

and the ratio

$$R = \frac{1}{100}\sum_{i=1}^{100} (y_i-\hat{y})'(y_i-\hat{y})/(N+k)\hat{\sigma}^2$$

where y is the (Nxl) vector of actual values of a dependent variable, over the estimation period, $\hat{y}$ the corresponding vector of nonstochastic single-equation predicted values, and y the vector of stochastic single-equation predicted values generated using the ith set of random coefficients and residuals. N and k are the number of observations used in estimating the equation in question and the number of independent variables, respectively. If the original coding was correct, $\hat{\sigma}^2$ should equal the residual variance reported in estimation. If our new coding is correct, the ratio R can be treated as F[(100)N,N-k].

In both models these statistics proved helpful in detecting and locating numerous errors that were bound to occur in a project of this size. For example, in a number of equations the residuals were improperly coded causing R to range as high as 1000.

Appendix 3

Generating Random Values of the Serial Correlation Coefficient

In the FRB-MIT model, a number of equations were corrected for serial correlation by taking partial first differences using an estimated first-order autocorrelation coefficient. Therefore, just as with all other estimated parameters, it was necessary to pick values of the autocorrelation coefficient consistent with the distribution implied by estimation.

Hildreth [5] has shown that the maximum likelihood estimator, $\hat{\rho}$, is asymptotically uncorrelated with all other estimated parameters, is asymptotically unbiased, and has asymptotic variance -- $(1-\hat{\rho}^2)/N$, where N is the number of observations. Based on that result and on the constraint that ρ lies in the interval (0,1), we constructed an approximate distribution for ρ as follows.

Define

$$\rho^* = \frac{1}{1+e^{A+BX}}$$

where X is distributed normally with mean zero and variance one. Clearly, ρ^* is confined to the interval (0,1). The problem is to find values of A and B such that, $E(\rho^*)=\hat{\rho}$ and $V(\rho^*)=(1-\hat{\rho}^2)/N$. To approximate such values, we used a series approximation to ρ^*, denoted r^*; where r^* consists of the first two terms of a Taylor expansion of ρ^* about the mean of X:

$$r^* = \frac{1}{1+e^A} - \frac{Be^A}{(1+e^A)^2} X + \frac{B^2 e^A (e^A - 1)}{2(1+e^A)^3} X^2$$

Since X is normal,

$$E(r^*) = \frac{1}{1+e^A} + \frac{B^2 e^A (e^A - 1)}{2(1+e^A)^3}$$

$$V(r^*) = \frac{(Be^A)^2}{(1+e^A)^4} + 2\left[\frac{B^2e^A(e^A-1)}{2(1+e^A)^3}\right]^2$$

Setting $E(r^*)$ equal to the estimated mean, $\hat{\rho}$, and $V(r^*)$ equal to the estimated variance, $(1-\hat{\rho}^2)/N$, the resulting equations can be solved for A and B.

The approximation was checked for different $\hat{\rho}$'s by drawing samples of 500 ρ^*'s and calculating sample means and variances. It was found that for $\hat{\rho}$ close to one, the approximation was poor; for $\hat{\rho}$'s greater than .9, the sample variances exceeded $(1-\hat{\rho}^2)/N$ by more than 20 percent. That led us to try a third-order Taylor expansion for ρ^*. With the third-order approximation, for $\hat{\rho}$ less than .98, sample means and variances differed from the actuals by less than 5 percent. However, for $\hat{\rho}$'s greater than .98, the approximation was still poor. Therefore, for the two equations with $\hat{\rho}$'s in excess of .98, we assumed zero variance as one would if first differences had been taken.

Table 1. Principal Exogenous Variables by Model

		Mich.	FRB-MIT
MONETARY	Narrowly Defined Stock of Money	--[a]	X[b]
	Interest Rate on 4-6 Month Commercial Paper	X	Y[c]
FISCAL	Ratio of Personal Tax Payments to Personal Income	X	X
	Ratio of Corporate Tax Liability to Before-Tax Profits	X	Y
	Transfer Payments to Persons	X	Y
	Federal Gov't Expenditures	X	X
GNP AND INCOME COMPONENTS	Capital Consumption Allowances	X	X
	Exports	X	X
	Imports	X	Y
	State and Local Gov't Expenditures	X	Y
	Farm Investment	Y	X
DEFLATORS	Gross Auto Product	X	Y
	Gross Farm Product	X	X
	Total Gov't Purchases	X	Y
	Exports	X	Y
	Imports	X	X
	Inventories	X	Y

[a] Not in the model.

[b] "X" stands for independent or exogenous.

[c] "Y" stands for dependent or endogenous.

[d] In the Michigan model, net exports and its deflator are exogenous variables.

Table 2. Michigan: Actuals, Forecast Errors, and Standard Errors of Forecast

	1968-1	1968-2	1968-3	1968-4	1969-1	1969-2	1969-3	1969-4	1970-1	1970-2	1970-3	1970-4	F(12,48)
1. Gross National	693.5	705.4	712.6	717.5	722.1	726.1	730.9	729.2	723.8	724.9	727.4	720.3	
Product ($1958)	2.4	7.0	13.7	18.5	23.0	26.2	29.6	26.6	22.5	20.5	18.1	12.4	.45
	(5.6)	(8.2)	(9.9)	(12.7)	(14.4)	(16.1)	(18.9)	(21.9)	(24.7)	(28.2)	(34.4)	(38.3)	
2. Implicit Deflator	120.4	121.7	122.9	124.3	125.7	127.2	129.0	130.5	132.6	134.0	135.5	137.4	
for GNP (1958=100)	.2	.1	-.1	.0	.3	.5	1.1	1.7	2.5	3.3	4.2	5.8	2.25*
	(.2)	(.4)	(.5)	(.7)	(.9)	(1.1)	(1.4)	(1.6)	(1.7)	(1.9)	(2.1)	(2.2)	
3. Consumption ($)	519.6	529.0	543.8	550.8	562.0	573.6	582.1	592.5	603.1	614.4	622.0	627.2	
	7.2	6.1	13.3	14.1	19.3	22.6	23.0	25.8	27.7	28.2	28.0	34.7	1.04
	(4.1)	(6.3)	(7.8)	(9.4)	(11.0)	(13.0)	(15.6)	(19.0)	(22.5)	(25.8)	(31.8)	(35.5)	
4. Corporate Before-	86.7	88.6	88.4	91.3	93.0	93.4	89.9	88.5	82.6	82.0	84.4	79.2	
Tax Profits ($)	-2.1	.1	2.5	3.4	5.0	5.9	5.9	2.3	-1.5	-.5	-.2	-.2	.50
	(3.0)	(4.0)	(4.8)	(6.0)	(6.7)	(7.5)	(8.5)	(9.5)	(10.3)	(11.5)	(13.8)	(15.0)	
5. Business Fixed	88.4	86.4	88.3	91.6	95.7	97.5	101.5	102.7	102.6	102.8	103.6	101.3	
Investment ($)	2.6	-.3	1.7	3.6	6.4	7.8	11.6	13.4	15.0	17.4	19.9	19.0	2.02*
	(1.5)	(2.6)	(3.8)	(5.3)	(6.9)	(8.5)	(10.1)	(11.9)	(13.7)	(15.5)	(17.2)	(18.8)	
6. Private Nonfarm	146.9	141.8	152.4	157.9	169.2	149.6	142.9	135.7	125.2	128.6	151.2	175.3	
Housing Starts	-6.3	-14.3	-8.7	-6.6	6.7	-.1	4.2	3.9	-.1	5.1	18.7	26.4	.68
(0,000's)	(8.0)	(10.5)	(13.4)	(15.0)	(17.6)	(21.2)	(21.8)	(22.2)	(23.6)	(24.7)	(27.2)	(27.3)	
7. Corporate AAA	6.1	6.3	6.1	6.2	6.7	6.9	7.1	7.5	7.9	8.1	8.2	7.9	
Interest Rate (%)	.0	.0	-.1	-.1	.1	.0	-.1	.2	.5	.7	.6	.7	2.27*
	(.1)	(.1)	(.2)	(.2)	(.2)	(.3)	(.3)	(.3)	(.3)	(.4)	(.4)	(.4)	
8. Unemployment	3.7	3.6	3.6	3.4	3.4	3.5	3.6	3.6	4.2	4.8	5.2	5.8	
Rate (%)	-.4	-.5	-.8	-1.4	-1.9	-2.2	-2.6	-3.1	-3.0	-2.9	-2.8	-2.6	.68
	(.3)	(.5)	(.7)	(.8)	(1.0)	(1.2)	(1.3)	(1.5)	(1.7)	(1.9)	(2.2)	(2.5)	
9. Change in Business	2.6	10.4	8.2	9.3	7.4	7.9	11.3	7.2	1.6	3.1	5.5	3.6	
Inventories ($)	-6.2	2.5	2.0	5.4	4.7	4.1	9.5	5.2	2.3	2.7	4.2	1.4	1.21
	(3.1)	(3.5)	(3.7)	(4.0)	(4.5)	(4.4)	(4.4)	(4.8)	(5.0)	(4.8)	(6.1)	(6.3)	
10. Output Per Manhour	132.4	133.7	134.2	134.6	134.1	134.0	134.2	134.3	133.4	134.7	136.1	137.2	
Nonfarm Index	-.5	-.8	-1.6	-2.7	-4.7	-6.5	-8.5	-10.6	-13.8	-15.5	-17.2	-17.9	.67
1957-1959=100	(1.1)	(1.5)	(1.9)	(2.4)	(2.9)	(3.4)	(4.1)	(4.8)	(5.7)	(6.6)	(7.5)	(8.6)	
11. Employment Rate	97.7	97.8	97.8	98.0	98.1	98.0	97.8	97.8	97.3	96.6	96.2	95.8	
of Males (20 Years	.2	.3	.6	1.2	1.8	2.1	2.4	2.9	3.0	2.8	2.7	2.7	.54
and Over (%)	(.3)	(.5)	(.7)	(.9)	(1.1)	(1.2)	(1.4)	(1.6)	(1.8)	(2.0)	(2.4)	(2.7)	
12. Residential	28.8	30.6	29.9	31.7	33.0	33.9	31.0	30.4	29.1	28.4	29.2	32.2	
Construction ($)	.1	.7	-.9	.0	.9	2.6	1.5	2.2	2.0	2.0	2.3	3.1	1.10
	(.7)	(1.2)	(1.7)	(2.2)	(2.6)	(3.0)	(3.5)	(3.7)	(4.0)	(4.2)	(4.6)	(4.9)	

