

Federal Reserve Bank of Minneapolis
Research Department Working Paper

UNEMPLOYMENT, THE VARIABILITY OF HOURS,
AND THE PERSISTENCE OF "DISTURBANCES":
A PRIVATE INFORMATION APPROACH

Bruce Smith*

Working Paper 279

Revised October 1985

*Federal Reserve Bank of Minneapolis. This paper owes a great intellectual debt to Ed Prescott and Tom Sargent. In addition, I would like to thank Bob King, Finn Kydland, and Will Roberds for helpful comments and conversations.

The views expressed herein are those of the author and not necessarily those of the Federal Reserve Bank of Minneapolis or the Federal Reserve System. The material contained is of a preliminary nature, is circulated to stimulate discussion, and is not to be quoted without permission of the author.

The thrust of recent research in macroeconomics has been to develop general equilibrium models that confront aggregate behavior. These developments have taken two primary forms. One has been to show that dynamic competitive equilibrium models, while incapable of confronting the apparent existence of unemployed labor, can account for most other salient aspects of business cycle behavior.^{1/} The other has been to develop models with privately informed agents to provide an explanation for the appearance of an "excess supply" of labor.^{2/} To date, little of the latter research has focused on any cyclical aspects of the behavior of aggregate labor markets or of the economy in general.^{3/} Hence these two strands of general equilibrium macro modelling have largely diverged in terms of the issues that have been addressed.

A natural question, then, is whether models of privately informed agents can explain observed unemployment and, at the same time, do as well as existing dynamic competitive equilibrium models in terms of confronting other important features of the business cycle? Moreover, in order for the comparison between the two approaches to be "fair," a model with private information should be constructed in which the level of abstraction and the degree of structure placed on the model is similar to what is standard in the dynamic competitive equilibrium approach. The purpose of this paper is to show that a simple, fully specified general equilibrium model with private information can confront the presence of unemployed labor, and simultaneously can confront essentially the same set of business cycle observations as can the best (currently) developed competitive models. Moreover, this is true even though a great deal of structure is placed on the model by requiring it to be consistent with a wide range of observations on aggregate behavior in the postwar U.S. economy.

What are the important cyclical features that the model here (as well as competitive models lacking private information) must confront, then? Considerable consensus seems to exist that a good cyclical model must be consistent with two salient facts. First, the observed variance of employment about either a trend or a conditional mean is quite high relative to the variance of real wage rates (or productivity) about trend (or about a conditional mean). This is a feature of observed behavior in a number of developed postwar economies.^{4/} Second, deviations of hours and output from trend display large positive serial correlation. Hence, business cycle models must address the variability of employment relative to the real wage, and the persistence of output and employment fluctuations.

Moreover, a good business cycle model should do this in a way which is consistent with micro evidence on the labor supply behavior of individuals. In particular, it has been widely remarked that micro evidence is inconsistent with the kinds of co-movements required in hours and real wages for consistency with aggregate data. As stated by Ashenfelter (1984, p. 150),

The average labor supply elasticity must apparently be quite large to square up these hours and wage rate movements, while the available estimates of its slope that I have surveyed are, in fact, very small. The basic empirical problem seems to be that within the life-cycle, the person-specific correlation between hours and wages is simply too small to explain the time series movements in average hours relative to the time-series movements in average wage rates. The intertemporal substitution hypothesis originally advanced by Lucas and Rapping was, of course, precisely the suspicion that this was not the case.

Of course, existing competitive models do largely rely on an intertemporal substitution mechanism. However, the existing evidence against such a mechanism is quite strong.^{5/} Thus, it should be the case that business cycle models

require neither large observed elasticities of labor supply, nor a strong intertemporal substitution mechanism to explain cyclical variation.

The analysis here will produce a model consistent with unemployment of labor, with the high relative variability of employment, and with observed persistence. Moreover, it will do so in a way that is consistent with the micro evidence cited above, and with the following additional cyclical regularities.

(1) There is a strong positive co-variation between aggregate hours movements and measured real wage movements.^{6/}

(2) Similarly, average productivity is pro-cyclical.^{7/}

(3) Wage dispersions (defined in almost any manner) decline at cyclical peaks.^{8/}

(4) Changes in relative employment across "sectors" are an observed cyclical phenomenon.^{9/}

Finally, any model of the cycle should be consistent with the secular observation that long periods of upward trends in real wages are observed while no similar trend (or perhaps a downward trend) is observed in hours worked per capita by labor force members. Also, a model with a heterogeneous work force should be consistent with the casually stated "observation" that relative wages across occupations are an important "determinant" of labor market behavior.^{10/}

Having stated the objectives of this exercise, we now briefly describe the form it takes. An overlapping generations model is presented in order to generate a time series. Within each generation there is a heterogeneous workforce and a set of firms with access to a technology for converting labor and capital into a single consumption good. Workers differ in terms

of their ability to convert labor and capital into this good. Each worker is privately informed about his own productive abilities. This, then, is a fairly standard adverse selection model with an additional twist: firms must make decisions about how to allocate capital among different workers.

As in other adverse selection settings^{11/} firms are viewed as imperfect Nash competitors in labor markets. As is well known, then, in equilibrium firms induce workers to self-select by offering a range of contracts for workers to choose among. In the sequel, contracts take either of two forms; either a contract specifies a wage rate and a level of employment or it specifies a wage rate and a probability of being employed "full-time" (with the alternative being unemployment). If the model is structured appropriately, the necessity of inducing self-selection will imply that some workers are either underemployed or unemployed (depending on the type of contract considered). Hence the analysis is consistent with the existence of unemployed labor.

In order to generate cyclical behavior, the technology of our economy is subject to some random aggregate disturbances. Thus the approach here is to produce a "real business cycle" model in the manner of Kydland and Prescott (1982). It is then shown that models in the class at hand can generate all of the cyclical behavior discussed above. Moreover, they can do so in an "empirically reasonable" manner, where something should be said about how this term is used. No estimation theory has been developed for the type of model under consideration here. Therefore, we proceed in the manner of Kydland and Prescott (1982). In particular, a fully articulated general equilibrium model is presented in a parametric form. A set of observations is then produced to be used in "calibrating" parameters of the model. These

observations, to be listed below, are similar in number to the number of model parameters, so that they substantially restrict the set of possible parameter choices. It is then shown that, with the model calibrated by using observations not related to intertemporal correlations, the model is consistent with a high degree of serial correlation in the deviations of per capita hours and output from trend. Finally, we then show that these results are not model specific, in that they will emerge from a fairly general class of economies. Moreover, we show that the model is not sensitive to assumptions about either the timing of certain events, or about what actions are generally observable by agents in the model.

As a final remark, it may seem strange to use an overlapping generations model to examine business cycles. This is a convenient modelling strategy here, since it permits us to avoid dealing with a multi-period incentive problem. However, we will show that here the overlapping generations structure is more than an analytical convenience. In particular, it will be demonstrated that one can, indeed, take each "period" in the model to be roughly a generation, and that the model is still consistent with all of the features of postwar business cycles cited above.

I. The Model

A. The Environment

We wish to consider a model which generates an arbitrarily long time series of observations, but at the same time to avoid the complication of multi-period incentive problems. Thus, we construct a model which consists of a sequence of two period lived overlapping generations. Within each generation there is a heterogeneous set of privately informed workers who work only when young, and are thus retired when old. Hence each individual is in the

workforce only once, so that multi-period incentive problems are, in fact, avoided here. Also, each generation contains a set of firms, who have access to a technology for converting labor and capital into a single produced consumption good.

Let time be indexed by $t = 0, 1, \dots$. At each date there is a working young generation and a retired old generation. Old agents have accumulated capital, which they rent to firms. In addition, each generation is identical in size and composition. Hence we focus on stationary states in what follows, and references to time will typically be omitted. Finally, while it is not necessary to be too specific on this point, to fix ideas we may think of there being a fixed and countable set of firms in each generation as well as a continuum of workers.^{12/} Let workers be divided into two types (to be described below), with type indexed by $i = 1, 2$. All workers of type i are identical, and let μ be the proportion of each generation's workforce that is of type 1.

Rather than laying out a general setting in the remainder of this section, we adopt the following strategy. A parametric example is described which has the following features. First, under full information, all economies in this class are incapable of generating nontrivial cyclical behavior in labor markets. Secondly, though, under private information the model performs roughly as well as comparable (but more complex) dynamic competitive models^{13/} at confronting salient features of postwar U.S. business cycles. The reasons for focusing first on a class of examples is two-fold. Obviously, this permits a simple presentation of the role of private information in generating interesting cyclical behavior here. Secondly, though, since the economies we consider are dynamic models with nontrivial randomness and capital accumula-

tion, except under special circumstances they would give rise to nontrivial cyclical behavior even under full information. It is useful to begin, then, by presenting special economies which avoid this in order to highlight the role of asymmetrically informed agents.

Prior to describing this class of examples three comments are in order. First, we make a very special assumption on the form of technology. This is not important for the results obtained, and permits considerable simplification. All that is really needed is that we specify a technology consistent with the observed empirical regularity that increases in the capital stock in specific sectors, *ceteris paribus*, reduce wage differentials [Reder (1962)]. Second, preferences are specified in a way which makes savings behavior very special. This provides considerable simplification (permitting explicit closed form solutions for the example economies), and is consistent with the empirical observation that constant *ex ante* real interest rates are a reasonable approximation to what we observe [Fama (1975)]. Third, it may seem unusual to build a business cycle model based on an overlapping generations structure. However, we argue below that this structure can be taken seriously here, so that one may, in fact, think of a period as a generation. With these comments in mind, we now proceed to a description of the economies we will be working with.

