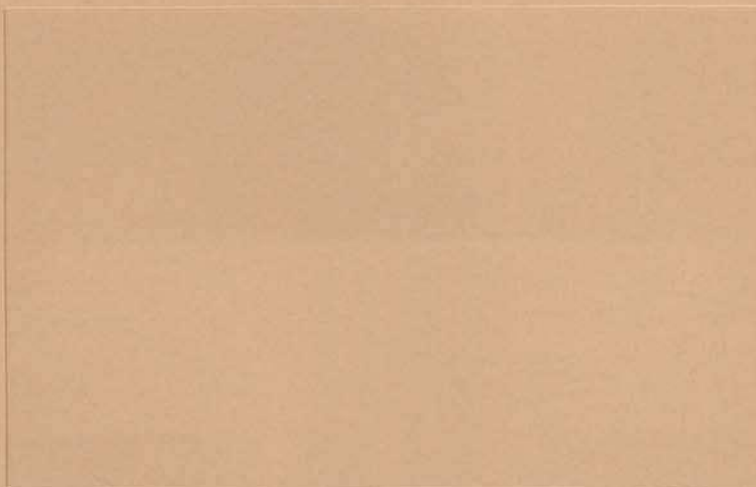*Working Paper*

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INTERNATIONAL COORDINATION OF FISCAL POLICY
IN LIMITING ECONOMIES

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ABSTRACT_____

This paper examines the limiting behavior of cooperative and noncooperative
fiscal policies as countries market power goes to zero. In the first part we
provide sufficient conditions for these policies to converge. In the second
part we provide examples where these policies diverge. Briefly, we show that
if there are unremovable domestic distortions then there can be gains to
coordination between countries even when countries have no ability to affect
world prices. These results are at variance with the received wisdom in the
optimal tariff literature. The key distinction is that we model explicitly
the spending decisions of the government while the optimal tariff literature
does not.

Frenkel and Razin (1985) recently called for "an analysis which would determine the optimal pattern of government spending along the lines of the optimal tariff literature." This paper is a first step in this direction. We consider a world economy composed of a number of countries in which governments choose policy to maximize the utility of consumers in their countries. With multiple policymakers we need first to take a stand on how they interact. We contrast two polar regimes. In one regime, policymakers act in a coordinated fashion, choosing policy cooperatively to maximize world welfare. In the other regime, they choose policies noncooperatively to maximize their own welfare. As has long been recognized, the consequences may be quite different in these two regimes. In particular, the literature on optimal tariffs shows that substantial distortions and reduction in world welfare can result if governments are unable to commit to cooperation. Here distortions arise from the monopoly power of large countries. A standard result is that if countries become small relative to the world economy, these distortions vanish and tariff policies in the two regimes converge.

We ask whether an analogous result holds for fiscal policy--whether cooperative and noncooperative fiscal policies converge as countries become small. Fiscal policies are modeled on choices of spending levels on public goods and their means of finance. Unlike the literature on optimal tariffs, we model explicitly the spending decisions of governments. This difference turns out to be crucial to the results.

In the first part of the paper we consider a model with lump-sum taxes. The choice of public good expenditures affects world relative prices even though these revenues are raised through lump-sum taxes rather than through price-distorting tariffs. As expected, the noncooperative equilibrium yields a lower level of welfare than the cooperative equilibrium. For this

model we show that the analogue of the standard tariff result holds: as countries become small the distortions vanish and policies in the two regimes converge.

We then consider a model with distorting taxes. In this case the tariff result does not go through: the cooperative and noncooperative policies are generally different, even in the limit. An implication of this is that if a tariff is the only source of revenue, the cooperative and noncooperative policies diverge even if the monopoly power of individual countries goes to zero. This suggests that if there are unremovable domestic distortions, there can be gains to cooperation between countries even in markets where they have no monopoly power.[1]

Since this result is at variance with standard results reported in the tariff literature, it is important to understand the intuition behind it. In a cooperative equilibrium consumers equate their marginal rates of substitution to the after tax price. Consequently, the world (or pretax) prices do not provide any country with the correct signal of the marginal rates of substitution of consumers in other countries. Hence, in the limiting noncooperative equilibrium, countries do not have the correct incentives to choose domestic spending appropriately. The resulting equilibrium is, in general, strictly worse than the cooperative equilibrium.

Throughout the paper we restrict ourselves to static models in order to avoid problems associated with the time inconsistency of optimal policy. Rogoff (1985) and Kehoe (1986b) have shown in dynamic settings that cooperative equilibria may be Pareto dominated by noncooperative equilibria. An essential ingredient for this nonoptimality result is that policy in the cooperative equilibrium be time inconsistent. In this paper we attempt instead to isolate and understand factors that cause noncooperative equilibria

to diverge from cooperative equilibria. Our main finding is that such a divergence result can hold in settings with distorting taxes. In particular, we show the divergence result can hold even in a static model. Of course, in dynamic models with distortions it is possible to have both of these results holding at the same time.

The paper is organized as follows: section 1 describes the basic model and establishes some preliminary results. Section 2 proves that in this model the two solutions converge as the economy is replicated. Sections 3 and 4 give examples in which these solutions diverge. In section 3 this happens because the economy is not replicated and in section 4 because of a tax distortion. In section 5 we summarize briefly and also suggest some extensions of the analysis.

1. Monopoly Distortions

Consider a world economy composed of a finite number of countries. Equilibria of this economy are compared under two regimes: in the first, governments set policy cooperatively; in the second, governments play a noncooperative game. There are two main results. First, noncooperative equilibria typically do not coincide with cooperative equilibria. Second, cooperative equilibria are optimal in a sense that noncooperative equilibria are not.

Under both regimes, governments take it as given that, for each setting of policy, private agents are in a competitive equilibrium. In both regimes, governments set policy to maximize the welfare of their residents. What drives the results is that for a fixed number of countries a change in policy by any country changes world prices. In a noncooperative equilibrium, it is optimal for each government to distort its decision away from the (world) social optimum in order to take advantage of its power to affect world prices, that is, its monopoly power. In a cooperative equilibrium, however,

it is not optimal for all governments to distort their decisions away from the social optimum.

The noncooperative and cooperative equilibria can be easily computed. Recall that for each setting of policy, private agents are in competitive equilibrium. We therefore solve first for the competitive equilibrium for an arbitrary setting of government policy. The competitive equilibrium allocations and prices are then used to express each government's objective function solely in terms of its policies and the policies of other governments. Finally, we solve for the governments' policies under the two regimes.

A. Competitive Equilibria for Private Agents

Consider a world economy composed of a finite number of countries I with both private and public goods. Each country is populated by a large number of identical consumers, say L, and a government. For ease of notation, let L equal 1. Consumers in each country have endowments of two private goods. The government of each country has access to a production technology that transforms the first of these private goods into a public good, which benefits only residents of the country. Each government pays for this public good by levying lump-sum taxes on its inhabitants.

In particular, a consumer of country i is endowed with a positive amount y_n^i of each (private) good n and is taxed τ^i units of good 1 for i = 1, ..., I, and n = 1, 2. This consumer chooses c_n^i units of good n and receives g^i units of the country-i-specific public good. Consumer i's preferences over the consumption bundle (c_1^i, c_2^i, g^i) are given by

$$(1.1) \qquad u^i(c_1^i, c_2^i, g^i).$$

We assume that each u^i is monotone, strictly concave, and twice continuously differentiable, and that the marginal utility of each good goes to infinity as the amount of each good goes to zero. This consumer's budget constraint is

(1.2) $\quad c_1^i + pc_2^i = y_1^i - \tau^i + py_2^i,$

where p denotes the price of good 2 relative to good 1. The consumer, taking as given the price p and the tax-spending policy (τ^i, g^i) of the government of country i, chooses private good consumption c_1^i and c_2^i to maximize (1.1) subject to (1.2). Let the demand functions for this consumer be denoted by $c_n^i(\tau^i, p)$ for n = 1, 2, where the explicit dependence of these functions on the endowments is suppressed.

The government of country i has access to a production technology that converts private good 1 into a country-i-specific public good. For notational simplicity only, let this production function be linear with a unit coefficient. The budget constraint for the government of country i is

(1.3) $\quad g^i = \tau^i.$

Since government spending always equals taxes, government i's policy is summarized by τ^i and referred to either as "spending" or "taxes."

