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PRIVATE INFORMATION, THE REAL
BILLS DOCTRINE, AND THE "QUANTITY THEORY":
AN ALTERNATE APPROACH

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ABSTRACT

Recent developments in the theory of economies with private information permit a re-examination of the issues raised in the "real bills-quantity theory" debate. A model is developed here in which there are banks, in which fiat money is present, and in which agents possess private information. Two regulatory regimes are then considered. In the first, banks are essentially unregulated. In the second, banks face 100 percent reserve requirements. Issues related to existence and optimality of equilibrium are addressed, and problems with existence are given an interpretation in terms of the "stability" of the banking system. Existence (stability) problems which arise under laissez-faire banking can be rectified by a 100 percent reserve requirement. However, unless there is private information regarding access to investment opportunities, there are typically better ways to accomplish this. Finally, it is shown that even in the presence of 100 percent reserve requirements banks are not simply "money warehouses." Bank deposits and money bear different (real) return streams, even under 100 percent reserves.

I have benefitted in writing this paper from discussions with David Laidler, Thomas Sargent, and Neil Wallace. None of these individuals bears responsibility for the contents, however.

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One of the longest continuing discussions in economics has involved the role which financial intermediaries (banks) play in allocating resources, and expediting the undertaking of "monetary" transactions. This discussion has most often manifested itself in the debates between adherents of the "real bills doctrine" and of the "quantity theory." The first of these positions has often been interpreted as an advocacy of "unfettered private intermediaries" (Sargent and Wallace [1982], p. 1212), and the second as an advocacy of 100 percent reserve requirements, or more generally, of legal restrictions on intermediaries which restrict their ability to "create money." As argued by Sargent and Wallace (p. 1212), these two views constitute "a useful way to organize the discussion of . . . issues" related to the theory of money and banking.

In fact, it will be noted that the two views suggest very different interpretations of the roles played by banks in the process of resource allocation. Smith (1776), for instance, clearly views banks as institutions which help to overcome economic frictions. On the other hand, Simons (1948) suggests that allowing investment to be intermediated through banks is destabilizing and results in (at times) socially suboptimal levels of investment. Thus, adherents of the real bills doctrine have stressed the economic benefits of banking, whereas quantity theory advocates have stressed problems introduced by banks.

Both of these aspects of banking have been addressed by recent developments which emphasize the presence of private information and its implications for financial intermediation. In particular, Boyd and Prescott (1984) discuss the role of intermediaries in efficiently allocating investment funds when borrowers know more than lenders about investment opportunities. Diamond and Dybvig (1983), Jacklin (1983), King and Haubrich (1983), and Smith

(1984) emphasize the role of intermediaries in creating insurance opportunities for lenders under private information, with all of these efforts but King and Haubrich also focusing on "stability" problems that arise for an unregulated banking system due to the frictions created by the presence of private information. Diamond and Dybvig (1983) and Smith (1984) go on to discuss some regulatory responses to this instability. Thus, recent developments would seem to provide an opportunity to re-examine the questions raised in the real bills-quantity theory debate.

However, existing models are not really equipped to do this for at least two reasons. One is that none of the models mentioned contains money, so that questions concerning the role of banks in a (potentially) monetary economy cannot be addressed. The second is that several of the models mentioned do not predict whether problems will arise in banking or not. In particular, Diamond and Dybvig (1983) and Jacklin (1983) construct models with multiple equilibria. Some equilibria result in Pareto optimal allocations, whereas in others "bank runs" arise. Hence, whether or not stability problems will arise under laissez-faire banking is not a question which these models can address.

This paper is an attempt to examine some of the questions involved in the real bills-quantity theory debate in the context of a model with private information. This seems appropriate, for several reasons. In particular, recent re-examinations of this debate, employing models which are essentially free from underlying economic frictions, conclude that there is no obvious reason to regulate banking. This is hardly surprising, of course, since in full information, complete markets settings there is nothing special about the activity of banking.^{1/} Arguments that intermediation is special (e.g., a special candidate for regulation), then, require some departure from

such settings. A simple method of introducing private information into a model of banking is therefore pursued below. In particular, the model synthesizes two features emphasized by Simons and Smith, respectively. One is that each individual who makes a deposit with a bank has a probability distribution over future withdrawal dates which is not known ex ante by the bank. The second is that, even so, if banks behave prudently, inflows of funds will match outflows of funds based on considerations of the law of large numbers (Smith [1776], p. 289). In addition, banks (and all agents) are endowed with a simple investment opportunity which generates a nonstochastic return stream.

The paper then investigates two different banking arrangements for economies with the features described. In one arrangement, banks are unrestricted in terms of behavior (with two inessential exceptions discussed below). In the other, banks are forced to hold 100 percent reserves of a safe, noninterest-bearing government liability against deposits. Several questions may then be raised regarding the two regimes. First, following Sargent and Wallace (1982), one might ask whether equilibria under one regime dominate, in any sense, equilibria under the other regime on the basis of some welfare criterion. This, of course, presupposes existence of an equilibrium in each case. Second, one could ask questions related to the existence of an equilibrium under either regime. Third, one could ask whether any economic interpretation could be placed on the failure of an equilibrium to exist with unregulated banking. Fourth, there are government interventions which produce an equilibrium when one otherwise fails to exist. Different interventions may be Pareto compared. What kinds of interventions seem socially desirable?

The analysis permits some conclusions to be drawn on these questions, and suggests what types of considerations are important when the analysis is inconclusive. Some of the results obtained are as follows:

- (1) Under the conditions outlined, when an equilibrium exists under each of the two regimes either may be "socially preferred." The 100 percent reserve regime seems most desirable when there is private information regarding access to investment opportunities.
- (2) Under either regime a Nash equilibrium (which is the equilibrium concept imposed) may fail to exist. When existence fails this is because no bank can structure deposit interest rates in such a way that its competitors are deterred from bidding away its most profitable depositors. This competition among banks for deposits was universally viewed as the reason for instability of the banking system prior to 1933. Hence, failure of an equilibrium to exist may be given an interpretation in terms of the "stability" of the banking system. When no equilibrium exists under laissez-faire banking, the banking system is unstable.
- (3) For some economies with no Nash equilibrium, an equilibrium can be caused to exist by imposing 100 percent reserve requirements. This can also be accomplished by imposing an interest rate ceiling.
- (4) Given the interpretation of nonexistence as "banking instability," and given that the two regulatory interventions in (3) above produce stability, one might attempt to Pareto rank the two interventions. There is some presumption in the model in favor of interest rate ceilings.

In addition, it will be seen that there are economies where fiat money fails to have value under either the laissez-faire banking regime, or under a regime where all banking is prohibited. Nevertheless, some such economies do have equilibria with valued fiat money under a 100 percent reserve regime. This indicates that in such a regime banks are not simply

"money warehouses." In particular, we will show that even under a 100 percent reserve regime, bank deposits and money bear different rates of return. Moreover, if there is private information regarding access to investment opportunities, we will see that more productive investors may prefer the 100 percent reserve regime to laissez-faire banking.

The scheme of the paper is as follows, then. In order to introduce the essential aspects of the model, and to develop the incentives which exist for banks to form, Section I presents a version of the model under full information. Section II introduces private information and examines the behavior of an (essentially) unregulated banking system. Section III describes how an equilibrium in the banking system is determined under 100 percent reserve requirements. Section IV then develops some of the results outlined above under the assumption that all agents have access to the same investment opportunities. Section V relaxes this assumption, permitting investment opportunities to vary across agents. This permits there to be uncertainty on the part of banks about an agent's characteristics both as a depositor, and as someone in whom the bank might invest. Section V then derives additional results under this assumption. Section VI concludes.

I. The Model Under Full Information

A. The Model Without Banking

The format of the paper is that the simplest possible monetary economy with private information will be examined. In order to introduce fiat money (which can potentially be valued in equilibrium), the economy will be given an overlapping generations structure. Thus, let time be discrete, and indexed by $t = 0, 1, \dots$. At $t = 0$ there is an initial old generation, which is endowed with the entire aggregate stock of fiat money, M . This stock

will then be held constant through time. At $t = 1$ this generation disappears. Also, at $t = 0$ there is an initial young generation which becomes old at $t = 1$, etc.

No further description will be required of the initial old generation. However, in order to introduce private information, each young generation will need to display some heterogeneity among its members. While this section does not deal explicitly with private information, in order to introduce the economic setup let each young generation consist of three types of agents, indexed by $i = 1, 2, 3$. Each generation is "large" (so that the population is infinite at each date), with proportion θ_i of type i agents. Type 1 and 2 agents face similar economic circumstances, whereas type 3 agents play a much different role in the analysis. Hence, we discuss these types separately.

Each young agent of type i ($i=1,2$) begins his first period of life with one unit of the single consumption good. However, while endowments of the good are received at the beginning of the first period, consumption of the good can occur only at the end of the first period, or at any time during the second period. Since endowments are not received by young agents at a time when consumption can occur, then, these agents have two options. One is to trade their endowment for money held by the current old, and then to use this money to later purchase the good themselves. Money acquired at the beginning of an agent's first period of life can be spent either at the end of that period, or when old.

The second option facing these agents is to place the good in "storage" at the beginning of the period, which yields the gross rate of return $Q_1 < 1$ if the good is removed from storage at the end of the same period, and return $Q_2 > 1$ if the good is removed from storage in the second period.

(Returns to storage are zero thereafter.) For the present, type 1 and 2 agents are assumed to have access to the same storage technology.

Let C_j denote consumption in period j for any agent, where $j = 1$ denotes youth and $j = 2$ denotes old age. Then, it is assumed that type 1 and 2 agents possess identical utility functions given by $U(C_1, C_2) = U(C_1 + C_2)^{2/}$, with $U'' < 0$. Thus, these agents are indifferent regarding the timing of consumption. In the absence of other considerations, then, they would simply consume when old in order to maximize the returns on their investments.

However, suppose there is a random shock, specific to each individual, so that in one state of nature a type i agent ($i=1,2$) is forced to consume when young.^{3/} The other possibility is that he can consume when old. Moreover, at the beginning of an agent's life, each agent has a probability distribution over which date he will be forced to consume in. In particular, a type i agent faces ex ante probability p_i of being forced to consume when young, where the values p_i are time invariant and obey $p_1 < p_2$. For the purposes of this section, each agent's type is publicly known.

Type 3 agents are less interesting, and are introduced for technical convenience (see below). These agents live two periods with certainty, and have preferences given by $V(C_1, C_2) = C_1 + C_2$. The other difference between them and agents of other types is that these individuals receive their endowment of a single unit of the good at the end of their first period of life. Investment in the storage technology can occur only at the beginning of an agent's youth, so that these agents have no access to a storage technology.^{4/}

Having described the economic environment, it will now be useful to consider two different trading arrangements in which risk sharing (the operation of a financial intermediary) is prohibited. The first is one where $M = 0$, so that monetary transactions are also prohibited. Then all agents face autarky, so that in particular,

- (1a) $C_{i1} = Q_1$ with probability p_i
 (1b) $C_{i1} = 0$ with probability $1 - p_i$
 (1c) $C_{i2} = Q_2$ with probability $1 - p_i$
 (1d) $C_{i2} = 0$ with probability p_i ,

where C_{ij} is the consumption of type i agents in period j ($i=1,2$). The autarky arrangement yields expected utility levels $U_i^A \equiv p_i U(Q_1) + (1-p_i)U(Q_2)$; $i = 1, 2$.