Technology

A type 1 worker produces $\pi_1(s)$ units of the consumption good per unit time, where s is the current period productivity shock, and $\pi_1(s)$ is a scalar. Notice, then, that capital does not augment the output of type 1 workers. Therefore, it will be convenient to measure the capital stock in terms of accumulated capital per type 2 worker. Let k be this quantity. Then

a type 2 worker providing L units of labor, and combined with k units of capital, produces the single consumption good according to the production function $\pi_2(s)k^\theta L^{1-\theta}$; $\theta \in (0,1)$. Again $\pi_2(s)$ is a scalar productivity parameter that varies with the current aggregate shock s . The current period state of the economy, then, is completely described by the vector (s,k) , the current shock along with the inherited capital stock.

It remains to describe the process generating s . We adopt here the simplest possible specification: the productivity shock is a two state Markov chain with transition matrix

$$\begin{bmatrix} p(1) & p(2) \\ 1-p(1) & 1-p(2) \end{bmatrix}.$$

Hence $s \in \{1,2\}$, and $p(s)$ is the probability that $s' = 1$ conditional on s , where s' denotes next period's state. It will also be convenient to have a notation for last period's state, which we denote \hat{s} .

Preferences and Endowments

Each worker has an endowment of one unit of time when young to be allocated between labor and leisure. Workers have no endowment of the good at any date, and no endowment of labor when old. Let L denote hours worked, $L \in [0,1]$, and let c_j denote age j consumption; $j = 1, 2$. When we want to distinguish between agents of different types, we will employ the notation c_{j1} and L_{j1} .

The preferences of type 2 workers are summarized by the utility function $U_2(c_1, c_2, L) = \ln c_1 + \ln(1-L)$. Hence type 2 agents care only about what happens to them when young. Type 1 agents have preferences described by $U_1(c_1, c_2, L) = c_1 + \beta c_2 + \phi \ln(1-L)$, and, of course, these agents have the endowments described above. ϕ is restricted by the following:

$$(1) \quad \frac{\phi}{1-L} > \frac{y}{1-L},$$

$L \in [0, 1/2]$, $y \leq \pi_1(s)$; $s = 1, 2$. This restriction guarantees that if any agents are "off of" their labor supply curves any equilibrium will be associated with unemployment of labor. (1) implies further that at any (y, L) pair such that $y \leq \pi_1(s)L$, and such that $L \leq 1/2$, type 1 agents value incremental leisure relatively more than do type 2 agents. Such an assumption is plausible under the following interpretation. Suppose the model here to be derived from an underlying model of home production. Hence workers have the option of producing either at home or in the marketplace. Suppose also that all workers have identical preferences over home-produced and market-produced commodities. Then (1) simply asserts that type 1 workers, who in equilibrium will be more productive in the workplace than type 2 workers, are also more productive at home over some relevant subset of income-leisure pairs.

Savings and Insurance

It is assumed that no storage of goods is possible except via capital accumulation. Also capital depreciates entirely in its period of use.^{14/} Hence the return to capital is simply the rental rate on capital. At the time savings decisions occur, then, the (uncertain) rental rate next period is $r(s', k')$, where k' denotes next period's capital stock. The function $r(s, k)$ is known to all, and savers behave competitively.

Borrowing and lending is not precluded here. Insurance markets are, for the following reason. We assume that young agents appear on the scene after the realization of the current period state. Hence insurance sales are precluded for the current period. Moreover, type 2 agents do not care about old age, and type 1 agents are risk neutral with respect to old age consump-

tion. Firms are also assumed to be risk neutral, so that again there is no role for insurance markets. Below we show that a lack of insurance markets is not crucial to the results we obtain.

Information

Each worker knows his own type, but this is private information ex ante. As already indicated, each worker's contribution to output is unobservable. At this point, we also assume that savings behavior is unobservable. This assumption is relaxed below. Finally, the current period state (s,k) , the Markov transition matrix, and all current prices are common knowledge at each date.

B. Agents' Behavior

Workers

There are two aspects to workers' behavior; an employment decision and a savings decision. It is best to describe employment decisions after describing firm behavior. However, savings decisions can be easily described. Clearly type 2 workers do not save (or borrow). Then all capital accumulation is carried out by type 1 workers. Let $\gamma(s,k)$ denote the capital accumulated by a representative type 1 agent as a function of the current period state. Then for any given level of current period income $y_1(s,k)$, $\gamma(s,k)$ solves the problem

$$\max c_1 + \beta E_s c_2(s',k')$$

subject to

$$c_1 + \gamma(s,k) \leq y_1(s,k)$$

$$c_2(s',k') \leq \gamma(s,k)r(s',k'),$$

where E_s denotes the conditional expectation taken with respect to s' . Writing the maximand as a function of $y_1(s,k)$, $\gamma(s,k)$, and $r(s',k')$, we see that $\gamma(s,k)$ is chosen to maximize the expression

$$y_1(s,k) + \beta\{p(s)r[1,k(s)] + [1-p(s)]r[2,k(s)]\}\gamma(s,k).$$

Hence in equilibrium, we must have

$$(2) \quad p(s)r[1,k(s)] + [1-p(s)]r[2,k(s)] = \beta^{-1}$$

where $k(s) \equiv \gamma(s,k)\left(\frac{1-\mu}{\mu}\right)$, i.e., $k(s)$ is the future capital stock (per type 2 worker) implied by $\gamma(s,k)$. (It should be clear from (2) that it is appropriate to write capital accumulation as a function of s alone.)

Firm Behavior

Recall that firms here have access to a technology for converting labor and capital into the consumption good. Moreover, there are constant returns to scale in a dual sense: the production function for each individual worker displays constant returns to scale, and output is additively separable across workers. This will make an equilibrium easy to characterize.

It is assumed here that firms behave competitively in a rental market for capital, i.e., firms make a capital rental decision taking $r(s,k)$ as parametric. In addition, firms are assumed to be imperfect competitors in labor markets. Hence we follow Hart (1982) in using a model of an imperfectly competitive labor market to examine macroeconomic issues. In the model here, however, the Nash equilibrium we examine coincides with a competitive equilibrium in the absence of private information.

Firms, then, are viewed as operating in the following manner in labor markets. Each firm, taking the actions of other firms as given, an-

nounces a set of contracts with each contract consisting of a wage-hours pair. In particular, if a firm's contract offer is accepted by any workers one of the following will occur: only type 1 workers accept the contract offered, only type 2 workers accept the contract, or workers of both types accept the contract. Hence we may focus on firms which offer (at most) two contracts denoted $[w_i(s,k), L_i(s,k)]$; $i = 1, 2$. A contract specifies a wage rate to be paid any worker accepting it, and a number of hours the worker will be employed. Without loss of generality the contract $[w_i(s,k), L_i(s,k)]$ will be accepted (if at all) by type i workers. Hence if a firm announces a contract pair such that $(w_1, L_1) \neq (w_2, L_2)$, it hopes to induce self-selection of workers by contract accepted. If a contract announcement is not meant to induce workers of different types to accept different contracts, we adopt the notational convention $(w_1, L_1) = (w_2, L_2)$.

Since firms do not directly observe the type of any worker, firm contract announcements must be incentive compatible, i.e., satisfy the self-selection conditions

$$(3) \quad U_2[w_2(s,k)L_2(s,k), 1-L_2(s,k)] > U_2[w_1(s,k)L_1(s,k), 1-L_1(s,k)]$$

$$(4) \quad w_1(s,k)L_1(s,k) + \phi \ln[1-L_1(s,k)] > w_2(s,k)L_2(s,k) + \phi \ln[1-L_2(s,k)],$$

$s \in \{1,2\}$, for all equilibrium values of k , where (4) is the appropriate self-selection condition in light of the assumed form of type 1 preferences, and where we have taken note of (2). Notice that contracts are announced with full knowledge by all parties of the current period state (s,k) . Finally, we impose one additional restriction on announced contracts which is quite common in analyses of this type.^{15/} In particular, we require that each announced contract at least break even given the set of workers accepting it. Hence contracts must satisfy

$$(5) \quad w_1(s,k) \leq \pi_1(s)$$

$$(6) \quad w_2(s,k)L_2(s,k) \leq \pi_2(s)k^\theta L_2(s,k)^{1-\theta} - r(s,k)k$$

if $(w_1, L_1) \neq (w_2, L_2)$. We discuss what occurs under a pooling contract below, but we may note now that in equilibrium announced contracts must induce self-selection. Hence (5) and (6) are the relevant restrictions here.

Finally, it will be noted that firms are restricted to pure strategies and that we do not allow firms to offer contracts which consist of wage-employment lotteries. Neither of these assumptions is important to the results obtained, and the latter is relaxed below.

We conclude by completing our description of worker behavior. Given the set of announced contracts each worker accepts the announced contract he most prefers within that set.

II. Equilibrium

A. Definition

As indicated above, firms operate in a competitive rental market for capital. Given the current rental rate on capital $r(s,k)$, then, each firm chooses a quantity of capital to be applied to each worker and announces a set of contracts subject to (3)-(6). At this point, we will assert that self-selection occurs in equilibrium (which will be discussed further below). Hence firms will apply no capital to type 1 workers. Further, let $\psi(s,r)$ denote the desired amount of capital firms wish to apply to type 2 workers. Then a stationary Nash equilibrium here may be defined as follows.

