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ALTERNATIVE CORE CONCEPTS WITH
PRIVATE INFORMATION: A COMMENT

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The recent growth of interest in economies with private information has resulted in a search for cooperative equilibrium concepts for these economies. Of these, three seem to have attracted considerable attention. Two of these have been advanced by Wilson (5), these being the coarse and the fine cores, and one by Srivastava (3), this being a more standard core concept. These alternative core concepts vary in the amount of information transmission envisioned between agents, with the coarse core allowing no such transmission, the fine core forcing complete transmission, and the core allowing arbitrary (feasible) forms of exchange of information.

Consideration of these alternate notions of the core has proceeded within the context of economies where trading occurs pre-state (Srivastava), and post-state (Wilson (5), Kobayashi (1)). When trading is pre-state, the coarse core, the fine core, and the core are nonempty. When trading occurs after realization of the state, the fine core may be empty. This note considers trading when the set of possible events has been reduced, but the state of nature has not been realized. A model along these lines is developed, and applied to the definition of the core. This is then compared with the coarse and fine cores. The result which emerges is that the coarse and fine cores may be nonempty for some such economies, while the core is empty.

There are several reasons for considering this notion of the core. The first is that the notions of the coarse and fine cores force agents to conceal, or to reveal information, even when it may not be in their best interests to do so. Thus, in some sense, these core concepts violate individual rationality with respect to information transmission.

The second is that existing analyses of these core concepts (Wilson (5), Kobayashi (1)) suggest that emptiness or nonemptiness of the core depends on whether or not certain kinds of opportunities exist to issue insurance.

The model developed here always allows for insurance of the same type, and thus indicates that the emphasis on opportunities for insurance is possibly misplaced.

Finally, it has been suggested "in general that the more communication is allowed the smaller is the resulting core."^{1/} Our definition of the core will allow no more information, and possibly strictly less information to be transmitted than does the definition of the fine core. Nevertheless, an economy is produced in which the fine core is nonempty, but the core is not.

The scheme of the paper is as follows. Section I outlines the economy considered, and defines the alternate core concepts outlined above. Section II considers the coarse and fine cores, while Section III considers the core. Section IV concludes.

I. The Model

The economy considered is the insurance model set forth by Rothschild-Stiglitz (2) and Wilson (4). The economy consists of a countably infinite set of agents, each of whom falls into one of two categories. These categories concern the probability of realizing a particular endowment of the single consumption good. In particular, any agent may realize an endowment of either e_1 or $e_2 < e_1$. In the latter case, we say a loss of $e_1 - e_2$ has occurred. The two categories of agents, then, differ with respect to the probability of realizing the endowment e_1 . Type 1 agents have a high probability of realizing e_1 (p_1), and type two agents have a low probability ($p_2 < p_1$) of realizing e_1 .

Let agents be indexed by $j = 1, \dots, \infty$. Let $t(j)$ be agent j 's type, let $e_s(j)$; $s = 1, 2$, be agent j 's endowment when there is no loss, and when

^{1/}Wilson (5), p. 814.

there is a loss respectively, and let x_j be agent j 's consumption. Finally, let θ denote the proportion of type 1 agents in the economy.

Trading in s -contingent claims occurs after $t(j)$ has been realized, but before s is known. Agents' types are private information. Finally, each agent has the same twice-continuously differentiable utility function $U(x)$ defined on R^2 , with $U' > 0$, $U'' < 0$.

Finally, a state of nature is an array $w \in \Omega$; $w = [t(1), t(2), \dots, t(\infty), e_s(1), e_s(2), \dots, e_s(\infty)]$. Thus a state of nature is a realization of a type, and an endowment for each agent. Let $x_j[t(j), s, w]$ be agent j 's consumption when his endowment is $e_s(j)$ if w is the realized state, and his type is $t(j)$.^{2/}

The information available to agent j ex ante consists of a partition \hat{P}_j of Ω consisting of the different events discernible by j in the absence of communication. The information available to j when trading occurs is denoted by P_j . Finally, denote by $\bigwedge_{j \in C} \hat{P}_j$ the coarsest partition that is refined by each of the \hat{P}_j for j belonging to some index set C , and by $\bigvee_{j \in C} \hat{P}_j$ the coarsest common refinement of the \hat{P}_j ; $j \in C$.

We are now prepared to define coalitions and blocking.