Now consider an economy in which $M > 0$. Let the consumption good be the numeraire, and let S_t denote the number of goods purchasable with a unit of money (the inverse price level) at t . Now young agents with $i = 1, 2$ face a nontrivial portfolio selection problem when young. In particular, they may choose to place a fraction $\lambda_i \in [0,1]$ of their goods in storage, and to use the remainder $1 - \lambda_i$ to purchase money from the current old. Hence, if a young type i agent acquires $1 - \lambda_i$ units of real balances when young, his consumption is given by (for $i=1,2$)

- (2a) $C_{i1} = \lambda_i Q_1 + (1-\lambda_i)$ with probability p_i
 (2b) $C_{i1} = 0$ with probability $1 - p_i$
 (2c) $C_{i2} = \lambda_i Q_2 + (1-\lambda_i) \left(\frac{S_{t+1}}{S_t}\right)$ with probability $1 - p_i$
 (2d) $C_{i2} = 0$ with probability p_i .

Hence, type i agents ($i=1,2$) select $\lambda_i \in [0,1]$ to maximize

$$p_i U(\lambda_i Q_1 + 1 - \lambda_i) + (1-p_i) U[\lambda_i Q_2 + (1-\lambda_i) \left(\frac{S_{t+1}}{S_t}\right)],$$

taking the sequence $\{S_t\}_{t=0}^{\infty}$ as given. The optimal choice of λ_i , $i = 1, 2$, along with (2) then dictates consumption for type i agents. In the sequel only steady states are considered. Since the economy does not vary over time, $S_{t+1}/S_t = 1$ in steady state. However, in places the notation S_{t+1}/S_t will be retained for clarity.

B. The Model With "Banks"

The object of this section is to demonstrate the incentives which exist in the economy just described for intermediaries to form. However, for such incentives to be present, it is necessary that $p_1Q_1 + (1-p_1)Q_2 > 1$. In fact, it is henceforth assumed that $p_2Q_1 + (1-p_2)Q_2 > 1$. The assumption of public knowledge of the type of each agent is still retained at this point.

The model is now augmented to contain a set of agents who act as bankers, with there being free entry into the activity of operating a bank. All banks have access to the storage technology described above, plus they can also acquire real balances if so desired. In addition, banks can borrow from (or lend to) type 3 agents at the end of period one.^{5/} Since type 3 agents supply their entire endowment of the good elastically at a gross rate of return equal to unity, so long as banks do not wish to borrow more than the total endowment of type 3 agents they will face an intertemporal (gross) rate of return equal to one.

It remains to describe bank behavior. It is assumed that banks announce gross rates of return R_{ij} to be paid to type i agents who withdraw their deposits at age j ; $i, j = 1, 2$. Since individual types are publicly known, this is informationally feasible. A Nash equilibrium concept can then be imposed, so that a vector of announcements $(R_{11}, R_{12}, R_{21}, R_{22})$ is an equilibrium if no bank has an incentive to announce a different vector of state contingent payoffs on deposits, given the announcements of other banks, and given the equilibrium sequence $\{S_t\}$.

Consider, then, the situation faced by type 1 and 2 agents when banks are present. These agents can place any nonnegative fraction of their goods in storage, in bank deposits, or they may acquire real balances. Let λ_{i1} be the fraction of goods deposited in a bank by a type i agent (each agent

need deal only with one bank), λ_{i2} be the fraction of real balances held by a type i agent, and λ_{i3} be the fraction of his portfolio held in storage, with $\lambda_{ik} \in [0,1]$; $i = 1, 2, k = 1, 2, 3$. Then, consumption for a type i agent is given by

$$(3a) \quad C_{i1} = \lambda_{i1} R_{i1} + \lambda_{i2} + \lambda_{i3} Q_1 \text{ with probability } p_i$$

$$(3b) \quad C_{i1} = 0 \text{ with probability } 1 - p_i$$

$$(3c) \quad C_{i2} = \lambda_{i1} R_{i2} + \lambda_{i2} \left(\frac{S_{t+1}}{S_t} \right) + \lambda_{i3} Q_3 \text{ with probability } 1 - p_i$$

$$(3d) \quad C_{i2} = 0 \text{ with probability } p_i.$$

The values λ_{ik} ; $k = 1, 2, 3$ are then chosen to solve the problem

$$(4) \quad \max p_i U(\lambda_{i1} R_{i1} + \lambda_{i2} + \lambda_{i3} Q_1) + (1 - p_i) U[\lambda_{i1} R_{i2} + \lambda_{i2} \left(\frac{S_{t+1}}{S_t} \right) + \lambda_{i3} Q_2]$$

subject to $\sum_k \lambda_{ik} = 1$, and $1 > \lambda_{ik} > 0 \forall k$. Then denote the optimal choice of the vector $(\lambda_{i1}, \lambda_{i2}, \lambda_{i3})$ by $(\lambda_{i1}, \lambda_{i2}, \lambda_{i3}) \equiv \Phi_i(R_{i1}, R_{i2}, Q_1, Q_2, \frac{S_{t+1}}{S_t}) \equiv [\Phi_{i1}(\cdot), \Phi_{i2}(\cdot), \Phi_{i3}(\cdot)]$.

Having described type 1 and 2 agents' portfolio choices, it is now possible to characterize an equilibrium for this economy. First, in light of the fact that there are large numbers of agents, banks face no uncertainty here. Therefore, in light of the assumption that there is free entry into banking, the equilibrium return vector must be such that banks earn zero profits. Here it should be noted, then, that banks can borrow from (and lend to) type 3 agents at a gross rate of return of unity, subject to one restriction which is now imposed. This is that for each withdrawal made from a bank by an agent only one period after making a deposit, the bank must remove one unit of the good from storage. This may be viewed as a legal restriction on the amount of indebtedness incurred by any bank. The reason for imposing this restriction is as follows. Given the linear preferences possessed by all

agents and the linearity of bank objective functions, in equilibrium banks must either face a gross intertemporal rate of return of one, or of Q_2 . In the absence of the requirement that one unit must be taken out of storage for each withdrawal by individuals of age one, the first situation (a unitary rate of return) would obtain if $\sum_{i=1}^2 p_i \theta_i Q_2 < \theta_3$, and the latter would obtain otherwise. Now it is a matter of considerable technical convenience here to have banks face an intertemporal rate of return of one. In particular, if they faced Q_2 , in order to guarantee a nontrivial role for banking here, it would be necessary to impose $cU''(c)/U'(c) < -1.6/$ This would preclude setting up examples in which it is possible to produce closed form solutions for equilibrium values. However, if $\sum_{i=1}^2 p_i \theta_i Q_2 < \theta_3$ held, and if banks faced no borrowing restrictions, no units would ever be withdrawn from storage after only one period. Then there would be no private information problem here. In light of these two considerations, then, it seems reasonable to impose a borrowing restriction on banks. Any such restriction would suffice. The assumption that one unit must be taken out of storage for each withdrawal is attractive, then, since under a restriction to be imposed shortly, it prevents banks from increasing, simply by their presence, the discounted present value of the aggregate wealth of any generation. This is important, as such a result would derive entirely from the presence of type 3 agents, who are introduced only to simplify the analysis. Thus, this assumption prevents banks from altering the aggregate set of resource feasible allocations for this economy. Finally, a borrowing restriction on banks is not unattractive, since in practice the amount of bank borrowing is closely monitored.

Given our assumption on bank borrowing, then, the zero profit conditions on return pairs offered to type i agents are

$$(5) \quad p_i R_{i1} + (1-p_i) R_{i2} = p_i Q_1 + (1-p_i) Q_2; \quad i = 1, 2,$$

where the fact that banks can borrow from type 3 agents at a zero net rate of interest has been used in (5). This requires that banks not wish to borrow more than type 3 agents can lend. Formally, the latter condition may be written as

$$(6) \quad \sum_{i=1}^2 \theta_i p_i (R_{i1} - Q_1) \phi_{i1} < \theta_3.$$

Below a parameter restriction is imposed which guarantees satisfaction of (6). Hence, it will be treated as not binding in the discussion that follows.

As argued above, any equilibrium return pair (R_{i1}, R_{i2}) must earn zero profits. It must also solve the problem

$$(7) \quad \max W_i(R_{i1}, R_{i2}, Q_1, Q_2, \frac{S_{t+1}}{S_t})$$

subject to (5), where

$$(8) \quad W_i(R_{i1}, R_{i2}, Q_1, Q_2, \frac{S_{t+1}}{S_t}) \equiv p_i U[R_{i1} \phi_{i1}(R_{i1}, R_{i2}, Q_1, Q_2, \frac{S_{t+1}}{S_t}) + \phi_{i2}(\cdot) + Q_1 \phi_{i3}(\cdot)] + (1-p_i) U[R_{i2} \phi_{i1}(\cdot) + \phi_{i2}(\cdot) (\frac{S_{t+1}}{S_t}) + \phi_{i3}(\cdot) Q_2].$$

The reason for this is easy to see. If (6) is not binding and all banks announce (R_{i1}, R_{i2}) pairs which solve (7) subject to (5), then no bank has an incentive to offer a different set of (R_{i1}, R_{i2}) pairs. In particular, if (R_{i1}^*, R_{i2}^*) , $i = 1, 2$, is a pair of announcements solving (7) subject to (5), the only way in which a bank can attract type i depositors (i.e., offer them a pair (R_{i1}, R_{i2}) such that $W_i(R_{i1}, R_{i2}, Q_1, Q_2, \frac{S_{t+1}}{S_t}) > W_i(R_{i1}^*, R_{i2}^*, Q_1, Q_2, \frac{S_{t+1}}{S_t})$) is to offer an (R_{i1}, R_{i2}) pair satisfying $p_i R_{i1} + (1-p_i) R_{i2} > p_i Q_1 + (1-p_i) Q_2$. But of course such an offer loses money, and so will not be made. Hence any arrangement with all banks offering (R_{i1}^*, R_{i2}^*) ; $i = 1, 2$, constitutes a Nash equilibrium.

It is also easy to see that this is the only Nash equilibrium for a given $\{S_t\}$ sequence. In particular, suppose to the contrary that (R_{i1}, R_{i2}) ; $i = 1, 2$ is a Nash equilibrium with $(R_{i1}, R_{i2}) \neq (R_{i1}^*, R_{i2}^*)$, and with (R_{i1}, R_{i2}) satisfying $p_i R_{i1} + (1-p_i)R_{i2} = p_i Q_1 + (1-p_i)Q_2$. Then clearly $W_i(R_{i1}^*, R_{i2}^*, Q_1, Q_2, \frac{S_{t+1}}{S_t}) > W_i(R_{i1}, R_{i2}, Q_1, Q_2, \frac{S_{t+1}}{S_t})$ for some i . Thus, there exists for this i a vector $(\hat{R}_{i1}, \hat{R}_{i2})$ with $W_i(R_{i1}^*, R_{i2}^*, Q_1, Q_2, \frac{S_{t+1}}{S_t}) > W_i(\hat{R}_{i1}, \hat{R}_{i2}, Q_1, Q_2, \frac{S_{t+1}}{S_t}) > W_i(R_{i1}, R_{i2}, Q_1, Q_2, \frac{S_{t+1}}{S_t})$, and such that $p_i \hat{R}_{i1} + (1-p_i)\hat{R}_{i2} < p_i Q_1 + (1-p_i)Q_2$. This offer attracts all type i agents, and earns a profit. Hence, the hypothesized arrangement is not an equilibrium, giving the desired contradiction.