Definition. A stationary Nash equilibrium is a set of announced contracts $\{[w_i(s,k), L_i(s,k)]\}; i = 1, 2$, satisfying (3)-(6), a set of values $r(s,k)$, and a set of values $k(s)$ such that

(i) $r(s,k)$ and $k(s)$ satisfy (2).

(ii) $r(s,k)$ is such that

$$(7) \quad \psi\{s, r[s, k(\hat{s})]\} = k(\hat{s})$$

* s, \hat{s} , i.e., capital markets clear.

(iii) Given the current value $r(s,k)$ and given the announced contracts of other firms, no firm has an incentive to change $\psi(s,r)$ or its own announced contracts (given that all announcements are subject to (3)-(6)).

In light of the constant returns to scale assumption, it is clear that any equilibrium must satisfy a number of no surplus conditions. These will be discussed below. Also, any equilibrium displays self-selection of workers by contract accepted. To see this fix the choice of capital rentals for each firm, and then notice that assumption (1) allows the standard Rothschild-Stiglitz (1976) argument to be employed that shows that any equilibrium must display self-selection of workers.

Two last points should be made here. First, given the assumed form of preferences in this section, it is a real restriction to prevent firms from employing contracts consisting of wage-employment lotteries.^{16/} In Section IV below, however, this restriction is relaxed to show that our results do not depend upon it. Second, as in Rothschild-Stiglitz (1976), no equilibrium in pure strategies need exist here. Existence issues are discussed below.

B. Equilibrium Contracts and Capital Stocks

Since any equilibrium must display self-selection, we begin by discussing what contracts, if any, will constitute equilibrium contracts. Section C then discusses when an equilibrium will exist.

As mentioned above, a number of "no surplus" conditions must be satisfied in equilibrium. Given that self-selection also occurs, it is easy to lay out these conditions. First, clearly (5) and (6) hold with equality. Then, in light of this fact, we claim that $[w_2(s,k), L_2(s,k)]$ and $\psi(s,r)$ solve the following problem:

$$\max U_2 [w_2(s,k)L_2(s,k), 1-L_2(s,k)]$$

subject to (6) by choice of $L_2(s,k)$ and $\psi(s,r)$, with $r(s,k)$ parametric. Given the assumed form of $U_2(-)$, and given that (6) holds with equality, this maximization problem may be written as

$$\max \ln[\pi_2(s)\psi^\theta L_2(s,k)^{1-\theta} - \psi r(s,k)] + \ln[1-L_2(s,k)].$$

This problem has the following solution:

$$(8) \quad L_2(s,k) = 1/2 \quad \forall s,k$$

$$(9) \quad \psi(s,r)r(s,k) = \theta\pi_2(s)\psi(s,r)^\theta L_2(s,k)^{1-\theta} = \theta\pi_2(s)(1/2)^{1-\theta}\psi(s,r)^\theta.$$

Also, since we have capital market clearing in equilibrium, we may rewrite (9) as

$$(9') \quad k(\hat{s})r(s,k) = \theta\pi_2(s)k(\hat{s})^\theta(1/2)^{1-\theta}.$$

This, of course, is equivalent to the standard marginal productivity condition

$$(9'') \quad r(s,k) = \theta \pi_2(s) (1/2)^{1-\theta} k^{\theta-1}.$$

Also, we may infer the wage rate received by type 2 agents from a standard marginal productivity relation:

$$(10) \quad w_2(s,k) = (1-\theta) \pi_2(s) (1/2)^{-\theta} k^{\theta}.$$

The reason why the firm's equilibrium choice of capital and its equilibrium contract solve the maximization problem just described is as follows. As should be clear, the self-selection condition (4) can never hold with equality in equilibrium so long as parameter values are chosen to satisfy

$$(11) \quad \pi_1(s) > (1-\theta) \pi_2(s) (1/2)^{-\theta} k(\hat{s})^{\theta}$$

* s, \hat{s} , with $k(\hat{s})$ chosen to satisfy (9'') and (2). This is because, under (11), $w_1(s,k) > w_2(s,k)$, and hence type 1 workers will have no incentive to select type 2 contracts. Then, since (4) does not bind on firms' choices of contracts, an absence of rent opportunities dictates that firms must choose capital inputs and $[w_2(s,k), L_2(s,k)]$ to maximize the utility of a type 2 worker born in state (s,k) . In particular, if this were not the case, some firm could make a (possibly) different choice of capital input and choose $[w_2(s,k), L_2(s,k)]$ so as to increase type 2 utility, and simultaneously to satisfy (6) with strict inequality. This would attract all type 2 workers and earn a profit. Hence any other choice of capital and wage-hours contract could not be an equilibrium choice.

Now consider $[w_1(s,k), L_1(s,k)]$. Since (5) holds with equality, $w_1(s,k) = \pi_1(s)$. Then we claim that $L_1(s,k)$ solves

$$\max \pi_1(s) L_1(s,k) + \phi \ln[1-L_1(s,k)]$$

subject to (3).^{17/} Given the assumed form of preferences, (3) may be written as

$$(12) \quad \ln(1/2)w_2(s,k) + \ln(1/2) \geq \ln[w_1(s,k)L_1(s,k)] + \ln[1-L_1(s,k)],$$

$w_1(s,k)$, where we have used (8), and where $w_2(s,k)$ is given by (10).

In the maximization problem just set out, (3) may or may not be binding. Then define

$$L_1^*(s) \equiv \operatorname{argmax} \pi_1(s)L + \phi \ln(1-L).$$

Then

$$(13) \quad L_1^*(s) = 1 - \frac{\phi}{\pi_1(s)},$$

where ϕ and the values $\pi_1(s)$ are chosen so that an interior solution obtains. Then using $w_1(s,k) = \pi_1(s)$, (10), and (12), (3) binds in our maximization problem iff

$$(14) \quad (1/4)(1-\theta)2^\theta \pi_2(s)k(\hat{s})^\theta < \phi - \frac{\phi^2}{\pi_1(s)},$$

where $k(\hat{s})$ satisfies (2) and (9''). (14) is henceforth assumed to hold for all s, \hat{s} (i.e., parameter values are selected to guarantee (14)).

If (14) holds then $[w_1(s,k), L_1(s,k)]$ is given by $w_1(s,k) = \pi_1(s)$, and by (12) at equality.^{18/} (12) at equality may be rewritten as ^{19/}

$$(15) \quad L_1(s,k) = (1/2) \left[1 - \left(1 - \frac{w_2(s,k)}{w_1(s,k)} \right)^{1/2} \right].$$

Again it is easy to see that the above must be the equilibrium type 1 contract. In particular, type 1 agents must receive the most preferred contract for them consistent with their employers earning zero profits, and

with self-selection constraints being satisfied. Otherwise, there would exist a preferred contract for type 1 agents which was consistent with self-selection, and which was profitable when offered.

Having described what equilibrium contracts and choices of capital inputs must be like, our description of equilibrium can be completed by stating what is required for (2) to be satisfied. Using (9'') in (2) we obtain

$$(16) \quad k(\hat{s})^{1-\theta} = \beta\phi\{p(\hat{s})\pi_2(1)+[1-p(\hat{s})]\pi_2(2)\} * \hat{s},$$

where $\phi \equiv \theta 2^{-(1-\theta)}$. This and $\psi(s,r) = k(\hat{s})$ completes our description of the equilibrium capital stock.

C. Existence of Equilibrium

The contracts described above are such that $[w_2(s,k), L_2(s,k)]$ (along with the choice $\psi(s,r)$) and $[w_1(s,k), L_1(s,k)]$ are the maximal contracts for type 2 and 1 agents (respectively) consistent with self-selection and with (5) and (6). Thus, there are no alternate contracts (or alternate choices of capital inputs plus contracts) which are consistent with self-selection, with (5) and (6), and which any workers prefer to the contracts derived above.

Now consider why an equilibrium might fail to exist. There are two reasons. First, clearly the values $k(\hat{s})$ which satisfy (16) must obey $(\frac{1-\mu}{\mu})\hat{k}(s) = \gamma(s,k) < w_1(s,k)L_1(s,k)$. Otherwise required equilibrium savings levels would result which violate type 1 agents' budget constraints. Parameter values are henceforth selected to guarantee that this condition is satisfied. Second, given that all firms announce the contracts described in the previous section, some firm may have an incentive to offer some other set of contracts (and possibly choose a different capital stock). Since the contracts derived above were maximal for each type of agent among the set of

contracts (satisfying (5) and (6)) that induce self-selection, any contract that is capable of attracting any workers must therefore be a pooling contract. We now describe what a pooling contract would involve, and what would be necessary for such a contract to attract any workers in a profitable manner.

Under a pooling contract obviously all workers work a common hours level $L(s,k)$ and receive a common wage rate $w(s,k)$. Moreover, as all workers appear identical from the point of view of a firm offering a pooling contract, all workers must be allocated the same quantity of capital. Let ψ denote the firm's choice for a level of capital input. Finally, since firms operate in competitive rental markets, we have the following. Suppose all firms initially offer the contracts and make the capital input decisions described in the previous section. Then the prevailing rental rate on capital is given by $r(s,k) = \theta\pi_2(s)(1/2)^{1-\theta}k^{\theta-1}$. A firm wishing to offer a different contract (and possibly to change its capital rental) takes this rental rate as given.

