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Monetary Targeting in a Dynamic Macro Model

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Abstract

The consequences of a straightforward monetary targeting scheme are examined for a simple dynamic macro model. The notion of "targeting" used below is the strategic one introduced by Rogoff (1985). Numerical simulations are used to demonstrate that for the model under consideration, monetary targeting is likely to lead to a deterioration of policy performance. These examples cast doubt upon the general efficacy of simple targeting schemes in dynamic rational expectations models.

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1. Introduction

A topic of considerable policy interest is whether monetary targeting improves upon or detracts from the overall performance of monetary policy. The theoretical debate on this subject has been quite intense, particularly after the Fed's 1979-82 targeting "experiment." While this debate continues to rage on many fronts, it has lately come to center on strategic issues.[1] That is, can monetary targeting improve upon the performance of monetary policy by improving the Fed's credibility?

A recent paper by Rogoff (1985) strongly suggests an affirmative answer to this question. In the context of a static macro model, Rogoff demonstrates that various intermediate targeting schemes may in fact be useful in overcoming the credibility problem inherent in a discretionary policy environment.[2]

The present paper could be viewed as a first attempt to examine the generality of Rogoff's results. In the analysis that follows, the consequences of a simple monetary targeting scheme are traced through for a dynamic macro model. Although analytical results are difficult to obtain, numerical simulations suggest that for the model under consideration, targeting is likely to lead to significant deterioration of policy performance. Nor does targeting lead to improved credibility of monetary policy. Instead, imposition of targeting often leads to erratic short term fluctuations in both the money stock and the price level.

The analysis below should not be construed as a general condemnation of all monetary targeting schemes. Indeed, recent work by Whiteman (1985) suggests that policy performance can always be improved by implementation of *some* targeting mechanism. However, the examples considered below suggest that simple, intuitively appealing targeting mechanisms can easily have the opposite of the desired effect.

2. The Model

The model considered can be derived from Cagan's (1956) demand function for real balances, i.e.

$$(1) \quad \log(M/P)_t = \alpha\pi_t + \xi y_t + \psi + U_t , \quad \alpha < 0 , \xi > 0$$

where M is the demand for nominal balances, P the price level, π_t the expected rate of inflation, y_t the log of real income, ψ a constant term, and U_t a stochastic error term. Following standard practice, the values of all variables will be interpreted as deviations about perfectly forecastible trends. In addition, the analysis below will abstract from all real effects by taking $\xi y_t + \psi$ to be identically zero. The process $\{ U_t \}$ will be assumed to follow the stationary first order autoregressive law

$$(2) \quad U_t = \gamma U_{t-1} + \alpha\epsilon_t , \; 0 < \gamma < 1$$

where ϵ_t is Gaussian white noise. Both private agents and policymakers are assumed to know the values of current and past realizations of U_t .

By imposing the rational expectations hypothesis and some relatively innocuous side conditions, the money demand equation (1) can be solved out as[3]

$$(3)\quad p_t = -\rho^{-1} \sum_{j=0}^{\infty} \rho^{-j} E_t \left(m_{t+j} + u_{t+j} \right)$$

where E_t denotes expectation conditional on current and past U_t's, p_t is the log of the price level, $m_t = \log(M_t)/\alpha$, $u_t = U_t/\alpha$, and $\rho = (\alpha-1)/\alpha$. Equation (3) reveals the dependence of the price level on current and all expected future values of nominal money demand. The problem of setting monetary policy in this model is thus an inherently dynamic one.

Throughout this paper it will be assumed that the Fed can completely control the nominal money stock, which will always be equal to nominal money demand in equilibrium. The objective of the Fed will be taken as to minimize a weighted average of the discounted sum of current and future fluctuations in the logarithms of the money supply and the price level, i.e. at time t the objective of the Fed will be given by

$$J_t \equiv E_t \sum_{j=0}^{\infty} \beta^j \left(1/2\, p_{t+j}^2 + \lambda/2\, m_{t+j}^2 \right) \quad , 1 > \beta > 0 \, , \lambda > 0$$

The objective of the Fed is thus taken as to stabilize the fluctuations of the money supply and price level about their long term trends, taking equation (3) as given.

3. Policy Rules Under Precommitment and Discretion[4]

As in virtually all rational expectations models, deriving "optimal" policy rules for the model described above requires a specification of the degree of precommitment of the policy authority. To begin, consider the case where the Fed can credibly commit itself to an infinite sequence of policies. In this case monetary policy is determined once and for all at time t=0, conditional on a given sequence of shocks $\{ \epsilon_t \}$. This sort of policy environment is sometimes referred to as a precommitment or "open loop" policy environment.