Definition. A viable coalition is a set of agents $C \subseteq \{1, \dots, \infty\}$, a set of allocations $x_j[t(j), s, w]$; $j \in C$, $s = 1, 2, w \in \Omega$, and a set of final informations P_j ; $j \in C$ satisfying

$$(1) \quad P_j \subseteq \hat{P}_j; P_j \supseteq \bigvee_{j \in C} \hat{P}_j \quad \forall j \in C$$

$$(2) \quad (w_1, w_2) \in P_j \text{ implies } x_j[t(j), s, w_1] = x_j[t(j), s, w_2] \quad \forall j$$

^{2/}Having s and $t(j)$ as arguments is redundant, but serves as a reminder that an agent's type matters in terms of exchanges which he desires, and of the types of exchanges which take place.

$$(3) \quad \sum_{j \in C} x_j[t(j), s, w] = \sum_{j \in C} e_s(j); \quad s = 1, 2; \quad w \in \Omega$$

$$(4) \quad E U\{x_j[t(j), s, w]\} \geq E U\{e_s(j)\} \quad \forall j \in C.$$

$P_j \subseteq \hat{P}_j$ is taken to indicate that P_j is no coarser than \hat{P}_j . These conditions state merely that (1) final information is feasible, (2) allocations are informationally feasible, (3) allocations are feasible within C , and (4) viable coalitions result in individually rational allocations.

Definition. An allocation list $\hat{x}_j[t(j), s, w]; j = 1, 2, \dots, \infty, s = 1, 2, w \in \Omega$ is blocked if there exists a viable coalition C with $x_j[t(j), s, w]$ and P_j such that $E U\{x_j[t(j), s, w]\} \geq E U\{\hat{x}_j[t(j), s, w]\} \quad \forall j \in C$, and with strict inequality for some $j \in C$.

These definitions of blocking are essentially Srivastava's (3). His core concept is then given by the following definition.

Definition. An allocation list $x_j[t(j), s, w,]; j = 1, \dots, \infty$, and information list $P_j; j = 1, \dots, \infty$, lies in the core if

$$(5) \quad P_j \supseteq \bigvee_{j=1}^{\infty} \hat{P}_j, \quad P_j \subseteq \hat{P}_j \quad \forall j.$$

$$(6) \quad \sum_{j=1}^{\infty} x_j[t(j), s, w] = \sum_{j=1}^{\infty} e_s(j); \quad s = 1, 2; \quad w \in \Omega$$

$$(7) \quad x_j[t(j), s, w]; \quad j = 1, \dots, \infty \text{ is not blocked.}$$

Expanding on these definitions, Wilson's (5) notions of the coarse and fine core are as follows.

Definition. The coarse core consists of all feasible allocations which are not blocked by any viable coalition, with (2) replaced by

$$(8) \quad P_j = \hat{P}_j.$$

Condition (8) is a no communication condition.

Definition. The fine core consists of all feasible allocations which are not blocked by any viable coalition C with (2) replaced by

$$(9) \quad P_j = \bigvee_{j \in C} \hat{P}_j$$

Condition (9) is a full communication condition.

With these alternate notions of blocking and the core advanced, we are now prepared to present our results.