The issue of existence of an equilibrium is now quite similar to that discussed by Rothschild and Stiglitz (1976). However, here there is one additional twist. In particular, the information generated by sorting permits capital to be allocated efficiently. If sorting does not occur then type 1 agents receive the same levels of capital inputs as do type 2 agents. In our context this amounts to a throwing away of resources. This will make existence easier to obtain here than it is in the Rothschild-Stiglitz setting.

Formally, then, consider a pooling contract $[w(s,k), L(s,k)]$ offered by a firm choosing per worker capital input ψ . Since workers are indistinguishable, ψ is allocated evenly among all workers. If the firm attracts workers in their population proportions, then, fraction μ of this capital

input is allocated to type 1 workers. This is completely unproductive. Fraction $(1-\mu)$ is allocated to type 2 workers. Hence with common hours level $L(s,k)$ and with a per worker capital stock of ψ , type 2 workers (per person) produce $\pi_2(s)[(1-\mu)\psi]^\theta L(s,k)^{1-\theta}$ units of output. Hence per worker output is given by $\mu\pi_1(s)L(s,k) + (1-\mu)\pi_2(s)[(1-\mu)\psi]^\theta L(s,k)^{1-\theta}$, and firm profits are nonnegative iff

$$(17) \quad \mu\pi_1(s)L(s,k) + (1-\mu)\pi_2(s)[(1-\mu)\psi]^\theta L(s,k)^{1-\theta} - r(s,k)\psi - w(s,k)L(s,k) > 0.$$

Now a pooling contract will attract type 1 workers only if it is preferred to the separating contract derived in the previous section. Here we wish to derive sufficient conditions for an equilibrium to exist. Hence we consider the maximal pooling contract for type 1 agents and examine when this contract is preferred by type 1 agents to the separating contract derived above. Now the optimal pooling contract for type 1 agents solves the problem

$$\max_{\substack{0 \leq L(s,k) \leq 1 \\ 0 \leq \psi}} w(s,k)L(s,k) + \phi \ln[1-L(s,k)]$$

subject to (17), where this is the appropriate maximand since the values $r(s,k)$ satisfy (2). Substituting (17) (at equality) into the maximand, then, we obtain the unconstrained problem

$$(P) \quad \max_{\substack{0 \leq L(s,k) \leq 1 \\ 0 \leq \psi}} \mu\pi_1(s)L(s,k) + (1-\mu)^{1+\theta}\pi_2(s)\psi^\theta L(s,k)^{1-\theta} - r(s,k)\psi + \phi \ln[1-L(s,k)].$$

The optimizing solution for ψ as a function of $L(s,k)$ and $r(s,k)$ is

$$\psi = L(s,k) \left[\frac{\theta(1-\mu)^{1+\theta} \pi_2(s)}{r(s,k)} \right]^{\frac{1}{1-\theta}}$$

Substituting (18) into the problem (P) gives us a maximization problem defining $L(s,k)$:

$$\begin{aligned} \max_{0 \leq L(s,k) \leq 1} \quad & L(s,k) \left\{ \mu \pi_1(s) + [(1-\mu)^{1+\theta} \pi_2(s) r(s,k)^{-\theta}]^{\frac{1}{1-\theta}} \left[\theta^{\frac{\theta}{1-\theta}} - \theta^{\frac{1}{1-\theta}} \right] \right\} \\ & + \phi \ln [1-L(s,k)] \end{aligned}$$

with $r(s,k) = \theta \pi_2(s) (1/2)^{1-\theta} k^{\theta-1}$. Let $L^*(s,k)$ denote the maximizing value of $L(s,k)$. Then $L^*(s,k)$ is the optimal pooling contract for type 1 agents which at least breaks even. Hence no pooling contract exists which can attract type 1 agents in a profitable manner if

$$(19) \quad \begin{aligned} \pi_1(s) L_1(s,k) + \phi \ln [1-L_1(s,k)] > L^*(s,k) \left\{ \mu \pi_1(s) + \left[\theta^{\frac{\theta}{1-\theta}} - \theta^{\frac{1}{1-\theta}} \right] \right. \\ \left. \cdot [(1-\mu)^{1+\theta} \pi_2(s)]^{\frac{1}{1-\theta}} [r(s,k)]^{-\frac{\theta}{1-\theta}} \right\} + \phi \ln [1-L^*(s,k)] \end{aligned}$$

* s and for all values $k(\hat{s})$ derived above. Thus satisfaction of (19) by the values $L_1(s,k)$ derived above is sufficient for the existence of an equilibrium.

III. A Calibrated Example

Given the assumed form of preferences and technology in section one, it is possible to explicitly solve for an equilibrium once given a set of values for the parameters of the model. We begin this section, then, by setting out the observations which guide our choices of free parameters. Prior to doing so, however, we will reduce the set of free parameters by one by imposing $\pi_1(1) = \pi_1(2) \equiv \pi_1$. Hence the set of parameters for the model is as follows:

preferences β, ϕ

technology $\theta, \pi_1, \pi_2(1), \pi_2(2), p(1), p(2)$

population μ

This gives a total of nine free parameters.

The purpose of this section is to lay out a set of observations which restricts choices of the above parameters, and then to examine the equilibrium derived from the calibrated model. This is done below. However, to begin we should discuss what has guided our choice of functional forms for preferences and technology, and why we have imposed $\pi_1(1) = \pi_1(2)$.

Preferences

We have made three important assumptions on preferences: type 2 agents care only about consumption in their youth, type 2 agents have logarithmic preferences, and type 1 agents have preferences that are linear in c_1 and c_2 . The first is an assumption of convenience which (in conjunction with the third) permits an explicit solution for the equilibrium capital stock at each point in time. Hence the second and third assumptions require some defense. First, then, if self-selection constraints bind on the determination of $L_1(s,k)$, then only the preferences of type 2 agents matter in the determination of hours. This is clear as $L_2(s,k) = \operatorname{argmax} U_2[w_2(s,k)L, 1-L]$ and as $L_1(s,k)$ is determined as the (smallest) solution to the relation $U_2[w_2(s,k)L_2(s,k), 1-L_2(s,k)] = U_2[\pi_1(s)L_1(s,k), 1-L_1(s,k)]$. Thus the form of type 2 preferences is the only crucial preference assumption for labor market behavior. Also, since $L_2(s,k) = \operatorname{argmax} U_2[w_2(s,k)L, 1-L]$, type 2 agents are always "on their labor supply function." Estimates using aggregate data under

this assumption favor logarithmic utility [Altug (1984)]. Hence there is some empirical support for such an assumption. In addition, it will be recalled that we desire our model specification to be consistent with the following observation. Over time trends are observed in real wage rates, but not in hours. Suppose we were to introduce a trend into our model by allowing type 1 output per unit of labor at time t to be given by $(1+n)^t \pi_1(s)$ and similarly letting the production function for a type 2 worker at t be given by $(1+n)^t \pi_2(s) k^{\theta} L^{1-\theta}$. Then let $w_{it}(s,k)$ denote time t wage rates in this amended model. What is the behavior of hours? Clearly $L_2(s,k) = 1/2 \neq s, k$ in any case. Moreover, we continue to have (so long as self-selection constraints bind)

$$L_1(s,k) = (1/2) \left[1 - \left(1 - \frac{w_{2t}(s,k)}{w_{1t}(s,k)} \right)^{1/2} \right].$$

Also, $w_{it}(s,k) = (1+n)^t w_i(s,k)$. Hence

$$L_1(s,k) = (1/2) \left[1 - \left(1 - \frac{w_2(s,k)}{w_1(s,k)} \right)^{1/2} \right]$$

as before. Hence introducing a trend in productivity would induce no trend in hours. This is another argument in favor of logarithmic preferences. And finally, of course, this preference assumption permits an explicit closed form solution of the model to be obtained.

It remains to consider type 1 agent preferences. It has already been noted that these allow for an explicit derivation of the capital stock at each point in time. Also, just as type 2 preferences dictate the behavior of hours, type 1 preferences dictate the behavior of savings and the ex ante real rate of interest. In particular, the assumed form of type 1 preferences implies a constant ex ante real interest rate. This is not violently at variance with observation [Fama (1975)].

Technology

There are two essential assumptions that have been made on technology. One is on the constancy of $\pi_1(s)$, and the other is that type 2 agents can productively employ capital while type 1 agents cannot. $\pi_1(s)$ has been assumed to be constant for a very simple reason. In particular, if $\pi_1(s)$ did vary across states, then even under full information our model would display cyclical variation of hours. We desire that all variation of hours in our model be due to the presence of private information. Hence we impose $\pi_1(1) = \pi_1(2)$.

The assumption that only type 2 workers can productively employ capital is a simplifying assumption that avoids considerable notational complexity. What we actually require is that a larger inherited capital stock, ceteris paribus, reduces the wage differential $w_1(s,k)/w_2(s,k)$. This is in accordance with observation [Reder (1962)]. Hence we have employed the simplest specification consistent with Reder's observation.

Having defended our specifications of preferences and technology, then, it remains to lay out the observations used to calibrate the model.

A. Observations

The following set of observations is used as guidance in the choice of parameter values:

- (i) Individuals in the workforce typically work about one third of available time.
- (ii) Postwar unemployment rates in the U.S. range between 4 and 10 percent.
- (iii) Relative wages between construction and manufacturing in the postwar U.S. range between .7 and .8.^{20/}

- (iv) Capital's share in total output is quite stable at approximately $1/3$.
- (v) The ratio of gross private saving plus corporate saving to GNP is quite stable over time at about $.15$.^{21/}
- (vi) The percentage standard deviation of the capital stock about trend is 1.2 .^{22/}

In addition to having these observations, let us recall our objectives and make some of them more specific. Our objectives were to produce a model consistent with the following.