The optimal precommitment monetary policy for this problem can be found using techniques outlined in Hansen, Epple, and Roberds (1985). Setting the Fed's discount factor β equal to one for convenience,[5] the appropriate Lagrangian for the time t=0 policy problem is

$$\mathcal{L} \equiv E_0 \sum_{t=0}^{\infty} \{ \tfrac{1}{2} [p_t^2 + \lambda m_t^2] \ +$$

$$\theta_t [p_t + \rho^{-1} E_t \sum_{j=0}^{\infty} \rho^{-j} (m_{t+j} + u_{t+j})] \}$$

where $\{ \theta_t \}$ is a sequence of random Lagrange multipliers. First order conditions for the precommitment problem are

$$(4) \qquad \lambda m_t + \rho^{-1} \sum_{j=0}^{t} \rho^{-j} \theta_{t-j} = 0$$

(5) $p_t + \theta_t = 0$

Equations (4) and (5) hold for $t \geq 0$. Solving out for m_t, we obtain

(6) $m_0 = (\lambda\rho)^{-1}\, p_0$

(7) $m_t = \rho^{-1}\, m_{t-1} + (\lambda\rho)^{-1}\, p_t \quad , t > 0$

The time inconsistency of the optimal precommitment policy is manifested in the fact that the representation for m_0 in equation (6) differs from the representation for m_t for positive t, given in equation (7). Equation (7) requires that m_t "feed back" on m_{t-1} after the initial period in which policy is set. If optimal policy were to be reset at some time $s > 0$, however, equation (6) would require that m_{s-1} be ignored in setting m_s. Thus, without some mechanism to guarantee that the Fed would always stick to its original plan, the precommitment policy is not a credible one. Nonetheless, it is useful to solve out for the precommitment policy as a benchmark to compare other policies against. In Appendix A, it is shown that the sequence of optimal policies follows the law

(8) $m_t = c_1 m_{t-1} + c_0 u_t , \quad t \geq 0$

where $1 > c_1 > 0$ and $c_0 < 0$, subject to the initial condition $m_{-1} = 0$.

We next consider a policy environment of pure discretion. In a discretionary policy environment, optimal policies are recomputed in every period, so that announcements about time t policy that are made before time t are not credible. Given this sort of policy environment, one could think of policy as being set by a sequence of Fed policymakers. The policymaker at time t has the authority to set time t policy only. Although a policymaker may predict what future policymakers will do, he cannot commit them to any predetermined course of action. Accordingly, the appropriate Lagrangian for the time t policymaker is

$$\mathcal{L}_t^{\star} \equiv E_t \sum_{j=0}^{\infty} \tfrac{1}{2} (p_{t+j}^2 + \lambda m_{t+j}^2) +$$

$$\theta_t [p_t + \rho^{-1} E_t \sum_{j=0}^{\infty} \rho^{-j} (m_{t+j} + u_{t+j})]$$

First order conditions for the time t policymaker are given by

(9) $\lambda m_t + \rho^{-1} \theta_t = 0$

(10) $p_t + \theta_t = 0$

Solving equations (8) and (9) out for m_t in turn yields

(11) $m_t = (\lambda\rho)^{-1} p_t$

The time consistency of monetary policy in this environment is manifested in the fact that the representation for optimal policy given in equation (11) holds for all t. In Appendix A, equation (11) is shown to imply the following feedback rule for policy:

$$(12) \quad m_t = f^{\star} u_t, \quad \text{where } f^{\star} = -[1-\lambda\rho(\gamma-\rho)]^{-1}$$

Since $f^{\star}$ lies between 0 and -1, optimal monetary policy in a discretionary environment consists of accommodating some fraction of the current money demand shock U_t .

As is true for most policy problems in a rational expectations setting, the performance of the discretionary policy rule given in equation (12) will be dominated by the performance of the precommitment policy rule given in equation (8). That is, the value of the Fed's loss function J_t will be greater under discretion than under precommitment.[6] However, as pointed out by Kydland and Prescott (1977), there is no way to recoup this difference in a discretionary policy environment. To improve the performance of policy, some sort of mechanism must be introduced that will augment the credibility of the policy authority. One candidate for such a mechanism is described in the next section.

4. A Simple Targeting Scheme

There are two major reasons for considering the targeting scheme described below. First, this targeting scheme constitutes a reasonable dynamic generalization of one proposed by Rogoff (1985) in the context of a static model, for the express purpose of overcoming a "policy credibility" problem similar to the one described above. Second, the targeting scheme considered below is designed to mimic, within the confines of the idealized model of this paper, several of the important aspects of monetary targeting as practiced under the Humphrey-Hawkins Act.

We should begin by explaining what is meant by "targeting" in a strategic policy environment. By requiring the Fed to "target" some aggregate variable, we mean to alter the Fed's objective function J_t so that the Fed is penalized according to deviations of that aggregate from its preannounced target value. The idea is that by altering the Fed's incentives, it may somehow be compelled to take policy actions that more closely approximate the precommitment policies that maximize its true objective J_t .