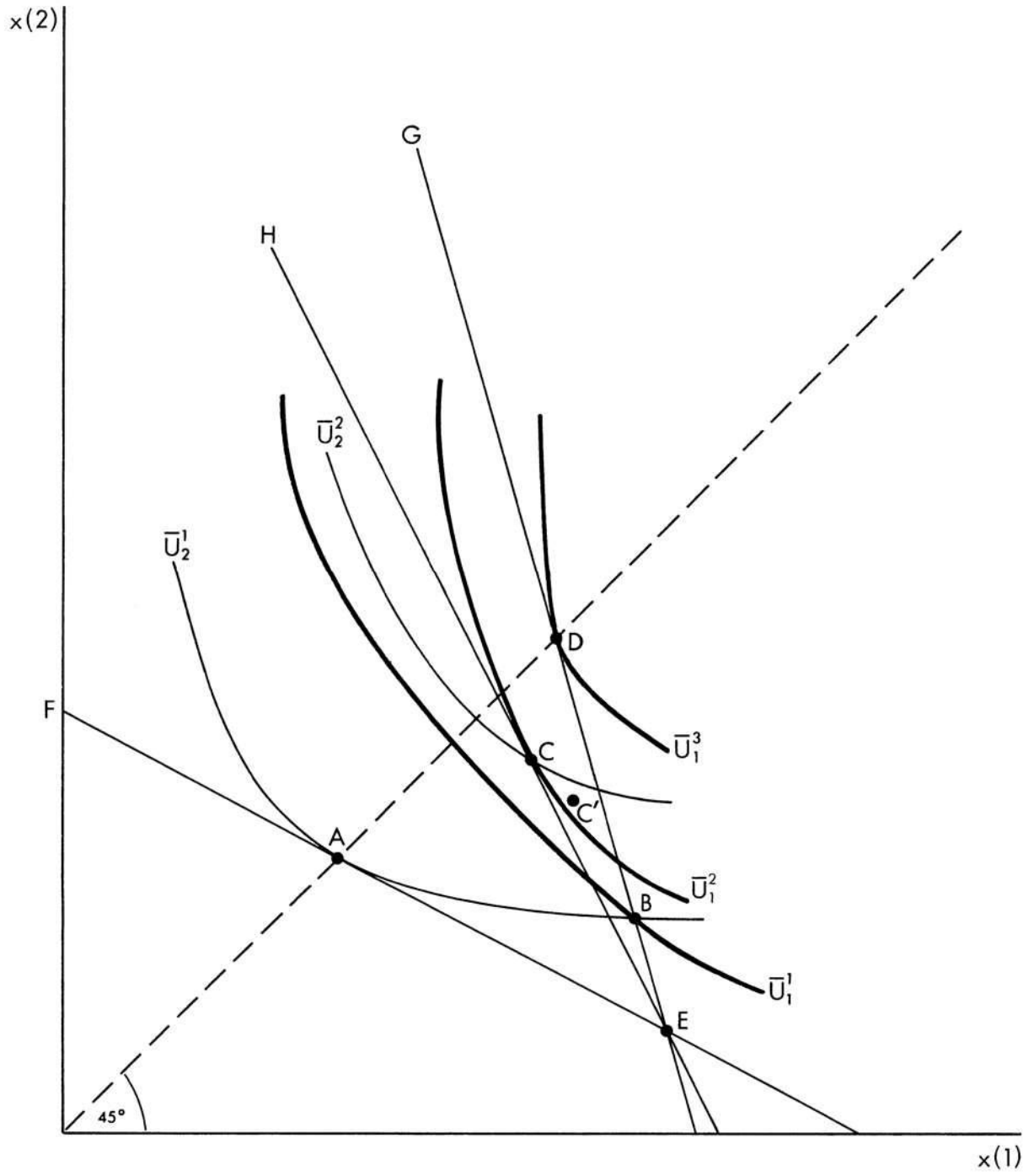
II. The Coarse and Fine Cores

As the purpose of this paper is to display an economy where the coarse and fine cores are nonempty but the core is empty, we need not strive for generality, even in this relatively simple context. Therefore, consider the economy depicted in Figure 1. Point E represents the endowment point with the axes representing consumption when $s = 1$ and $s = 2$. The locus labelled EF is the upper boundary of the feasible consumption set for a large number of type 2 agents. The locus labelled EG is the boundary for type 1 agents, and the locus EH is the boundary of the feasible set when all agents are "pooled," so that no distinction of type is permitted. The loci labelled \bar{U}_2^1 and \bar{U}_2^2 are indifference curves for type 2 agents, and the loci labelled \bar{U}_1^1 , \bar{U}_1^2 , and \bar{U}_1^3 are indifference curves for type 1 agents ($\bar{U}_1^3 > \bar{U}_1^2 > \bar{U}_1^1$).

Recall that the initial information of agents concerns only their own type. Then, since consideration of the coarse core imposes $P_j = \hat{P}_j \forall j$, we may write $x_j[t(j),s,w]$ as $x_j[t(j),s]$ in what follows.

Proposition 1. The coarse core of this economy is not empty.

Figure 1



Proof. Suppose that it is. Then there exists a viable coalition that can block the allocation list $\{x_j[t(j),1], x_j[t(j),2]\} = \{\tilde{x}(1), \tilde{x}(2)\} \forall j$, where $\{\tilde{x}(1), \tilde{x}(2)\}$ is the solution to

$\max p_1 U[x(1)] + (1-p_1)U[x(2)]$ subject to

$$(10) \quad [p_1\theta + p_2(1-\theta)]x(1) + [(1-p_1)\theta + (1-p_2)(1-\theta)]x(2) \leq$$

$$[p_1\theta + p_2(1-\theta)]e(1) + [(1-p_1)\theta + (1-p_2)(1-\theta)]e_2.$$

Such a coalition may take one of six forms.

Case 1. The blocking coalition consists of a finite number of agents of each type. But in this case, the law of large numbers does not operate, so that the feasible set of allocations for this coalition obeys $\forall j \in C$,

$$(11) \quad \sum_{s=1}^2 \sum_{j \in C} x_j[t(j),s,w] = \sum_{j \in C} e_s(j) \quad \forall w \in \Omega$$

$$(12) \quad x_j[t(j),s] = x_k[t(k),s] \quad \forall j, k \in C; s = 1, 2$$

$$(13) \quad [p_1\theta + p_2(1-\theta)] x_j[t(j),1] + [(1-p_1)\theta + (1-p_2)(1-\theta)] x_j[t(j),2] \leq$$

$$[p_1\theta + p_2(1-\theta)]e_1 + [(1-p_1)\theta + (1-p_2)(1-\theta)]e_2$$

((11) is a resource constraint for each state, and (12) and (13) are informational restrictions. If (13) was not a constraint, then members of the blocking coalition would know that the coalition consisted of a higher percentage of type 1 agents than the population of the economy.)