Notes: (a) Forecast Error = Actual - Mean Forecast.

(b) The F statistics are by variable over 12 quarters. Here and in subsequent tables, F values in excess of the relevant .05 critical values are starred.

Table 3. FRB-MIT: Actuals, Forecast Errors, and Standard Errors of Forecast

		1968-4	1969-1	1969-2	1969-3	1969-4	1970-1	1970-2	1970-3	1970-4	F(9,48)
1.	Gross National	721.8	722.0	726.2	730.7	729.3	723.6	724.7	727.3	720.5	
	Product ($1958)	9.8	11.8	8.9	9.0	8.6	2.2	-3.5	-5.5	-19.5	.98
		(5.3)	(10.1)	(14.1)	(16.9)	(19.9)	(21.8)	(22.7)	(24.0)	(25.3)	
2.	Implicit Deflator	123.5	125.7	127.2	129.0	130.5	132.6	134.0	135.5	137.4	
	for GNP (1958=100)	.7	2.3	3.4	4.7	5.8	7.4	8.5	9.7	11.4	6.75*
		(.3)	(.5)	(.6)	(.8)	(1.0)	(1.1)	(1.3)	(1.4)	(1.6)	
3.	Consumption ($)	550.8	561.8	573.3	582.1	592.6	603.1	614.4	622.1	627.0	
		3.1	6.7	10.4	11.5	16.6	19.8	21.8	20.8	17.5	.37
		(4.6)	(7.1)	(8.7)	(10.5)	(12.6)	(13.9)	(14.8)	(16.5)	(18.0)	
4.	Dividend Price	2.9	3.1	3.1	3.3	3.4	3.6	4.0	4.0	3.6	
	Ratio (%)	-.3	-.2	-.2	-.1	-.1	.1	.5	.5	.1	1.22
		(.2)	(.3)	(.3)	(.4)	(.4)	(.4)	(.4)	(.5)	(.5)	
5.	Commercial Paper	6.0	6.7	7.5	8.5	8.6	8.6	8.2	7.8	6.3	
	Interest Rate (%)	.8	1.6	2.3	2.6	3.2	3.9	3.6	3.4	1.8	2.45*
		(.6)	(.8)	(.9)	(1.2)	(1.2)	(1.1)	(1.1)	(1.0)	(1.1)	
6.	Corporate AAA	6.2	6.7	6.9	7.1	7.5	8.0	8.1	8.2	7.9	
	Interest Rate (%)	.4	.8	.9	.8	1.4	2.0	2.2	2.4	2.1	5.18*
		(.2)	(.3)	(.3)	(.4)	(.4)	(.4)	(.4)	(.5)	(.5)	
7.	Deposits at	132.1	134.1	135.3	136.0	136.2	136.8	139.2	143.1	147.4	
	S&Ls ($)	.0	.8	.8	.4	.2	-.9	-1.2	.5	3.0	.63
		(1.0)	(2.1)	(3.1)	(4.0)	(4.8)	(5.5)	(6.1)	(6.7)	(7.3)	
8.	Corporate Before-	95.7	93.0	93.4	89.9	88.5	82.6	82.3	84.3	76.3	
	Tax Profits ($)	8.7	10.9	8.6	6.1	7.2	2.2	3.1	2.4	-8.8	.82
		(4.8)	(7.3)	(9.2)	(9.8)	(10.7)	(10.9)	(11.0)	(11.6)	(12.7)	
9.	Residential	31.7	33.0	33.9	31.0	30.4	29.1	28.4	29.2	32.2	
	Construction ($)	1.6	3.0	4.6	2.5	3.1	2.5	1.0	-.2	.2	2.05
		(1.0)	(2.4)	(3.9)	(4.9)	(5.3)	(5.6)	(5.8)	(6.0)	(6.7)	
10.	Producer	61.3	63.1	65.2	66.3	67.5	66.9	67.5	68.6	66.6	
	Durables ($)	-.6	1.5	4.5	6.8	9.6	11.3	14.0	16.7	15.2	1.23
		(1.1)	(1.8)	(2.8)	(4.0)	(5.1)	(6.2)	(7.1)	(7.7)	(8.1)	
11.	Nonresidential	30.3	32.6	32.3	35.2	35.1	35.7	35.3	35.0	34.7	
	Structures ($)	1.2	3.4	3.1	6.1	6.2	7.2	7.2	7.5	7.5	3.40*
		(.7)	(.8)	(1.0)	(1.3)	(1.8)	(2.2)	(2.6)	(3.0)	(3.3)	
12.	Change in Business	9.7	7.3	7.6	10.8	6.5	.9	2.6	5.0	3.0	
	Inventories ($)	.4	.6	3.1	7.1	4.6	1.2	4.1	5.3	2.2	.44
		(2.7)	(4.0)	(4.9)	(4.9)	(5.4)	(5.6)	(5.5)	(5.9)	(5.9)	
13.	State & Local	104.6	107.6	110.0	111.7	114.3	117.4	118.6	122.4	125.0	
	Purchases ($)	7.5	10.4	11.3	11.4	11.4	11.6	7.7	11.0	11.1	4.01*
		(1.6)	(2.1)	(2.5)	(2.8)	(3.3)	(3.5)	(3.8)	(4.1)	(4.5)	
14.	Employed Civilian	76.4	77.4	77.6	78.1	78.6	79.0	78.5	78.5	78.6	
	Labor Force (mil.)	.3	1.4	1.5	2.0	2.5	3.1	2.7	2.7	2.7	1.72
		(.4)	(.8)	(1.2)	(1.6)	(2.0)	(2.3)	(2.6)	(2.8)	(3.0)	
15.	Unemployment	3.4	3.4	3.5	3.6	3.6	4.1	4.8	5.2	5.8	
	Rate (%)	-.4	-.7	-.9	-1.0	-1.3	-1.2	-.7	-.5	.0	1.07
		(.2)	(.4)	(.6)	(.8)	(1.0)	(1.2)	(1.3)	(1.4)	(1.6)	
16.	Federal	187.0	197.2	202.5	200.8	202.0	195.9	196.6	194.9	191.7	
	Taxes ($)	5.6	17.7	19.4	17.2	18.2	17.2	15.2	16.7	10.8	1.63
		(3.4)	(5.0)	(6.5)	(7.4)	(8.2)	(9.0)	(8.9)	(9.2)	(9.7)	

Table 4. Michigan: A Submatrix from the Correlation Matrix
-- Real GNP and the Deflator