Market clearing in good markets 1 and 2 requires

(1.4) $\quad \displaystyle\sum_{i=1}^{I} c_1^i + \sum_{i=1}^{I} \tau^i = \sum_{i=1}^{I} y_1^i$

and

(1.5) $\quad \displaystyle\sum_{i=1}^{I} c_2^i = \sum_{i=1}^{I} y_2^i.$

Let $\tau = (\tau^1, \ldots, \tau^I)$ and $c_n = (c_n^1, \ldots, c_n^I)$ for n = 1, 2.

A <u>competitive equilibrium</u> is an allocation of private consumption (c_1, c_2), a price p, and a vector of government tax-spending policies τ such that these conditions hold:

- <u>Feasibility</u>. The consumption and government spending vectors satisfy (1.4) and (1.5).

- <u>Consumer maximization</u>. The consumption allocations c_1^i and c_2^i maximize (1.1) subject to (1.2) given τ^i and p for each i = 1, ..., I.

Notice several features of this equilibrium, which we will use later. First, given the government's budget, we can express the maximized value of consumer i's utility as

$$(1.6) \qquad V^i(\tau^i, p) = u^i\left[c_1^i(\tau^i, p), c_2^i(\tau^i, p), \tau^i\right].$$

Next, for any given vector τ, the market clearing conditions together with the consumer demand functions implicitly define the equilibrium price as a function of τ, say

$$(1.7) \qquad p = p(\tau).$$

Finally, notice that for a given vector τ of government policies, the private consumption allocations and prices in the above equilibrium with public goods are identical to those in an economy with only private goods in which country i consumers' private good endowments are $y_1^i - \tau^i$ and y_2^i, respectively and τ enters the utility function as a fixed parameter. Because of this, it is clear that the competitive equilibrium is Pareto optimal in the class of allocations (c_1, c_2, τ) that satisfy (1.4) and (1.5) and that take τ as given.

B. Noncooperative and Cooperative Equilibria

In the last section government policies were arbitrary. In this section we consider policies that are outcomes of either a noncooperative or a cooperative game among governments. In both of these games, governments maximize the utility of their residents taking as given that these private agents are in a competitive equilibrium. This fact can be used to express the objective function solely in terms of government policies.

For a given policy τ^i and price p, the maximized value of utility of a consumer in country i is given by (1.6). We can combine (1.6) with (1.7) to express the objective function of the government of country i solely as a function of τ, say as $R^i(\tau)$, where

$$(1.8) \qquad R^i(\tau) = V^i\left[\tau^i, p(\tau)\right].$$

Given these objective functions, we define a noncooperative equilibrium for governments to be a vector τ of government policy such that for each country i, τ^i maximizes (1.8) given $\tau^{-i} = (\tau^1, \ldots, \tau^{i-1}, \tau^{i+1}, \ldots, \tau^I)$. A noncooperative equilibrium is a vector of government policy τ, an allocation of private consumption (c_1, c_2) and a price p such that these conditions hold:

- Noncooperative equilibrium for governments. The vector τ constitutes a noncooperative equilibrium for governments.

- Competitive equilibrium for private agents. Given τ, the allocation (c_1, c_2) and the price p constitute a competitive equilibrium for private agents.

In a noncooperative equilibrium each government chooses policy separately to maximize its country's objective function. In a cooperative equilibrium, governments instead choose policy jointly to maximize a world

objective function. We will assume the world objective function is a weighted average of the individual country's objective functions. For an arbitrary vector of nonnegative weights λ, we define a <u>cooperative equilibrium for governments relative to λ</u> to be a vector τ of government policies that maximize

$$(1.9) \qquad \sum_{i=1}^{I} \lambda^i R^i(\tau).$$

A complete definition of a cooperative equilibrium includes an allocation (c_1, c_2) and a price system p such that these conditions hold:

- <u>Cooperative equilibrium for governments</u>. The vector τ constitutes a cooperative equilibrium for governments relative to λ.

- <u>Competitive equilibrium for private agents</u>. Given τ, the allocation (c_1, c_2) and price p constitute a competitive equilibrium for private agents.

We have defined cooperative equilibria for arbitrary weights, but are interested in cooperative equilibria relative to particular values of these weights. Such weights respect private ownership in the sense that they set to zero an excess savings function associated with a planning problem in which both private consumption and government spending are chosen. We will show that cooperative equilibria relative to such weights solve a planning problem. To this end, consider the following planning problem: For a given vector $\lambda = (\lambda^1, \ldots, \lambda^I)$ of nonnegative weights let

$$(1.10) \qquad W(\lambda) = \max_{\{c_1, c_2, \tau\}} \left\{ \sum_{i=1}^{I} \lambda^i \left[u^i(c_1^i, c_2^i, \tau^i) \right] \right\}$$

subject to

$$\sum_{i=1}^{I} c_1^i + \sum_{i=1}^{I} \tau^i = \sum_{i=1}^{I} y_1^i$$

and

$$\sum_{i=1}^{I} c_2^i = \sum_{i=1}^{I} y_2^i.$$

Let p_1 and p_2 denote the Lagrange multipliers on these constraints, and let $p = p_2/p_1$ be the normalized Lagrange multiplier. Write the solution to this problem as $\{c_1(\lambda), c_2(\lambda), \tau(\lambda), p(\lambda)\}$ and call it a (world) social optimum relative to λ.

For each country i define the excess savings function $s^i(\lambda)$ to be

(1.11) $s^i(\lambda) = \left[y_1^i - c_1^i(\lambda) - \tau^i(\lambda) \right] + p(\lambda) \left[y_2^i - c_2^i(\lambda) \right].$

Let S denote the set of weights that yields excess savings of zero in each country, that is,

(1.12) $S = \left\{ \lambda \epsilon R_+^I \mid s^i(\lambda) = 0 \text{ for } i = 1, \ldots, I \right\}.$

Call S the set of weights that respect private ownership, call a social optimum relative to some λ in S a social optimum that respects private ownership, and call a cooperative equilibrium relative to some λ in S a cooperative equilibrium that respects private ownership. We then have:

Proposition 1: A cooperative equilibrium that respects private ownership is a social optimum that respects private ownership.

The proof of this proposition is in the appendix. A slightly more precise way to state the proposition is: for any λ in S, the set of cooperative equilibria relative to λ coincides with the set of social optima relative

to the same λ. The intuition behind the proposition runs something like this: Think of the cooperative maximization problem as a search across policies (and therefore across competitive equilibria given these policies) for the one that yields the highest value of the objective function. We know that the private consumption allocations of these competitive equilibria are optimal given the government policy. The only circumstance, then, that could render the cooperative equilibria nonoptimal is that government policy is not chosen optimally.

To understand how government spending could be chosen suboptimally, consider a cooperative equilibrium for an arbitrary vector of weights λ. One might think that for any such λ, a cooperative equilibrium relative to λ is a social optimum relative to λ. It is easy to see, however, that this is not true. Recall that in our cooperative equilibrium the only choice governments make is the level of government spending. Suppose, instead, we considered a cooperative equilibrium in which governments not only choose spending but also make lump-sum transfers between residents of each country. In this equilibrium, for any vector for weights, the governments will set government spending optimally and then will use a separate set of instruments--the lump-sum transfers--to achieve the optimal income distribution across countries. In contrast, in our cooperative equilibrium these two goals must be achieved by a single set of instruments, the levels of government spending. If the weights chosen do not respect the initial distribution of income, the government spending decisions are distorted. Basically, countries that are assigned higher (lower) weights than their endowments justify are compensated in utility terms by inefficiently high (low) levels of government spending. In the proof of the proposition, we establish that the set of weights that respects this initial distribution of endowments is nonempty, and that the amount of

government spending for a cooperative equilibrium relative to such weights is optimal.

We next show that with a fixed number of countries, the cooperative equilibria typically do not coincide with the noncooperative equilibria. To demonstrate this, compare the first-order conditions of the noncooperative equilibria with those of the cooperative equilibria. In a noncooperative equilibrium, the government of country k chooses spending τ^k to satisfy

$$(1.13) \qquad \frac{\partial R^k}{\partial \tau^k} = \frac{\partial V^k}{\partial \tau^k} + \frac{\partial V^k}{\partial p} \frac{\partial p}{\partial \tau^k} = 0.$$

This is easily transformed into

$$(1.14) \qquad -1 + \frac{u_\tau^k}{u_1^k} + (y_2^k - c_2^k) \frac{\partial p}{\partial \tau^k} = 0$$

by using the definition of R in equation (1.8). We call the first term in equations (1.13) and (1.14) the <u>direct</u> effect of a change in policy and the second term the <u>indirect</u> (or general equilibrium) effect. The direct effect measures the impact of a change in policy by a government on that country's residents at a given world price p. Note, however, that with a finite number of countries, a change in spending by one government also affects this world price. The indirect effect measures the impact on residents of a change in government spending solely in terms of changing this world price.