The targeting scheme we consider proceeds as follows. For convenience the duration of the time period in the model is taken as six months. At the beginning of each year, i.e. in every even numbered period, the Fed is required to submit a target value of the nominal money stock six months hence, i.e. in the subsequent odd-numbered period. In even numbered periods the Fed is free to set the nominal money stock at its discretion, and its one-period loss function is the same as the one given above:

$$L_e(p_t, m_t) \equiv \tfrac{1}{2}(p_t^2+\lambda m_t^2)$$

During odd periods, the Fed feels some pressure to meet its preannounced money supply target, so that its one-period loss function becomes

$$L_o(p_t, m_t, m_t^\star) \equiv \tfrac{1}{2}[p_t^2+\lambda m_t^2 + \tau(m_t - m_t^\star)^2] , \tau > 0$$

where $m_t^{\star}$ represents the logarithm of the preannounced monetary target, divided by the parameter α.[7] The objective(s) of the Fed are taken as to minimize

$$K_t \equiv E_t \sum_{j=0}^{\infty} \beta^j L_{t+j}$$

where $L_{t+j} = L_o(\cdot)$ for t+j odd, and $L_{t+j} = L_e(\cdot)$ for t+j even, taking into account the private sector's reaction function given by equation (3). In even periods, the Fed maximizes K_t by choice of two policy instruments: the current value of the logged money stock m_t, and the choice of a target for the next period, $m_{t+1}^{\star}$. During odd periods, the Fed can only set one instrument, the current value of the money stock.

Several features of this targeting model deserve discussion. First, it should be emphasized that under the targeting model, the Fed is still operating in a policy environment of pure discretion, although its objective function is changed. The monetary targets $m_t^{\star}$ cannot be interpreted as either binding promises or optimal predictions. Except in the limiting case where $\tau = \infty$ there is no constraint that targets be met exactly. Nor is there an explicit requirement that $m_{t+1}^{\star}$ represent an optimal time t prediction of m_{t+1}, i.e. that $E_t\, m_{t+1} = m_{t+1}^{\star}$. Some pressure to target accurately does exist, however, because the Fed wishes to diminish the penalty term $\tau/2\ (m_{t+1} - m_{t+1}^{\star})^2$ associated with deviations from the targeted money stock. The exact nature of this penalty is left to the reader's imagination, while the parameter τ is assumed to be exogenously determined by the institutional setting under which monetary policy is set.

Another important feature of the targeting model is that deviations from target are only subject to penalty at midyear, i.e. in the odd numbered periods. As will be seen in the next section, this feature is important methodologically, since it allows the model to be solved using a simple recursive algorithm. More importantly, this feature is meant to reflect the "conical" shape of the target bands that are actually announced for monetary aggregates. The intuition is that deviations from target at midyear are penalized more heavily than deviations at yearend. This notion is captured in an abstract setting by imposing positive costs to such deviations at midyear, while assigning zero costs to yearend deviations. In the terminology of Broaddus and Goodfriend (1984), yearend "rebasing" of the money stock carries no explicit penalty.

5. Equilibrium with Targeting

A computationally convenient way of deriving equilibrium policies in a discretionary policy environment is to use the notion of "feedback" or recursive equilibrium of dominant player dynamic games, as defined in Kydland (1977). Before using this equilibrium concept to solve out for equilibrium policies under targeting, it is perhaps instructive to reconsider the problem of setting discretionary policy without targeting, i.e. when $\tau = 0$.

We begin by noting that for the model without targeting, there is only one dynamic state variable, i.e. the money demand shock u_t . We wish to consider feedback policies for the Fed, of the form $m_t = f(u_t)$.

Given the linear-quadratic-Gaussian setup of the model, we can restrict our attention to linear feedback rules of the form $m_t = fu_t$. Let f_0 be our initial guess as to the value of the optimal feedback parameter $f^\star$. If, at time period t, private agents believe that policy in all future periods will be set according to the rule $m_t = f_0 u_t$, then equation (3) may be evaluated as

$$(13) \quad p_t = [\ (1 + \rho^{-1}\gamma f_0) \ / \ (\gamma-\rho)\ ]\ u_t - \rho^{-1} m_t$$

Now define the Fed's value function $V(u_t)$ as the value of the Fed's objective J_t when the optimal feedback parameter $f^\star$ is used in the current and all future periods. In equilibrium, the optimal feedback parameter $f^\star$ must satisfy, for any value of u_t, the requirement that $\underline{m}_t = f^\star u_t$, where $\underline{m}_t$ solves

$$(14) \qquad \min_{m_t} [\ \tfrac{1}{2}(p_t^{\,2} + \lambda m_t^{\,2}) + \beta E_t V(u_{t+1})\,] \quad \text{s.t. eqs. (2) and (12)}$$

where in equilibrium, $f^\star = f_0$. In Appendix B, it is shown that solving program (14) and imposing the condition that $f^\star = f_0$ yields a feedback rule $f^\star$ identical to that shown in equation (12).