		REAL GNP 68-1	68-2	68-3	68-4	69-1	69-2	69-3	69-4	70-1	70-2	70-3	70-4	DEFLATOR 68-1	68-2	68-3	68-4	69-1	69-2	69-3	69-4	70-1	70-2	70-3	70-4
REAL GNP	68-1	1.00	.78	.62	.51	.44	.40	.36	.34	.31	.26	.24	.22	-.24	-.26	-.12	.05	.20	.29	.33	.35	.36	.35	.32	.31
	68-2		1.00	.86	.75	.67	.62	.57	.54	.50	.46	.44	.42	-.14	-.30	-.20	-.03	.18	.30	.36	.41	.44	.43	.41	.39
	68-3			1.00	.91	.83	.80	.74	.69	.64	.59	.55	.53	-.16	-.33	-.34	-.19	.05	.20	.28	.37	.41	.41	.39	.38
	68-4				1.00	.94	.89	.82	.77	.71	.67	.63	.61	-.17	-.31	-.33	-.25	-.03	.15	.24	.34	.39	.39	.39	.38
	69-1					1.00	.94	.88	.83	.77	.73	.68	.65	-.15	-.30	-.33	-.28	-.10	.07	.16	.27	.33	.34	.34	.33
	69-2						1.00	.94	.89	.84	.79	.74	.71	-.17	-.31	-.36	-.32	-.16	-.02	.08	.19	.26	.28	.29	.29
	69-3							1.00	.96	.91	.86	.80	.76	-.14	-.28	-.33	-.30	-.16	-.03	.04	.15	.23	.24	.27	.27
	69-4								1.00	.96	.91	.85	.81	-.11	-.25	-.30	-.26	-.13	.00	.06	.16	.23	.24	.27	.27
	70-1									1.00	.97	.92	.89	-.11	-.21	-.26	-.23	-.11	.01	.07	.16	.22	.22	.26	.26
	70-2										1.00	.97	.94	-.10	-.20	-.23	-.21	-.10	.02	.08	.17	.22	.21	.26	.26
	70-3											1.00	.99	-.07	-.16	-.17	-.16	-.06	.04	.10	.18	.23	.22	.26	.27
	70-4												1.00	-.06	-.14	-.16	-.14	-.05	.05	.11	.19	.23	.22	.26	.27
DEFLATOR	68-1													1.00	.73	.58	.49	.38	.29	.24	.20	.18	.15	.15	.14
	68-2														1.00	.87	.73	.55	.42	.34	.27	.23	.19	.18	.18
	68-3															1.00	.91	.74	.61	.51	.42	.37	.33	.30	.30
	68-4																1.00	.92	.81	.71	.61	.55	.51	.48	.46
	69-1																	1.00	.96	.89	.81	.76	.71	.67	.65
	69-2																		1.00	.97	.92	.88	.83	.80	.77
	69-3																			1.00	.98	.94	.90	.87	.85
	69-4																				1.00	.98	.96	.93	.91
	70-1																					1.00	.98	.96	.94
	70-2																						1.00	.99	.97
	70-3																							1.00	.99
	70-4																								1.00

Table 5. FRB-MIT: A Submatrix from the Correlation Matrix
-- Real GNP and the Deflator

REAL GNP									DEFLATOR										
68-4	69-1	69-2	69-3	69-4	70-1	70-2	70-3	70-4	68-4	69-1	69-2	69-3	69-4	70-1	70-2	70-3	70-4		
1.00	.77	.60	.50	.43	.36	.29	.25	.23	-.33	-.13	.02	.15	.18	.21	.22	.21	.18	68-4	REAL GNP
	1.00	.88	.73	.64	.55	.44	.36	.30	-.23	-.16	.03	.21	.26	.33	.35	.35	.33	69-1	
		1.00	.90	.80	.72	.60	.49	.41	-.15	-.12	-.01	.20	.28	.37	.41	.41	.40	69-2	
			1.00	.94	.86	.75	.64	.54	-.12	-.10	-.05	.09	.19	.31	.37	.39	.39	69-3	
				1.00	.95	.85	.75	.65	-.13	-.10	-.08	.04	.11	.25	.32	.35	.37	69-4	
					1.00	.94	.85	.74	-.11	-.10	-.11	-.01	.05	.16	.24	.29	.33	70-1	
						1.00	.95	.85	-.10	-.10	-.13	-.06	-.01	.08	.13	.20	.27	70-2	
							1.00	.95	-.11	-.11	-.16	-.11	-.07	.02	.05	.10	.18	70-3	
								1.00	-.11	-.12	-.17	-.14	-.12	-.05	-.02	.03	.10	70-4	
									1.00	.73	.58	.48	.44	.36	.33	.30	.31	68-4	DEFLATOR
										1.00	.85	.74	.67	.57	.51	.47	.44	69-1	
											1.00	.89	.83	.74	.67	.62	.58	69-2	
												1.00	.93	.87	.79	.74	.69	69-3	
													1.00	.94	.89	.83	.79	69-4	
														1.00	.95	.91	.87	70-1	
															1.00	.97	.93	70-2	
																1.00	.98	70-3	
																	1.00	70-4	

Table 6. Joint Test Results

Michigan		FRB-MIT	
M	F(11M,48)	M	F(16M,48)
1	2.23*	1	3.89*
4	2.62*	4	4.63*
8	2.26*	8	5.81*
12	3.90*	9	5.79*

Table 7. Characteristic Roots and Vectors of the Covariance Matrix of Corporate Before-Tax Profits from the Michigan Model

Root as a Fraction of the Sum	Vector (element i multiplies quarter i value)											
	1	2	3	4	5	6	7	8	9	10	11	12
.845	.04	.07	.11	.15	.18	.22	.26	.31	.34	.38	.46	.49
.089	.14	.21	.30	.35	.35	.34	.31	.24	.05	-.11	-.34	-.45
.023	.21	.34	.32	.34	.21	-.01	-.26	-.40	-.37	-.22	.11	.40
.011	-.43	-.51	-.23	.09	.39	.34	.19	-.14	-.29	-.20	-.02	.20
.007	-.17	-.11	-.01	.26	.26	-.02	-.35	-.42	.16	.59	.11	-.37
.006	.42	.23	-.34	-.45	.09	.46	.15	-.25	-.26	.19	.16	-.14
.004	-.25	.00	.26	.21	-.51	.10	.28	-.01	-.44	.09	.44	-.27
.004	-.06	-.09	.31	-.09	-.38	.41	.12	-.51	.47	-.09	-.23	.14
.003	-.51	.40	.09	-.33	.16	.24	-.35	.13	.17	-.32	.31	-.16
.002	-.38	.44	-.02	-.18	.17	-.40	.54	-.28	-.02	.17	-.19	.09
.002	.23	-.13	-.15	.12	.15	-.28	.28	-.27	.35	-.46	.49	-.26
.002	.13	-.36	.66	-.51	.29	-.17	.05	.01	-.13	.08	.09	-.07

Table 8. Michigan: Test Statistic and Root as a Fraction of the Sum

1. Gross National Product ($1958)	.68 .87	3.03 .08	.06 .02	.38 .01	.04 .01	.25 .00	.34 .00	.00 .00	.01 .00	.00 .00	.51 .00	.12 .00
2. Implicit Deflator for GNP (1958=100)*	3.00 .90	5.37* .07	6.05* .02	2.15 .01	2.47 .00	1.55 .00	3.46 .00	.99 .00	1.23 .00	.11 .00	.13 .00	.25 .00
3. Consumption ($)	1.37 .91	1.43 .06	.34 .01	.29 .01	.00 .00	.04 .00	1.64 .00	.52 .00	.09 .00	.93 .00	.52 .00	5.33* .00
4. Corporate Before-Tax Profits ($)	.02 .84	.69 .09	.02 .02	3.08 .01	.18 .01	.04 .01	1.06 .00	.00 .00	.07 .00	.59 .00	.14 .00	.09 .00
5. Business Fixed Investment ($)*	1.21 .95	.01 .04	.45 .01	.03 .00	.01 .00	1.06 .00	.48 .00	5.95* .00	.40 .00	2.48 .00	.00 .00	12.11* .00
6. Private Nonfarm Housing Starts (000's)	.11 .73	.92 .14	.01 .06	4.16* .02	.49 .01	.20 .01	1.07 .01	.78 .01	.33 .01	.02 .01	.01 .00	.02 .00
7. Corporate AAA Interest Rate (%)*	1.18 .86	7.50* .07	.18 .03	.18 .01	6.50* .01	3.74 .01	5.02* .00	.34 .00	1.85 .00	.11 .00	.57 .00	.00 .00
8. Unemployment Rate (%)	2.39 .90	1.98 .07	1.47 .02	.03 .01	.03 .00	.01 .00	.59 .00	.97 .00	.01 .00	.00 .00	.38 .00	.33 .00
9. Change in Business Inventories ($)	1.08 .41	1.58 .19	.03 .11	.31 .08	.04 .04	.64 .03	2.29 .03	.54 .02	1.15 .02	3.65 .02	.09 .02	3.09 .01
10. Output Per Manhour Nonfarm Index 1957-1959=100	5.07* .92	.13 .04	1.19 .01	.03 .01	.50 .01	.22 .00	.45 .00	.08 .00	.23 .00	.10 .00	.02 .00	.01 .00
11. Employment Rate of Males (20 Years and Over (%))	2.00 .90	1.24 .06	1.78 .02	.10 .01	.00 .00	.06 .00	1.01 .00	.16 .00	.01 .00	.01 .00	.09 .00	.11 .00
12. Residential Construction ($)	.32 .79	.01 .12	.02 .05	.88 .01	.43 .01	.36 .01	4.12* .00	.40 .00	.10 .00	.18 .00	.88 .00	5.55* .00

Note: Here and in Tables 9, 10, and 11, a "*" is affixed to any variable for which the M-period F statistic (see Tables 2 and 3) exceeds the .05 critical value.