It is not difficult to see, then, that since $p_i Q_1 + (1-p_i)Q_2 > 1 \forall i$, and since banks can offer return vectors obeying $R_{i1} = R_{i2}$, that both money and storage are dominated here by bank deposits. Hence, $\Phi_{i2} = \Phi_{i3} = 0$ for this economy. Therefore (R_{i1}, R_{i2}) solves

$$\max p_i U(R_{i1}) + (1-p_i)U(R_{i2})$$

subject to (5). This optimization problem yields first-order conditions which can be manipulated to obtain

$$(9) \quad \frac{p_i U'(R_{i1})}{(1-p_i)U'(R_{i2})} = \frac{p_i}{1-p_i},$$

or $R_{i1} = R_{i2}$. Hence in equilibrium $R_{ij} = R_{ij}^* = p_i Q_1 + (1-p_i)Q_2$. Thus, under full information, banks provide full insurance. Finally, it remains to impose parameter restrictions which imply (6). In light of the fact that $R_{ij}^* = p_i Q_1 + (1-p_i)Q_2$, (6) is implied by

$$(10) \quad p_2 \theta_2 [p_2 Q_1 + (1-p_2)Q_2 - Q_1] + p_1 \theta_1 [p_1 Q_1 + (1-p_1)Q_2 - Q_1] =$$

$$(Q_2 - Q_1) \sum_{i=1}^2 p_i \theta_i (1-p_i) < \theta_3.$$

It remains to explain the role played by type 3 agents here, since their presence is clearly somewhat artificial. Type 3 agents, then, add to the analysis in two important ways. One is that banks obviously act so as to provide insurance for "investors." In order for there to be an insurance motive here, clearly $Q_2 > Q_1$ must hold. However, if there were no agents of type 3 in the model, there would be no opportunity for banks to borrow at gross rate of interest one. Then, as is easily verified, banks would not provide the complete insurance derived above. This provision of complete insurance greatly simplifies subsequent analysis, so that the presence of type 3 agents is helpful in this respect.

Second, considerable attention has been devoted to equilibria which display "bank runs" in models of the type considered here.^{7/} Suppose that a bank could not observe whether an agent had to consume at the end of his initial period or not, and hence could not make individual payments contingent on this event. In the absence of type 3 agents, banks could still offer the rate of return pairs (R_{i1}^*, R_{i2}^*) (or their analog when they cannot borrow from type 3 agents). However, now "run equilibria" could occur in the following way. If all agents conjecture that all other agents are planning to make withdrawals at the end of their initial period, banks may be bankrupted. Hence, those who do not attempt to withdraw at this time earn a zero rate of return. Thus, all agents will withdraw at the end of their initial period, causing a "run."

Now consider the model with type 3 agents present. If the claims of depositors take precedence over the claims of type 3 lenders on banks, a "run" cannot occur. To see this, notice that for every agent who does not make a withdrawal at the end of their initial period, one unit of the good remains in storage. Since (as is easily verified) $R_{i2}^* < Q_2$, and since depositor claims

on the bank enjoy a preferred status, a bank can always pay off depositors who do not make "premature" withdrawals. Thus, no agent has an incentive to make such a withdrawal, and no "run" can occur. Then, notice that bank debts to type 3 agents are always honored in equilibrium.^{8/}

One final point about type 3 agents is worthy of note. It was assumed above that once a unit was placed in storage, its ownership could not be transferred. However, it is reasonable to ask what would happen if type 1 and 2 agents who learn they must complete their consumption at the end of their initial period could sell their investment to type 3 agents (who are the only agents that could make such a purchase)? The answer is that this does not affect any of the above analysis if

$$(11) \quad (p_1\theta_1 + p_2\theta_2) > \theta_3.$$

To see this, notice that type 3 agents would not give up more than Q_2 units of current consumption for a claim to a unit in storage. If asked to give up exactly Q_2 units for a claim to a unit in storage, type 3 agents (in per capita terms) would supply any amount of current consumption in the interval $[0, \theta_3]$. Finally, if asked to surrender less than Q_2 units for such a claim, the aggregate (per capita) supply of current consumption by type 3 agents would be θ_3 . Type 1 or 2 agents who were willing to sell such claims, however, would obviously not surrender them for less than Q_1 units of current consumption. At Q_1 units of current consumption per unit in storage, aggregate supply of such claims could take any value in the interval $[0, \theta_1 p_1 + \theta_2 p_2]$, and at any amount greater than Q_1 , the supply of such claims would be $\theta_1 p_1 + \theta_2 p_2$. Then the supply of, and the demand for claims to future consumption are as depicted in Figure 1. Under assumption (11), the equilibrium price of a unit in storage would be Q_1 , as shown, so that type 1 and 2 agents would not

benefit from being able to sell such claims. Of course, type 3 agents would benefit from being able to buy them at this price, but this would not affect the arguments above regarding the incentives for banks to form. Henceforth, then, to maintain maximum simplicity, the assumption that units already in storage are not transferable is retained.

To summarize, then, there is an incentive for banks to form here in order to provide a source of insurance for investors. Attention may now be directed to provision of such insurance under private information.

II. Private Information and Laissez-Faire Banking

A. Description

In this section the operation of an (essentially) unregulated banking system under private information is considered. Two minor restrictions are imposed on banks, however. One is the borrowing restriction discussed above: for each unit withdrawn a unit of the good must be removed from storage. A second is a technical condition imposed to simplify the analysis, which is that banks are not permitted to lose money on any deposit payoff pair, i.e.,

$$(12a) \quad p_i R_{i1} + (1-p_i) R_{i2} \leq p_i Q_1 + (1-p_i) Q_2; \quad i = 1, 2,$$

if type i deposits are held only by type i agents. If agents of both types were to make the same types of deposits (in their population proportions), then the relevant condition would be

$$(12b) \quad (\theta_1 p_1 + \theta_2 p_2) R_1 + [\theta_1 (1-p_1) + \theta_2 (1-p_2)] R_2 \leq$$

$$(\theta_1 p_1 + \theta_2 p_2) Q_1 + [\theta_1 (1-p_1) + \theta_2 (1-p_2)] Q_2.$$

This is a standard assumption in private information settings of the type now described.^{9/}

The economy has all of the features described in Section II, except that now each agent's type is unobservable, ex ante. Thus, depositors are possessed of private information regarding their probability distribution over future dates of withdrawal. As above, banks announce deposit payoff vectors (R_{i1}, R_{i2}) ; $i = 1, 2$ in order to compete for depositors, and type 1 and 2 agents face the portfolio choices described above given the announcements of banks. Hence, their portfolio behavior is essentially as described above, with one difference. Now type 2 agents could, in principle, make type 1 deposits and vice versa.

It remains to describe what actions are observable here. It is assumed that all economic actions taken by all agents are common knowledge. The only such actions here are portfolio choices, and the type of deposit selected. Hence, if type 2 agents wish to select type 1 deposits (or conversely), they must choose the same portfolio as type 1 agents. Also, under the assumption that all portfolio choices are observable, it is easy to show^{10/} that each depositor need make only one deposit here.

Finally, then, any bank may pursue either of two courses of action. It may announce values $(R_{11}, R_{12}) \neq (R_{21}, R_{22})$ hoping to induce self-selection of depositor types by deposit selected (without loss of generality, hoping to induce type 1 agents to select type 1 deposits, etc.), or it may set $(R_{11}, R_{12}) = (R_{21}, R_{22})$ and forego the opportunity to price discriminate. The former arrangement is referred to as a separating arrangement, and the latter is termed pooling. If a bank does wish to induce self-selection, its announcements must be consistent with type 1 and only type 1 agents holding type 1 deposits. Define

$$(13) \quad V_i(R_{k1}, R_{k2}, Q_1, Q_2, \frac{S_{t+1}}{S_t}) \equiv p_i U(R_{k1} \phi_{k1} + \phi_{k2} + Q_1 \phi_{k3}) + \\ (1-p_i) U[R_{k2} \phi_{k1} + \phi_{k2} (\frac{S_{t+1}}{S_t}) + Q_2 \phi_{k3}] \text{ for } k \neq i.$$

Then the occurrence of self-selection requires that the following incentive compatibility conditions hold if $(R_{11}, R_{12}) \neq (R_{21}, R_{22})$;

$$(14) \quad W_i(R_{i1}, R_{i2}, Q_1, Q_2, \frac{S_{t+1}}{S_t}) > V_i(R_{k1}, R_{k2}, Q_1, Q_2, \frac{S_{t+1}}{S_t});$$

$i = 1, 2; k = 1, 2; i \neq k$. If $(R_{11}, R_{12}) = (R_{21}, R_{22})$, then type 2 agents are in a sense mimicking type 1 agents, and they must make the same portfolio choices.

As before, a Nash equilibrium concept is imposed on the game played by banks here. Hence, an equilibrium under laissez-faire banking (LF) must satisfy the following

Definition. An equilibrium is a set of announcements $(R_{11}, R_{12}, R_{21}, R_{22})$ and a sequence $\{S_t\}$ such that

- (i) given the sequence $\{S_t\}$, no bank has an incentive to announce a different set of deposit payoff vectors (with announcements subject to (12) and (14)).
- (ii) $(R_{i1}, R_{i2}); i = 1, 2$, satisfy (12).
- (iii) if $(R_{11}, R_{12}) \neq (R_{21}, R_{22})$, then $(R_{11}, R_{12}, R_{21}, R_{22})$ satisfies (14) given $\{S_t\}$.
- (iv) the money market clears, i.e.,

$$(15) \quad \sum_{i=1}^2 \theta_i \phi_{i2} [R_{i1}, R_{i2}, Q_1, Q_2, \frac{S_{t+1}}{S_t}] = S_t M \quad \forall t \geq 0.$$

As a convention, type 3 agents hold only as much money as other agents wish to sell them at the end of their first period.^{11/} Also, throughout attention is confined to pure strategies on the part of banks.

B. Some Features of Equilibrium

This section provides some heuristic characterization of features of the equilibrium discussed above. Two features, which will be familiar from Rothschild and Stiglitz (1976) or Wilson (1977), are that (a) no equilibrium need exist, and (b) if an equilibrium does exist it involves separation of type 1 and 2 agents by deposits selected. The second feature is a well-known aspect of adverse selection settings, such as this one, and so need not be discussed further. Given that any equilibrium must involve such separation, however, it will be useful to discuss determination of an equilibrium here, and to briefly discuss issues concerning existence.

If an equilibrium exists, then, it must display certain features. One is that in equilibrium $R_{2j} = R_{2j}^*$; $j = 1, 2$, where it will be recalled that (R_{21}^*, R_{22}^*) solves the problem (7) subject to (5) for type 2 agents. The reason for this is as follows. First, since type 2 agents are "higher risk" agents from the point of view of a bank ($p_2 > p_1$), it is easy to show that (14) holds with strict inequality for $i = 1$. Thus, competition among banks for type 2 agents (who must be distinguishable in equilibrium) will dictate that (R_{21}, R_{22}) solve (7) subject to (5) for exactly the same reasons as in Section II. In short, then, the presence of private information does not impinge upon the equilibrium returns earned by type 2 agents (if an equilibrium exists).