In a cooperative equilibrium that respects private ownership, the sum of the indirect effects is zero. For each country k government spending τ^k must satisfy $-1 + u_\tau^k / u_1^k = 0$. The wedge between these two first-order conditions is the term

$$(1.15) \qquad (y_2^k - c_2^k) \, \frac{\partial p}{\partial \tau^k},$$

which we call the monopoly distortion.

In the cooperative allocation, the first-order conditions require that government spending be chosen optimally, that is, to equate marginal rates of substitution with marginal rates of transformation. In the noncooperative allocation, however, the monopoly distortion drives a wedge between the optimal decision and the noncooperative decisions. Basically, in the noncooperative allocation each government takes into account its effect on world prices and chooses a policy not only to balance these marginal rates, but also to influence prices in a direction that benefits its residents. In particular, suppose that at the cooperative level of spending country k is a net exporter of good 1. At this allocation, a noncooperative government of this country would have an incentive to raise its spending a little, thereby decreasing the net private supply of private good 1 and raising the relative price of exports (lowering the relative price p of imports) and in the process make itself better off. Likewise, if at the cooperative level of spending country k is a net importer of good 1, then a noncooperative government of this country would have an incentive to lower its spending a little, thereby increasing the net private supply of private good 1 and lowering the relative price of imports (raising the relative price of exports).

In general, then, the noncooperative and cooperative equilibria do not coincide when there is a finite number of countries because of monopoly distortions. Indeed the only type of a cooperative equilibrium which could also be a noncooperative equilibrium is one in which there is no trade. In this special case monopoly distortions disappear and governments have no incentives to distort spending decisions to affect world prices.

2. Convergence in Replica Economies

In section 1 we showed that, given a finite number of countries, noncooperative and cooperative equilibria typically do not coincide. This is so because in the noncooperative equilibria it is in each government's interest to take advantage of its monopoly power and distort its decision away from the social optimum. In this section, we show that if the economy of section 1 is replicated, then the monopoly distortions go to zero, and the noncooperative and cooperative allocations converge.

Consider replicating the economy of section 1 a fixed number of times, say J. Eventually we will let J go to infinity. The Jth replica economy has countries indexed by ij for $i = 1, \ldots, I$ and $j = 1, \ldots, J$, where i refers to the type of country and j refers to the replication number. All J consumers of type i have the same utility functions and endowments:

(2.1) $\quad u^{ij} = u^{i1}$ and $y^{ij} = y^{i1}$ for all $j = 1, \ldots, J$.

The demand function of consumer ij for good n is denoted by $c_n^{ij}(\tau^i, p)$ for $n = 1, 2$. Market clearing for good 1 then requires

(2.2) $\quad \sum_j \sum_i c_1^{ij}(\tau^{ij}, p) + \sum_j \sum_i \tau^{ij} = \sum_j \sum_i y_1^{ij}.$

This condition implicitly defines the equilibrium price as a function of government spending. Let us write this function as

(2.3) $\quad p = p(T^J)$

where $T^J = (\tau^{11}, \ldots, \tau^{I1}; \ldots; \tau^{1J}, \ldots, \tau^{IJ})$. The objective function of the government of country ij is

(2.4) $\quad R^{ij}(T^J) = V^{ij}[\tau^{ij}, p(T^J)],$

where V^{ij} is defined analogously to (1.6).

For the replica economy, noncooperative and cooperative equilibria are defined as in the last section. We focus on equilibria that are symmetric in the sense that all countries of the same type choose the same policy, that is, $\tau^{ij} = \tau^{i1}$ for all i and j. From now on, this symmetry requirement will be understood. We then have:

Proposition 2: As the number of replications goes to infinity, the noncooperative equilibria converge to cooperative equilibria that respect private ownership.

The proof of this proposition is a straightforward application of the definition of a replica economy, together with a little price theory. For any given number of replications, the noncooperative solution clearly coincides with the cooperative solution if and only if the monopoly distortions are zero. In the Jth replica economy, the monopoly distortion for country k1 (the first replica of type k) is

(2.5) $\quad (y_2^{k1} - c_2^{k1}) \dfrac{\partial p(T^J)}{\partial \tau^{k1}}.$

The proposition is proved by showing that this distortion goes to zero as J goes to infinity for each type k of country.

Consider first an economy with J equal to 1, the original economy. In this economy, the market clearing condition,

(2.6) $\quad \sum_{i=1}^{I} c_1^{i1} + \sum_{i=1}^{I} \tau^{i1} = \sum_{i=1}^{I} y_1^{i1},$

defines the equilibrium price function $p(T^1)$ and the private consumption allocations $\{c^{i1}|i=1,\ldots,I\}$. To evaluate how a change in spending by the government of a country of type k affects the equilibrium price, differentiate (2.6) to obtain

$$(2.7) \qquad \frac{\partial p(T^1)}{\partial \tau^{k1}} = \frac{1 - \partial c_1^{k1}/\partial \tau^{k1}}{\sum\limits_{i=1}^{I} \partial c_1^{i1}/\partial p}.$$

Now consider an economy with J greater than 1. In such an economy, the market clearing condition,

$$(2.8) \qquad \sum_{j=1}^{J} \sum_{i=1}^{I} c_1^{ij} + \sum_{j=1}^{J} \sum_{i=1}^{I} \tau^{ij} = \sum_{j=1}^{J} \sum_{i=1}^{I} y_1^{ij},$$

defines the equilibrium price function $p(T^J)$ and the private consumption allocations $\{c^{ij}|i=1,\ldots,I;j=1,\ldots,J\}$. To evaluate how a change in spending by a government of a country of type k, say country k1, affects this price, differentiate (2.8) to obtain

$$(2.9) \qquad \frac{\partial p(T^J)}{\partial \tau^{k1}} = \frac{1 - \partial c_1^{k1}/\partial \tau^{k1}}{\sum\limits_{j=1}^{J} \sum\limits_{i=1}^{I} \partial c_1^{ij}/\partial p}.$$

From the definition of a replica economy, $c_1^{ij}(p,\tau^{ij}) = c_1^{i1}(p,\tau^{i1})$ for all i and j, and by our symmetry assumption $\tau^{ij} = \tau^{i1}$ for all i and j. Thus in an equilibrium of the Jth replica economy, we can write (2.8) as

$$(2.10) \qquad J \sum_{i=1}^{I} c_1^{i1}(p,\tau^{i1}) + J \sum_{i=1}^{I} \tau^{i1} = J \sum_{i=1}^{I} y_1^{i1},$$

which is equivalent to (2.6). That is, the competitive equilibria of the Jth replica economy are simply the competitive equilibria of the original economy replicated J times. In particular, with concave utility functions all consumers of the same type get the same allocation. This implies

$$(2.11) \qquad \sum_{j=1}^{J} \sum_{i=1}^{I} \frac{\partial c_1^{ij}}{\partial p} = J \sum_{i=1}^{I} \frac{\partial c_1^{i1}}{\partial p}.$$

Combining (2.7), (2.9), and (2.11) gives

$$(2.12) \qquad \frac{\partial p(T^J)}{\partial \tau^{k1}} = \frac{1}{J} \frac{\partial p(T^1)}{\partial \tau^{k1}}.$$

Using (2.12) and the fact that the equilibria in the replica economy are the replicated equilibria of the original economy, we have that as J goes to infinity the monopoly distortion (2.5) goes to zero for each country. The noncooperative equilibria thus converge to the cooperative equilibria.

3. Divergence in Nonreplica Economies

In the last section, replication was shown to cause the cooperative and noncooperative equilibria to converge. The process of replication implies that countries become small in two ways. First, each country's endowment, as a fraction of the world endowment, converges to zero. Second, each country's socially optimal level of government spending, as a fraction of the world endowment, converges to zero. In this section we present a parametric example of a nonreplica economy in which these conditions fail and in which the two solutions diverge. [This example is closely related to Devereux (1986).]