Table 9. FRB-MIT: Test Statistic and Root as a Fraction of the Sum

1.	Gross National Product ($1958)	.00 .81	1.84 .13	.02 .03	.87 .01	.60 .01	3.81 .00	.31 .00	1.35 .00	.01 .00
2.	Implicit Deflator for GNP (1958=100)*	49.41* .88	.02 .08	2.13 .02	1.88 .01	.59 .01	2.83 .00	.29 .00	.06 .00	3.49 .00
3.	Consumption ($)	1.75 .86	.12 .08	.28 .03	.68 .01	.23 .01	.00 .01	.01 .00	.12 .00	.17 .00
4.	Dividend Price Ratio (%)	.13 .72	1.76 .15	.47 .16	1.82 .02	4.59 .01	.01 .01	1.03 .01	.84 .01	.54 .01
5.	Commercial Paper Interest Rate (%)	11.37* .63	.10 .18	.14 .08	4.02 .03	1.53 .02	.23 .02	1.59 .02	.62 .01	2.44 .01
6.	Corporate AAA Interest Rate (%)*	22.60* .77	3.60 .08	.90 .05	11.52 .03	.07 .02	5.09* .02	2.22 .02	.05 .01	.46 .01
7.	Deposits at S&Ls ($)	.01 .88	.06 .09	1.98 .02	2.17 .00	.44 .00	.02 .00	.46 .00	.33 .00	.22 .00
8.	Corporate Before-Tax Profits ($)	.13 .62	1.27 .22	.18 .07	2.62 .03	.22 .02	2.15 .02	.39 .01	.02 .01	.42 .01
9.	Residential Construction ($)	.14 .67	.42 .25	.50 .06	.68 .01	2.34 .00	1.62 .00	.23 .00	8.75* .00	3.73 .00
10.	Producer Durables ($)	4.14* .90	.01 .07	.00 .01	.11 .00	1.52 .00	.88 .00	1.09 .00	3.01 .00	.27 .00
11.	Nonresidential Structures ($)	8.53* .85	9.32* .07	2.47 .03	.04 .01	.04 .01	.66 .01	1.14 .01	8.30* .01	.07 .00
12.	Change in Business Inventories ($)	1.02 .40	.08 .23	.17 .09	.00 .07	.08 .06	2.45 .05	.04 .04	.08 .04	.03 .03
13.	State & Local Purchases ($)*	11.28* .82	13.58* .08	4.31* .03	.01 .02	.94 .01	.21 .01	5.47* .01	.24 .01	.04 .01
14.	Employed Civilian Labor Force (mil.)	1.28 .91	.66 .06	.02 .01	.26 .00	1.54 .00	.81 .00	2.74 .00	1.83 .00	6.09* .00
15.	Unemployment Rate (%)	.37 .85	2.64 .11	.32 .02	.18 .01	2.23 .00	3.11 .00	.62 .00	.02 .00	.03 .00
16.	Federal Taxes ($)	5.11* .70	2.78 .14	2.72 .05	3.01 .02	.24 .02	.04 .02	.59 .02	.17 .01	.04 .01

Table 10. Michigan: High and Low Variance Test Statistics by Variable

		K	High Variance Test Statistic	Low Variance Test Statistic
1.	Gross National Product ($1958)	2	1.86	.17
2.	Implicit Deflator for GNP (1958=100)*	2	4.18*	1.84
3.	Consumption ($)	2	1.40	.97
4.	Corporate Before-Tax Profits ($)	2	.36	.53
5.	Business Fixed Investment ($)*	1	1.21	2.09*
6.	Private Nonfarm Housing Starts (000's)	3	.35	.79
7.	Corporate AAA Interest Rate (%)*	2	4.34*	1.85
8.	Unemployment Rate (%)	2	2.19	.38
9.	Change in Business Inventories ($)	4	.75	1.44
10.	Output Per Manhour Nonfarm Index 1957-1959=100	1	5.07*	.27
11.	Employment Rate of Males (20 Years and Over (%))	2	1.62	.33
12.	Residential Construction ($)	3	.12	1.43

Table 11. FRB-MIT: High and Low Variance Test Statistics by Variable

		K	High Variance Test Statistic	Low Variance Test Statistic
1.	Gross National Product ($1958)	2	.92	1.00
2.	Implicit Deflator for GNP (1958=100)*	2	24.72*	1.61
3.	Consumption ($)	2	.93	.21
4.	Dividend Price Ratio (%)	3	.79	1.47
5.	Commercial Paper Interest Rate (%)*	4	3.91*	1.28
6.	Corporate AAA Interest Rate (%)*	3	9.03*	3.24*
7.	Deposits at S&Ls ($)	2	.04	.80
8.	Corporate Before-Tax Profits ($)	4	1.05	.64
9.	Residential Construction ($)	3	.35	2.89*
10.	Producer Durables ($)	2	2.07	.99
11.	Nonresidential Structures ($)*	2	8.92*	1.82
12.	Change in Business Inventories ($)	4	.32	.54
13.	State & Local Purchases ($)*	2	12.43*	1.60
14.	Employed Civilian Labor Force (mil.)	2	.97	2.00
15.	Unemployment Rate (%)	2	1.50	.93
16.	Federal Taxes ($)	3	3.54*	.68

Table 12. Annual Joint Test Results

Michigan		FRB-MIT	
Forecast Span	F	Forecast Span	F
1968	1.67	1969	8.26*
1968-69	1.26	1969-70	8.41*
1968-70	2.12*		

Table 13. Michigan: "Point Forecasts," Points Minus Means, and Standard Errors of the Discrepancy

	1968-1	1968-2	1968-3	1968-4	1969-1	1969-2	1969-3	1969-4	1970-1	1970-2	1970-3	1970-4
1. Gross National Product ($1958)	691.1	698.8	699.4	699.5	699.6	700.6	702.4	704.0	703.1	705.8	710.0	708.4
	.0	.3	.5	.4	.5	.7	1.1	1.5	1.8	1.4	.8	.5
	(.3)	(.5)	(.6)	(.7)	(.8)	(.4)	(1.1)	(1.3)	(1.4)	(1.6)	(2.0)	(2.2)
2. Implicit Deflator for GNP (1958=100)	120.2	121.6	123.0	124.2	125.4	126.6	127.8	128.6	129.8	130.4	130.9	131.1
	.0	.0	.0	.0	-.1	-.1	-.2	-.2	-.3	-.3	-.4	-.5
	(.0)	(.0)	(.0)	(.0)	(.1)	(.1)	(.1)	(.1)	(.1)	(.1)	(.1)	(.1)
3. Consumption ($)	512.3	523.1	530.7	537.2	543.3	551.6	559.8	567.3	575.8	586.4	592.9	590.6
	-.1	.2	.2	.5	.6	.6	.6	.6	.4	.2	-1.2	-1.9
	(.2)	(.4)	(.5)	(.5)	(.6)	(.8)	(.9)	(1.1)	(1.3)	(1.5)	(1.8)	(2.1)
4. Corporate Before-Tax Profits ($)	88.8	88.9	86.2	87.9	88.3	87.8	84.4	86.7	84.7	83.0	84.7	79.2
	.0	.3	.3	.1	.3	.2	.5	.5	.7	.4	.1	.2
	(.2)	(.2)	(.3)	(.3)	(.4)	(.4)	(.5)	(.6)	(.6)	(.7)	(.8)	(.9)
5. Business Fixed Investment ($)	85.8	86.8	86.9	88.3	89.7	90.3	90.5	89.8	88.0	85.7	83.8	82.1
	.0	.1	.3	.3	.4	.6	.7	.5	.4	.3	.1	-.2
	(.1)	(.2)	(.2)	(.3)	(.4)	(.5)	(.6)	(.7)	(.8)	(1.0)	(1.0)	(1.1)
6. Private Nonfarm Housing Starts (0,000's)	153.7	155.2	158.0	158.3	157.8	142.7	133.0	126.3	120.6	116.0	130.7	146.0
	.5	-.9	-3.0	-6.1	-4.7	-7.0	-5.8	-5.5	-4.6	-7.4	-1.8	-3.0
	(.5)	(.6)	(.8)	(.9)	(1.0)	(1.2)	(1.3)	(1.3)	(1.4)	(1.4)	(1.6)	(1.6)
7. Corporate AAA Interest Rate (%)	6.1	6.3	6.2	6.3	6.6	6.9	7.2	7.3	7.4	7.5	7.6	7.2
	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0
	(.0)	(.0)	(.0)	(.0)	(.0)	(.0)	(.0)	(.0)	(.0)	(.0)	(.0)	(.0)
8. Unemployment Rate (%)	4.0	4.1	4.3	4.8	5.2	5.6	6.1	6.5	7.0	7.4	7.8	8.2
	.0	.0	-.1	-.1	-.1	-.1	-.1	-.2	-.2	-.3	-.3	-.3
	(.0)	(.0)	(.1)	(.1)	(.1)	(.1)	(.1)	(.1)	(.1)	(.1)	(.1)	(.2)
9. Change in Business Inventories ($)	8.9	8.0	6.5	4.1	2.9	3.8	1.8	2.2	-.3	.3	1.2	1.8
	.1	.1	.3	.2	.1	.0	.0	.2	.4	-.1	-.2	-.4
	(.2)	(.2)	(.2)	(.2)	(.3)	(.3)	(.3)	(.3)	(.3)	(.3)	(.4)	(.4)
10. Output Per Manhour Nonfarm Index 1957-1959=100	132.8	134.5	135.7	137.1	138.5	140.0	142.1	144.2	146.1	148.8	151.4	152.8
	-.1	.0	-.1	-.2	-.4	-.5	-.6	-.8	-1.0	-1.4	-1.9	-2.3
	(.1)	(1.0)	(.1)	(.1)	(.2)	(.2)	(.2)	(.3)	(.3)	(.4)	(.4)	(.5)
11. Employment Rate of Males (20 Years and Over (%))	97.6	97.6	97.3	96.9	96.4	96.0	95.5	95.1	94.6	94.1	93.8	93.4
	.0	.0	.1	.1	.1	.1	.1	.2	.2	.3	.3	.3
	(.0)	(.0)	(.0)	(.1)	(.1)	(.1)	(.1)	(.1)	(.1)	(.1)	(.1)	(.2)
12. Residential Construction ($)	28.7	29.9	30.6	31.1	31.2	30.3	28.6	27.2	26.2	25.4	26.1	28.6
	.0	.0	-.3	-.7	-.9	-.9	-1.0	-1.0	-.9	-1.1	-.8	-.5
	(.0)	(.1)	(.1)	(.1)	(.2)	(.2)	(.2)	(.2)	(.2)	(.2)	(.3)	(.3)