Private information does impinge upon the returns earned by type 1 agents, however. To see this, suppose that $R_{1j} = R_{1j}^*$; $j = 1, 2$ held. Since $R_{11}^* = R_{12}^* = p_1 Q_1 + (1-p_1) Q_2 > 1$, type 1 agents would hold all of their portfolios in the form of bank deposits (and similarly for type 2 agents).

Then equilibrium consumption levels for type i agents would be (independent of the date at which consumption occurred); $c_i = p_i Q_1 + (1-p_i)Q_2$. But then $c_1 > c_2$, implying that $p_2 U(c_1) + (1-p_2)U(c_1) > p_2 U(c_2) + (1-p_2)U(c_2)$; i.e., that (14) is violated. Hence, (14) must hold with equality in equilibrium.

This places the following restriction on (R_{11}, R_{12}) then. Since $R_{21}^* = R_{22}^* = p_2 Q_1 + (1-p_2)Q_2 > 1$, type 2 agents hold only deposits in a steady state equilibrium. Since they also receive full insurance, they receive expected utility $U(R_{21}^*) = U[p_2 Q_1 + (1-p_2)Q_2]$. Now the expected utility these agents could realize by mimicking type 1 agents cannot exceed this level (or self-selection could not occur). Of course, if type 2 agents were to mimic type 1 agents, they would have to acquire the same portfolio as these agents. Their expected utility would then be given by $V_2(R_{11}, R_{12}, Q_1, Q_2, \frac{S_{t+1}}{S_t})$ (with V_2 given by (13)), so that (14) at equality would be

$$(16) \quad U[p_2 Q_1 + (1-p_2)Q_2] = V_2(R_{11}, R_{12}, Q_1, Q_2, \frac{S_{t+1}}{S_t}).$$

This along with (5) must be satisfied by any equilibrium pair (R_{11}, R_{12}) . Then competition for type 1 depositors implies that the (R_{11}, R_{12}) pair satisfying (16) and (5) which is most preferred by type 1 agents will be the one offered in equilibrium.

The determination of an equilibrium under LF banking (if one exists) is easy to depict graphically under the assumption that $\phi_{12}(R_{11}, R_{12}, Q_1, Q_2) = 0$, i.e., that type 1 agents do not hold money in equilibrium. This will be the case for all of the analysis below under LF banking. Henceforth, then, $\frac{S_{t+1}}{S_t}$ will be notationally suppressed in discussions of LF banking. As will be seen momentarily, $\phi_{13}(\) = 0$ will also hold. Hence, determination of an equilibrium can be depicted in Figure 2, in which R_1 and R_2 (rates of return in youth and old age) appear on the axes. The loci labelled $\pi_i = 0$ are the

zero profit loci for deposit payoff vectors when type i deposits are held only by type i agents. And finally, the loci labelled $EU_i = k_i$ are type i indifference curves in this space.

As argued above, since the rates of return offered type 2 agents are not affected by considerations of self-selection, (R_{21}, R_{22}) occurs at point B in the figure, where a type 2 indifference curve is tangent to the locus $\pi_2 = 0$. Then any pairs (R_{11}, R_{12}) on or below $EU_2 = k_2$ are consistent with self-selection, and such pairs along $\pi_1 = 0$ break even. The most preferred such pair (from the point of view of type 1 agents) occurs at point A, where $EU_2 = k_2$ intersects $\pi_1 = 0$. Notice, then, that $\phi_{13} = 0$, i.e., type 1 agents make no use of storage themselves. This is the case since (as is readily verified) the locus $\pi_1 = 0$ and the locus $\pi_2 = 0$ intersect at the point (Q_1, Q_2) . Then point A dominates storage, and also it clearly dominates convex combinations of itself and storage. Thus, no agents other than banks make use of storage opportunities.

Having discussed what an equilibrium looks like (if it exists), it is now appropriate to devote some attention to existence issues. The economy depicted in Figure 2 is one in which an equilibrium exists. To see this, note that since all banks announce the deposit payoff vectors denoted A and B, no single bank could offer a profitable deposit payoff vector which attracts any depositors. In particular, clearly no payoff vector exists which is preferred to B by type 2 agents, and which earns a profit when it attracts only those agents. Also, any payoff vector which is profitable when accepted by only type 1 agents, and which is preferred by such agents to A, also attracts type 2 agents. Hence, if any incentive exists to offer payoff vectors other than A and B, this involves offering a payoff vector which attracts all agents (and in particular, all agents in their population proportions). However, in order

for such a payoff vector to (at least) break even, it must lie on or below $\bar{\pi} = 0$ (which is the locus of (R_1, R_2) pairs satisfying (12b)). But since A is preferred to all such payoff vectors,^{12/} there is no alternate payoff vector which attracts any agents, and which earns a profit given the agents it attracts. Hence, the situation depicted is an equilibrium.

The situation just described is reversed in Figure 3. In particular, in Figure 3 the type 1 indifference curve through point A now intersects the locus $\bar{\pi} = 0$. Hence there are now points such as C which are preferred to A and B by all agents, and which at least break even given the agents they attract. Therefore, A and B no longer constitute Nash equilibrium payoff vectors.

Figure 4 is an enlargement of the area around point C. If any equilibrium could arise with pooling of agents, it would have to involve all banks offering a payoff vector which occurred where a type 1 indifference curve is tangent to the locus $\bar{\pi} = 0$.^{13/} However, no such point could be an equilibrium. In particular, if all banks were to offer C, then some bank could offer a payoff vector such as D. This offer attracts only type 1 agents. Hence, if D is selected sufficiently close to C it earns a profit (since C breaks even when it attracts all agents). Therefore, no Nash equilibrium exists here.

Prior to concluding this section, it will be worthwhile to note two things. First, when no Nash equilibrium exists this is because competition among banks for depositors (through deposit interest rates) prevents any bank from structuring its set of deposit interest rates in such a way that it can prevent other banks from attracting away its "best" depositors, and thereby causing it to "fail." This is the description of bank "instability" provided by contemporaries during all banking panics in the U.S. from 1857 onward.

Hence, the failure of a Nash equilibrium to exist under LF banking coincides with the description of banking system instability current prior to 1933. Nonexistence, then, may be interpreted as "instability of the banking system."^{14/}

Second, this "instability" can be overcome by the imposition of a ceiling on interest rates. In particular, let R_c be the level of R_2 associated with point C. If $R_{ij} \leq R_c \forall i, j$, (when nonexistence is a problem) then a Nash equilibrium (in pure strategies) always exists here.^{15/} Such a ceiling results in point C being an equilibrium when bank instability arises, and in the standard Nash equilibrium otherwise (since no ceiling is imposed). However, there are other regulatory interventions, such as 100 percent reserve requirements, which can also overcome existence problems. Below some welfare comparison of such interventions is undertaken.

III. Banking Under a Quantity Theory Regime

This section examines banking when banks are required to hold 100 percent reserves against deposits in the form of noninterest-bearing government liabilities (fiat money). Throughout the focus is on steady states, so $S_{t+1} = S_t$, and S_{t+1}/S_t is henceforth suppressed in notation. Also, in order to distinguish equilibrium interest rates under a quantity theory (QT) regime from LF interest rates, here (r_{i1}, r_{i2}) denotes the deposit payoff vector offered to type i agents.

The economy is exactly as described above, except now when agents deposit some units of the good with a bank, the bank uses this good to purchase money from the current old or from other banks. The assumption that all payoff vectors must at least break even individually is retained, so the analog of (12) here is

$$(17a) \quad p_i r_{i1} + (1-p_i) r_{i2} \leq 1; \quad i = 1, 2, \text{ if } (r_{11}, r_{12}) \neq (r_{21}, r_{22})$$

$$(17b) \quad (\theta_1 + \theta_2)^{-1} \{ (\theta_1 p_1 + \theta_2 p_2) r_{i1} + [\theta_1 (1-p_1) + \theta_2 (1-p_2)] r_{i2} \} \leq 1 \text{ otherwise.}$$

Again banks may attempt to induce either separation or pooling of depositors by types. The former requires

$$(18) \quad W_i(r_{i1}, r_{i2}, Q_1, Q_2) \geq V_i(r_{k1}, r_{k2}, Q_1, Q_2); \quad i, k = 1, 2, \quad i \neq k$$

whenever $(r_{11}, r_{12}) \neq (r_{21}, r_{22})$, where $W_i(\)$ and $V_i(\)$ are as previously defined. Finally, a Nash equilibrium concept is again imposed on the game played by banks. Hence,

Definition. A steady-state Nash equilibrium is a set of announcements $(r_{11}, r_{12}, r_{21}, r_{22})$ and a constant sequence $\{S_t\}$ such that

(v) given the sequence $\{S_t\}$, no bank has an incentive to offer a different return vector.^{16/}

(vi) The return vector satisfies (17) and (18).

(vii) The money market clears, i.e.,

$$(19) \quad \sum_{i=1}^2 \theta_i [\phi_{i1}(\) + \phi_{i2}(\)] = S_t M \quad \forall t \geq 0.$$

(19) represents money market clearing since for each unit deposited with a bank (ϕ_{i1}) , a unit of real balances must be acquired by the bank, so that $\sum \theta_i \phi_{i1}$ represents per capita bank demand for real balances. Again, as a convention type 3 agents only hold money sold to them by agents at the end of their first period.

The features of this equilibrium are much as they were in Section III. Specifically, similar existence problems arise, and also any equilibrium must involve separation of types. However, there are some differences, the

most important of which is that (18) can hold with strict inequality $\forall i, k$, i.e., it is possible that private information does not impinge upon the determination of equilibrium return streams.

Because this implies that a more diverse set of possible outcomes can arise in equilibrium, as an expositional device it is perhaps easiest to present an example in which private information does matter. A more general graphical analysis follows.

A. An Example

For notational purposes, let $\mu_i = \theta_i / (\theta_1 + \theta_2)$; $i = 1, 2$. Then let $\mu_1 = .84$ and $\mu_2 = .16$. (θ_3 can be chosen to guarantee satisfaction of (10).) Preferences of type 1 and 2 agents are given by $U(c) = \ln c$, while $p_2 = 1/2$, $p_1 = 1/5$, $Q_1 = 3/4$, and $Q_2 = 2$. For purposes of exposition, it will be useful in this first example to describe determination of a QT equilibrium in some detail. This determination begins by describing how type 1 and 2 agents choose their portfolios given rates of return on their various investment opportunities. Now type i agents receive the state contingent consumption levels given by (3). Hence, the ϕ_{ik} ; $k = 1, 2, 3$, solve the problem

$$(20) \quad \max p_i \ln[\phi_{i1}r_{i1} + \phi_{i2} + \phi_{i3}Q_1] + (1-p_i) \ln[\phi_{i1}r_{i2} + \phi_{i2} + \phi_{i3}Q_2]$$

subject to $\sum_{k=1}^3 \phi_{ik} = 1$ and $0 \leq \phi_{ik} \leq 1$, where the fact that $S_{t+1} = S_t$ in steady state has been employed.

