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THE COMPETITIVE ALLOCATION OF LABOR MARKET RISK: AN EXAMPLE

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ABSTRACT

This note presents a model whose competitive equilibrium can be consistent with the observation that current labor market conditions affect the well-being of new entrants more than they do that of senior workers. The model uses the notion that new entrants are not around soon enough to participate in risk-sharing contingent on the shocks that determine the equilibrium marginal products of first-period employment. This timing notion is formalized using a stochastic overlapping generations model.

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Casual observation suggests that current labor market conditions affect the well-being of new entrants and less senior workers much more than they do that of more senior workers. To a degree, more senior workers seem to be insulated from current labor market conditions. In this note, we describe a simple model whose competitive equilibrium can be consistent with this observation. The crucial feature of the model is a timing assumption that prevents new entrants from participating in some markets. We assume that new entrants are not around soon enough to participate in risk-sharing contingent on the shocks that determine the equilibrium marginal products of first-period employment.

A simple way to formalize not-being-around-soon-enough is to use an overlapping generations model. Indeed, stochastic versions of such models *require* assumptions about when outcomes get revealed relative to when new generations appear. As is well known, both the nature of the competitive equilibria for such models and the kind of welfare analysis that is appropriate for them depend on those assumptions.

The particular model we use has exogenous and random factor supplies--random labor supply and nonrandom land supply--and a constant-returns-to-scale and random technology for producing a single consumption good at each date. In addition, there can be enough intra-generation diversity of factor ownership to produce nontrivial market risk sharing, particularly of the consequences of the labor supply shock. An example shows that such risk sharing is consistent with equilibria in which more senior workers are insulated from current labor-market conditions.

The competitive equilibrium we study is one with complete contingent markets, with participation limited only by the timing assumption about when new generations appear. Because we assume complete markets, we cannot explain observed contractual arrangements, such as those between workers and firms. Partly for that reason, we judge whether more senior workers are insulated from current labor market conditions on the basis of consumption patterns, not wage rates.

I. An Overview of the Model

The model is a discrete-time, overlapping generations model with the initial date labeled $t = 1$. There is a single, nonstorable consumption good at each date. People derive utility only from their own consumption of this good at the different dates they are alive.

The amount of time t good available for the economy as a whole is the output from a production process with labor and land as inputs. Since labor and land are supplied perfectly inelastically, total output of time t good is determined exogenously in any equilibrium by total time t factor supplies and the production process. The amount of time t labor, denoted $L(t)$, is random in a way to be described below, while the amount of land, denoted K, is constant over time. In any equilibrium, output of time t good is $u(t)F(L(t),K)$, where $u(t)$ is a productivity shock and F, a function, is homogeneous of degree one, and twice differentiable with positive and diminishing marginal products.

At any date t, three groups are present in the economy. There are N members of generation t - 1 who are in the second and last period of their lives and who in the aggregate supply N/2 units of time t labor; there are N two-period lived members of generation t who in the aggregate supply N/2 units of time t labor (and who will supply N/2 units of time t + 1 labor); and there are n(t) one-period lived members of generation t who in the aggregate supply n(t) units of time t labor only, where n(t) is a random variable. Thus, the total supply of time t labor, L(t), is N + n(t).

As noted above, there are K units of land in the economy. Some of the land, bK units, is marketable. It is owned at t = 1, the first date, by the members of generation 0 who are present at t = 1. By convention, this land is sold ex-dividend, after the time t rental from it is obtained. The rest of the land, (1-b)K units, is never sold; the time t rentals from it are part of the endowments of the people alive at t. This splitting up of the land serves two purposes. The presence of some marketable land produces equilibrium interest rates that are sufficiently high to insure optimality. The presence of some nonmarketable land, with rentals assigned as endowments, allows us to have intra-generation diversity of factor ownership in every generation.

Because there is only one good at each date and because factor supplies are exogenous, the model has a recursive structure; the competitive equilibrium spot-market factor rentals do not depend on preferences or individual endowments. The per unit rentals are simply the respective physical marginal products,

$u(t)F_1(N+n(t),K)$ for labor and $u(t)F_2(N+n(t),K)$ for land, where F_i is the derivative of F with respect to its ith argument. We exploit this structure in the next section, where we treat the model as a pure exchange model, the endowments of which are to be interpreted as the equilibrium factor rentals that arise from a given pattern of factor service ownership.