In the previous section, the original economy had I types of countries and two private goods. At each stage of the replication process, the number of types of countries and goods remained fixed as replicas were

added. Each country thus became small relative to the number of countries of the same type. In contrast, the present example has only one country of each type, and at each stage we add a country of a different type, which has monopoly power over a new good. This difference in how we add countries and goods is crucial to the results.

In particular, let there be I countries (indexed $i=1,\ldots,I$) and I private goods (indexed $n=1,\ldots,I$). Consumers in country i own the world endowment of good i and own no other goods. Only the government of country i has access to a production technology that converts private good i into a country-specific good at a one-to-one rate.

In addition, let c_n^i denote the consumption of private good n by consumers in country i, let y_i^i denote the country i consumer endowment of good i, and let τ_i^i denote the amount of private good i that is converted by the government of country i into a public good. Let each type of country be symmetric in the sense that the utility functions and endowments are symmetric:

$$u^i(c_1^i,\ldots,c_I^i,\tau_i^i) = \sum_{n=1}^{I} \frac{1}{I} \ln c_n^i + \ln \tau_i^i$$

and

$$y_i^i = y_1^1 \text{ for all } i = 1, \ldots, I.$$

Let $p = (p_1,\ldots,p_I)$ denote the prices of the private goods. Consumers in country i solve the problem

$$(3.1) \qquad v^i(\tau^i,p) = \max_{\{c^i\}} \left\{ \sum_{n=1}^{I} \frac{1}{I} \ln c_n^i + \ln \tau_i^i \right\}$$

subject to

$$\sum_{n=1}^{I} p_n c_n^i = p_i(y_i^i - \tau_i^i),$$

where $c^i = (c_1^i, \ldots, c_I^i)$ and the consumer and the government budget constraints are already combined. The resulting demand functions are

$$(3.2) \qquad c_n^i = \frac{1}{I} \frac{p_i(y_i^i - \tau_i^i)}{p_n}.$$

Market clearing requires

$$(3.3) \qquad \sum_{i=1}^{I} c_n^i + \tau_n^n = y_n^n \text{ for } n = 1, \ldots, I.$$

Substituting the demand functions into the market clearing conditions gives the equilibrium prices,

$$(3.4) \qquad p_n = \frac{y_1^1 - \tau_1^1}{y_n^n - \tau_n^n} \text{ for } n = 1, \ldots, I,$$

where we normalized prices by setting $p_1 = 1$. Substituting these prices into the demand functions gives the equilibrium allocations

$$(3.5) \qquad c_n^i = (y_n^n - \tau_n^n)/I \text{ for } n = 1, \ldots, I, \text{ and } i = 1, \ldots, I.$$

Given these, the objective function of the government of country i can be written as

$$(3.6) \qquad R^i(\tau_1^1, \ldots, \tau_I^I) = \sum_{n=1}^{I} \frac{1}{I} \ln \frac{y_n^n - \tau_n^n}{I} + \ln \tau_i^i.$$

The first-order conditions for the noncooperative equilibrium are

$$(3.7) \qquad -\frac{1}{I} \frac{1}{y_i^i - \tau_i^i} + \frac{1}{\tau_i^i} = 0 \text{ for } i = 1, \ldots, I,$$

which, given our symmetry assumption, implies that the noncooperative level of government spending is

$$(3.8) \qquad \tau_i^i = \left(\frac{I}{I+1}\right) y_1^1.$$

Consider next the cooperative solution. Given the symmetry of the example, any vector of weights that places an equal weight on each country will respect private ownership. Such a cooperative solution maximizes with respect to $\tau = (\tau_1^1, \ldots, \tau_I^I)$:

$$(3.9) \qquad \sum_{i=1}^{I} \lambda^i R^i(\tau) = \sum_{i=1}^{I} \sum_{n=1}^{I} \ln \frac{y_n^n - \tau_n^n}{I} + \ln \tau_i^i.$$

The first-order conditions for this problem are

$$(3.10) \qquad -\frac{1}{y_i^i - \tau_i^i} + \frac{1}{\tau_i^i} = 0 \text{ for } i = 1, \ldots, I.$$

If we impose symmetry, the cooperative level of government spending is given by

$$(3.11) \qquad \tau_i^i = \frac{y_1^1}{2}.$$

Thus, as the number of countries goes to infinity, the cooperative and non-cooperative solutions diverge.

Even though the number of countries in this example goes to infinity, each type of country maintains monopoly power over a good. A given country i has two sources of monopoly power over private good i. First, it has monopoly power in endowments--it is the only country with endowments of good i. Second, it has monopoly power in production--it is the only country that can convert private good i into a public good. Neither of these sources

of monopoly power goes to zero as new types of countries are added. It is possible to construct examples in which either source of monopoly power alone causes the two solutions to diverge, but the algebra is somewhat tedious.

4. Divergence in an Economy With Tax Distortions

In section 1 we considered a simple model in which the only distortion was the monopoly power distortion. In section 2 as we replicated the economy the distortion went to zero and the two solutions converged. In the last section the monopoly distortion did not go to zero and the solutions diverged. In this section and the next we describe economies with other features that lead to inefficient equilibria. Because of these features, the two solutions do not converge even if monopoly distortions go to zero. Indeed, in our examples the solutions diverge. In the example in this section inefficiency results from a distortionary tax, and in the next from the overlapping generations structure.

Consider an economy identical to the one in section 1 except that there are distortionary taxes instead of lump-sum taxes. For simplicity let all countries be identical. Since there is only one type of country, think of an economy with J such countries as the Jth replica of an original economy with one country. Finally, let the distortionary tax be a linear tax on the consumption of the first good.

A representative consumer in country j (j=1,...,J) solves the problem

$$(4.1) \quad \max_{\{c^j\}} \{u^j(c_1^j, c_2^j, g^j)\}$$

subject to

$$(1+\tau^j)c_1^j + pc_2^j = y_1^j + py_2^j,$$

where τ^j is the consumption tax imposed by the government of country j on its residents' consumption of good 1. This problem yields demand functions $c_n^j(\tau^j,p)$ for n = 1, 2. The government of country j chooses taxes τ^j and government spending g^j to satisfy its budget constraint

$$(4.2) \qquad g^j = \tau^j c_1^j.$$

We define a competitive equilibrium as in section 1. The market clearing conditions implicitly define the equilibrium price as a function of the tax policies, say $p = p(\tau)$.

To define the objective functions of governments, first substitute the consumer's demand functions and the equilibrium value of the government's budget constraint into the consumer's utility function and obtain

$$(4.3) \qquad V^j(\tau^j,p) = u^j\left[c_1^j(\tau^j,p),c_2^j(\tau^j,p),\tau^j c_1^j(\tau^j,p)\right].$$

Given this substitution, define the objective function of the government of country j to be

$$(4.4) \qquad R^j(\tau) = V^j\left[\tau^j,p(\tau)\right].$$

The first-order conditions for the noncooperative level of taxes are

$$(4.5) \qquad \frac{\partial R^k}{\partial \tau^k} = \frac{\partial V^k}{\partial \tau^k} + \frac{\partial V^k}{\partial p}\frac{\partial p}{\partial \tau^k} = 0 \quad \text{for } k = 1, \ldots, J.$$

Again the first-order conditions are the sum of direct and indirect effects. The direct effects can be written as

$$(4.6) \qquad \frac{\partial V^k}{\partial \tau^k} = u_1^k \frac{\partial c_1^k}{\partial \tau^k} + u_2^k \frac{\partial c_2^k}{\partial \tau^k} + u_\tau^k \frac{\partial g^k}{\partial \tau^k}\Bigg|_{p \text{ constant}}$$

$$= u_1^k c_1^k \left[- \frac{1}{(1+\tau^k)} + \frac{u_\tau^k}{u_1^k} \left(1 + \frac{\tau^k}{c_1^k} \frac{\partial c_1^k}{\partial \tau^k} \right) \right]$$

and the indirect effects as

$$(4.7) \qquad \frac{\partial V^k}{\partial p} \frac{\partial p}{\partial \tau^k} = u_1^k \frac{\partial c_1^k}{\partial p} + u_2^k \frac{\partial c_2^k}{\partial p} + u_\tau^k \frac{\partial g^k}{\partial p} \frac{\partial p}{\partial \tau^k}$$

$$= \frac{u_1^k}{(1+\tau^k)} (y_2^k - c_2^k) \frac{\partial p}{\partial \tau^k} + u_\tau^k (\tau^k \frac{\partial c_1^k}{\partial p}) \frac{\partial p}{\partial \tau^k}.$$

where, from the market clearing conditions, we have

$$(4.8) \qquad \frac{\partial p}{\partial \tau^k} = - \frac{[c_1^k + (1+\tau^k) \partial c_1^k / \partial \tau^k]}{\sum\limits_{j=1}^{J} (1+\tau^j) \partial c_1^j / \partial p}.$$