The first point of note is that $\phi_{i1}\phi_{i2} = 0$; $i = 1, 2$. To see this, suppose otherwise and notice that the first-order conditions associated with (20) if $\phi_{i1}\phi_{i2} > 0$ can be manipulated to obtain

$$(21) \quad \frac{r_{i1} - Q_1}{1 - Q_1} = \frac{Q_2 - r_{i2}}{Q_2 - 1}.$$

(21), in conjunction with (17a) (recall that separation of types must result in equilibrium), implies that $r_{i2} = 1$ (and hence that $r_{i1} = 1$). Hence, for $\phi_{i1}\phi_{i2} > 0$ to obtain, it is necessary that banks call out return vectors identical to the return on money. Since bank deposits and real balances would then be perfect substitutes, without loss of generality $\phi_{i2} = 0$; $i = 1, 2$, could be chosen as normalization. In addition, since banks can call out return vectors at least weakly preferred by all agents to the return on real balances, $\phi_{i2} = 0$; $i = 1, 2$ in equilibrium. Thus, all money is held by banks here.

Given that $\phi_{i2} = 0$ will hold for any equilibrium set of returns on deposits, the first-order condition for selection of ϕ_{i3} (which implies ϕ_{i1} , since $\phi_{i1} = 1 - \phi_{i3}$) is

$$(22) \quad \frac{p_i(r_{i1} - Q_1)}{\phi_{i3}Q_1 + (1 - \phi_{i3})r_{i1}} = \frac{(1 - p_i)(Q_2 - r_{i2})}{\phi_{i3}Q_2 + (1 - \phi_{i3})r_{i2}}; \quad i = 1, 2.$$

This can be manipulated to obtain

$$(23) \quad \phi_{i3}(r_{i1}, r_{i2}, Q_1, Q_2) = \frac{(1 - p_i)Q_2 r_{i1} + p_i Q_1 r_{i2} - r_{i1} r_{i2}}{(Q_2 - r_{i2})(r_{i1} - Q_1)},$$

where throughout an interior optimum has been assumed.

Having derived optimal portfolios, the next step is to substitute (23) (using $\phi_{i2} = 0$ and $\phi_{i1} = 1 - \phi_{i3}$) into utility functions to derive the functions $W_i(\)$ defined by:

$$(24) \quad W_i(r_{i1}, r_{i2}, Q_1, Q_2) = p_i \ln\{[1 - \phi_{i3}(r_{i1}, r_{i2}, Q_1, Q_2)]r_{i1} + \phi_{i3}(r_{i1}, r_{i2}, Q_1, Q_2)Q_1\} + (1 - p_i) \ln\{[1 - \phi_{i3}(r_{i1}, r_{i2}, Q_1, Q_2)]r_{i2} + \phi_{i3}(r_{i1}, r_{i2}, Q_1, Q_2)Q_2\}; \quad i = 1, 2.$$

Now for the same reasons as previously, competition among banks for deposits, and the fact that considerations of private information do not impinge upon selection of (r_{21}, r_{22}) , imply that (r_{21}, r_{22}) maximizes (24) (with $i = 2$) subject to (17a). Defining $\psi_i \equiv Q_2 \left(\frac{1-p_i}{p_i} \right) + Q_1$ and $\phi_i \equiv \frac{1}{p_i} - Q_1$, and given the form of $\Phi_{i3}(\cdot)$ in (23), maximization of (24) subject to (17a) is equivalent to the unconstrained problem

$$(25) \quad \max \quad \ln\left(\frac{Q_2}{p_2} - \psi_2 r_{22}\right) - p_2 \ln(Q_2 - r_{22}) - (1-p_2) \ln\left[\phi_2 - \left(\frac{1-p_2}{p_2}\right) r_{22}\right]$$

(subject, of course, to $r_{22} \in [0, \frac{1}{1-p_2}]$). It is tedious but straightforward to show that at any value of r_{22} such that the first-order condition for r_{22} holds with equality, the second-order condition fails. Hence, a corner solution emerges for r_{22} , and it is easy to check that type 2 agents prefer $r_{22} = 0$ to $r_{22} = \frac{1}{1-p_2}$. Thus, if an equilibrium exists, it has $(r_{21}, r_{22}) = (\frac{1}{p_2}, 0)$. 17/

Next, it can be checked that considerations of self-selection do impinge on the determination of equilibrium values for (r_{11}, r_{12}) . To see this, suppose the contrary. Using (23) in (24), using (17a), and supposing that (18) holds with strict inequality implies that r_{12} would solve

$$(26) \quad \max \quad \ln\left(\frac{Q_1}{p_1} - \psi_1 r_{12}\right) - p_1 \ln(Q_1 - r_{12}) - (1-p_1) \ln\left[\phi_1 - \left(\frac{1-p_1}{p_1}\right) r_{12}\right]; \quad r_{12} \in [0, \frac{1}{1-p_1}].$$

As before, a corner solution always results with $r_{12} = 0$. Then $r_{11} = \frac{1}{p_1}$, $r_{12} = 0$ would obtain. Consider portfolio choices, then. Using (23), $\Phi_{13} = .941$, and $\Phi_{23} = .8$. Then, type 2 agents receive state contingent consumption levels

$$c_{21} = (.8)Q_1 + (.2)r_{21} = (.8)(3/4) + (.2)(1/p_2) = 1$$

$$c_{22} = (.8)Q_2 = 1.6.$$

On the other hand, if these agents were to make type 1 deposits (which would require making type 1 portfolio choices), they would receive state contingent consumption levels

$$c_{21} = (.941)Q_1 + (.059)r_{11} = (.941)(3/4) + (.059)\left(\frac{1}{p_1}\right) = 1$$

$$c_{22} = (.941)Q_2 = 1.882.$$

Clearly, then, type 2 agents prefer to make type 1 deposits, contradicting that (18) holds.

(r_{11}, r_{12}) must be chosen so that type 2 agents do not prefer mimicking type 1 agents to making type 2 deposits, then. Using (23) and the fact that $r_{22} = 0$, it is easy to check that (for any parameter values) if type 2 agents make type 2 deposits, then $c_{21} = 1$, and $c_{22} = \phi_{23} Q_2$. Therefore, the utility they realize in equilibrium is given by $p_2 \ln(1) + (1-p_2) \ln(\phi_{23} Q_2)$. Similarly, given (23) for $i = 1$, and given that mimicking type 1 agents involves type 2 agents making the same portfolio choices as do type 1 agents, state contingent consumption for both type 1 agents and type 2 agents who make type 1 deposits is given by (if $\phi_{13} < 1$)

$$(26a) \quad c_{11} = \frac{p_1(Q_2 r_{11} - Q_1 r_{12})}{Q_2 - r_{12}}$$

$$(26b) \quad c_{12} = \frac{(1-p_1)(Q_2 r_{11} - Q_1 r_{12})}{r_{11} - Q_1}.$$

Then for sorting to occur, it is necessary that

$$(27) \quad (1-p_2) \ln [Q_2 \Phi_{23}(\cdot)] \geq p_2 \ln \left[\frac{p_1(Q_2 r_{11} - Q_1 r_{12})}{Q_2 - r_{12}} \right] \\ + (1-p_2) \ln \left[\frac{(1-p_1)(Q_2 r_{11} - Q_1 r_{12})}{r_{11} - Q_1} \right]$$

(again if $\Phi_{13} < 1$). Since if (27) did not hold with equality the contradiction derived above would occur, (27) at equality and (17a) for $i = 1$ determines the set of possible (r_{11}, r_{12}) pairs. The most preferred of these for type 1 agents is the equilibrium level of (r_{11}, r_{12}) if an equilibrium exists. Finally, it has been noted that only banks hold money. Hence, (19) requires that

$$(28) \quad \sum_{i=1}^2 \theta_i \Phi_{i1}(r_{i1}, r_{i2}, Q_1, Q_2) = \sum_{i=1}^2 \theta_i [1 - \Phi_{i3}(r_{i1}, r_{i2}, Q_1, Q_2)] = S_t M \neq t < 0.$$

For the example, then, multiplying both sides of (27) by 2 and exponentiating both sides, the resulting equation can be manipulated to obtain

$$(29) \quad (1.6)(2-r_{12})(r_{11}-3/4) = (4/25)[4r_{11}^2 + (9/16)r_{12}^2 - 3r_{11}r_{12}].$$

Using (17a), which for the parameter values of the example becomes $r_{12} = (1-p_1)^{-1} - p_1 r_{11} / (1-p_1) = 1.25 - (.25)r_{11}$, the expression in (29) can be solved for $r_{11} = 3.426$. Using the zero profit condition, $r_{12} = .393$. Then, $\Phi_{13}(3.426, .393, .75, 2) = .975 \frac{18}{\dots}$

Finally, it remains to say something about existence issues. The same reasoning as in the previous section implies that an equilibrium will fail to exist iff there exists a pooling arrangement which type 1 agents prefer to the separating payoff vectors discussed above. The return vector under pooling most preferred by type 1 agents maximizes $W_1(r_1, r_2, Q_1, Q_2)$ subject to (17b). As was the case for separating return vectors the most preferred return vector under pooling involves a corner solution, i.e., sets

$r_1 = \bar{p}^{-1}$, $r_2 = 0$, where $(r_1, r_2) \equiv (r_{11}, r_{12}) = (r_{21}, r_{22})$, and where $\bar{p} \equiv \mu_1 p_1 + (1 - \mu_1) p_2$. Then using the portfolio choices dictated by (23), and for the parameter values above, $\phi_{13}(\bar{p}^{-1}, 0, Q_1, Q_2) = .9828$, which implies (common) consumption values $c_1 = (.75)(.9828) + (.0172)\bar{p}^{-1} = .8065$, and $c_2 = 2(.9828) = 1.9656$. This compares with consumption levels $c_{11} = .817$, $c_{12} = 1.96$ under the separating arrangement. Now, $p_1 \ln(.817) + (1 - p_1) \ln(1.96) = .4979 > .4976 = p_1 \ln(.8065) + (1 - p_1) \ln(1.9656)$. Thus, for this example an equilibrium exists under 100 percent reserve requirements.

Before going on to discuss a more general graphical analysis, one further point about the example is worthy of note. This is that a 100 percent reserve requirement does not simply convert banks into "money warehouses." In particular, while banks are forced to hold portfolios consisting of only money, deposits do not bear the same returns as money. In other words, bank deposits and money do not become redundant assets under 100 percent reserves, or put yet another way, 100 percent reserve requirements do not eliminate the importance of banks in the resource allocation process when private information is present.

B. A Graphical Exposition

This section presents a graphical analysis of a QT equilibrium. The presentation of this analysis should then make clear that the main features of Example 1 are completely general when self-selection constraints bind on banks' choices of values (r_{11}, r_{12}) . Consider Figure 5, then. Consumption for type i agents ($i=1,2$) when young (c_1) appears on the horizontal axis, and old age consumption (c_2) on the vertical axis. The point labelled (Q_1, Q_2) represents consumption under autarky (returns to storage), and the remainder of the diagram is interpreted as follows. The dashed lines labelled $\pi_i = 0$ are zero profit loci for payoff vectors on deposits held (only) by type i agents.

Thus, the $\pi_i = 0$ loci are combinations of (r_{i1}, r_{i2}) pairs satisfying (17a). Note, in particular, that these loci intersect the horizontal axis at the points $(p_i^{-1}, 0)$. Finally, the loci labelled \bar{U}_i are type i indifference curves.