We call the realization for the pair $(u(t),n(t))$ the time t state and we assume a finite number of possible time t states. Our crucial timing assumption is that the time t state is revealed simultaneously with (or just prior to) the appearance of generation t.

Factor markets aside, post-state at t, then, the following actions occur in a competitive equilibrium. The members of generation t - 1 fulfill previous (contingent) obligations and in the aggregate supply bK units of land perfectly inelastically. The n(t) one-period lived members of generation t consume their endowment, which is assumed to be the spot market rental of the labor they supply. Only the N two-period lived members of generation t make choices. The endowment of any such person consists of some time t good, the amount of which depends on the t state, and a vector of time t + 1 goods indexed by the time t + 1 state. This endowment is determined by the assumed underlying pattern of factor service ownership. The person can trade claims on time t + 1 good contingent on the time t + 1 state and can purchase land, which the person plans to sell at t + 1. In equilibrium, in addition to the factor service markets, both the markets in contingent claims and that in land must clear.

II. The Pure Exchange Equilibrium

Here we treat the model as one of pure exchange. We describe equilibrium conditions and briefly discuss existence and Pareto optimality of equilibrium. From now on, we use the term equilibrium to refer to a perfect foresight (rational expectations), competitive equilibrium.

Choice and Demands

The budget set for a two-period lived member of generation t in state j can be written

$$(1) \qquad c_t^h(t,j) \leq w_t^h(t,j) - \textstyle\sum_{j'} p_{tj}(t+1,j')q_{tj}^h(t+1,j') - P(t,j)k_{tj}^h$$

$$(2) \qquad c_t^h(t+1,j') \leq w_t^h(t+1,j') + q_{tj}^h(t+1,j') + k_{tj}^h[P(t+1),j')+f_2(j')]; \; j' = 1, 2, \ldots J$$

Here $c_s^h(t,j)$ ($w_s^h(t,j)$) is consumption (endowment) of time t state j good of member h of generation s, $q_{tj}^h(t+1,j')$ is contingent claims on time t+1 state j' good purchased by h and $p_{tj}(t+1,j')$ is the per unit price of such claims, k_{tj}^h is the amount of land purchased by h and $P(t,j)$ is its per unit price, $f_2(j')$ is the spot factor market rental of land at t + 1 in state j', and J is the number of possible time t + 1 states. Note that the w's are to be interpreted as the equilibrium spot market rentals of the factor services (land and labor) owned by h.

Equation (1) says that first period consumption is no greater than the first period endowment less expenditures on assets, while equation (2) says that second period consumption in state j' is no greater than the corresponding endowment plus the payoffs from the assets purchased earlier.

We write the utility function of h in generation t as $u_t^h(c_t^h(t,j), c_t^h(t+1))$, where $c_t^h(t+1)$ is the J-element vector with typical element $c_t^h(t+1,j')$. We assume that u is twice differentiable, quasiconcave and increasing in each of its arguments. Person h chooses nonnegative consumption, nonnegative land purchases, and q's, which are unrestricted as to sign, to maximize u_t^h subject to (1) and (2).

Because the q's are unrestricted as to sign, (1) and (2) constrain consumption bundles exactly as does the single constraint obtained by solving each equation of (2) for $q_{tj}^h(t+1,j')$ and substituting the result into (1). This constraint is

$$(3) \qquad c_t^h(t,j) + p_{tj}(t+1)c_t^h(t+1) \leq w_t^h(t,j) + p_{tj}(t+1)w_t^h(t+1)$$
$$- k_{tj}^h\{P(t,j) - p_{tj}(t+1)[P(t+1) + f_2]\}$$

where $w_t^h(t+1)$, $p_{tj}(t+1)$, $P(t+1)$ and f_2 are the obvious J-element vectors. It follows that the coefficient of k_{tj}^h in (3) must be zero in any equilibrium or that prices must satisfy the present value formula

$$(4) \qquad P(t,j) = p_{tj}(t+1)[P(t+1) + f_2]$$

in any equilibrium. If (4) does not hold, then (3) implies that the demand for land is either zero or infinity, neither of which is consistent with equilibrium.