Recall that the direct effects measure how a change in government policy affects that country's residents at a given world price, while indirect effects measure how a policy change affects residents by affecting the world price. With distortionary taxes, both effects are changed. The direct effects no longer imply that the marginal rate of substitution should be equated with the marginal rate of transformation. Rather, these terms are modified by the elasticity of consumption with respect to the distortionary tax. The indirect effects are now composed of two terms. The first term in (4.7) is analogous to the indirect effect in (1.14)—both represent monopoly distortions. The second term in (4.7), called the _tax distortion effect_, measures how much the price changes that result from a tax change affect

utility by changing the level of public goods provided. Note that if consumption of good 1 were completely inelastic with respect to its price, this second distortion would be zero and only the monopoly distortion would be left.

Compare this solution with the cooperative solution. Given the symmetry of the example, equal weights respect private ownership. We will consider a symmetric solution in which all policies are equal. The first-order conditions for the cooperative allocation are[2]

$$(4.9) \qquad \sum_{j=1}^{J} \frac{\partial R^j}{\partial \tau^k} = \frac{\partial V^k}{\partial \tau^k} + \sum_{j=1}^{J} \frac{\partial V^j}{\partial p} \frac{\partial p}{\partial \tau^k} = 0.$$

In contrast to the model in section 1, the extra distortion that results from taxes causes the indirect effects not to cancel. Indeed, the sum of indirect effects is

$$(4.10) \qquad \sum_{j=1}^{J} \frac{\partial V^j}{\partial p} \frac{\partial p}{\partial \tau^k} = \sum_{j=1}^{J} \left[u_\tau^j \tau^j \frac{\partial c_1^j}{\partial p} \right] \frac{\partial p}{\partial \tau^k}.$$

It is this sum of the tax distortions that causes the two solutions to diverge. To see this, let $p(T^J)$ represent the equilibrium price function with J identical countries. As in Proposition 2

$$(4.11) \qquad \frac{\partial p(T^J)}{\partial \tau^k} = \frac{1}{J} \frac{\partial p(T^1)}{\partial \tau^k}.$$

Imposing symmetry and using (4.11), we define the noncooperative solution by

$$(4.12) \qquad - \frac{1}{1+\tau} + \frac{u_\tau}{u_1} \left(1 + \frac{\tau}{c_1} \frac{\partial c_1}{\partial \tau} \right) + \frac{1}{J} \left[\frac{u_\tau}{u_1} \frac{\tau}{c_1} \frac{\partial c_1}{\partial p} \right] \frac{\partial p(T^1)}{\partial \tau} = 0,$$

while the cooperative solution is defined by

$$(4.13) \qquad - \frac{1}{1+\tau} + \frac{u_\tau}{u_1}\left(1+ \frac{\tau}{c_1}\frac{\partial c_1}{\partial \tau}\right) + \left[\frac{u_\tau}{u_1}\frac{\tau}{c_1}\frac{\partial c_1}{\partial p}\right]\frac{\partial p(T^1)}{\partial \tau} = 0.$$

The wedge between these solutions is

$$(4.14) \qquad \frac{J-1}{J}\left[\frac{u_\tau}{u_1}\frac{\tau}{c_1}\frac{\partial c_1}{\partial p}\right]\frac{\partial p(T^1)}{\partial \tau}.$$

We can use (4.8) to show that, in general, this wedge is nonzero and thus these two solutions will diverge as the number of countries J goes to infinity. It is worth pointing out that in the special case of log utility the relevant income and substitution effects cancel and this wedge is zero.

The intuition for this result is as follows: We can use (4.8) and (4.13), (see also footnote 2) to see that in a cooperative equilibrium, the marginal of substitution between private and government consumption is equated to the marginal rate of transformation. Thus, the cooperative equilibrium yields the same solution as the equilibrium with lump sum taxes. Denote the relative price in the cooperative equilibrium with lump sum taxes by q and the price in the cooperative equilibrium with distorting taxes by p. We have that p = q(1+τ). Consider now the problem faced by one country in the limiting noncooperative equilibrium. To see that, in general, the solutions are different, suppose that the world price it faces is given by p, the cooperative equilibrium price with distorting taxation. It is easy to see that _even if_ this country can use lump sum taxation it will not choose the same level of spending as in the cooperative equilibrium. This is because the world price p does not signal the correct marginal rate of substitution for consumers in other countries. Of course, since this country has access only to distorting taxes, the problem is further magnified. Hence, in general, the limiting noncooperative equilibrium and the cooperative equilibrium do not coincide.

It is worth noting that all results in this section hold if the instrument available to governments is a tariff rather than a consumption tax. We considered a consumption tax rather than a tariff for two reasons. First, we wanted to examine a model with identical countries for notational convenience. Obviously, a tariff cannot raise revenues if there is no trade. Second, note that with identical countries, there is no monopoly distortion effect [see (2.5)]. Consequently, the only source of distortion lies in the way taxes distort private decisions. Since we wanted to focus on this issue, we considered a consumption tax. In appendix B we construct an environment where the only incentive feasible means of raising revenues is through a linear tariff. This appendix is intended to provide one possible motivation for investigating the properties of economies with distortionary taxes. The appendix is also of independent interest because it provides a simple environment where only linear taxes can be levied.

5. Conclusion

We have extended the analysis of tariff policy to simple models of fiscal policy. We show first that if lump sum taxes are available, then the basic results on tariff policy carry over to fiscal policy: as each country becomes small in the world economy, the noncooperative allocations converge to the cooperative allocations. Second, if revenues must be raised through distorting taxes, in general, these solutions do not converge.

We have made these points in simple models, but the intuition behind them is general. In the limiting noncooperative equilibrium each country uses a distorting tax to attempt to achieve two conflicting goals. It seeks to provide an optimal level of government spending and at the same time to equate the marginal rates of substitution of its consumers to the world price.

However, since other countries also must use distorting taxes, the world price does not reflect the marginal rates of substitution of consumers in other countries. Thus, in general there is a loss of efficiency relative to the cooperative equilibrium. Similar results may hold for other types of distortions, such as incomplete markets.

This paper is related to several strands of literature.[3] First, it is related to other analyses of fiscal policy in a world economy. In terms of strategic analyses of fiscal policy, we unify the results of Backus, Devereux, and Purvis (1986), Devereux (1986), Hamada (1986), and Kehoe (1986a). We have, however, limited our attention to static models in order to avoid issues concerning the time inconsistency of tax-spending policy of the type considered by Lucas and Stokey (1983) and by Persson and Svensson (1986). Once these simple models are well understood, it would be interesting to explore dynamic models of policy in which a key ingredient is the interaction between time inconsistency and cooperation. Rogoff (1985) and Kehoe (1986b) are examples of this type of analysis. Interesting nonstrategic models of these interactions have been developed by Frenkel and Razin (1985 and 1986).

Footnotes

[1]Given that our results with distorting taxes are somewhat at odds with received wisdom, it is natural to ask whether other sources of inefficiency lead to similar results. In an earlier version of the paper we showed that for an overlapping generations economy with an inefficient competitive equilibrium, the noncooperative policies do not converge to the cooperative policies.

[2]Using (4.8) it is easy to show that the cooperation solution coincides with the equal weight social optimum. That is, the cooperation solution with proportional consumption taxes equals what would be the cooperative solution with lump sum taxes. This special feature of the cooperative equilibrium arises because there is symmetry and there is no production. If we change either of the assumptions this result will not hold. However, the algebra of the rest of the derivations is somewhat tedious. The details of the more general case are available on request.