Consider first the determination of (r_{21}, r_{22}) . Type 2 agents realize a return of (Q_1, Q_2) on storage, and competition will force banks to call out a return (r_{21}, r_{22}) on the locus $\pi_2 = 0$. Money earns return $(1, 1)$, which is where $\pi_1 = 0$ intersects $\pi_2 = 0$. Suppose, then, that type 2 agents do not hold money. Their state contingent consumption levels can then be any convex combination of (Q_1, Q_2) and (r_{21}, r_{22}) . The return vector (r_{21}, r_{22}) most preferred by type 2 agents, then, is the one that gives them the largest feasible consumption set. Since the relevant portion of this feasible set is just the line connecting (Q_1, Q_2) and (r_{21}, r_{22}) , clearly these agents will prefer $(r_{21}, r_{22}) = (p_2^{-1}, 0)$ to all other return vectors. Hence, competition among banks produces this result in equilibrium. Notice also that the return on money $(1, 1)$ lies in the interior of the feasible set for type 2 agents, so money is not held in their portfolios.

Given the announced returns on bank deposits, then, type 2 agents choose a portfolio such that a type 2 indifference curve is tangent to the feasibility frontier (the solid locus connecting (Q_1, Q_2) with $(p_2^{-1}, 0)$). This is the point labelled (c_{21}, c_{22}) , which represents the equilibrium level of state contingent consumption values for type 2 agents.

Now consider type 1 agents. It is easy to check that if $\phi_{11}(\) > 0$, and if $r_{11} = p_1^{-1}$ then $c_{11} = 1$. Similarly, if $\phi_{21}(\) > 0$, then $c_{21} = 1$. Hence, if $(r_{11}, r_{12}) = (p_1^{-1}, 0)$ (as type 1 agents would prefer under full information) it is clear from the diagram than $c_{12} > c_{22}$, so that (18) would be violated.^{19/} Hence, (r_{11}, r_{12}) must be chosen as follows. In order for

(r_{11}, r_{12}) to be consistent with self-selection, (c_{11}, c_{12}) must lie on (or below) \bar{U}_2 . Also, the feasibility locus for type 1 agents will be the line connecting (Q_1, Q_2) and (r_{11}, r_{12}) . These agents will then choose a portfolio such that a type 1 indifference curve is tangent to the feasibility locus. Thus, (r_{11}, r_{12}) must be chosen so that a tangency between a type 1 indifference curve and the feasibility locus occurs along \bar{U}_2 . Such a choice then results in the state contingent consumption pair (c_{11}, c_{12}) for type 1 agents as shown. Notice that (r_{11}, r_{12}) will always be selected (if $\phi_{11}(\cdot) > 0$) so that r_{11} occurs to the southeast of the intersection of the line connecting (Q_1, Q_2) and $(p_2^{-1}, 0)$ with the locus $\pi_1 = 0$. Hence, $(1, 1)$ lies strictly in the interior of the feasibility set for type 1 agents, so that again these agents do not hold real balances.

Existence issues could also be discussed here, but these are essentially the same as in the previous section. Thus a discussion of existence is omitted here.

IV. A Comparison of LF and QT Banking

There are a number of bases on which LF and QT regimes might be compared. One is simply on the basis of a Pareto criterion. Another is on the basis of the possibility that 100 percent reserve requirements could correct problems of banking "instability." Both are considered here. Finally, it will be shown that even when 100 percent reserve requirements do overcome stability problems, there may be superior means of accomplishing this.

A. A Pareto Comparison: an Example

As a reader of Simons (1948) will recognize, the analysis here contains a number of elements mentioned by Simons in his arguments in favor of a 100 percent reserve regime. Nevertheless, there is no presumption here in

favor of QT banking on the basis of a Pareto criterion. This is now demonstrated through presentation of an example in which (except for the initial old) all agents prefer the LF regime, and in which an equilibrium exists under each arrangement.

Example 2. The economy is identical to that of Example 1, except that $\mu_2 (= \theta_2 / (\theta_1 + \theta_2)) = .5$. From previous arguments, under the LF equilibrium (if it exists) $R_{21} = R_{22} = p_2 Q_1 + (1-p_2) Q_2 = 1.375$. Then, the self-selection condition is

$$(1/2) \ln R_{11} + (1/2) \ln R_{12} = \ln(1.375).$$

Multiplying both sides by two, and then exponentiating both sides, the self-selection condition becomes $R_{11} R_{12} = (1.375)^2$. Also (R_{11}, R_{12}) must satisfy the zero profit condition, which requires $p_1 R_{11} + (1-p_1) R_{12} = p_1 Q_1 + (1-p_1) Q_2$, or

$$R_{12} = \left(\frac{1}{1-p_1} \right) [p_1 Q_1 + (1-p_1) Q_2] - \left(\frac{p_1}{1-p_1} \right) R_{11}.$$

Solving this along with $R_{11} R_{12} = (1.375)^2$ results in $R_{11} = .973$, $R_{12} = 1.944$. These are the candidate values for a separating equilibrium. Finally, notice that this equilibrium does exist. To see this, note that the most preferred pooling arrangement for type 1 agents solves

$$(30) \quad \max (1/5) \ln(R_1) + (4/5) \ln(R_2)$$

subject to

$$(31) \quad \bar{p} R_1 + (1-\bar{p}) R_2 = \bar{p} Q_1 + (1-\bar{p}) Q_2,$$

where $\bar{p} = \mu_1 p_1 + \mu_2 p_2$. The solution has $R_1 = .89$ and $R_2 = 1.92$. (At these returns it is easy to check that type 1 agents do not hold money; i.e., $\phi_{11} = 1$.) Then clearly the candidate separating arrangement dominates any pooling arrangement (from the viewpoint of type 1 agents), so that an equilibrium exists as claimed.

Now consider a QT regime. The candidate values for a separating equilibrium were computed in Example 1 above. These were $(r_{21}, r_{22}) = (2, 0)$, and $(r_{11}, r_{12}) = (3.426, .393)$. Notice that this arrangement dominates autarky, as $\phi_{i1} > 0$; $i = 1, 2$. Finally, an equilibrium exists here, since as is easily checked, for $\mu_2 > 2/9$ the most preferred pooling arrangement for type 1 agents is the autarky arrangement. Thus, an equilibrium exists under both LF and QT regimes here.

Now compute expected utilities under the LF and QT arrangements. These are

$$W_2(R_{11}, R_{12}, Q_1, Q_2) = \ln(1.375)$$

$$W_2(r_{11}, r_{12}, Q_1, Q_2) = (1/2) \ln[2\phi_{23}(\)] = \ln(1.265)$$

$$W_1(R_{11}, R_{12}, Q_1, Q_2) = (1/5) \ln(.973) + (4/5) \ln(1.944) = .527$$

$$W_1(r_{11}, r_{12}, Q_1, Q_2) = (1/5) \ln(.817) + (4/5) \ln(1.96) = .498.$$

Thus, all agents except the initial old prefer the LF regime.^{20/} Then, ignoring the initial old, LF banking Pareto dominates QT banking.

B. Banking System Stability

It is the case, however, that even for an economy quite similar to that of Example 2, 100 percent reserve requirements can eliminate instability in the banking system. This is the point of

Example 3. The economy is identical to that of Example 1. It has already been shown that an equilibrium exists with 100 percent reserve requirements. Also, the discussion of Example 2 computed the candidate for a separating equilibrium under LF banking. Now, however, with $\mu_2 = .16$, the solution to (30) subject to (31) has $R_1 = 1.36$ and $R_2 = 1.798$. This results in expected utility (for type 1 agents) $p_1 \ln(1.36) + (1-p_1) \ln(1.798) = .531 > p_1 \ln(R_{11}) + (1-p_1) \ln(R_{12}) = (1/5) \ln(.973) + (4/5) \ln(1.944) = .527$. Hence, the situation depicted in Figure 3 exists here, and no equilibrium exists under LF banking. Given the interpretation of such nonexistence as "banking instability," then, a 100 percent reserve regime can eliminate such instability, as Simons suggested.

C. A Comparison of Regulatory Regimes

As pointed out in Section II, this instability could also be corrected by imposition of an interest rate ceiling. Then $(R_{i1}, R_{i2}) = (1.36, 1.798)$; $i = 1, 2$, would result under an appropriately set ceiling. As the situation in Example 3 under LF is depicted in Figure 3, clearly $W_i(1.36, 1.798, Q_1, Q_2) > W_i(R_{i1}, R_{i2}, Q_1, Q_2)$; $i = 1, 2$, where (R_{i1}, R_{i2}) , $i = 1, 2$, are the values derived in Example 2 above. Also, as shown in the discussion of Example 2, $W_i(R_{i1}, R_{i2}, Q_1, Q_2) > W_i(r_{i1}, r_{i2}, Q_1, Q_2)$; $i = 1, 2$. Hence, ignoring the initial old,^{21/} the regulatory intervention of setting an interest rate ceiling Pareto dominates that of setting 100 percent reserve requirements as a means of eliminating bank instability for this example.^{22/}

V. An Economy with Investment Opportunities Varying Across Types

Suppose now that there is some relationship between the probability of first period withdrawal and access to investment opportunities. In parti-

cular, a type i agent (and only a type i agent) can invest in storage which yields gross return Q_{i1} if goods are withdrawn from storage in period 1, and Q_{i2} if they are stored two periods. Again $Q_{i1} < 1 < Q_{i2}$; $i = 1, 2$, and all other features of the economy are as previously. Then the analysis done above goes through as previously with an obvious modification of notation, and a few minor modifications of results. The most obvious of these are that now under LF banking type 1 agents can be forced into an autarky situation, as will be seen below, and that self-selection constraints need not bind in a QT equilibrium even if type 1 agents do make deposits.

Also, this change in economic environment permits some modification of the way in which banking is interpreted here. In particular, under LF banking banks could be thought of as accepting deposits from type i agents, and the investing in type i (storage) "projects." Hence, for a type 2 agent to claim to have a type 1 project in which a bank might invest, for instance, it is necessary that the type 2 agent also make a type 1 deposit. Notice, then, that in a sense there is now private information here about both investment opportunities and an agent's characteristics as a depositor. Thus, private information now impinges upon investment, although in a very simple way.

An example is now produced in which the following two possibilities are demonstrated: (a) type 1 agents prefer the QT regime to the LF regime, and (b) money is valued under a QT regime, but money does not have value either under LF banking, or if banking is prohibited altogether. Notice, then, that in this economy there can be no role for fiat money unless there are banks (which face nontrivial reserve requirements.)

Example 4. Let preferences be as in Example 1, and let $p_1 = 1/4$, $p_2 = .35$, $Q_{21} = 1/2$, $Q_{22} = 5/2$, $Q_{11} = 9/10$, and $Q_{12} = 16/3$. There are now three cases to consider.