- (i) Unemployed labor.
- (ii) The high variability of hours (employment) relative to real wages or productivity.
- (iii) High serial correlation of fluctuations in output and employment.
- (iv) Strong positive co-variation between aggregate employment and aggregate real wages.
- (v) (iv) should be produced in a manner consistent with micro evidence, i.e., individual labor supply elasticities are low and intertemporal substitution is not strong.
- (vi) Wage dispersions decline at cyclical peaks.
- (vii) Relative wages across occupations are an important "determinant" of labor market behavior.^{23/}
- (viii) Secular trends in real wages are not accompanied by similar trends in hours.
- (ix) Business cycle movements are accompanied by changes in the sectoral composition of employment.^{24/}

While some of these are obviously qualitative (and we have seen that others are accomplished, such as (viii) above), many are quantifiable. We now elaborate on these.

- (i) The percentage standard deviation of hours about trend is 2 percent.^{25/}
- (ii) The percentage standard deviation of productivity about trend is 1 percent.^{26/}
- (iii) The percentage standard deviation of output about trend is 1.8 percent.^{27/}

All of these relate to the variability of employment and output relative to productivity. The two-to-one ratio of variability in hours relative to productivity is quite important as such a ratio also applies to employment and real wages, and obtains across a range of postwar economies [Geary and Kennan (1984)] and in the prewar U.S. economy [Bernanke and Powell (1984)].

In addition to these observations, there are a number of observations related to the serial correlation properties of employment and output. Discussion of these will be deferred until after the calibrated model is presented so as to make interpretation easier.

B. The Example

It will be recalled that we have nine parameters to specify. These are as follows.^{28/}

preferences $\beta = .579, \phi = 6$

technology $\pi_1(1) = \pi_1(2) = \pi_1 = 8.6$

$\pi_2(1) = 7, \pi_2(2) = 6.7$

$\theta = 1/2, p(1) = 2/3, p(2) = 1/3$

population $\mu = 2/3$

We will now display the equilibria of this economy under full information and private information respectively. We begin by specifying the equilibrium behavior of the capital stock and relative wage rates which are the same in either case.

Recall that the rental rate on capital is given by $r(s,k) = \theta\pi_2(s)k^{\theta-1}L_2(s,k)^{1-\theta}$. It is also easy to see that $L_2(s,k) = 1/2$ in equilibrium (under either full or private information). Also, we have equation (2):

$$\beta\{p(s)r(1,k')+[1-p(s)]r(2,k')\} = 1; s = 1, 2.$$

Using the form of $r(s,k)$, along with $L_2(s,k) = 1/2$, we obtain the equilibrium inherited capital stocks (as a function of last period's state) $k(1) = 2$, $k(2) = 1.942$. The implied percentage standard deviation of the capital stock about trend (mean) is 1.5 percent. (Contrast with the actual value of 1.2 percent.)

Having obtained equilibrium capital stocks, it is easy to obtain equilibrium wage rates (again under either full or private information). Obviously $w_1(s,k) = \pi_1$, and $w_2(s,k) = (1-\theta)\pi_2(s)k^{\theta}L(s,k)^{\theta}$. Now define $w_i(s,\hat{s}) \equiv w_i[s,k(\hat{s})]$, use $L(s,k) = 1/2$, and the equilibrium values for $k(\hat{s})$ to obtain

$$\frac{w_2(1,1)}{w_1(1,1)} = .814 \quad \frac{w_2(1,2)}{w_1(1,2)} = .802$$

$$\frac{w_2(2,1)}{w_1(2,1)} = .779 \quad \frac{w_2(2,2)}{w_1(2,2)} = .768.$$

These are essentially in the range for relative wages set out above of [.7,.8].

Full Information

We now derive the remainder of the equilibrium for this economy under full information. It is easy to verify that the Nash equilibrium for this economy corresponds to the full information competitive equilibrium. Hence wages are as above, and

$$L_2(s,k) = \operatorname{argmax} U_2[w_2(s,k)L_2(s,k), 1-L_2(s,k)]$$

$$L_1(s,k) = \operatorname{argmax} \pi_1 L_1(s,k) + \phi \ln[1-L_1(s,k)].$$

Hence, $L_2(s,k) = 1/2$, and $L_1(s,k) = L_1^*(s,k) = 1 - \frac{\phi}{w_1(s,k)} = 1 - \frac{\phi}{\pi_1}$. Thus employment is constant in this economy under full information. It will therefore be clear that all cyclical variation of employment in this economy in the presence of private information will be due to informational frictions.

Private Information

It is easy to check that (14) is satisfied by the economy set out above, i.e., that self-selection constraints bind in the determination of $L_1(s,k)$. Hence, following the determination of equilibrium discussed above, $L_2(s,k) = 1/2$, and $L_1(s,k)$ is given by

$$L_1(s,k) = (1/2) \left[1 - \left(1 - \frac{w_2(s,k)}{w_1(s,k)} \right)^{1/2} \right].$$

Using the relative wage rates determined above, we can derive the following, where $L_i(s, \hat{s}) \equiv L_i[s, k(\hat{s})]$:

$$L_1(1,1) = .284 \quad L_1(1,2) = .278$$

$$L_1(2,1) = .265 \quad L_1(2,2) = .259.$$

This completes our description of the equilibrium behavior of individual hours and real wages. We now turn our attention to aggregates. Define aggregate per capita hours by

$$\hat{L}(s, \hat{s}) \equiv \mu L_1(s, \hat{s}) + (1-\mu)L_2(s, \hat{s}),$$

and define average (hours weighted) real wages (and productivity) by

$$\hat{w}(s, \hat{s}) \equiv \mu \left[\frac{L_1(s, \hat{s})}{\hat{L}(s, \hat{s})} \right] w_1(s, \hat{s}) + (1-\mu) \left[\frac{L_2(s, \hat{s})}{\hat{L}(s, \hat{s})} \right] w_2(s, \hat{s}).$$

Finally, per capita output is given by

$$\hat{y}(s, \hat{s}) \equiv \mu \pi_1 L_1(s, \hat{s}) + (1-\mu) \pi_2(s) [k(\hat{s})]^\theta L_2(s, \hat{s})^{1-\theta}$$

and the unemployment rate is

$$u(s, \hat{s}) = \frac{[L_1^*(s, \hat{s}) - L_1(s, \hat{s})] \mu}{\mu L_1^*(s, \hat{s}) + (1-\mu)L_2(s, \hat{s})},$$

since type 2 agents are not unemployed. Then we have the following behavior of aggregate hours per capita in equilibrium.^{29/}

$$\hat{L}(1,1) = .356 \quad \hat{L}(1,2) = .352$$

$$\hat{L}(2,1) = .343 \quad \hat{L}(2,2) = .339.$$

It will be noted that, on average, people work roughly a third of available time here. Also, the implied percentage standard deviation of per capita hours about trend (mean) here is 2.2 percent. (Contrast with an actual value of 2 percent.) Finally, we can compute the first order autocorrelation of hours per capita here, which is .547.

The behavior of average per capita wages is as follows:

$$\hat{w}(1,1) = 7.851 \quad \hat{w}(1,2) = 7.795$$

$$\hat{w}(2,1) = 7.686 \quad \hat{w}(2,2) = 7.627.$$

The implied percentage standard deviation about trend (mean) is 1.3 percent. (Contrast with an actual value of 1 percent.) Similarly, per capita output is

$$\hat{y}(1,1) = 3.961 \quad \hat{y}(1,2) = 3.894$$

$$\hat{y}(2,1) = 3.752 \quad \hat{y}(2,2) = 3.686.$$

The implied percentage standard deviation about trend (mean) is 3.2 percent, which is too large relative to the observed value of 1.8 percent. The first order autocorrelation coefficient of per capita output is .73. Finally we may compute unemployment rates. $L_1^*(s, \hat{s}) = 1 - (\phi/\pi_1) = .302$, so that

$$u(1,1) = 3.3\% \quad u(1,2) = 4.3\%$$

$$u(2,1) = 6.7\% \quad u(2,2) = 7.9\%$$

Hence unemployment rates lie in the appropriate range.

It remains to say something about capital's share in total output and about savings behavior for this economy. Capital's share here varies between .294 and .299. Hence this is appropriate. Also the ratio of total savings (as defined above) to GNP here is given by $(1-\mu)k(s)/\hat{y}(s, \hat{s})$, since $(1-\mu)k(s)$ is the aggregate level of capital accumulation. This ratio here varies between .168 and .175, which is appropriate relative to the David-Scadding (1973) value of about .15.

Remarks

A number of remarks are now in order. Clearly the calibrated model generates the desired high variability of hours relative to real wages (productivity). As we will discuss shortly, it is also consistent with the observed persistence of disturbances. However, we might now comment on some other features of the model.

First type 1 workers, who are high productivity workers (in equilibrium) are the workers who experience unemployment here. At first glance, this may seem strange. However, the relative wages of workers in manufacturing and construction were used in calibrating the model. If we continue with this interpretation, construction workers are relatively highly paid, but they experience high rates of unemployment and work fewer hours on average than workers in manufacturing. Hence this feature of the model is actually an appropriate one.