For more complex models, it is often difficult to solve out for equilibrium feedback rules directly. However, the recursive character of feedback equilibrium suggests a natural algorithm for numerical computation of feedback rules. That is, given an initial guess f_0 for $f^\star$, find the feedback rule f_1 that solves program (14), take f_1 as the next guess for $f^\star$, and so on. The recursive nature of feedback equilibrium also guarantees that equilibrium policies will be time consistent: in solving the program (14) at time t, note that the Fed is constrained to take all future policies as given.

We now consider the problem of setting discretionary policy under the targeting scheme described in the previous section. Under targeting, it will be important to distinguish between yearend (even) and midyear (odd) periods. In even periods, as in the model without targeting, there is only one state variable that influences the Fed's one-period loss function, i.e. the shock u_t . In odd periods, however, the previously announced logged money stock target $m_t^\star$ must be added to the list of state variables. Two decision variables, the current logged money stock m_t and the midyear target $m_{t+1}^\star$, must be set in even periods, while only the current money stock is set at midyear. Consequently we consider policies of the form

$$\left.\begin{aligned} &(15)\quad m_t = f_0\, u_t \\ &(16)\quad m_{t+1}^\star = f_1\, u_t \end{aligned}\right\}\qquad \text{for even } t$$

$$(17)\quad m_t = g_0\, u_t + g_1\, m_t^\star \qquad \text{for odd } t$$

In Appendix C, it is shown that when equations (15), (16), and (17) hold, equation (3) may be evaluated as

$$(18)\quad p_t = d_0 u_t + d_1 m_t + d_2 m_{t+1}^\star, \quad \text{for } t \text{ even}$$

(19) $p_t = b_0 u_t + b_1 m_t$, for t odd

where the b's and d's are complicated functions of f_0, f_1, g_0, g_1, λ, ρ, and γ. Under targeting, equilibrium feedback rules are determined by a four-tuple $(f_0^\star, f_1^\star, g_0^\star, g_1^\star)$ such that when t is even,

(20) $\underline{m}_t = f_0^\star u_t$ and $\underline{m}_{t+1}^\star = f_1^\star u_t$

and when t is odd

(21) $\underline{m}_t = g_0^\star u_t + g_1^\star \underline{m}_t^\star$

where the $\underline{m}_t$'s in turn solve the program

(22) $$\min_{m_t, m_{t+1}^\star} \left\{ L_e(p_t, m_t) + \beta E_t \left[\min_{m_{t+1}} L_o(p_{t+1}, m_{t+1}, m_{t+1}^\star) \right] + \beta^2 E_t W(u_{t+2}) \right\}, \quad t \text{ even}$$

subject to constraints (2), (18), and (19), where $W(u_t)$ represents the value of K_t for t even when optimal policies are in effect, and the b's and d's in (18) and (19) are evaluated at $(f_0^\star, f_1^\star, g_0^\star, g_1^\star)$. The equilibrium feedback parameters can be numerically determined, given values for ρ, γ, and λ , by the iterative procedure outlined in Appendix C. The idea of this procedure is to take an initial guess (f_0, f_1, g_0, g_1)

for the feedback parameters, use these values to obtain equations (18) and (19), and then a solution to program (22). The feedback rules implied by this solutions are in turn used to generate updated versions of equations (18) and (19), and so on, until an approximate fix point is reached.

6. Numerical Simulations

Because of the somewhat complicated nature of the program (22), analytical the targeting equilibrium are somewhat difficult to obtain. For this reason, numerical simulations were performed to obtain some idea of the performance of policy under targeting. The results of three representative sets of simulations are reported in Table 1 below.

In each of the simulations, a random number generator was used to create artificial time series of length T = 1000 for the money demand shock process { u(t) }. Arbitrary values were assumed for γ, ρ, and γ, and the model was simulated under precommitment, discretionary, and various targeting environments. The discount factor β was taken as equal to one, and the policy objective was reinterpreted as an average cost objective. As an approximation to $2T^{-1}J_t$, the statistic S was computed for every simulation, where

$$S \equiv \text{svar}(p) + \lambda \text{svar}(m)$$

and "svar" means sample variance. For each simulation, the performance index $P \equiv 100(S/S_d)$ was calculated, where S_d represents the value of S for the same parameter values, given a discretionary policy environment without targeting, i.e. where $\tau = 0$. P thus gives the sample performance of policy in a given environment, as a percentage of the performance of the best consistent policy without targeting . A value of P under 100 indicates improvement.