(a) LF banking. LF banking here works largely as before. Private information does not impinge on the deposit payoff vector faced by type 2 agents. Hence, these agents receive complete insurance, i.e., $R_{21} = R_{22} = p_2 Q_{21} + (1-p_2) Q_{22} = 1.8$. Notice, then, that these agents do not hold money. It will also be noted that type 2 agents receive expected utility equal to $\ln(1.8)$ under LF banking. Now $p_2 \ln(Q_{11}) + (1-p_2) \ln(Q_{12}) = (.35) \ln(9/10) + (.65) \ln(16/3) > .588 = \ln(1.8)$. Therefore, type 2 agents would claim to be type 1 agents if they could receive even their autarky returns. Hence, type 1 agents cannot deal with banks in any LF equilibrium (if one exists). Type 1 agents might conceivably hold money, however. Let τ_1 denote the fraction of their portfolio held in storage. Then (since real balances would earn return (1,1) in steady state), τ_1 solves

$$\max_{0 \leq \tau_1 \leq 1} p_1 \ln[\tau_1 Q_{11} + (1-\tau_1)] + (1-p_1) \ln[\tau_1 Q_{12} + (1-\tau_1)].$$

Then

$$(32) \quad \tau_1 = \frac{(1-p_1)(Q_{12}-1) - p_1(1-Q_{11})}{(1-Q_{11})(Q_{12}-1)}$$

if the right-hand side lies in the unit interval, and $\tau_1 = 1$ if the right-hand side exceeds one. For the parameters of the example this is the case, so $\tau_1 = 1$. Finally, notice that there is some critical value μ_2^* here such that an LF equilibrium exists if $\mu_2 \geq \mu_2^*$, and fails to exist if $\mu_2 < \mu_2^*$. If an LF equilibrium does exist, money fails to have value under it.

(b) Banking prohibited. Suppose all banking is prohibited. Then type 1 agents, as noted above, do not hold money. The analog of (32) for type 2 agents is

$$(33) \quad \tau_2 = \frac{(1-p_2)(Q_{22}-1) - p_2(1-Q_{21})}{(1-Q_{21})(Q_{22}-1)}$$

if the right-hand side lies in the unit interval, etc., where τ_2 is the fraction of the portfolio of type 2 agents held in storage. Again, for the parameters of the example the right-hand side of (33) exceeds one, so $\tau_2 = 1$ here. Thus, even under a situation where banking is prohibited altogether, money does not have value in this economy.

(c) QT banking. Again, QT banking works much as previously. In particular, $r_{21} = p_2^{-1}$, $r_{22} = 0$. Using expression (23), then, the fraction of type 2 portfolios held in storage is

$$\phi_{23} = \frac{(1-p_2)Q_{22}r_{21}}{Q_{22}(r_{21}-Q_{21})} = .787.$$

Then type 2 agents receive state contingent consumption levels

$$c_{21} = (.787)(1/2) + (1-.787)\left(\frac{1}{.35}\right) = 1$$

$$c_{22} = (.787)(5/2) = 1.9675.$$

This results in expected type 2 utility $(.65)\&n(1.9675) = .44$.

Now suppose that self-selection conditions do not bind on the determination of (r_{11}, r_{12}) . Then, as previously, $r_{11} = p_1^{-1}$, $r_{12} = 0$ would result. Thus (23) would imply that

$$\phi_{13} = \frac{(1-p_1)r_{11}Q_{12}}{Q_{12}(r_{11}-Q_{11})} = .968.$$

Then if type 2 agents were to mimic type 1 agents, they would obtain state contingent consumption levels

$$c_1 = (.968)(1/2) + 4(1-.968) = .612$$

$$c_2 = (.968)(5/2) = 2.42.$$

This results in expected type 2 utility $(.35)\ln(.612) + (.65)\ln(2.42) = .402$. Hence, it is incentive compatible to set $(r_{11}, r_{12}) = (p_1^{-1}, 0)$, so that private information does not affect the determination of deposit payoff vectors in a QT equilibrium. Further, this implies that a QT equilibrium exists here.

Several features of the example can now be noted. First, if an equilibrium exists under LF banking, the equilibrium has the feature that money does not have value. Similarly, if banking is prohibited money does not have value. Thus, a banking system with reserve requirements is a prerequisite for the valuation of fiat money here. Or, put otherwise, there can be no role in this economy for money unless there are banks. This can also be viewed as a validation of the claim that without reserve requirements the government will "lose control" of the process of money creation.

Second, if $\mu_2 < \mu_2^*$ no equilibrium exists under LF banking. Thus, 100 percent reserve requirements serve to stabilize the banking system here, as the magnitude of μ_2 is irrelevant to existence of an equilibrium under QT banking.

Third, it is clearly the case that type 1 agents prefer the QT equilibrium to autarky (as autarky is feasible for them). Since these agents face autarky under LF banking, they therefore prefer the QT regime to the LF regime. So do the initial old, since their money holdings have value under a

QT regime and only under a QT regime. Thus, there are now agents with a preference for 100 percent reserve requirements (in contrast to the earlier examples).

VI. Conclusions

Recent literature^{23/} has emphasized that, in the absence of frictions, banking is not a special activity. In particular, banking is not a special candidate for government regulation. In order to make sense of the arguments of Smith (1776) or Simons (1948), for instance, that various kinds of banking activities should be encouraged or discouraged, then, it is necessary to introduce frictions of some sort. In light of recent developments in the theory financial intermediation, it is natural to introduce private information into a model with a real role for such intermediaries. Moreover, private information as a source of frictions seems consistent with the discussions of Smith (1776) and Simons (1948).

When there is a real role for financial intermediaries in the resource allocation process, a 100 percent reserve regime does not simply convert banks into "money warehouses." It has been seen that even under a 100 percent reserve regime, bank deposits and money bear different rates of return. Hence, such a proposal does not eliminate an economically meaningful role for banks. Moreover, when investors possess private information about the nature of their projects (as in Section V above), investors with access to the most lucrative investment opportunities may prefer a 100 percent reserve regime. Hence, economic arguments may be made in favor of such a intervention. The analysis above suggests that these need to be based on the possession of private information by both depositors (ultimate lenders) and investors (ultimate borrowers).

More generally, though, when only depositors are possessed of private information it is more difficult to construct a case for a 100 percent reserve regime. For instance, it is true that for some economies an equilibrium exists under QT but not under LF banking. Moreover, this may be given an interpretation in terms of the "stability" of the banking system. However, there are always other regulatory interventions which result in existence of an equilibrium, and which result in a Pareto superior (except for the initial old in the model) allocation of resources. This is the case even though the model seems to possess the important features emphasized by Simons (1948) in his discussion of 100 percent reserve requirements.

Finally, one conclusion that emerges from the analysis is that it matters a great deal for different regulatory proposals who is endowed with private information. In particular, as seen in Section V, even a very simple introduction of private information regarding access to investment opportunities changes conclusions related to the desirability of a 100 percent reserve regime. While such an introduction has been accomplished in only the most rudimentary way here, the results strongly suggest that future analyses of the real bills-quantity theory debate should consider different possibilities regarding the presence of private information and the specific manner in which private information enters the analysis.

Footnotes

1/See e.g., Kareken and Wallace (1978) or Fama (1980).

2/Preferences of this or of similar form are widely employed in the literature on banking with private information. See e.g., Diamond and Dybvig (1983), King and Haubrich (1983), or Smith (1984).

3/Say, for instance, an individual learns he will only live one period at the end of the period.

4/It is also assumed that once goods are in storage, they cannot be transferred between agents.

5/Subject to a restriction to be discussed below.

6/See e.g., Diamond and Dybvig (1983).

7/Diamond and Dybvig (1983) or Jacklin (1983).

8/This argument requires that type 3 agents have enough resources in the aggregate so that they can lend banks enough to tide them over even if all agents (except one) choose to withdraw at the end of their initial period. The condition which would permit this is (under full information) $[(1-p_1)\theta_1 + (1-p_2)\theta_2](Q_2 - Q_1) \leq \theta_3$. This condition in turn implies (6). Under private information below an even weaker condition suffices to prevent runs, so that these can never occur in the model if the condition above is assumed.

9/See e.g., Rothschild and Stiglitz (1976) or Wilson (1977).

10/See Smith (1984) for a formal demonstration.

11/In steady state these agents are indifferent regarding their portfolio composition, so this convention is just a device for avoiding price level indeterminacy.

12/Formally, $W_1(R_{11}, R_{12}, Q_1, Q_2) > W_1(R_1, R_2, Q_1, Q_2)$ for any value (R_1, R_2) satisfying (12b).

13/For the same reason as before, $\phi_{i3} = 0$; $i = 1, 2$ (i.e., there is no private storage). Also, previous assumptions on parameters imply that $\phi_{12} = 0$, i.e., type 1 agents do not hold money here. Thus, the discussion in the text is accurate.

14/A more detailed version of this argument appears in Smith (1984).

15/See Smith (1984).

16/Subject, of course, to (17) and (18).

17/The reason for this will be easy to see when a graphical version of the analysis is presented below.

18/Of course, there is always one other equilibrium in which $S_t = 0 \forall t$. Then agents face the autarky situation described in Section II. This equilibrium is not of interest here, however, and is not discussed further.

19/This is an argument that (18) holds with equality unless some set of agents prefer autarky to making deposits under the QT regime. This result depends upon the assumption of logarithmic utility, however.

20/The initial old prefer the QT regime since under it their money holdings are valued. Under LF, $S_t = 0 \forall t$.

21/Whose money holdings would have value under a QT regime.

22/It is straightforward to show that, so long as an equilibrium exists under the QT regime, this result is general in the model where all agents have identical investment opportunities.

23/See e.g., Kareken and Wallace (1978) or Fama (1980).