Second, continuing with this interpretation, the model suggests that over the cycle the sectoral composition of the workforce will change. This is in accordance with observation [Lilien (1982), Samson (1985)].

Third, we should comment on the consistency of behavior here with micro evidence. As indicated above, existing micro evidence suggests very low individual correlations between hours and real wages. In our model parameters for each type of agent are chosen to be consistent with this evidence. In particular, for type 2 agents hours will be constant independent of the wage rate while for type 1 agents wages are constant while hours vary over the cycle.

Fourth, it should be noted that relative wages are an important "determinant" of labor market behavior here. This is clear since $L_2(s,k)$ is constant and $L_1(s,k)$ is given by

$$L_1(s,k) = (1/2) \left[1 - \left(1 - \frac{w_2(s,k)}{w_1(s,k)} \right)^{1/2} \right].$$

This should also make clear (as do the values for relative wages reported above) that wage differentials decline at cyclical peaks.

Finally,

$$\frac{w_2(s,k)}{w_1(s,k)} = \frac{(1-\theta)\pi_2(s)2^{\theta}k^{\theta}}{\pi_1(s)}.$$

Hence increases in the capital stock, *ceteris paribus*, reduce wage differentials, as we observe. This is, in fact, what leads to "persistence" here.

Interpreting the Serial Correlation Parameters

We have seen that our calibrated model generates persistence in the deviations of output and hours from trend. The question addressed here is how this is to be interpreted. Either of two interpretations is possible. However, prior to suggesting these interpretations, it will be useful to have some "observed" values for purposes of comparison.

To this end, then, we begin by noting that in our model the deviations of output and hours from trend can be represented as a first order Markov process. For purposes of comparison, then, suppose we estimate the relation

$$y_t^* = \rho_1 y_{t-1}^* + \epsilon_{1t}$$

using quarterly data, where

$$y_t^* = y_t - \bar{y} - \alpha_1 t,$$

and y_t is actual GNP, i.e., y_t^* is the deviation of output from its mean and a linear trend.^{30/} The estimated relations are

$$y_t - \bar{y} = \frac{(1.959)t}{(6.966)} + u_{1t}$$

$$R^2 = .243$$

where the t-statistic appears below the coefficient estimate, and

$$y_t^* = \frac{(.995)}{(289.56)} y_{t-1}^* + \epsilon_{1t}$$

$$R^2 = .998.$$

Thus $\hat{\rho}_1 = .995$ is an estimated first order autocorrelation coefficient for output (in this detrended form). Similarly, let L_t be hours, where here we use nonagricultural employee hours (in billions), let $L_t^* = L_t - \bar{L} - \alpha_2 t$, and estimate the relations

$$L_t - \bar{L} = \frac{(.148)t}{(6.688)} + u_{2t}$$

$$R^2 = .238$$

$$\begin{aligned} L_t^* &= \hat{\rho}_2 L_{t-1}^* + \epsilon_{2t} \\ &= \frac{(.992)}{(205.65)} L_{t-1}^* + \epsilon_{2t} \end{aligned}$$

$$R^2 = .996.$$

Then $\hat{\rho}_2 = .992$ is an estimated first order autocorrelation coefficient for hours.

With these "observed" values we can now discuss the two interpretations of the first order autocorrelation coefficients for hours and output in the model, which were .547 and .73 respectively. The first interpretation is as follows. Nothing prevents us from regarding the model as if each "period"

in the model were a quarter. Then the degree of "persistence" in the model is low relative to observed values. However, we could go further and contrast the persistence generated by this model with far more sophisticated dynamic competitive models employing an infinitely lived agent paradigm. Hence in these models the "quarterly interpretation" should be more reasonable. If we contrast our first order autocorrelation coefficient for output with that of the model produced by Kydland and Prescott (1982), then, they are virtually identical: .73 for this model versus .71 for the Kydland-Prescott model. (Kydland and Prescott do not report autocorrelations for hours.) Hence taken as a quarterly model the framework of this paper does as well in generating persistence as do more sophisticated dynamic competitive models.

Of course, there are a number of reasons to be unhappy with an interpretation of our model as a quarterly one. In addition to the obvious fact that individuals are two period lived in the model, there would be the additional troublesome assumption that capital depreciates completely in one period, as well as the discount rate of $\beta = .579$. Therefore, it is reasonable to suggest an alternate interpretation.

Suppose, then, that we take the overlapping generations structure of the model seriously, i.e., take a "period" to be twenty years. Then setting $\beta = .579$ corresponds to an annual discount rate of .973, and it is far more reasonable to assume that capital depreciates completely in one period. Thus, the troublesome assumptions are no longer troublesome, and we need only ask how to interpret the variances and autocorrelations produced above.

With respect to variance measures, it clearly does not matter how we interpret a period, since variance measures do not depend on the frequency of sampling. Obviously, though, autocorrelations do depend on this. Suppose we

perform the following thought experiment, then. Let τ be the number of quarters in one of our periods. If deviations of output or hours from trend are generated by the estimated first order Markov processes reported above, then the τ -th order autocorrelation coefficient for output (hours) would be $\hat{\rho}_1^\tau (\hat{\rho}_2^\tau)$. Then set

$$\hat{\rho}_1^{\tau_1} = (.995)^{\tau_1} = .73$$

$$\hat{\rho}_2^{\tau_2} = (.992)^{\tau_2} = .547,$$

where .73 and .547 are the first order autocorrelation coefficients for our model for output and hours respectively. Then τ_1 and τ_2 correspond to the implied number of quarters in one of our periods. Solving the above for τ_1 and τ_2 we obtain $\tau_1 = 62.8$ and $\tau_2 = 75.11$. Using τ_1 , we would infer a period length of 62.8 quarters (15.7 years), and using τ_2 we would infer a period length of 75.11 quarters (18.8 years). Thus the autocorrelation coefficients generated by the model are consistent with the estimated processes for hours and output, and with the overlapping generations structure employed.

To summarize, then, if we take the overlapping generations nature of the model seriously, the serial correlation coefficients generated by our model are consistent with observed (high) levels of persistence in output and hours.

Existence of Equilibrium

Recall that an equilibrium exists if the contract just derived is preferred by type 1 agents to any pooling contract that at least breaks even for any firm offering it. Recall also that we denoted the optimal (zero profit) level of hours in a pooling contract (for type 1 agents) by $L^*(s,k)$.

For the parameters of our example it is straightforward to verify that $L^*(s,k) = 0$. Since it is incentive compatible to set $L_1(s,k) = 0$ and this is not done, clearly type 1 agents prefer the contract discussed above to any pooling contract. Hence this equilibrium does, in fact, exist.

IV. A Linear Model

The analysis of the previous sections is obviously special in a number of ways. In this section, we present a class of economies in which all agents have linear preferences. The purpose of this presentation is to demonstrate that a class of models exists where all economies in the class display the major qualitative features of the economy in Section III. In addition, the analysis above used a number of assumptions that one might wish to relax. Most obvious among these were that (i) savings is unobservable, (ii) wage-employment lotteries were ruled out, (iii) insurance markets were ruled out by an assumption about when random events occurred, and (iv) agents of different types had different functional forms for preferences, and in particular, for indirect utility functions over income and labor. In this section all these assumptions are relaxed. In particular, savings is observable (as is borrowing and lending behavior), firms do offer contracts specifying wage-employment lotteries, insurance markets are permitted, and all workers have linear preferences.

A. The Model

The model is essentially as previously. All workers are endowed with one unit of time when young and have no endowment of the good. As before $w_i(s,k)$ is the wage rate paid to a type i agent and $L_i(s,k)$ is the number of hours worked by a type i agent. $(1-L_i(s,k))$ is therefore leisure consump-

tion.) Type 2 workers, who as before will be low productivity workers in equilibrium, have preferences

$$u_2(c_1, c_2, 1-L) = c_1$$

and type 1 workers have preferences

$$u_1(c_1, c_2, 1-L) = c_1 + \beta c_2.$$

Thus workers do not value leisure, and all workers supply their unit of time inelastically.

Technology is identical here to the technology of the previous section: type 1 workers produce $\pi_1(s)L$ units if they work L hours, and a type 2 worker working L hours, combined with k units of capital, produces $\pi_2(s)k^\theta L^{1-\theta}$ units of the single good. k continues to denote the capital stock per type 2 worker, and as before the productivity shock constitutes a two state Markov chain with transition matrix

$$\begin{array}{cc} & \begin{array}{cc} s = 1 & s = 2 \end{array} \\ \begin{array}{c} s' = 1 \\ s' = 2 \end{array} & \begin{array}{cc} p(1) & p(2) \\ 1 - p(1) & 1 - p(2) \end{array} \end{array}$$

As before, we let $r(s,k)$ denote the rental rate on capital, and capital depreciates completely in one period of use. Let $\gamma(s,k)$ denote the fraction of income saved by a young type 1 agent. $\gamma(s,k)$ is chosen to maximize

$$w_1(s,k)L_1(s,k)[1-\gamma(s,k)] + \beta E_s r(s,k) w_1(s,k)L_1(s,k)\gamma(s,k),$$

where as before E_s denotes the conditional expectation taken over future states of nature. Hence the values $r(s,k)$ must satisfy (2) as before:

$$\beta\{p(s)r(1,k')+[1-p(s)]r(2,k')\} = 1 \quad \forall s.$$

The structure of information is also largely as before. Each agent knows his own type, which is unknown to others. However, now savings, borrowing and lending, etc. are observable by all.