Table 14. FRB-MIT: "Point Forecasts," Points Minus Means, and Standard Errors of the Discrepancy

	1968-4	1969-1	1969-2	1969-3	1969-4	1970-1	1970-2	1970-3	1970-4
1. Gross National Product ($1958)	712.7	711.7	718.1	722.1	720.4	720.6	726.3	730.4	736.1
	.8	1.5	.9	.3	-.3	-.8	-1.9	-2.4	-3.9
	(.3)	(.6)	(.8)	(1.0)	(1.2)	(1.3)	(1.3)	(1.4)	(1.5)
2. Implicit Deflator for GNP (1958=100)	122.9	123.6	124.1	124.6	125.1	125.6	125.8	126.1	126.3
	.1	.2	.3	.3	.3	.4	.3	.3	.3
	(.0)	(.0)	(.0)	(.0)	(.1)	(.1)	(.1)	(.1)	(.1)
3. Consumption ($)	548.9	556.6	563.8	571.5	577.3	584.6	593.4	601.7	608.9
	1.3	1.5	1.0	.9	1.3	1.3	.8	.4	-.7
	(.3)	(.4)	(.5)	(.6)	(.7)	(.8)	(.9)	(1.0)	(1.0)
4. Dividend Price Ratio (%)	3.2	3.3	3.3	3.4	3.5	3.5	3.6	3.6	3.6
	.0	.0	.0	.0	.0	.0	.0	.1	.1
	(.0)	(.0)	(.0)	(.0)	(.0)	(.0)	(.0)	(.0)	(.0)
5. Commercial Paper Interest Rate (%)	5.3	5.2	5.3	6.1	5.5	4.7	4.6	4.4	4.5
	.1	.1	.1	.1	.1	.1	.0	.0	.0
	(.0)	(.1)	(.1)	(.1)	(.1)	(.1)	(.1)	(.1)	(.1)
6. Corporate AAA Interest Rate (%)	5.9	5.9	6.0	6.3	6.1	5.9	5.9	6.0	5.9
	.0	.0	.0	.0	.0	.0	.0	.0	.0
	(.0)	(.0)	(.0)	(.0)	(.0)	(.0)	(.0)	(.0)	(.0)
7. Deposits at S&Ls ($)	132.3	133.6	134.7	135.7	136.0	137.5	140.0	142.0	143.4
	.1	.3	.2	.0	.0	-.2	-.4	-.6	-1.0
	(.1)	(.1)	(.2)	(.2)	(.3)	(.3)	(.4)	(.4)	(.4)
8. Corporate Before-Tax Profits ($)	88.0	83.6	85.8	84.9	82.2	81.6	79.9	82.2	85.0
	.9	1.5	.9	1.0	.9	1.2	.7	.2	-.2
	(.3)	(.4)	(.5)	(.6)	(.6)	(.6)	(.6)	(.7)	(.7)
9. Residential Construction ($)	30.1	30.3	29.7	28.8	27.2	25.9	26.2	27.9	30.2
	.1	.2	.4	.3	-.1	-.7	-1.2	-1.5	-1.9
	(.1)	(.1)	(.2)	(.3)	(.3)	(.3)	(.3)	(.4)	(.4)
10. Producer Durables ($)	62.1	62.1	61.3	60.3	58.7	56.3	54.1	52.4	51.5
	.2	.4	.6	.8	.8	.7	.6	.5	.1
	(.1)	(.1)	(.2)	(.2)	(.3)	(.4)	(.4)	(.4)	(.5)
11. Nonresidential Structures ($)	29.1	29.2	29.1	29.1	29.0	28.6	28.1	27.5	27.1
	.0	.0	.0	.0	.1	.1	.0	.0	-.1
	(.0)	(.1)	(.1)	(.1)	(.1)	(.1)	(.2)	(.2)	(.2)
12. Change in Business Inventories ($)	9.3	7.4	5.2	3.9	2.1	.1	-1.2	-.3	.8
	-.1	.7	.7	.2	.2	.4	.3	.0	.0
	(.2)	(.2)	(.3)	(.3)	(.3)	(.3)	(.3)	(.3)	(.3)
13. State & Local Purchases ($)	97.5	97.5	98.9	100.4	102.9	105.7	110.7	111.1	113.4
	.4	.3	.2	.1	.0	-.1	-.2	-.3	-.6
	(.1)	(.1)	(.1)	(.2)	(.2)	(.2)	(.2)	(.2)	(.3)
14. Employed Civilian Labor Force (mil.)	76.1	76.1	76.1	76.2	76.1	75.9	75.8	75.7	75.7
	.0	.1	.1	.1	.0	.0	-.1	-.1	-.1
	(.0)	(.1)	(.1)	(.1)	(.1)	(.1)	(.2)	(.2)	(.2)
15. Unemployment Rate (%)	3.8	4.1	4.3	4.5	4.8	5.2	5.4	5.6	5.8
	.0	-.1	-.1	-.1	-.1	-.1	-.1	.0	.0
	(.0)	(.0)	(.0)	(.0)	(.1)	(.1)	(.1)	(.1)	(.1)
16. Federal Taxes ($)	182.0	180.7	184.0	184.1	184.5	179.4	181.6	178.2	180.4
	.6	1.2	.9	.5	.8	.7	.3	.0	-.9
	(.2)	(.3)	(.4)	(.4)	(.5)	(.5)	(.5)	(.5)	(.6)

Table 15. Michigan: Sequence of One-Quarter Forecasts: Actuals, Forecast Errors, and Standard Errors of Forecast