Before describing the results of the simulations, it may be useful to describe the effects of variations on the parameters λ, ρ, and γ on the potential gains in policy performance due to precommitment. First, setting $\lambda = 0$ allows the Fed to costlessly offset money demand surprises (if targeting is not in effect) , so that the global minimum of $J_t = 0$ can be attained in a discretionary policy environment without targeting. Accordingly, one would expect the gains from precommitment to be small when λ is close to zero. A similar conclusion holds when γ is close to zero. This is because in the limiting case that $\gamma = 0$, the dynamic policy game inherent in the model reduces to a sequence of repeated static games, which by definition are immune to *dynamic* consistency problems.[8] Finally, equation (3) reveals that when ρ becomes large, p_t is driven to zero. In the limiting case that $\rho = \infty$, the problem of stabilizing p_t becomes trivial. Hence the effects of precommitment are likely to be reduced when ρ is relatively large.

For the first set of simulations, the parameter values $\lambda = 1.0$, $\rho = 1.5$, and $\gamma = .95$ were assumed. The performance index P for the ideal precommitment environment indicates that the potential gains to precommitment for this example are significant: perfect credibility entails about a 23% decrease in the policy loss function. However, attempts to increase policy credibility via targeting were not successful. For the positive values of τ that were tried, the implementation of targeting resulted in a deterioration of policy performance, i.e. values of P over 100 percent. This deterioration is apparently increasing in the "strictness" τ of the targeting mechanism.

In the second set of simulations, the parameter values $\lambda = 10.0$, $\rho = 1.1$, and $\gamma = .95$ were assumed. As might be inferred from the discussion above, increasing the value of λ and decreasing the value of ρ, relative to the first set of parameter values, results in an even greater potential gain in policy performance from precommitment. The value of the performance index P under precommitment is 37.87 for this example, implying a 62% decrease in the policy loss function under full credibility. Again, attempts to recoup this gain under targeting only resulted in deterioration of policy performance, with the degree of deterioration increasing in τ.

For some of the numerical examples considered, implementation of targeting did lead to gains in policy performance. Typical of these examples is the third set of simulations given in Table 1, for which the parameter values $\lambda = .1$, $\rho = 2.0$, and $\gamma = .5$ were assumed. As seen from the performance index column for the table, taking $\tau = .05$ in this example results in a decrease of about .5% in the policy objective function. Larger values of τ again lead to deterioration of policy performance. However the value of P for the precommitment case (94.08)

Table 1

Parameter Values			Policy Environment	Performance Index
λ	ρ	γ		P (%)
1.0	1.5	.95	Precommitment	76.57
			Discretion with:	
			$\tau = 0$	100.00
			$\tau = .1$	100.03
			$\tau = 1.0$	102.91
			$\tau = 10.0$	114.51
10.0	1.1	.95	Precommitment	37.87
			Discretion with:	
			$\tau = 0$	100.00
			$\tau = 1.0$	102.44
			$\tau = 10.0$	116.22
.1	2.0	.5	Precommitment	94.08
			Discretion with:	
			$\tau = 0$	100.00
			$\tau = .05$	99.46
			$\tau = .1$	102.48

reveals that this example is one for which the magnitude of the dynamic consistency problem is not large. Even in an environment of perfect credibility, only about a 6% gain in policy performance can be attained.

In summary, the numerical simulations reveal that the effect of targeting on policy performance is somewhat ambiguous. For some parameter values, targeting resulted in gains in policy performance, while losses occurred for other values. The magnitude of the gains (under 2% in all the examples tried) tended to be quite small relative to the magnitude of the potential losses (sometimes over 50%). Moreover, the gains were always present in examples for which the dynamic consistency problem was relatively unimportant, i.e. examples for which the values of λ, ρ, or γ were "close" to regions where the dynamic consistency problem does not exist. The larger losses were present in examples where potential gains due to increases in credibility were quite large.

Some intuition concerning the failure of the targeting scheme considered in this paper is offered by Figures 1 and 2. These figures depict the responses of m(t) and p(t) to a -.1 standard deviation shock $\epsilon(t)$, corresponding to a 1 standard deviation shock to money demand, where the parameter values $\lambda = 10$, $\rho = 1.1$, and $\gamma = .95$ are assumed. Responses are plotted for the precommitment case, and for the discretionary case where $\tau = 0$ (no targeting) and $\tau = 10$ (targeting).

Figure 1 shows the response of m(t) (here equal to -.1 times the response of the logged money stock) under the three environments. The optimal precommitment response is seen to require an initial rapid series of increases in m(t), followed by a series of gradual decreases. The discretionary response without targeting consists of an initial rapid increase, followed by a series of gradual decreases of m(t). The effect of the targeting scheme considered is to introduce oscillations into the response of m(t). During the midyear (odd numbered) periods when targeting is in effect, m(t) is biased towards zero, while during even periods m(t) is very close to its values under discretion without targeting. Figure 2 shows that similar, if somewhat less extreme oscillations are introduced into the p(t) process under targeting.