Since we need demands only at prices that can potentially be equilibria, we face h only with prices that satisfy

(4). It follows from (3) that h's consumption demands at such prices can be obtained by constraining consumption choices by

$$(5) \qquad c_t^h(t,j) + p_{tj}(t+1)c_t^h(t+1) \leq w_t^h(t,j) + p_{tj}(t+1)w_t^h(t+1)$$

Maximization of u_t^h subject to (5) gives rise to unique demands in the usual way. We write the demand vector for $c_t^h(t+1)$ as $d_{tj}^h[p_{tj}(t+1)]$ and the summation of these demands over the N two-period lived members of generation t as $D_{tj}[p_{tj}(t+1)]$.

Equilibrium

In terms of D_{tj}, a condition for equilibrium is

$$(6) \qquad D_{tj}[p_{tj}(t+1)] - W_t(t+1) = bK[P(t+1)+f_2]$$

where $W_t(t+1)$ is the sum of $w_t^h(t+1)$ over the N two-period lived members of generation t. An equilibrium, then, consists of non-negative sequences for contingent claims prices and land prices that satisfy (4) and (6) for all $t \geq 1$.

One way to prove that an equilibrium exists is to mimic the proof in Wallace [1981]. The idea is to use (4) and (6) to define a mapping from land prices at t + 1 to land prices at t and to show, using properties of that mapping, that there exists at least one sequence of land prices with the property that every pair of adjoining terms and an associated set of contingent claims prices satisfy (4) and (6). Here is an outline of the ingredients of the argument.

First define a compact set to serve as the domain for bKP(t). One possible choice is the set of nonnegative vectors bounded above by $W_t(t) \equiv \sum_h w_t^h(t)$, this being the vector of aggregate endowments of time t good of the two-period lived members of generation t. Call this set A_t. One can choose this set, or something bigger, because $W_t(t)$ is an upper bound on equilibrium bKP(t). To see this, sum equation (1) over h and use the fact that the sum of the q's is zero in equilibrium.

Next, for any P(t+1) in A_{t+1}, define a composite mapping to P(t) in two steps. First, associate with any P(t+1) in A_{t+1} one or more $p_{tj}(t+1)$ vectors that satisfy (6). Second, use (4) to associate a set of P(t)'s with the given P(t+1) and the set of $p_{tj}(t+1)$ vectors given by the first step. In order to apply the procedure used in Wallace (1981), one must prove that this composite mapping is such that nonempty compact subsets of A_{t+1} are mapped into nonempty compact subsets of A_t. Since (4) gives P(t) as an explicit continuous function of $p_{tj}(t+1)$ and P(t+1), one has to show only that the first step has the requisite properties.

To do this, it suffices to show that for any P(t+1) in A_{t+1} there exists at least one $p_{tj}(t+1)$ vector satisfying (6) and that the correspondence from P(t+1) to $p_{tj}(t+1)$ so defined is upper semi-continuous. The upper semi-continuity is trivial since the demands, the $D_{tj}(\cdot)$, are continuous functions of $p_{tj}(t+1)$. A proof of existence may be constructed by considering a corresponding pure-exchange economy consisting of the N two-period lived members of generation t and one other person who has the RHS of (6) as an endowment and who wants to consume only time t good. The existence of a strictly positive equilibrium price-vector for this (j+1)-good pure exchange economy is immediate, because it satisfies standard assumptions.

Optimality

From the point of view of optimality questions, our model so closely resembles others in the literature that only a few summary remarks are called for (see in particular, Peled (1982), and also Muench (1977) and Cass and Shell (1981)). There are two potential sources of nonoptimality of the equilibria of our model: insufficient risk-sharing and interest rates that are too low. We have already noted that the presence of marketable land rules out the second.

As regards risk sharing, our timing assumption is such that there is no inter-generation risk sharing in equilibrium. If one judges the equilibria using a criterion of individual well-being which calls for such risk-sharing, then one will conclude that they are nonoptimal. One such criterion is the expected value of u_t^h, the expectation being taken over the time t states. If, alternatively, one uses u_t^h itself as a criterion of individual well-being, then our conjecture--a well-founded conjecture given Peled's result--is that our equilibria are Pareto optimal.