	1968-1	1968-2	1968-3	1968-4	1969-1	1969-2	1969-3	1969-4	1970-1	1970-2	1970-3	1970-4
1. Gross National Product ($1958)	693.5 2.8 (5.3)	705.4 -.2 (5.0)	712.6 11.9 (4.5)	717.5 9.1 (4.8)	722.1 7.6 (4.7)	726.1 6.6 (5.3)	730.9 11.2 (5.8)	729.2 7.7 (5.8)	723.8 8.3 (7.1)	724.9 6.4 (6.6)	727.4 13.4 (9.1)	720.3 1.4 (5.3)
2. Implicit Deflator for GNP (1958=100)	120.4 .2 (.2)	121.7 .0 (.2)	122.9 -.2 (.2)	124.3 .2 (.2)	125.7 .0 (.2)	127.2 -.3 (.2)	129.0 .0 (.3)	130.5 -.2 (.3)	132.6 .0 (.3)	134.0 -.1 (.3)	135.5 .2 (.3)	137.4 1.2 (.2)
3. Consumption ($)	519.6 7.5 (3.9)	529.0 -.5 (3.7)	543.8 9.2 (3.3)	550.8 4.3 (3.6)	562.0 7.0 (3.5)	573.6 5.9 (3.8)	582.1 4.8 (4.3)	592.5 5.3 (4.2)	603.1 7.7 (6.2)	614.4 2.1 (4.9)	622.0 9.5 (8.0)	627.2 7.2 (4.0)
4. Corporate Before-Tax Profits ($)	86.7 2.0 (2.9)	88.6 -1.7 (3.0)	88.4 4.7 (3.0)	91.3 3.5 (3.0)	93.0 1.7 (3.3)	93.4 .0 (3.3)	89.9 .1 (3.3)	88.5 -3.7 (3.5)	82.6 -4.0 (4.0)	82.0 -3.7 (4.0)	84.4 1.4 (4.6)	79.2 -.8 (4.7)
5. Business Fixed Investment ($)	88.4 2.6 (1.4)	86.4 -3.9 (1.6)	88.3 3.1 (1.7)	91.6 4.2 (1.7)	95.7 1.2 (1.5)	97.5 -1.3 (1.6)	101.5 1.7 (1.6)	102.7 .3 (1.9)	102.6 .6 (2.0)	102.8 .8 (2.1)	103.6 2.4 (2.1)	101.3 -.6 (2.2)
6. Private Nonfarm Housing Starts (0,000's)	146.9 -5.6 (7.9)	141.8 -9.9 (7.9)	152.4 2.8 (8.1)	157.9 3.6 (7.9)	169.2 11.3 (8.4)	149.6 -1.7 (10.4)	142.9 4.0 (9.2)	135.7 1.0 (8.5)	125.2 -2.6 (8.0)	128.6 -4.4 (8.4)	151.2 11.3 (9.2)	175.3 16.0 (8.5)
7. Corporate AAA Interest Rate (%)	6.1 .0 (.1)	6.3 .0 (.1)	6.1 -.1 (.1)	6.3 .1 (.1)	6.7 .1 (.1)	6.9 -.1 (.1)	7.1 -.1 (.1)	7.5 .4 (.1)	8.0 .3 (.1)	8.1 .2 (.1)	8.2 .0 (.1)	7.9 .1 (.1)
8. Unemployment Rate (%)	3.7 -.4 (.3)	3.6 .1 (.3)	3.6 -.4 (.3)	3.4 -.6 (.3)	3.4 -.5 (.3)	3.5 -.4 (.3)	3.6 -.5 (.3)	3.6 -.8 (.3)	4.2 -.5 (.3)	4.8 -.2 (.3)	5.2 -.5 (.4)	5.8 -.2 (.3)
9. Change in Business Inventories ($)	2.6 -6.0 (3.1)	10.4 2.9 (3.0)	8.2 .2 (2.9)	9.3 2.5 (3.0)	7.4 .2 (3.4)	7.9 .2 (3.7)	11.3 6.7 (3.6)	7.2 1.3 (3.7)	1.6 1.6 (4.1)	3.1 4.1 (3.7)	5.5 7.2 (4.5)	3.6 3.6 (4.0)
10. Output Per Manhour Nonfarm Index 1957-1959=100	132.4 -.3 (1.1)	133.7 -1.2 (1.1)	134.2 -.2 (1.1)	134.6 -.4 (1.2)	134.1 -1.7 (1.1)	134.0 -1.9 (1.1)	134.2 -1.4 (1.3)	134.3 -1.2 (1.2)	133.4 -2.3 (1.5)	134.7 -1.5 (1.5)	136.1 -.4 (1.6)	137.2 -.7 (1.3)
11. Employment Rate of Males (20 Years and Over (%))	97.7 .2 (.3)	97.8 -.2 (.3)	97.8 .4 (.3)	98.0 .6 (.3)	98.1 .6 (.3)	98.0 .4 (.3)	97.8 .5 (.3)	97.8 .8 (.3)	97.3 .6 (.4)	96.6 .3 (.4)	96.2 .5 (.4)	95.8 .4 (.3)
12. Residential Construction ($)	28.8 .2 (.7)	30.6 1.5 (.7)	29.9 1.0 (.7)	31.7 1.5 (.8)	33.0 1.8 (.7)	33.9 1.9 (.9)	31.0 1.2 (.8)	30.4 1.5 (.8)	29.1 1.1 (.7)	28.4 .9 (.8)	29.2 .8 (.8)	32.2 .2 (.9)

Table 16. FRB-MIT: Sequence of One-Quarter Forecasts: Actuals, Forecast Errors, and Standard Errors of Forecast

		1968-4	1969-1	1969-2
1.	Gross National Product ($1958)	721.8 9.1 (5.0)	722.0 16.4 (6.2)	726.2 14.4 (6.3)
2.	Implicit Deflator for GNP (1958=100)	123.5 .7 (.3)	125.7 1.6 (.4)	127.2 1.2 (.4)
3.	Consumption ($)	550.8 2.5 (4.8)	561.8 8.6 (4.9)	573.3 10.7 (4.9)
4.	Dividend Price Ratio (%)	2.9 -.3 (.2)	3.1 .0 (.2)	3.1 -.3 (.2)
5.	Commercial Paper Interest Rate (%)	6.0 .7 (.7)	6.7 1.4 (.6)	7.5 1.6 (.7)
6.	Corporate AAA Interest Rate (%)	6.2 .3 (.2)	6.7 .7 (.2)	6.9 .5 (.3)
7.	Deposits at S&Ls ($)	132.1 -.1 (1.0)	134.1 1.2 (1.3)	135.3 1.1 (1.3)
8.	Corporate Before-Tax Profits ($)	95.7 8.1 (4.7)	93.0 .0 (12.6)	93.4 3.0 (12.6)
9.	Residential Construction ($)	31.7 1.7 (1.0)	33.0 1.8 (1.2)	33.9 2.2 (1.2)
10.	Producer Durables ($)	61.3 -.8 (1.2)	63.1 .1 (1.2)	65.2 .4 (1.2)
11.	Nonresidential Structures ($)	30.3 1.2 (.7)	32.6 2.2 (.8)	32.3 .7 (.8)
12.	Change in Business Inventories ($)	9.7 .1 (2.5)	7.3 -1.4 (2.7)	7.6 -.7 (2.7)
13.	State & Local Purchases ($)	104.6 7.3 (1.5)	107.6 10.3 (1.7)	110.0 9.8 (1.8)
14.	Employed Civilian Labor Force (mil.)	76.4 .3 (.3)	77.4 1.3 (.4)	77.6 .1 (.4)
15.	Unemployment Rate (%)	3.4 -.4 (.2)	3.4 -.5 (.2)	3.5 -.4 (.2)
16.	Federal Taxes ($)	187.0 5.0 (3.4)	197.2 13.4 (5.0)	202.5 13.9 (4.9)

Table 17. Michigan: Ratios of Residual Standard Errors to Standard Errors of Forecast

	1968-1	1968-2	1968-3	1968-4	1969-1	1969-2	1969-3	1969-4	1970-1	1970-2	1970-3	1970-4
1. Gross National Product ($1958)	.85	.75	.68	.64	.61	.58	.58	.54	.51	.48	.43	.39
2. Implicit Deflator for GNP (1958=100)	.95	.91	.88	.86	.81	.75	.72	.68	.66	.65	.62	.60
3. Consumption ($)	.79	.71	.66	.61	.58	.54	.53	.50	.47	.45	.40	.39
4. Corporate Before-Tax Profits ($)	.81	.76	.70	.67	.62	.56	.58	.55	.52	.50	.45	.42
5. Business Fixed Investment ($)	.91	.87	.78	.73	.67	.60	.56	.52	.49	.45	.42	.39
6. Private Nonfarm Housing Starts (000's)	.96	.87	.82	.82	.75	.68	.64	.61	.59	.60	.59	.54
7. Corporate AAA Interest Rate (%)	.90	.93	.83	.76	.74	.69	.66	.61	.53	.53	.51	.50
8. Unemployment Rate (%)	.90	.82	.76	.71	.68	.66	.63	.58	.54	.51	.46	.43
9. Change in Business Inventories ($)	.87	.81	.75	.76	.80	.81	.79	.81	.81	.79	.69	.62
10. Output Per Manhour Nonfarm Index 1957-1959=100	.88	.77	.71	.60	.54	.47	.42	.38	.36	.30	.28	.25
11. Employment Rate of Males (20 Years and Over (%))	.90	.80	.74	.68	.63	.61	.58	.54	.51	.47	.43	.40
12. Residential Construction ($)	.94	.89	.82	.78	.75	.70	.65	.61	.58	.57	.56	.54

Table 18. FRB-MIT: Ratios of Residual Standard Errors to Standard Errors of Forecast

		1968-4	1969-1	1969-2	1969-3	1969-4	1970-1	1970-2	1970-3	1970-4
1.	Gross National Product ($1958)	.84	.78	.72	.74	.73	.72	.77	.75	.76
2.	Implicit Deflator for GNP (1958=100)	.79	.81	.76	.67	.67	.70	.69	.71	.71
3.	Consumption ($)	.82	.73	.73	.74	.72	.73	.77	.74	.75
4.	Dividend Price Ratio (%)	.73	.67	.78	.81	.82	.88	.84	.83	.86
5.	Commercial Paper Interest Rate (%)	.97	.93	1.01	.95	.94	.91	.91	.88	.91
6.	Corporate AAA Interest Rate (%)	.95	.88	.97	.90	.92	.86	.88	.85	.86
7.	Deposits at S&Ls ($)	.86	.74	.73	.74	.79	.78	.80	.79	.80
8.	Corporate Before-Tax Profits ($)	.86	.84	.79	.85	.88	.91	.91	.85	.88
9.	Residential Construction ($)	.78	.69	.71	.75	.77	.76	.75	.73	.72
10.	Producer Durables ($)	.99	.83	.76	.73	.69	.68	.71	.71	.72
11.	Nonresidential Structures ($)	.93	.95	.88	.77	.69	.70	.71	.70	.73
12.	Change in Business Inventories ($)	.99	.97	.86	.93	.92	.86	.94	.83	.91
13.	State & Local Purchases ($)	.70	.63	.61	.61	.59	.59	.64	.61	.62
14.	Employed Civilian Labor Force (mil.)	.74	.68	.65	.64	.64	.61	.61	.60	.59
15.	Unemployment Rate (%)	.82	.78	.71	.70	.71	.71	.73	.73	.71
16.	Federal Taxes ($)	.95	.87	.78	.79	.79	.79	.81	.82	.85