In the context of the model considered in this paper, Whiteman (1986) has shown that discretionary policy (without targeting) will dominate a passive policy of always setting m(t) = 0. Hence it is not surprising that targeting, which seems to bias policy responses towards this passive policy, results in a worsening of policy performance. This bias towards zero is a direct result of the targeting mechanism, which assigns positive costs to active policy responses. Of course, these costs are assigned with the idea that they will be more than offset by a resultant increase in credibility. However, the parameter values in this example cause money demand shocks to have very persistent effects, so that the marginal benefit of a one-period-ahead commitment on the part of the Fed is quite small.

7. Summary and Conclusion

The consequences of a simple monetary targeting mechanism have been considered for a dynamic model of price stabilization under rational expectations. Through the use of numerical examples, the effect of this targeting mechanism on policy performance in this model has been shown to be in general ambiguous, and to be negative for examples in which policy credibility is an important problem.

These highly stylized examples cannot provide a general answer concerning the usefulness of targeting mechanisms in setting governmental policies. However, given that many rational expectations policy problems closely resemble vectorizations of the one considered above, it seems unlikely that arbitrarily applied targeting schemes will always yield improvements in policy performance. Instead, the examples above strongly suggest that more research is needed on the strategic effects of monetary targeting mechanisms.

Appendix A

Derivation of the Optimal Policy Rule (8)

Substituting the portfolio balance schedule (3) into the Fed's first order condition (7) yields

$$\text{(A1)} \quad -\lambda(L-\rho)\, m_t = (L^{-1}-\rho)^{-1}(E_t m_t + E_t u_t)$$

where the operator L is defined as $L(E_t m_t) \equiv m_{t-1}$, and L^{-1} as $L^{-1}(E_t m_t) \equiv E_t m_{t+1}$. Equation (A1) can be solved using the method outlined in Sargent (1979). Operating on (A1) with $(L^{-1}-\rho)$, we obtain the second order expectational difference equation

$$\text{(A2)} \quad [\,-\lambda(L^{-1}-\rho)(L-\rho)-1\,]\,E_t m_t = u_t$$

Applying Sargent's technique then yields the solution for m_t

$$\text{(A3)} \quad m_t = c_1 m_{t-1} + [c_3^{-1}/(1-c_2\gamma)\,]\,u_t$$

where $-\lambda(z^{-1}-\rho)(z-\rho)-1$ can be factored as $c_3(1-c_1 z)(1-c_2 z^{-1})$, $c_3 < 0$ and c_1 , $c_2 \in (0,1)$. Equation (8) of the text follows if we substitute c_0 for $[c_3^{-1}/(1-c_2\gamma)]$ and note that the first order condition for the initial period (6) may be written as (7), subject to the initial condition $m_{-1}=0$.

Derivation of the Consistent Policy Rule (11)

Using the portfolio balance schedule (3) to eliminate p(t) from the first order condition (11) yields

(A4) $[\lambda\rho L^{-1} - (\lambda\rho^2+1)]E_t m_t = u_t$

Defining $d_0 \equiv -(1+\lambda\rho^2)$ and $d_1 \equiv \lambda\rho/(1+\lambda\rho^2)$, equation (A4) can be solved using Sargent's (1979) technique to yield

(A5) $m_t = d_0^{-1} u_t/(1-d_1\gamma)$

The feedback parameter f* may be found by evaluating $d_0^{-1}/(1-d_1\gamma)$ and simplifying.

Appendix B

Alternative Derivation of f Under Feedback Equilibrium*

Begin by writing constraint (13) in abbreviated form as

(B1) $p_t = a_0 u_t + a_1 m_t$

Since the Fed's value function $V(u_{t+1})$ does not depend on m_t, solving program (14) is equivalent to solving the simpler program

(B2) $\min_{m_t} \frac{1}{2}[\lambda m_t^2 + p_t^2]$ subject to (B1)

The first order condition for program (B2) is given by

(B3) $(\lambda + a_1^2) m_t + (a_0 a_1) u_t = 0$

Substituting for the a's in (B3) and solving for m_t yields

$$\text{(B4)} \quad m_t = \frac{\rho^{-1}(1+\rho^{-1}\gamma f_0)}{(\lambda+\rho^{-2})(\gamma-\rho)} u_t$$

Imposing the conditions $m_t = f^\star u_t$, $f^\star = f_0$, and dividing (B4) by u_t yields

$$\text{(B5)} \quad f^\star = -[1-\lambda\rho(\gamma-\rho)]^{-1}$$

Appendix C

Calculation of Feedback Equilibrium Under Targeting

We begin by evaluating the public's portfolio balance schedule (3) when policies are set using the linear decision rules (15), (16), and (17). Using prediction formulas from Hansen and Sargent (1980), it can be shown that equation (3) may be evaluated for even t as

$$\text{(C1)} \quad p_t = a^\star u_t$$

where

$$\text{(C2)} \quad a^\star = \frac{-\rho^{-1}f_0 - \gamma g_0 - \rho g_1 f_1 + \gamma + \rho}{\rho^2 - \gamma^2}$$

When t is odd, equation (3) can be evaluated as

$$\text{(C3)} \quad p_t = b_0 u_t + b_1 m_t$$

where $b_0 = \rho^{-1}(a^\star\gamma-1)$, and $b_1 = -\rho^{-1}$.