The use of u_t^h can be defended on several grounds. For us, the most convincing is that u_t^h is what h maximizes in a competitive equilibrium. One would not expect an equilibrium to be Pareto-optimal if one judges individual well-being according to a criterion that is different from the one the individual attempts to maximize. Finally, note that use of u_t^h as a criterion of individual well-being implies that many allocations are Pareto-optimal. In particular, many that are characterized by more inter-generation sharing of risks than occurs in our competitive equilibrium are optimal.

III. An Example

Let there be M + N two-period lived members of each generation, M land owners and N identical workers. The former are endowed only with the rentals of the nonmarketed land, the fraction 1-a when young and the fraction "a" when old; the latter are endowed only with labor. We assume that each such person of generation t maximizes

$$(7)\qquad c_t^h(t,j) + \sum_{j'} \theta_{j'} v[c_t^h(t+1,j')]$$

where $\theta_{j'}$ is the probability that state j' occurs at t + 1 and where v is twice differentiable, strictly increasing, and concave and satisfies $-xv''(x)/v'(x) \leq 1$.

If v and endowments are such that $c_t^h(t,j) > 0$, a condition which is easy to satisfy, then the following first-order conditions for a maximum of (7) subject to (5) hold in equilibrium:

$$(8)\qquad p_{tj}(t+1,j') + \theta_{j'} v'[c_t^h(t+1,j')] = 0, \text{ for each } j'.$$

It follows that in equilibrium, $c_t^h(t+1,j')$ does not depend on h and, hence, by (6) satisfies

$$(9)\qquad (M+N)c_t^h(t+1,j') = W_t(t+1,j') + bK[P(t+1,j') + f_2(j')]$$

where

$$(10)\qquad W_t(t+1,j') = (N/2)u_{j'}F_1(N+n_{j'},K)+a(1-b)Ku_{j'}F_2(N+n_{j'},K)$$

Substituting the solution for $p_{tj}(t+1,j')$ from (8) into the RHS of (4), we have

$$(11) \qquad P(t,j) = \sum_{j'} \theta_{j'} v'[c_t^h(t+1,j')]$$

Using the assumptions about v and the fact that $W_t(t+1,j')$ depends only on j', it can be shown that the unique solution of (9) and (11) for land prices is

$$(12) \qquad P(t,j) = P^* \text{ for all } (t,j)$$

It follows from (9) and (10), then, that second period consumption at date t is given by

$$(13) \qquad c_{t-1}^h(t,j) = \{bKP^* + (N/2)u_j F_1(N+n_j,K) + u_j K[b+a(1-b)]F_2(N+n_j,K)\}/(N+M)$$

We will compare the variation of $c_{t-1}^h(t,j)$ across states j at t to that of the consumption of workers who are young at t.

By (9) and (12), $c_t^h(t+1,j')$ does not depend on the state at t. It follows, therefore, from (8) that $p_{tj}(t+1,j')$ does not depend on the state at t. But, then from (5), if j and k are two alternative time t states,

$$(14) \qquad c_t^h(t,j) - c_t^h(t,k) = w_t^h(t,j) - w_t^h(t,k)$$

Then, with all workers equally endowed--each with 1/2 unit of labor in each period--the first-period consumption of each worker varies across states according to

$$(15) \qquad c_t^h(t,j) - c_t^h(t,k) = [u_j F_1(N+n_j,K) - u_k F_1(N+n_k,K)]/2$$

We compare this with the difference $c^h_{t-1}(t,j) - c^h_{t-1}(t,k)$ implied by (13) by examining

$$\rho_{jk} \equiv \left|[c^h_{t-1}(t,j)-c^h_{t-1}(t,k)]/[c^h_t(t,j)-c^h_t(t,k)]\right|$$

We say that consumption of old workers fluctuates less than consumption of young workers if $\rho_{jk} < 1$.