Figure 1. MICHIGAN: FORECAST DISTRIBUTIONS OF REAL GNP

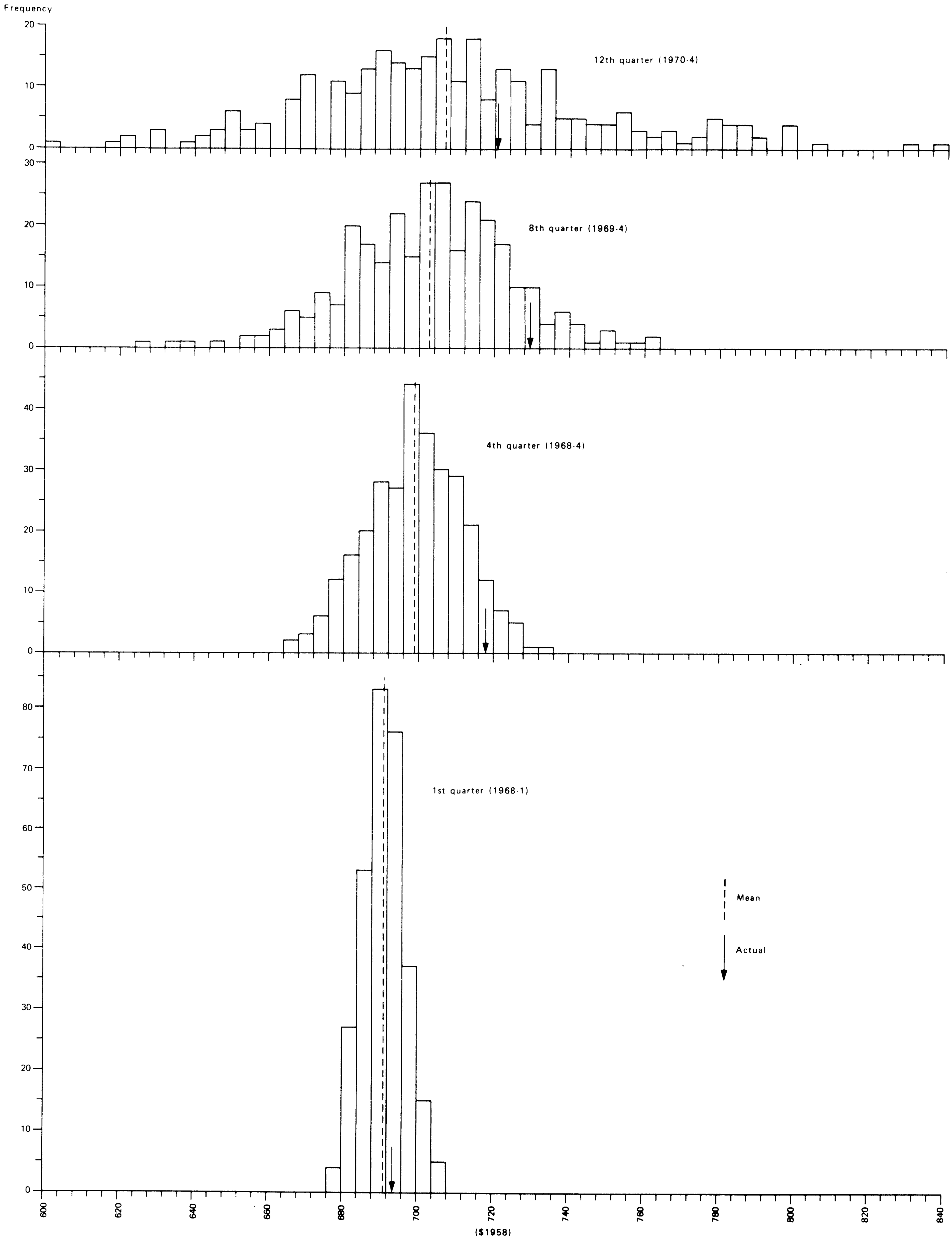


Figure 2. MICHIGAN: FORECAST DISTRIBUTIONS OF THE GNP DEFLATOR

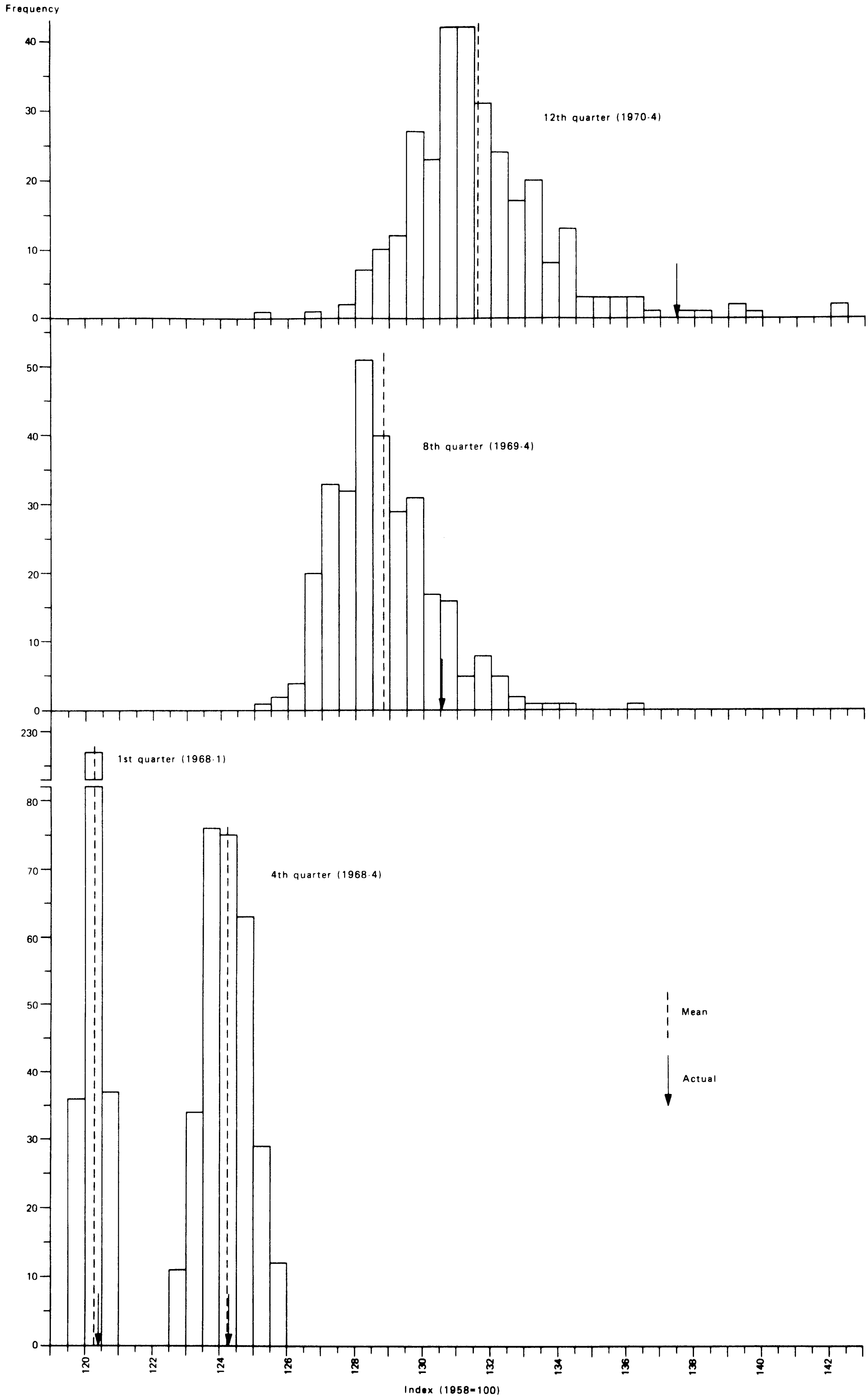


Figure 3. FRB-MIT: FORECAST DISTRIBUTIONS OF REAL GNP

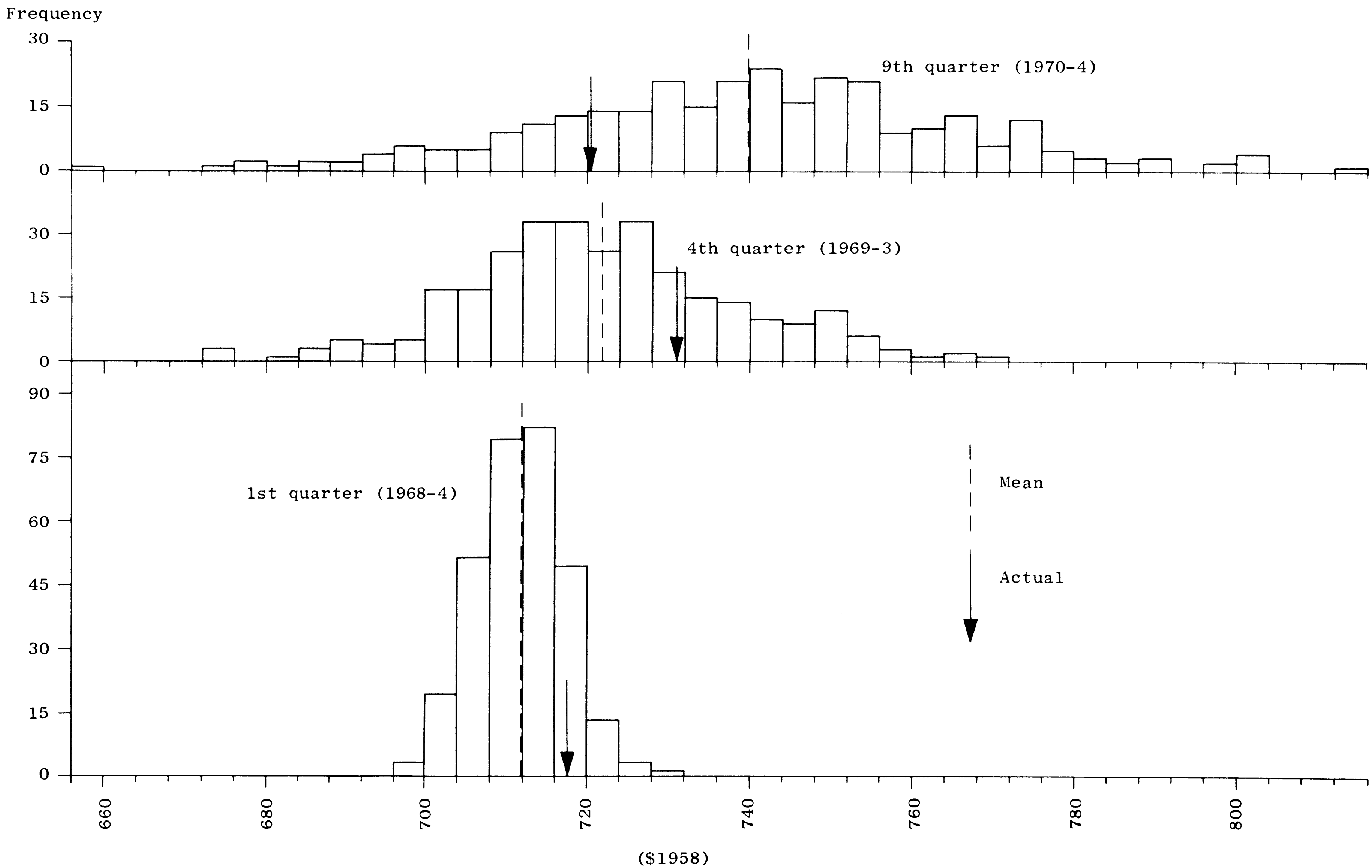
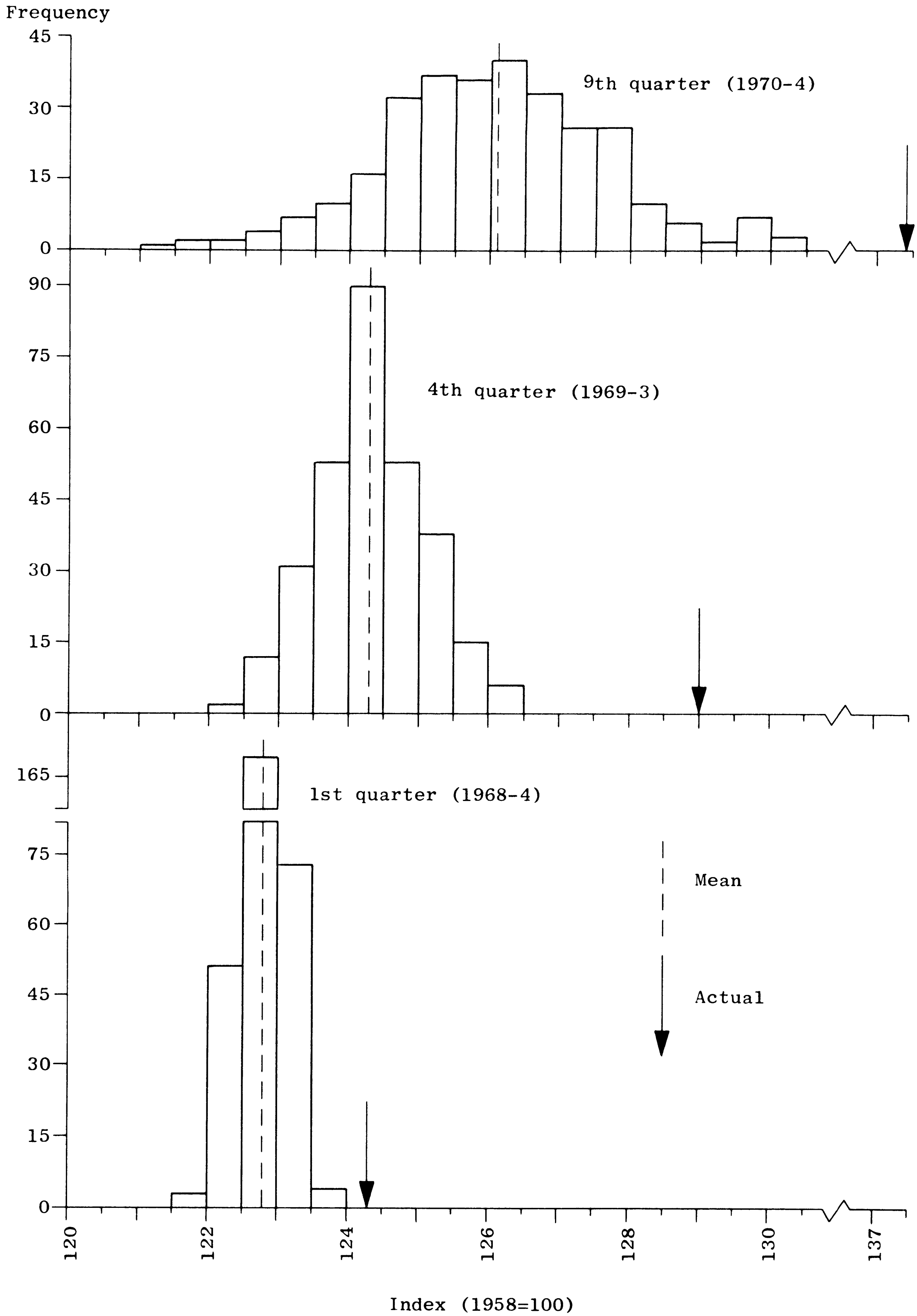


Figure 4. FRB-MIT: FORECAST DISTRIBUTIONS OF THE GNP DEFLATOR



Footnotes

1/ The Michigan model is described in [6]. The version of the FRB-MIT model we test has not been published. Versions much like it are described in [2] and [3].

2/ We altered two equations in the FRB-MIT model, those for capacity utilization and the unemployment rate. In both cases it was an alteration of form only, one that constrained the variables to their economically meaningful ranges, roughly speaking (0,1). In both cases, residual standard errors for the variables themselves were lower for our forms than for those originally in the model.

3/ In the next section we attempt to justify the distribution assumption and the use of D for a test for structural change.

4/ This follows from the assumed independence of disturbances across structural equations.

5/ The elements of v are drawn from a truncated normal distribution. Let x be a zero-one normal random variable. We draw values of x and accept only those for which $|x|<2$. The accepted x's have mean zero and variance $(.88)^2$, so that $v=(1.137)x$ has mean zero and variance one, the desired distribution. We choose v's from a truncated distribution, because most parameters and disturbances do not a priori have infinite range.

The above description applies to all parameters except first-order serial correlation coefficients in the FRB-MIT model. For their distribution, see Appendix 3.