[3]This paper is also related to a large literature in mathematical economics which characterizes Walrasian equilibria as the limit of noncooperative equilibria. [See, for example, the recent symposium in the _Journal of Economic Theory_ (1980).] To help clarify this relationship, consider the following two-stage manipulation game in a pure exchange economy inhabited only by private agents. In stage 1 these agents decide how much of their endowments to destroy. In stage 2, given the remaining endowments, they participate as price-takers in a competitive equilibrium. This manipulation game is closely related to the games studied in this paper. Indeed there may be a way to adapt some of the results in this literature to prove a more general version of proposition 2, which would cover certain types of nonreplica economies.

Appendix A
Proof of Proposition 1

The basic line of argument is as follows: First, we consider a cooperative equilibrium in which governments are allowed to make transfers between countries. We show in Lemma 1 that for any vector of weights this equilibrium is a social optimum. In Lemma 2, we show that a nonempty set of weights exists for which the optimal transfers in such a cooperative equilibrium are zero. Combining these two lemmas gives us proposition 1. (Let us remark that we view these equilibria with transfers simply as a convenient construct for proving proposition 1; we are not particularly interested in them in their own right.)

To set up lemma 1, we need several definitions. We first define a cooperative equilibrium with transfers relative to any nonnegative vector of weights λ: for brevity, call this a λ-cooperative equilibrium with transfers. This equilibrium is composed of a competitive equilibrium for private agents and a cooperative equilibrium for governments. We begin with the competitive equilibrium. Let x^i denote the amount of good 1 that each agent in country i transfers to the rest of the world. Let $x = (x^1, \ldots, x^I)$ with $x^I = -\sum_{i=1}^{I-1} x^i$ be the vector of such transfers. For a given vector of government spending τ and transfers x, a competitive equilibrium is an allocation of private consumption (c_1, c_2) and a price p such that these conditions hold.

• <u>Market clearing</u>. The consumption and government spending vectors satisfy

$$(A1) \qquad \sum_{i=1}^{I} c_1^i + \sum_{i=1}^{I} \tau^i = \sum_{i=1}^{I} y_1^i$$

and

(A2) $\qquad \sum_{i=1}^{I} c_2^i = \sum_{i=1}^{I} y_2^i.$

- Consumer maximization. The consumption vector satisfies

$$V^i(\tau^i, x^i, p) = \max_{\{c_1^i, c_2^i\}} u^i(c_1^i, c_2^i, \tau^i)$$

subject to

(A3) $\qquad c_1^i + p c_2^i = y_1^i - \tau^i - x^i + p y_2^i.$

Substituting the resulting demand functions into the market clearing conditions gives the equilibrium price as a function of government spending and transfers, say $p = p(\tau, x)$. Let $R^i(\tau, x) = V^i[p(\tau, x), \tau^i, x^i]$ and define a λ-cooperative equilibrium with transfers for governments to be a vector (τ, x) that solves

$$R(\lambda) = \max_{\{\tau, x\}} \sum_{i=1}^{I} \lambda^i R^i(\tau, x).$$

We then have: a λ-cooperative equilibrium with transfers is a vector (τ, x, c_1, c_2, p) such that these conditions hold.

- Cooperative equilibrium for governments. The vector (τ, x) constitutes a λ-cooperative equilibrium with transfers for governments.

- Competitive equilibrium for private agents. Given (τ, x), the vector (c_1, c_2, p) constitutes a competitive equilibrium for private agents.

Next, a λ-social optimum is a vector (τ, c_1, c_2, p) that solves

$$W(\lambda) = \max_{\{\tau, c_1, c_2\}} \sum_{i=1}^{I} \lambda^i u^i(c_1^i, c_2^i, \tau^i)$$

subject to

(A4) $\qquad \sum_{i=1}^{I} c_1^i + \sum_{i=1}^{I} \tau^i = \sum_{i=1}^{I} y_1^i$

(A5) $\qquad \sum_{i=1}^{I} c_2^i = \sum_{i=1}^{I} y_2^i$

where p denotes the normalized Lagrange multiplier for these constraints.

Notice that a vector (τ, x, c_1, c_2, p) is a λ-cooperative equilibrium with transfers only if it satisfies (A1)-(A3) and (A6)-(A8) where

(A6) $\qquad \dfrac{u_2^k}{u_1^k} = p$

(A7) $\qquad \sum_{i=1}^{I} \lambda^i \dfrac{\partial R^i}{\partial \tau^k} = \lambda^k [u_\tau^k - u_1^k] + \sum_{i=1}^{I} \lambda^i u_1^i (y_2^i - c_2^i) \dfrac{\partial p}{\partial \tau^k} = 0$

(A8) $\qquad \sum_{i=1}^{I} \lambda^i \dfrac{\partial R^i}{\partial x^k} = -\lambda^k u_1^k + \lambda^I u_1^I + \sum_{i=1}^{I} \lambda^i u_1^i (y_2^i - c_2^i) \dfrac{\partial p}{\partial x^k} = 0$

for $k = 1, \ldots, I$. Notice also that a vector (τ, c_1, c_2, p) is a λ-social optimum if and only if it satisfies (A4)-(A5) and (A9)-(A11) where

(A9) $\qquad \dfrac{u_2^k}{u_1^k} = p$

(A10) $\qquad u_\tau^k = u_1^k$

(A11) $\lambda^k u_1^k = \lambda^I u_1^I$

for k = 1, ..., I.

      With these definitions, it is straightforward to establish the first lemma.

<u>Lemma 1</u>: For any nonnegative λ, a λ-cooperative equilibrium with transfers is a social optimum.

<u>Proof</u>: Since the cooperative equilibrium problem $R(\lambda)$ is simply the social optimum problem $W(\lambda)$ combined with the constraint that private agents are in a competitive equilibrium, we have $W(\lambda) \geq R(\lambda)$ for any given λ. Next, by concavity the λ-social optimum is unique. Thus, if we can choose transfers such that the λ-social optimum together with the transfers is a λ-cooperative equilibrium, we are then done. To this end, let $(\hat{\tau}, \hat{c}_1, \hat{c}_2, \hat{p})$ be the unique λ-social optimum. We claim $(\hat{\tau}, \hat{x}, \hat{c}_1, \hat{c}_2, \hat{p})$ is a λ-cooperative equilibrium where

(A12) $\hat{x}^k = y_1^k - \hat{c}_1^k - \hat{\tau}^k + \hat{p}(y_2^k - \hat{c}_2^k)$ for k = 1, ..., I.

To see this note that if the social optimum satisfies (A4)-(A5) and (A9)-(A11), then the social optimum plus the transfers satisfy (A1)-(A3) and (A6)-(A8). Two details are worth noting. First, (A11) requires that $\lambda^k u_1^k$ be constant for all i in the social optimum. Using (A5), we can see that the bracketed terms in (A7) and (A8) are zero, thus (A7) and (A8) are equivalent to (A10) and (A11), respectively. Second, the definition of transfers (A12) implies that the private agents' budget constraints are satisfied. ◊

      In the next lemma, we show that the set of weights that respect private ownership is nonempty. These will turn out to be exactly the set of weights for which the optimal transfers of lemma 1 are zero.

Lemma 2: There exists a nonempty set S of nonnegative weights λ such that for each λ in S the excess savings of each consumer is zero.

Proof: The proof is a fairly standard application of a fixed point theorem along the lines of Negishi (1960) and Mantel (1974). Recall that the excess savings function of the ith country is

$$(A13) \qquad s^i(\lambda) = \left[y_1^i - c_1^i(\lambda) - \tau^i(\lambda)\right] + p(\lambda)\left[y_2^i - c_2^i(\lambda)\right]$$

and is defined for all λ in Δ where

$$(A14) \qquad \Delta = \left\{\lambda \epsilon R^I \,\middle|\, \lambda^i \geq 0 \text{ for all } i \text{ and } \sum_{i=1}^{I} \lambda^i = 1\right\}.$$

These excess savings functions have several properties we will exploit in the proof. First, given our assumptions on utility functions, the maximum theorem guarantees they are continuous functions of λ. Second, feasibility implies they sum to zero. Third, they are positively homogeneous of degree zero. This follows from the fact that if we multiply the vector of weights λ by a positive scalar, the allocations and normalized Lagrange multiplier that result from the social optimum problem are unchanged. Finally, these functions satisfy

$$(A15) \qquad \text{if } \lambda \text{ is in } \Delta \text{ and } \lambda^i = 0, \text{ then } s^i(\lambda) \geq 0,$$

that is, if consumer i receives a zero weight in the social optimum, then consumer i's excess saving is nonnegative.