For states j and k that differ only with regard to labor supply $(u_j=u_k, n_j \neq n_k)$, we find, using the mean value theorem, that

$$(16) \qquad \rho_{jk}(n) = [N/(N+M)] \left| 1 - 2[(N+\bar{n})/N][b+a(1-b)] \right|$$

where $\bar{n}$ is a point intermediate between n_j and n_k.[1/] If $\bar{n}$ is small relative to N, then we are virtually guaranteed that $\rho_{jk}(n) < 1$.

For states j and k that differ only with regard to factor neutral productivity $(u_j \neq u_k, n_j=n_k=n)$, we have directly from (13) and (15) that

$$(17) \qquad \rho_{jk}(u) = [(N/(N+M)]\{1+2[b+a(1-b)](K/N)(F_2/F_1)\}$$

Letting $\alpha = KF_2(N+n,K)/(N+n)F_1(N+n,K)$, land's share relative to labor's share, we have

$$(18) \qquad \rho_{jk}(u) = [N/(N+M)]\{1+2[b+a(1-b)]\alpha(N+n)/N\}$$

Clearly, there is no presumption that $\rho_{jk}(u) < 1$.

Since the labor supply shock, n, generates a negative correlation between the marginal products of labor and land and the factor neutral productivity shock, u, generates a positive correlation between them, it is not surprising that for most parameter values (16) implies $\rho_{jk}(n)-1 < 0$, while (18) says nothing about the sign of $\rho_{jk}(u) - 1$. What may be surprising is that the difference in the origins of the shocks does not show up in a simpler way in (16) and (18). The reason for this is that more than static risk-sharing is occurring in these equilibria; there is also intertemporal trade.

IV. Concluding Remarks

As noted at the outset, our model is silent as regards the nature of contractual arrangements. For example, it makes no predictions relating features of the physical environment--for example, the kind of shocks present--to features of wage contracts. It does make predictions relating features of the physical environment to consumption allocations. If the model does well explaining consumption allocations, then its failure to explain contractual arrangements does not seem important.

A desirable feature of our set-up is that we are sure that the equilibrium allocation exhausts all the feasible and beneficial possibilities for trade. In set-ups with general diversity among agents, it would seem difficult to formulate "labor contracts" that by themselves accomplish this. If labor contracts do not exhaust all the possibilities for beneficial trade, then one must decide which asset markets to include. If a rich set is included, then contractual indeterminacy will be implied. If a rich set is not allowed, then, unless features of the environment imply the set of markets, one is left ruling out seemingly feasible and beneficial trades. In our set-up, the assumptions about when new generations and new shocks appear rationalize the market structure.

Footnotes

[1]To get (16), we use the mean value theorem and the homogeneity of F to express F_2 as a function of F_1. Define ϕ and x_j by $F_1(N+n_j,K) = F_1[(N+n_j)/K,1] \equiv \phi[(N+n_j)/K] \equiv x_j$. Then, $F_2(N+n_j,K) - F_2(N+n_k,K) = F_2[\phi^{-1}(x_j),1] - F_2[\phi^{-1}(x_k),1] = (x_j-x_k)F_{21}(\phi^{-1})'$, where the last equality uses the mean value theorem. But $(x_j-x_k)F_{21}(\phi^{-1})' = (x_j-x_k)F_{12}(\phi^{-1})' = (x_j-x_k)F_{12}/F_{11} = -(x_j-x_k)(N+\bar{n})/K \equiv -[F_1(N+n_j,K)-F_1(N+n_k,K)](N+\bar{n})/K$, where $(N+\bar{n})/K$ corresponds to the point, say $\bar{x}$, between x_j and x_k at which the derivative, $F_{21}(\phi^{-1})'$, is evaluated--i.e., $(N+\bar{n})/K = \phi^{-1}(\bar{x})$--and where $F_{12}/F_{11} = -(N+\bar{n})/K$ is a consequence of the homogeneity of F. Upon substituting this expression for $F_2(N+n_j,K) - F_2(N+n_k,K)$ into the expression for $c^h_{t-1}(t,j) - c^h_{t-1}(t,k)$ implied by (13), we can factor out the difference, $F_1(N+n_j,K) - F_1(N+n_k,K)$, which also appears in (15). In this way, we obtain (16).

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