Contracts

Firm behavior is also essentially as before. Firms behave competitively in a rental market for capital, and are Nash imperfect competitors in labor markets. In particular firms announce contracts, and then given the constant returns to scale nature of technology, accept all applicants for employment. Here, however, since workers do not value leisure, contracts must take a different form than they did above. Since workers supply labor inelastically, we now let a contract specify a wage offer and a probability of being employed. Let $q_i(s,k)$ denote the probability that a type i worker is employed in state (s,k) . Then a contract offered to a type i agent (which is offered after the state has been observed by all) is a pair $[w_i(s,k), q_i(s,k)]$. If $[w_1(s,k), q_1(s,k)] \neq [w_2(s,k), q_2(s,k)]$ contracts must be structured so as to induce self-selection. If a pooling contract is offered, we use the notational convention $[w_1(s,k), q_1(s,k)] = [w_2(s,k), q_2(s,k)]$. Finally, we continue to require that any offered contract (at least) break even given the workers accepting it. Thus if $[w_1(s,k), q_1(s,k)] \neq [w_2(s,k), q_2(s,k)]$ we require

$$(20.a) \quad w_1(s,k) \leq \pi_1(s)$$

$$(20.b) \quad w_2(s,k) \leq \pi_2(s)\psi^\theta - r(s,k)\psi$$

where we have used $L_2 = 1$ if a type 2 worker is employed, and where ψ is the firm's choice of capital input.

It remains to discuss what is required to induce self-selection (if a firm wishes to do this). Thus if contracts do induce sorting, a type 2 agent receives the wage rate $w_2(s,k)$ and works one unit if employed, and receives nothing otherwise. Moreover, the probability of being employed is $q_2(s,k)$. Hence, this contract yields expected utility $q_2(s,k)w_2(s,k)$. On the other hand, if a type 2 agent takes a type 1 contract, he is employed with probability $q_1(s,k)$, in which case he receives the wage rate $w_1(s,k)$. Also, since savings behavior is observable, in this case the type 2 worker must save fraction $\gamma(s,k)$ of his income. Recall that type 2 agents do not value old age consumption. Hence self-selection occurs iff

$$(21) \quad q_2(s,k)w_2(s,k) > q_1(s,k)[1-\gamma(s,k)]w_1(s,k).$$

Similarly, sorting requires that

$$(22) \quad q_1(s,k)w_1(s,k) > q_2(s,k)w_2(s,k).$$

(21) and (22) are the self-selection conditions for this version of the model.

It remains to say something about the form of a pooling contract. Since such a contract does not attempt to induce sorting, competition among firms for workers would imply that such a contract would have to set $q(s,k) \equiv 1$. Also, all workers would work one unit under such a contract. Finally, under a pooling contract workers are not distinguishable by type. Thus, if a firm rents ψ units of capital, this is allocated among all employees. Since type 1 workers cannot productively use capital, this implies that fraction

$(1-\mu)$ of the rented capital is effectively wasted. Then a firm offering a pooling contract and renting ψ units of capital (at least) breaks even if its offered wage rate $w(s,k)$ obeys

$$(20.c) \quad \mu\pi_1(s) + (1-\mu)\pi_2(s)[(1-\mu)\psi]^\theta - r(s,k)\psi \geq w(s,k).$$

B. A Separating Equilibrium

Since workers do not value leisure here, standard arguments which rule out an equilibrium with pooling are not applicable. However an efficient allocation of capital continues to require sorting, so we continue to focus on equilibria in which self-selection occurs. Let us now define an equilibrium, and then examine the properties of an equilibrium which induces self-selection.

Definition. A stationary (separating) Nash equilibrium is a set of announced contracts $[w_i(s,k), q_i(s,k)]$; $i = 1, 2$, which satisfy (20) and (21)-(22), and a set of values $r(s,k)$, $k(s)$, and $\psi(s,r)$ such that

(i) $r(s,k)$ and $k(s)$ satisfy (2)

(ii) the rental market clears, i.e.,

$$\psi(s,r) = k(\hat{s}).$$

(iii) given the announced contracts of all other firms, no firm has an incentive to offer an alternate contract, with all offers subject to (20).

Having defined an equilibrium, then, we begin by characterizing the set of contracts consistent with self-selection that can be equilibrium contracts. We then discuss when such an equilibrium will exist.

To begin, then, for the same reason as before any contract $[w_2(s,k), q_2(s,k)]$ must be maximal for type 2 workers among the set of all contracts satisfying (20.b) and (21)-(22). Then clearly $q_2(s,k) = 1 \forall (s,k)$, (20.b) holds with equality, and ψ is chosen to maximize profits. Hence

$$r(s,k) = \theta \pi_2(s) \psi^{\theta-1}$$

Using $k(\hat{s}) = \psi(s,r)$, we have then that

$$(23) \quad r(s,k) = \theta \pi_2(s) k^{\theta-1}.$$

Therefore, from (20.b),

$$(24) \quad w_2(s,k) = (1-\theta) \pi_2(s) k^\theta$$

as before.

Now consider the contracts offered to type 1 agents. If self-selection constraints did not bind then obviously we would have $q_1(s,k) = 1$ and $w_1(s,k) = \pi_1(s)$. Moreover, type 1 workers would save fraction $\gamma(s,k)$ of their income. Thus aggregate savings would equal $\mu \gamma(s,k) \pi_1(s)$, and the aggregate capital stock is, of course, $(1-\mu)k$ (since k is measured as capital per type 2 agent). Since these must be equal we have, then, that $\gamma(s,k)$ is given by

$$(25) \quad \gamma(s,k) = \left(\frac{1-\mu}{\mu} \right) \frac{k(s)}{\pi_1(s)}$$

Moreover, using (23) and (2), $k(s)$ must satisfy

$$(26) \quad \theta \beta \{ p(s) \pi_2(1) k(s)^{\theta-1} + [1-p(s)] \pi_2(2) k(s)^{\theta-1} \} = 1$$

* $s = 1, 2$. Then (25) and (26) give $\gamma(s,k)$, and henceforth $k(s)$ will be understood to be the value of k satisfying (26).

Having derived $\gamma(s,k)$, it is now possible to say when self-selection conditions will bind on the choice of $q_1(s,k)$. This will occur if the contract above is not incentive compatible, i.e., if

$$(27) \quad \left[1 - \frac{(1-\mu)k(s)}{\mu \pi_1(s)}\right] \pi_1(s) > (1-\theta) \pi_2(s) k(\tilde{s})^\theta.$$

This condition is assumed to hold for the remainder of the discussion.

If (27) holds, then, a contract inducing self-selection sets $w_1(s,k) = \pi_1(s)$, and chooses $q_1(s,k)$ so that (21) holds with equality, i.e.,

$$(28) \quad q_1(s,k) = \frac{w_2(s,k)}{w_1(s,k)} [1 - \gamma(s,k)]^{-1},$$

where we must now re-derive $\gamma(s,k)$. As before, each employed type 1 worker saves fraction $\gamma(s,k)$ of his income. However, as the probability of employment is independent across workers, and as the number of these workers is large, only a fraction $q_1(s,k)$ of type 1 workers are employed. Hence total savings is given by $\mu q_1(s,k) \gamma(s,k) \pi_1(s)$. This must equal $(1-\mu)k(s)$, where $k(s)$ is as above. Thus now

$$(29) \quad \gamma(s,k) = \left(\frac{1-\mu}{\mu}\right) \frac{k(s)}{q_1(s,k) \pi_1(s)}.$$

Using this expression in (28) gives the following expression for $q_1(s,k)$:

$$q_1(s,k) = \left[\frac{w_2(s,k)}{w_1(s,k)}\right] \left[\frac{\mu q_1(s,k) \pi_1(s)}{\mu q_1(s,k) \pi_1(s) - (1-\mu)k(s)}\right]$$

Rearranging terms, then, we have

$$(30) \quad q_1(s,k) = \frac{w_2(s,k)}{w_1(s,k)} + \left(\frac{1-\mu}{\mu}\right) \frac{k(s)}{\pi_1(s)},$$

where we have used $w_1(s,k) = \pi_1(s)$, and where $k(s)$ is given by (26). Thus, as before, relative wages dominate hours behavior, and it should be clear that if a separating equilibrium exists here it will retain the qualitative features of the equilibrium examined in Section III above.

It should also be clear that if all firms offer the contracts just described, no firm can profitably offer an alternate set of contracts consistent with (20) and self-selection. Hence the above will be an equilibrium if no firm can offer a pooling contract and choose a value ψ such that $q(s,k) = 1$, $w(s,k)$ satisfies (20.c) at equality, and such that $w(s,k) > q_1(s,k)w_1(s,k)$. We now examine what this entails.