6/ We performed checks on both the input and the output; the output was checked for oscillatory within-run behavior, (see Appendix 1), while the input was checked for coding errors (see Appendix 2).

7/ In terms of the statistic D, the t^2 statistic for variable i in quarter j is found by using for C the relevant row of an identity matrix of order Mn: namely, the row with unity in the $[(i-1)M+j]$th column. The F statistic for the ith variable is found by using for C the rows obtained by letting $j=1,2,\ldots,M$.

8/ It may also be of interest to note that the FRB-MIT model does poorly predicting the corporate AAA interest rate, but does well predicting the dividend-price ratio, variable 4, even though the former is an important determinant of the latter.

9/ This statistic is a special case of D, since if C is chosen to be a subset of the characteristic vectors of Σ, then $C\Sigma C'$ is a diagonal matrix with the corresponding roots as diagonal entries. ($F_{.05}(1,48)=4.04$.)

[10] For these computations, each variable was expressed as a ratio to its corresponding mean forecast, so that variances become coefficients of variation, etc.

[11] Because we were missing data for many of the endogenous variables for the FRB-MIT model for the period 1969-2 through 1970-4, we could not perform the entire sequence of one-period simulations.

[12] The residual variances were calculated from a set of simulation experiments similar in all respects to those underlying the statistics in Tables 2 and 3, except that parameters were held fixed at their point estimates.

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