Now consider the Fed's optimization problem at some odd time t, i.e. the inner minimization problem of program (22). Because the next (even) period's value function $W(u_{t+1})$ does not depend on the choice of m_t, this minimization problem is equivalent to the simpler program

(C4) $$\min_{m_t} \tfrac{1}{2} [p_t^2 + \lambda m_t^2 + \tau (m_t - m_t^*)^2] \quad \text{s.t. (C3)}, m_t^* \text{ given}$$

Solving program (C4) yields a solution for m_t

(C5) $$m_t = (\tau m_t^* - b_1 b_0 u_t) / (\lambda + \tau + b_1^2)$$

Solution (C5) in turn implies the following values for g_0 and g_1

(C6) $$g_0' = -b_0 b_1 / (\lambda + \tau + b_1^2)$$

(C7) $$g_1' = \tau / (\lambda + \tau + b_1^2)$$

Now consider the Fed's optimization problem at some even time t, i.e. the outer minimization problem of program (22). Since the public knows that in the next period policy will be set according to a rule of the form (17), the Fed should take into account the impact of its target announcement on the public's expectation of m_{t+1}. Substituting (17) into equation (3) and taking expectations then yields

(C8) $$p_t = d_0 u_t + d_1 m_t + d_2 m_{t+1}^*$$

where $d_0 = -\rho^{-1}\{1 + \rho^{-1}[(1 + g_0 - \gamma a^*)\gamma]\}$, $d_1 = -\rho^{-1}$, and $d_2 = -\rho^{-2} g_1$. Also, the Fed should take into account the impact of its target announcement on its time t+1 loss function, via the decision rule (17). Substituting (17) and (C3) into the time t+1 policy loss function yields

$$\text{(C9)} \quad L_o(p_{t+1}, m_{t+1}, m^{\star}_{t+1}) = C(m^{\star}_{t+1}, u_{t+1})$$

where

$$C(m^{\star}_{t+1}, u_{t+1}) \equiv \tfrac{1}{2}(c_1 u_{t+1}^2 + c_2 m^{\star 2}_{t+1} + 2c_3 u_{t+1} m_{t+1})$$

and

$$c_2 = \lambda g_1^{\,2} + (b_1 g_1)^2 + \tau (g_1 - 1)^2$$

$$c_3 = \lambda g_0 g_1 + (b_0 + b_1 g_0)(b_1 g_1) + \tau g_0 (g_1 - 1)$$

Since policy decisions made at time t (even) do not affect the Fed's value function $W(u_{t+2})$ at time t+2, the outer minimization problem of program (22) reduces to the following problem:

$$\text{(C10)} \quad \min_{m_t, m^{\star}_{t+1}} \; \tfrac{1}{2}(\lambda m_t^{\,2} + p_t^{\,2}) + E_t C(m^{\star}_{t+1}, u_{t+1}) \quad \text{s.t. (C9)}$$

Necessary first order conditions for program (C10) are given by

$$\text{(C11)} \quad \begin{bmatrix} (\lambda + d_1^{\,2}) & d_1 d_2 \\ d_1 d_2 & (c_2 + d_2^{\,2}) \end{bmatrix} \begin{bmatrix} m_t \\ m^{\star}_{t+1} \end{bmatrix} = \begin{bmatrix} -d_1 d_0 \\ -(c_3 \gamma + d_2 d_0) \end{bmatrix} u_t$$

which we abbreviate as $Dm = du_t$. Substituting for m_t and $m^{\star}_{t+1}$ using equations (15) and (16) and dividing (C11) by u_t then implies the following values for f_0 and f_1:

$$(C12) \quad \begin{bmatrix} f_0' \\ f_1' \end{bmatrix} = D^{-1} d$$

A feedback equilibrium can be calculated by taking some initial guess for the parameters of the equilibrium feedback laws (20) and (21), then iterating on equations (C6), (C7), and (C12), until convergence is reached. In practice convergence was quite rapid from essentially arbitrary starting values, for each of the examples reported above. The convergence criterion was that maximal differences between successive approximations be no greater than 10^{-7} in absolute value. For some unreported simulations, convergence was not obtained for large values of τ. Similar convergence problems are reported by Kydland and Prescott (1977) for simulations of a policy game in a discretionary environment, suggesting that such problems are not uncommon to this type of model.

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Notes

1. See McCallum (1985) for a survey of the literature on monetary targeting.

2. A similar result is shown in Canzoneri (1985).

3. See Whiteman (1983) and Watson (1985) for a discussion of the solution of equation (1).

4. The policy problems considered in this Section were first proposed and analyzed by Whiteman (1986), using techniques different from the ones employed here.