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Figure 1

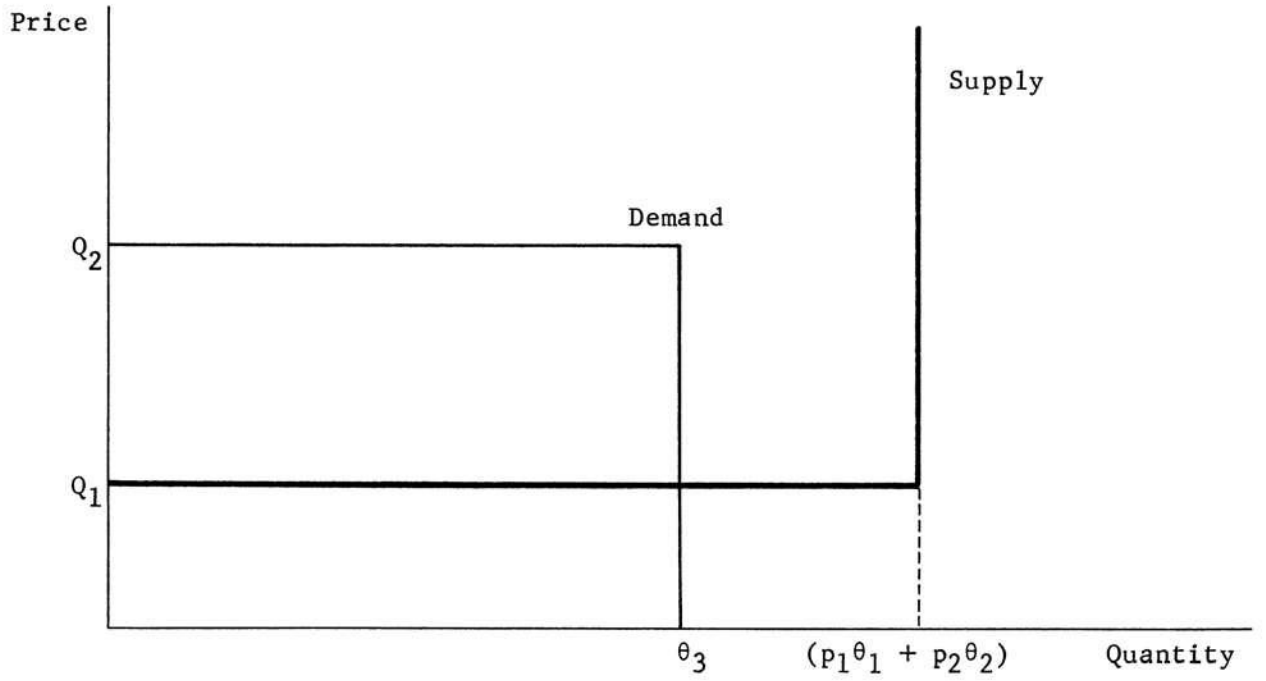


Figure 2

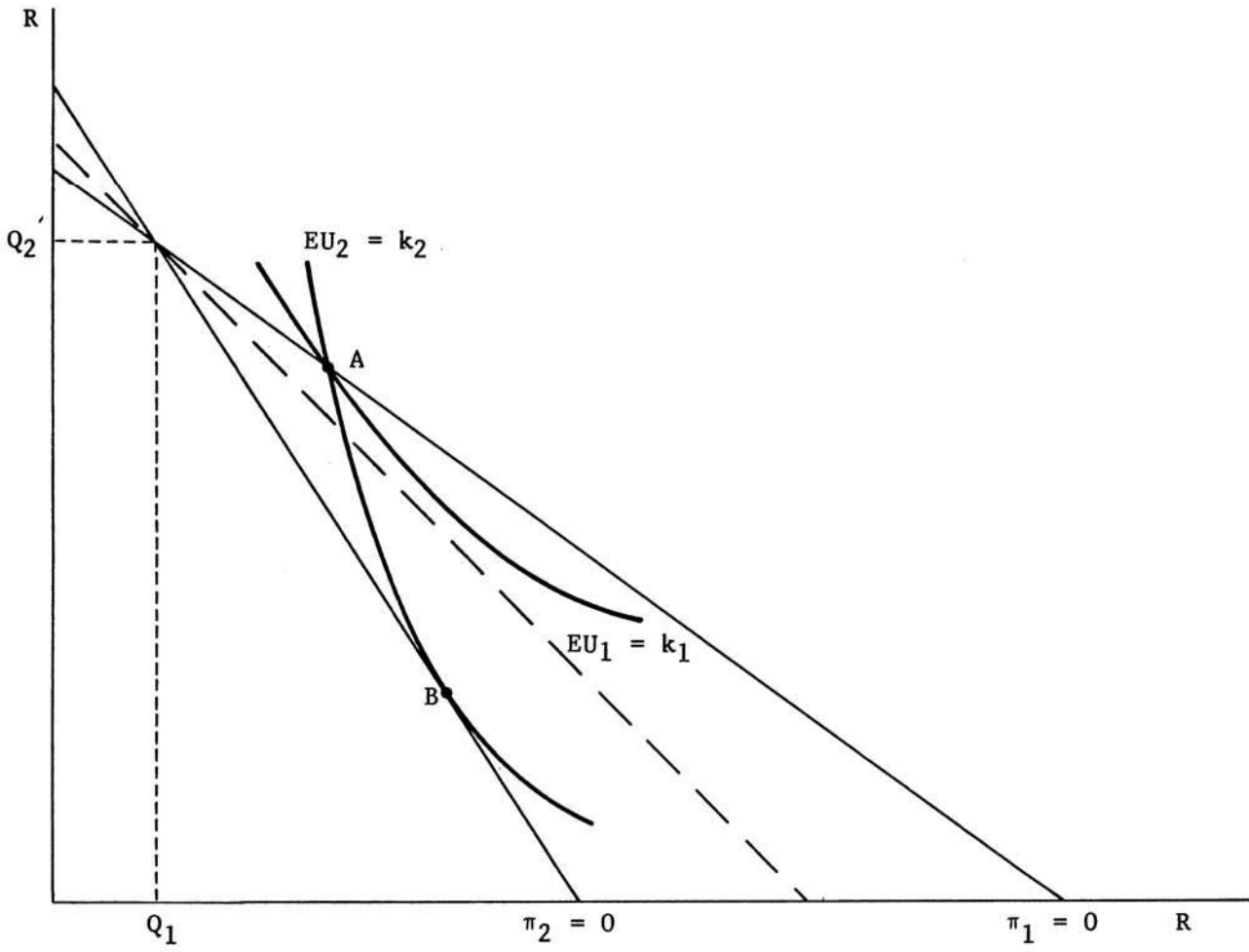


Figure 3

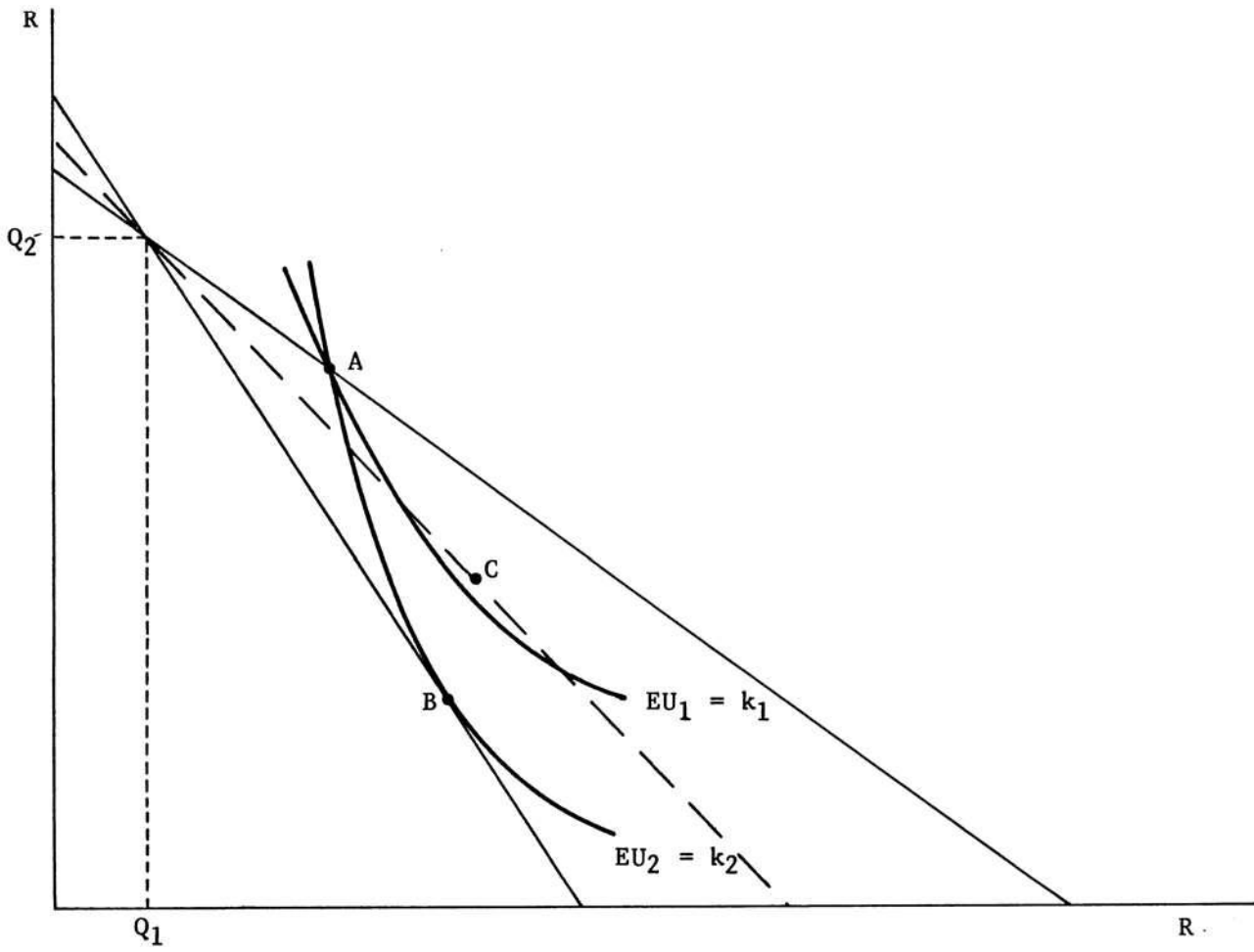


Figure 4

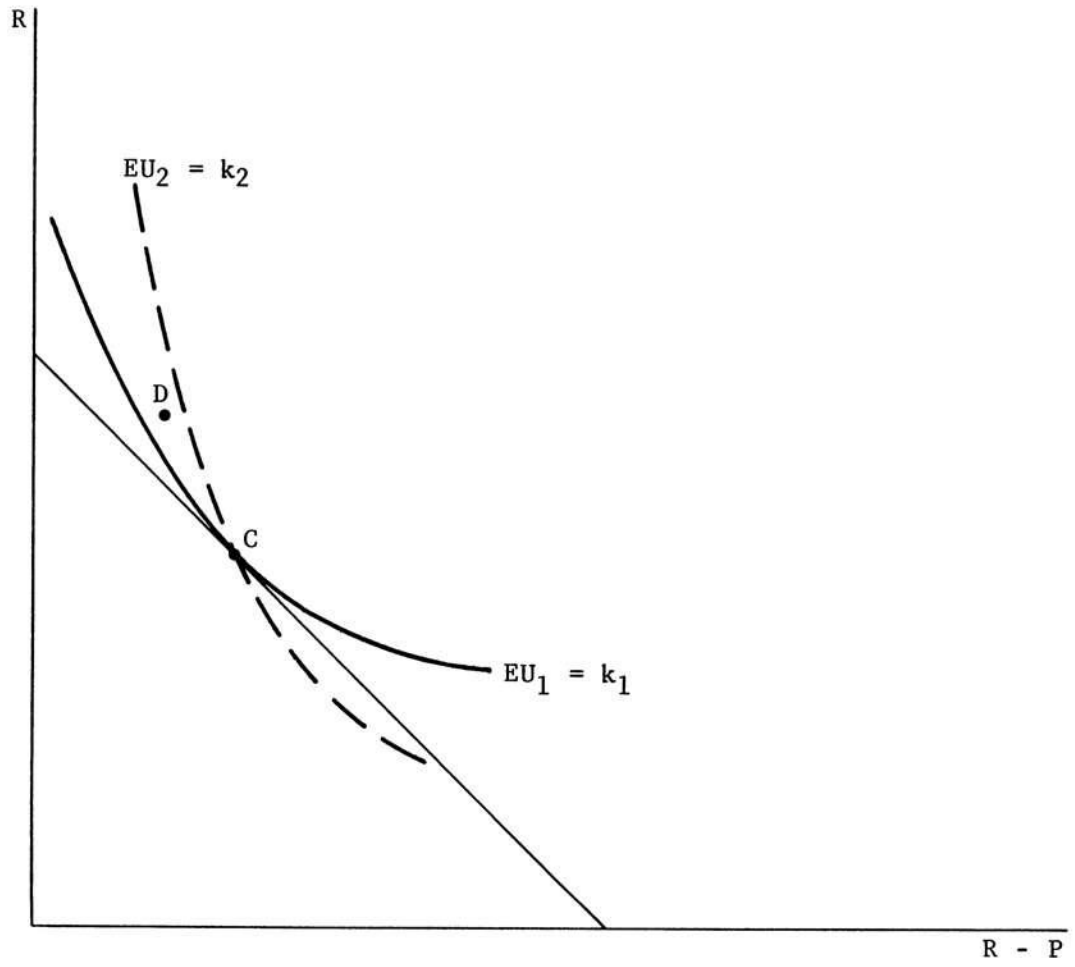


Figure 5