It is readily verified that (11)-(13) define a strict subset of the consumption pairs defined by (10). Thus type 1 agents in the coalition cannot be made better off. This, in turn, contradicts the assumption that there can be a blocking coalition of this type.

Case 2. The proposed allocation list is blocked by an infinite number of type 2 agents. Then $\forall j \in C$ (the blocking coalition), $\{x_j(2,1), x_j(2,2)\}$ is constrained by

$$(14) \quad p_2 x_j[2,1] + (1-p_2) x_j[2,2] \leq p_2 e_1 + (1-p_2) e_2.$$

But from Figure 1, the maximal point for type 2 agents in this set (point A) is not preferred to the proposed allocation (point C). Therefore, this assumption entails a contradiction.

Case 3. The proposed allocation list is blocked by an infinite number of type 1 agents. But for this to be informationally feasible, $\forall j \in C \{x_j[t(j),1], x_j[t(j),2]\}$ must satisfy (10). Then clearly no agents in the coalition are made better off.

The next two cases are obviously not blocking coalitions: coalitions of finite numbers of a single type of agent. Finally, we have the grand coalition. But this is also not a blocking coalition for the following reason. Type 1 agents cannot be made better off in the set defined by (10). Since type 1 and 2 agents must receive the same allocations, neither can type 2 agents. Thus, there exists no coalition which blocks the proposed allocation list. \square

Consider the fine core, then. Since each agent knows his own type, $\bigvee_{j=1}^{\infty} \hat{P}_j$ results in the type of each agent being public information. Then we have

Proposition 2. The fine core of this economy is not empty.

Proof. It is obvious that given $P_j = \bigvee_{j \in C} \hat{P}_j$ for any viable blocking coalition C, no coalition can block the points A and D. \square

III. The Core

A. Communication

In keeping with the notion that arbitrary feasible exchanges of information are to be allowed, we think at this point of members of a coalition either agreeing to communicate, or not to communicate types. If members of a coalition C agree not to communicate type, then $P_j = \hat{P}_j \forall j \in C$. If they agree to communicate, then each agent in the coalition simply announces his type. In addition, coalition formation must be informationally feasible. In other words, a coalition cannot be formed consisting only of type 1 agents if type 2 agents will not permit their type to be revealed; i.e., if they wish to belong to the coalition.

Finally, consider any coalition C in which members have agreed to communicate. Then consider agent j's announcement of type, $j \in C$. Let $a(j)$ be j's announced type. An incentive compatibility restriction is placed on j's announcement:

$$(15) \quad p_{t(j)} U\{x[a(j),1,w]\} + [1-p_{t(j)}] U\{x[a(j),2,w]\} > \\ p_{t(j)} U\{x[t(j),1,w]\} + [1-p_{t(j)}] U\{x[t(j),1,w]\}$$

for the allocation list of coalition C.

B. Emptiness of the Core

With this notion of communication, we have

Proposition 3. The core of this economy is empty.

Proof. With reference to Figure 1, suppose that an allocation list lies in the core, and has $P_j \neq \hat{P}_j$ (which will then be true $\forall j$). Then type 1 agents' allocations will lie along EG, and type 2 agents' allocations along EF. Type 2 agents cannot do better than point A. Clearly, type 1 agents may not do better than point B, then, since if they did, type 2 agents would announce type 1, and it would not be informationally feasible to form a coalition which could attain a better point.

However, points A and B are clearly blocked by the grand coalition agreeing not to communicate, and having each agent's allocation be point C. Moreover, any allocation list attained without communication is also blocked. To see this, note that point C is each agent's allocation in the coarse core. But all type 1 agents can agree to communicate type, and offer each member of a coalition a point such as C'. Type 2 agents will not join such a coalition, so that it will be informationally feasible to form the blocking coalition, and C' is compatible with the resource constraint of the blocking coalition. Thus, C is blocked as well, and the core is empty. \square

It will be recognized by those familiar with Rothschild-Stiglitz (2) or Wilson (4) that the core is empty whenever a Nash equilibrium fails to exist. This fact, together with Propositions 1 and 2, implies that nonemptiness of the coarse and fine cores does not imply nonemptiness of the core.

IV. Implications

The implications of Propositions 1-3 have already been mentioned. To recap, the definition of the core allows no more, and possibly less communication than does that of the fine core. Nevertheless, the core is empty while the fine core is not. Thus it is not generally true that the more communication is permitted, the smaller is the resulting "core."

In addition, the economy presented makes clear that the emptiness or nonemptiness of these alternative versions of the "core" is not based on there being different opportunities for insurance with different degrees of communication. In each case, essentially the same insurance opportunities were present, and yet some core concepts resulted in emptiness while others did not.

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