Next, define the fixed point map $g: \Delta \rightarrow \Delta$ where $g = (g^1, \ldots, g^I)$ and

$$(A16) \qquad g^i(\lambda) = \frac{\max\left[0, \lambda^i + s^i(\lambda)\right]}{\sum_{j=1}^{I} \max\left[0, \lambda^j + s^j(\lambda)\right]}.$$

Notice the denominator in (A16) is always positive. This is true since $\sum_j [\lambda^j + s^j(\lambda)] = 1$ implies $[\lambda^j + s^j(\lambda)] > 0$ for some j, which implies $\sum_j \max[0, \lambda^j + s^j(\lambda)] > 0$. Then since the savings functions are continuous and the $\max(\cdot, \cdot)$ function is continuous, the function g is continuous. Since the $g^i(\lambda)$ are nonnegative and sum to 1, we know that $g(\lambda)$ is in Δ. Thus, f is a continuous function that maps the compact convex set Δ into itself. So by Brouwer's theorem we know there is a nonempty set S of weights such that $g(\lambda) = \lambda$ for all λ in S.

To finish the proof we must show that a fixed point of f is a zero of s, that is, $g^i(\lambda) = \lambda^i$ for all i implies $s^i(\lambda) = 0$ for all i. If λ is a fixed point of g, then

$$(A17) \qquad a\lambda^i = \max[0, \lambda^i + s^i(\lambda)] \text{ for all i,}$$

where $a = \sum_j \max[0, \lambda^j + s^j(\lambda)]$. This implies

$$(A18) \qquad a\lambda^i = \lambda^i + s^i(\lambda) \text{ for all i,}$$

since from (A15) we know that if $\lambda^i = 0$, then $s^i(\lambda) \geq 0$. Summing (A18) over all consumers gives

$$(A19) \qquad a \sum_i \lambda^i = \sum_i \lambda^i + \sum_i s^i(\lambda).$$

Since the sum of these savings functions is zero, we have $a = 1$. Thus by (A18) we have $s^i(\lambda) = 0$ for all i. ◊

Combining these two lemmas gives us:

Proposition 1: A cooperative equilibrium that respects private ownership is a social optimum that respects private ownership.

Proof: A more precise statement of the proposition is: for any λ in S, a λ-cooperative equilibrium (without transfers) is a λ-social optimum. Comparing (A12) and (A13), we see that the transfers used to support a given λ-social optimum are simply the excess savings that result from that optimum. Thus, by lemma 2 for any λ in S, these optimal transfers are zero, so for such a λ, a λ-cooperative equilibrium with transfers is a λ-cooperative equilibrium (without transfers). Then, by Lemma 1, such a cooperative equilibrium is optimal. $\Diamond$

Appendix B

We construct an environment for which the only incentive feasible means of raising revenues is through a tariff. Thus we provide one possible motivation for investigating the properties of economies with certain types of distortionary taxes.

Our environment and analysis related to the work of Hammond (1987). Two features of the environment play a crucial role. First, we assume that governments can only observe trades between domestic residents and foreigners, or between itself and other agents. In particular, it cannot observe trades among domestic residents. Second, we assume that all trades are anonymous, so that individual specific taxes cannot be levied by the government.

B.1. Closed Economy

We begin by describing a closed economy in which there are unobserved trades among domestic residents and observed trades between the government and the domestic residents. We will show in Section B2 how the model can be reinterpreted as an open economy.

There are three commodities: the first two are private consumption goods and the third is government consumption. There is a continuum of agents distributed on the unit interval according to the Lebesque measure λ. The preferences of agents are identical and are represented by a strictly increasing quasiconcave utility function. For agent i the utility of the consumption bundle $(c_1(i), c_2(i), c_3(i))$ is $U(c_1(i), c_2(i), c_3(i))$.

Consumers' endowments of the private consumption goods are independent draws from a continuous distribution function $G(y_1, y_2)$ defined on the compact interval $[0, \bar{y}_1] \times [0, \bar{y}_2]$. [We will use the results of Judd (1985) and

Uhlig (1987) for the laws of large numbers with a continuum of independent random variables.] The technology set for the economy is

$$Z = \{(z_1, z_2, z_3) \mid R(z_1, z_2, z_3) \le 0\}.$$

The function R describes the production technology, available only to the government, for transforming private goods into a public good.

An allocation for this economy is a triple of Lebesque measurable functions $c = (c_1, c_2, c_3)$ and a triple of production decisions $z = (z_1, z_2, z_3)$. An allocation is feasible if $z \in Z$,

(B.1) $\int c_1(i)\lambda(di) = z_1 + \int y_1(i)dG(y)$

(B.2) $\int c_2(i)\lambda(di) = z_2 + \int y_2(i)dG(y)$ and

$c_3(i) = z_3$ all i.

The information structure and the trading arrangements of the economy are as follows. Preferences, technology and the distribution governing endowments are common knowledge. Individual agents' endowments and consumption decisions, however, are privately observed. Individuals can conduct trades either with the government or privately amongst themselves in a competitive market place. Private trades are not observed by the government.

The government trades the two private goods with agents at the production location. During the trading period agents can trade as many times as they wish either at the competitive marketplace or with the government at the production location. Each time an individual arrives at the production location the government observes the amount of private goods he wishes to trade. However, the government cannot observe the identity of the trader and, in particular, it does not know if the individual has traded with it previously during the trading period.

Applying the Revelation Principle [see Myerson (1979), Harris and Townsend (1981), and Dasgupta, Hammond, and Maskin (1982)] to this environment it follows that the Bayesian Nash equilibrium outcome of any mechanism is also a truth-telling Bayesian Nash equilibrium of the revelation mechanism.

In the revelation mechanism each consumer reports his endowment vector. Clearly, incentive feasibility requires that allocations depend only upon the endowments of consumers (and not upon their "names"). Thus, an allocation is a pair of functions c_1 and c_2 which map the endowments of each consumer into his consumption of private goods, a scalar c_3, and a triple of production decisions $z = (z_1, z_2, z_3)$. This allocation is resource feasible if $z \in Z$,

(B.3) $\qquad \int c_n(y) dG(y) = z_n + \int y_n dG(y)$ for $n = 1, 2$ and $c_3 = z_3$.

It will be useful to rewrite an allocation as follows. Let $t_1(y)$ and $t_2(y)$ denote the net trade functions of the government with a consumer who reports an endowment vector y. The consumption of private goods of a consumer with endowment y is

$$c_1(y) = y_1 + t_1(y) \text{ and } c_2(y) = y_2 + t_2(y).$$

An allocation can be represented as a pair of net trade functions $t_1(y)$, $t_2(y)$ and a triple of production decisions $z = (z_1, z_2, z_3)$ (where z_3 is understood to be each agent's consumption of the public good). This allocation is feasible if $z \in Z$ and for $n = 1, 2$

(B.4) $\qquad \int t_n(y) dG(y) = z_n$.

Now let p denote the price of the second good relative to the fist in the competitive market place.

A consumer with an endowment $y = (y, y_2)$ can trade in the private market to any endowment $\hat{y} = (\hat{y}_1, \hat{y}_2)$ that satisfies

(B.5) $\qquad \hat{y}_1 + p\hat{y}_2 = y_1 + py_2,$

and report $\hat{y}$ as his true endowment.   Thus, incentive compatibility requires that

(B.6) $\qquad U(y_1 + t_1(y), y_2 + t_2(y), y_3) \geq U(y_1 + t_1(\hat{y}), y_2 + t_2(\hat{y}), y_3)$

for all $\hat{y}$ that satisfy (B.5).   Consequently, incentive compatible net trade functions can vary only with the value of consumer's income evaluated at private market prices.

Next we claim that incentive compatibility requires that the net trade functions satisfy

(B.7) $\qquad t_1(y) + pt_2(y) = 0$ all y.

To see this, suppose first that for some y, $t_1(y) + pt_2(y) > 0$.   Then any consumer who either starts with or who trades on the private market to the vector y can trade once with the government and leave with a strictly higher income.   The consumer can repeatedly show up and conduct this same trade each time and increase his income to infinity.   Obviously, such a mechanism will not be both incentive compatible and resource feasible.   Suppose next, that for some y, $t_1(y) + pt_2(y) < 0$.   Then any consumer with endowment y will be strictly better off by reporting an endowment of zero to the government and conducting all trades in the private market place.   This contradicts the requirement that truth-telling be an equilibrium.   Thus (B.7) holds.