Begin, then, by considering the selection of ψ which maximizes $w(s,k)$, taking $r(s,k)$ as parametric (at its candidate equilibrium value of $r(s,k) = \theta\pi_2(s)k(\hat{s})^{\theta-1}$). Since $w(s,k)$ is given by (20.c) at equality, the maximizing choice of ψ is

$$\psi = \left[\frac{\theta(1-\mu)^{1+\theta} \pi_2(s)}{r(s,k)} \right]^{\frac{1}{1-\theta}}.$$

This implies the following value for $w(s,k)$:

$$w(s,k) = \mu\pi_1(s) + (1-\mu)^{1+\theta} \pi_2(s) \left[\frac{\theta(1-\mu)^{1+\theta} \pi_2(s)}{r(s,k)} \right]^{\frac{\theta}{1-\theta}} \\ - r(s,k) \left[\frac{\theta(1-\mu)^{1+\theta} \pi_2(s)}{r(s,k)} \right]^{\frac{1}{1-\theta}}.$$

Rearranging terms and using $r(s,k) = \theta\pi_2(s)k(\hat{s})^{\theta-1}$, we obtain

$$w(s,k) = \mu\pi_1(s) + (1-\theta)(1-\mu)^{\frac{1+\theta}{1-\theta}} \pi_2(s)k(\hat{s})^{\theta}.$$

Thus a separating equilibrium exists if $q_1(s,k)w_1(s,k) > w(s,k)$, or if

$$(31) \quad w_2(s, k) + \left(\frac{1-\mu}{\mu}\right)k(s) > \mu\pi_1(s) + (1-\theta)(1-\mu) \left(\frac{1+\theta}{1-\theta}\right) \pi_2(s)k(\hat{s})^\theta$$

* s, \hat{s} . If (31) is satisfied, then, (as it obviously will be for a nontrivial set of parameters) the sorting contracts (along with the values $k(s)$) derived above do constitute an equilibrium. Moreover, this equilibrium will display qualitatively the same features as the equilibrium of the economy examined in Section III.

V. Conclusions

The preceding sections have produced a private information model that explains the existence of unemployed labor, the high variability of employment relative to real wages/productivity, and the persistence of disturbances to output and employment. Moreover, it has done so in a way that is consistent with existing micro evidence on individual "labor supply" behavior, and with a number of other important cyclical features. Finally, this has been done using a model where a number of parameter choices have been heavily guided by empirical observation. Even so the model "fits the facts well" with fewer free parameters than the sum of (i) observations used to calibrate and (ii) facts to be fit.

Having summarized the capabilities of this modelling approach, then, we will conclude by anticipating an objection to the approach employed here and responding to it. In particular, it has been common to object to business cycle models which ignore choices of hours at the "extensive margin" [Heckman (1984)]. In particular, it is frequently objected that much employment variation is due to variation in the number of individuals employed as opposed to employment per individual. In the model of Sections I-III, employment variation was due entirely to variation in hours per person. Does this invalidate the analysis?

The answer is no. In the model of Section IV all variation in employment is due to variation in the number (fraction) of employed individuals. Nevertheless, the derivation of equilibrium contracts proceeded far enough in that setting to indicate that the qualitative behavior of the model would be the same regardless of how the variation in employment occurred (i.e., whether it occurred at an "intensive" or an "extensive" margin). Hence this objection does not invalidate the approach taken here. In fact, it suggests that this approach may be a useful one in reconciling observed aggregate behavior with existing evidence on the employment behavior of individuals.

Footnotes

1/ See, e.g., Kydland and Prescott (1982).

2/ A complete list of contributions in this area would be extremely long. A partial survey of this literature appears in Hart (1983).

3/ Some exceptions are Holmstrom and Weiss (1985) and Smith (1984a,b).

4/ On the first point see Hodrick and Prescott (1981) or Geary and Kennan (1984). On the second point see Geary and Kennan. This is also apparently the case for the pre-war U.S. economy. On this point, see Bernanke (1984) or Bernanke and Powell (1984).

5/ For aggregate evidence see Altonji (1983) or Altug (1984). The micro evidence on this point is surveyed by Ashenfelter (1984).

6/ See Prescott (1983).

7/ Prescott (1983).

8/ Reder (1962).

9/ Lilien (1982) or Samson (1985).

10/ See Dunlop (), Keynes (1936), and Solow (1980) for a statement of the importance of this in macroeconomics.

11/ See, e.g., Rothschild and Stiglitz (1976).

12/ In particular, if each worker's contribution to output is not directly observable, this means that any individual has negligible incremental effect on a firm's output. Thus no worker's contribution to output can be inferred.

13/ E.g., Kydland and Prescott (1982) or Hansen (1984).

14/ This is reasonable if we do interpret a period as a generation here.

15/See Rothschild and Stiglitz (1976) or Wilson (1977).

16/The role of wage-employment lotteries in settings of this type is discussed by Prescott and Townsend (1984a,b).

17/This is the appropriate maximand, as (2) holds in equilibrium.

18/This is the solution to the relevant maximization problem because of assumption (1).

19/Again there are two solutions. (1) implies that this is the solution preferred by type 1 agents.

20/Council of Economic Advisers (1982), p. 276.

21/David and Scadding (1973).

22/Prescott (1983).

23/See, e.g., Dunlop (); Keynes (1936), and Solow (1980).

24/Lilien (1982) and Samson (1985).

25/Hodrick and Prescott (1981).

26/Hodrick and Prescott (1981).

27/Hodrick and Prescott (1981).

28/Recall that we are taking the overlapping generations structure of the model seriously and thinking of a period as roughly twenty years. Then setting $\beta = .579$ corresponds to an annual discount rate of .973.

29/Computation of serial correlation values below requires that these values be computed to more than three decimal places. We report to only three decimal places here for brevity.

30/This is not entirely consistent with our model, since if we write output for the model as a function of lagged output the derived relation will be nonlinear. For the type of "back of the envelope" calculations being performed here we ignore this.

References

- Altonji, Joseph G., "The Intertemporal Substitution Model of Labour Market Fluctuations: an Empirical Analysis," Review of Economic Studies, 49, 1982, p. 783-824.
- Altug, Sumru, "Gestation Lags and the Business Cycle: an Empirical Analysis," unpublished, 1984.
- Ashenfelter, Orley, "Macroeconomic and Microeconomic Analyses of Labor Supply," Carnegie-Rochester Conference Series on Public Policy, 21, 1984, p. 117-156.
- Bernanke, Ben S., "Employment, Hours, and Earnings in the Depression: An Analysis of Eight Manufacturing Industries," unpublished, 1984.
- Bernanke, Ben S., and James L. Powell, "The Cyclical Behavior of Industrial Labor Markets: a Comparison of the Pre-War and Post-War Eras," unpublished, 1984.
- Council of Economic Advisers, Economic Report of the President, U.S. Government Printing Office, Washington, 1982.
- David, Paul A., and John L. Scadding, "Private Savings: Ultrarationality, Aggregation, and 'Denison's Law,'" Journal of Political Economy, 82, 1974, p. 225-249.
- Dunlop, John, Wage Determination Under Trade Unions.
- Fama, Eugene F., "Short-Term Interest Rates as Predictors of Inflation," American Economic Review, 65, 1975, p. 269-282.
- Gear, Patrick T., and John Kennan, "Intertemporal Substitution and the Phillips Curve: International Evidence," unpublished, 1984.
- Hansen, Gary, "Indivisible Labor and the Business Cycle," unpublished, 1984.

- Hart, Oliver D., "A Model of Imperfect Competition with Keynesian Features," Quarterly Journal of Economics, 97, 1982, p. 109-138.
- Hart, Oliver D., "Optimal Labour Contracts Under Asymmetric Information: an Introduction," Review of Economic Studies, 50, 1983, p. 3-36.
- Heckman, James, "Comments on the Ashenfelter and Kydland Papers," Carnegie-Rochester Conference Series on Public Policy, 21, 1984, p. 209-224.
- Hodrick, Robert J., and Edward C. Prescott, "Post-war U.S. Business Cycles: an Empirical Investigation," unpublished, 1981.
- Holmstrom, Bengt, and Laurence Weiss, "Managerial Incentives, Investment, and Aggregate Implications," Review of Economic Studies, 52, 1985.
- Keynes, John Maynard, The General Theory of Employment, Interest, and Money, Harcourt Brace and World, New York, 1936.
- Kydland, Finn E., and Edward C. Prescott, "Time to Build and Aggregate Fluctuations," Econometrica 50, 1982, p. 1345-1370.
- Lilien, David M., "Sectoral Shifts and Cyclical Unemployment," Journal of Political Economy, 90, 1982, p. 777-793.
- Prescott, Edward C., and Robert M. Townsend, "Pareto Optima and Competitive Equilibria with Adverse Selection and Moral Hazard," Econometrica, 52, 1984, p. 21-45.
- Prescott, Edward C., and Robert M. Townsend, "General Competitive Analysis in an Economy with Private Information," International Economic Review, 25, 1984, p. 1-20.
- Prescott, Edward C., "Can the Cycle be Reconciled with a Consistent Theory of Expectations?" unpublished, 1983.
- Reder, Melvin W., "Wage Differentials: Theory and Measurement," in Aspects of Labor Economics (H. Gregg Lewis, ed.), Princeton University Press, Princeton, 1962.

- Rothschild, Michael, and Joseph Stiglitz, "Equilibrium in Competitive Insurance Markets: an Essay on the Economics of Imperfect Information," Quarterly Journal of Economics, 90, 1976, p. 629-650.
- Samson, Lucie, "A Study of the Impact of Sectoral Shifts on Aggregate Unemployment in Canada," Canadian Journal of Economics, 18, 1985, p. 518-530.
- Smith, Bruce D., "A Business Cycle Model with Private Information," unpublished, 1984a.
- Smith, Bruce D., "A Model of Nominal Contracts," unpublished, 1984b.
- Solow, Robert M., "On Theories of Unemployment," American Economic Review, 70, 1980, p. 1-11.
- Wilson, Charles, "A Model of Insurance Markets with Incomplete Information," Journal of Economic Theory, 16, 1977, p. 167-207.