5. Setting the Fed's discount factor equal to one does not affect the qualitative properties of the models studied below. The government's objective is still well defined if we reinterpret J_t as an "average cost" objective, as in Bertsekas (1976). Average cost objectives are convenient for the numerical simulations reported in Section 6, since they allow estimation of the Fed's objective using sample moments.

6. See Corollary 3.2 of Whiteman (1986).

7. This transformation (division of the logged monetary target by α) is done purely for notational convenience.

8. Any potential credibility problems arising in a static context are

assumed away in the models of this paper, so as to concentrate on dynamic credibility issues. This assumption seems warranted, given that dynamic credibility issues were the main focus of Kydland and Prescott's (1977) original critique.

Fig. 1 - Responses of m_t to a unit shock to U_t

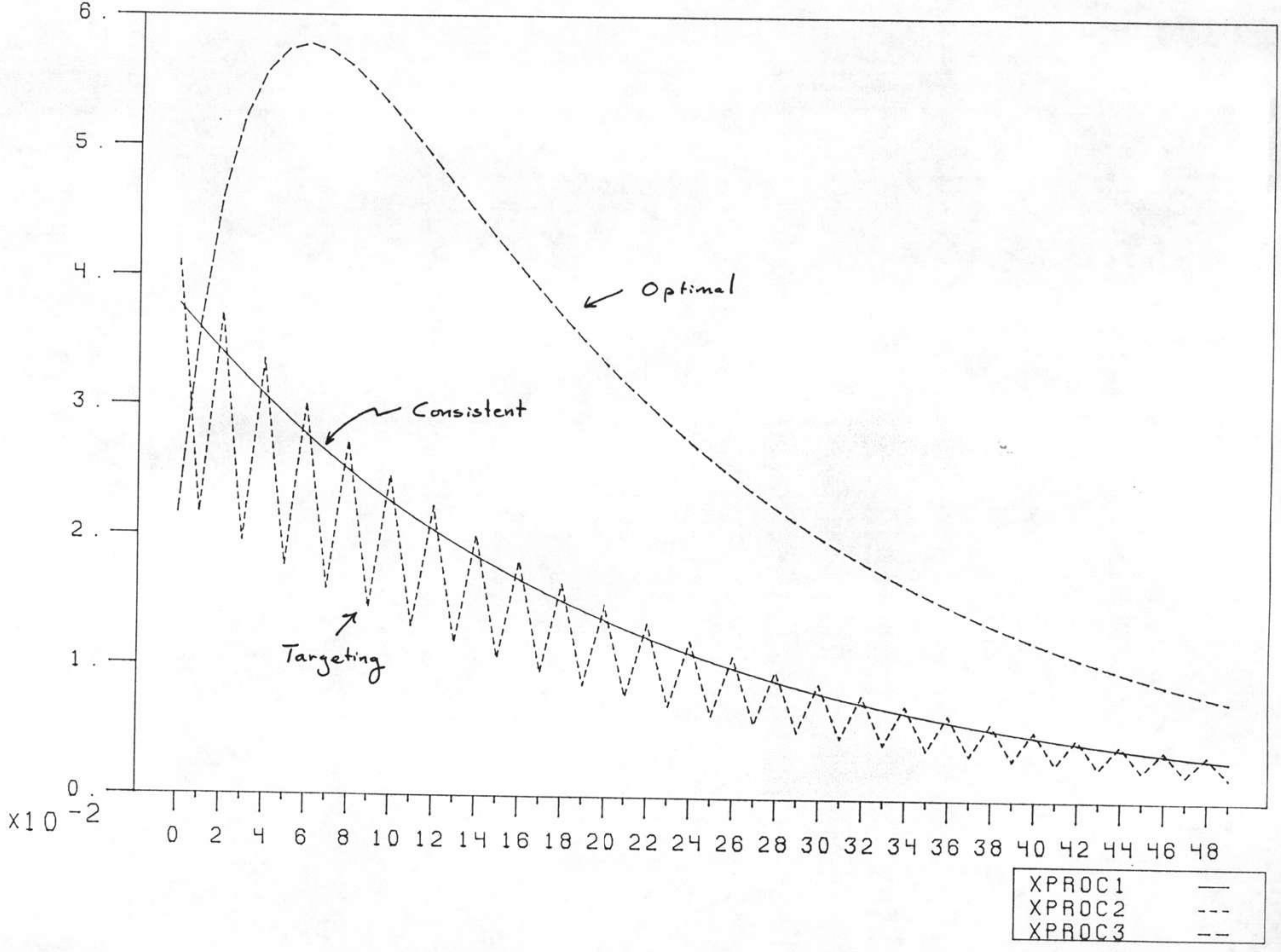

Fig. 2 - Responses of p_t to a unit shock to u_t