Finally, since any consumer can undertake trades in the private market place, any incentive compatible net trade functions must satisfy

(B.8) $U(y_1+t_1(y),y_2+t_2(y),y_3) = \max_{c_1,c_2} U(c_1,c_2,y_3)$

subject to

$$c_1 + pc_2 \le y_1 + t_1(y) + p(y_2+t_2(y)).$$

Using (B.6), the constraints on consumption allocations imposed by (B.8) reduce to

(B.9) $\dfrac{U_2}{U_1} = p$

and

(B.10) $c_1(y) + pc_2(y) = y_1 + py_2$

where the derivatives in (B.9) are evaluated at $(c_1(y),c_2(y),y_3)$.

Thus, the constraints on allocations by resource feasibility and incentive compatibility can be summarized by $z \in Z$

(B.11) $z_n = \int (y_n-c_n(y)) \, dG(y)$ for $i = 1, 2,$

and $z_3 = y_3$, together with (B.9) and (B.10).

B2. Open Economy

We consider a variant of the model in section 4 in which there are a continuum of consumers in each country and in which the only source of revenues for each government is a tariff. We demonstrate the equivalence between the set of outcomes of a tariff equilibrium and the set of outcomes induced by incentive feasible mechanisms. We will consider the limiting replica economy in which each country takes world prices as given. It will be clear how to generalize the analysis to characterize the whole sequence of replica economies.

Consider a specific country. In it there are a continuum of consumers with the same endowments and preferences as in section (B.1). Let q denote the world relative price of the second good to the first. Assuming the government raises revenues through a tariff τ, the domestic price p satisfies $p = q(1-\tau)$. Let the government have sole access to a technology that transforms the first private good into a country specific public good at a one-for-one rate.

Now letting $c_1(y)$, $c_2(y)$ denote the consumption of a consumer with endowments $y = (y_1, y_2)$ we can write the budget constraint of the government as

$$(B.12) \quad g \leq q\tau \int \left(y_2 - c_2(y)\right) dG(y).$$

The problem faced by a consumer with endowments y is to choose $c_1(y)$, $c_2(y)$ to solve

$$\max U\left(c_1(y), c_2(y), g\right)$$

subject to

$$(B.13) \quad c_1(y) + q(1-\tau)c_2(y) = y_1 + q(1-\tau) y_2.$$

The solution to this problem is summarized by the budget constraint (B.13) and the first order condition

$$(B.14) \quad \frac{U_2}{U_1} = q(1-\tau).$$

The government chooses a tariff rate to maximize social welfare subject to the constraints imposed by the competitive equilibrium. Clearly the government's problem is equivalent to choosing a tariff rate τ, a spending level g, and allocation rules $c_1(y)$ and $c_2(y)$ to maximize welfare subject to (B.12)-(B.14).

We claim that the constraints on the government's problem are equivalent to those imposed by a truth-telling Bayesian Nash equilibrium of a revelation mechanism. To see this, specialize the technology set of the previous section to

$$Z = \{(z_1, z_2, z_3) \mid z_3 \leq z_1 + qz_2\}.$$

In section (B.1) we established that the constraints imposed by resource feasibility and incentive compatibility are $z \in Z$, $z_3 = y_3$ and (B.9)-(B.11). Obviously (B.9) and (B.10) coincide with (B.13) and (B.14). Next using (B.11) and $p = q(1-\tau)$ we can write the technology constraint $z_3 \leq z_1 + qz_2$ as

$$(B.15) \quad z_3 \leq \int (c_1(y)-y_1)dG(y) + \frac{p}{(1-\tau)} \int (c_2(y)-y_2)dG(y).$$

Now add and subtract the term

$$p \int (c_2(y)-y_2)dG(y)$$

from the right-hand side of (B.15). Then, using the budget constraint (B.13) to cancel terms, we can write the resulting equation as

$$z_3 \leq \frac{p}{(1-\tau)} \int (c_2(y)-y_2)dG(y) - p \int (c_2(y)-y_2)dG(y).$$

Collecting terms, identifying z_3 with g, and using $q = p/(1-\tau)$ we can rewrite this equation as

$$g \leq q\tau \int (c_2(y)-y_2)dG(y).$$

Thus, the two sets of constraints are equivalent.

So far we have considered arbitrary utility functions. Suppose now that the utility function of each consumer is

$$u(c_1, c_2, g) = v(c_1, c_2) + w(g)$$

where v is homogenous of degree one. Recall that any homothetic utility function can be represented by a linear homogenous utility function. As will become clear the key property we use in the following is homotheticity; we assume homogeneity for convenience.

Let the government maximize an equally weighted sum of consumer's utility. We claim that in any incentive feasible mechanism such a government will choose the same policies as a government which maximizes the "representative" agent's utility function. That is, we show in any incentive feasible mechanism

(B.16) $\int \left[v(c_1(y), c_2(y)) + w(g) \right] dG(y) = v\left(\int c_1(y), \int c_2(y) \right) + w(g).$

Recall that in any incentive feasible mechanism the marginal rate of substitution between the private consumption goods is equal for all consumers. From homotheciticity this implies

$c_1(y)/c_2(y) = k$ for all y.

Thus we can write the left-side of (B.16) as

$v(k,1) \int c_2(y) dG(y) + w(g).$

We can write the right-side of (B.16) as

$v\left(\dfrac{\int c_1}{\int c_2}, 1 \right) \int c_2(y) dG(y) + w(g).$

Since $\int c_1 / \int c_2 = k \, (\int c_2 / \int c_2) = k$, we are done.

References

Backus, D., Devereux, M., and Purvis. D. 1986. A positive theory of fiscal policies in open economies. Forthcoming in <u>International Aspects of Fiscal Policy</u> edited by J. Frenkel.

Dasgupta, P., Hammond, P., and Maskin E. 1979. The implementation of social choice rules: Some general results on incentive compatibility. <u>Review of Economic Studies</u> 46: 185-216.

Devereux, M. 1986. The international coordination of fiscal policy and the terms of trade: An example. mimeo, University of Toronto.

Frenkel, J., and Razin, A. 1985. Government spending, debt, and international economic interdependence. <u>The Economic Journal</u> no. 379 (September): 619-37.

_____. 1986. Fiscal policies in a world economy. Forthcoming in <u>International Aspects of Fiscal Policy</u> edited by J. Frenkel.

Hamada, K. 1986. Strategic aspects of international fiscal interdependence. Vol. 37, no. 2: 165-80.

Hammond P. 1987. Markets as constraints: Multilateral incentive compatibility on continuum economics. <u>Review of Economic Studies</u> LIV: 399-412.

Harris, M. and Townsend, R. 1981. Resource allocation under asymmetric information. <u>Econometrica</u> 49:33-64.

Judd, K.L. 1985. The law of large numbers with a continuum of iid random variables. <u>Journal of Economic Theory</u> 35: 19-25.

Kehoe, P. 1986a. Coordination of fiscal policies in a world economy. Research Department Staff Report 98. Minneapolis: Federal Reserve Bank of Minneapolis (forthcoming <u>Journal of Monetary Economics</u> 1987).

_____. 1986b. International policy cooperation may be undesirable. Research Department Staff Report 103. Minneapolis: Federal Reserve Bank of Minneapolis.

Lucas, R., and Stokey, N. 1983. Optimal fiscal and monetary policy in an economy without capital. <u>Journal of Monetary Economics</u> no. 12: 55-93.

Mantel, R. 1971. The welfare adjustment process: Its stability properties. <u>International Economic Review</u> 12: 415-30.

Myerson, R. 1979. Incentive Compatibility and the bargaining problem. <u>Econometrica</u> 47: 61-74.

Negishi, T. 1960. Welfare economics and existence of an equilibrium for a competitive economy. <u>Metroeconomica</u> 12: 92-7.

Persson, T., and Svensson, L. 1986. International borrowing and time-consistent fiscal policy. <u>The Scandinavian Journal of Economics</u> vol. 88, no. 1: 273-95.

Rogoff, K. 1985. Can international monetary policy cooperation be counterproductive? <u>Journal of International Economics</u> vol. 18, no. 3-4 (May): 199-217.

Uhlig, H. 1987. A law of large numbers for large economies. Federal Reserve Bank of Minneapolis, Working Paper no. 342.