Federal Reserve Bank of Minneapolis
Research Department Working Paper

ADVERSE SELECTION, AGGREGATE UNCERTAINTY,
AND THE NATURE OF EQUILIBRIUM CONTRACTS

Bruce D. Smith and Michael J. Stutzer*

Working Paper 344

March 1987

ABSTRACT

A standard adverse selection environment is modified by the addition of aggregate uncertainty. With this simple alteration of a basic adverse selection environment, Nash equilibrium contracts take on a fairly rich structure. In particular, mutual and nonmutual forms of organization coexist or, under an alternative interpretation, contingent and uncontingent debt coexist in equilibrium. In addition, the introduction of aggregate uncertainty has implications for the existence of Nash equilibria and for the nature of Pareto optimal allocations in adverse selection settings.

*Smith, Carnegie-Mellon University and University of California at Santa Barbara; Stutzer, Federal Reserve Bank of Minneapolis.

Since their introduction, the adverse selection environments (and related equilibrium concepts) developed by Rothschild and Stiglitz [1976] and Wilson [1977], and the signaling models of Spence [1973, 1974] have attracted continuing attention on the part of the economics profession. This attention has largely taken two forms. First, there has been considerable interest in altering the equilibrium concepts and/or the nature of the games employed in these contexts in order to overcome problems of nonexistence or multiplicity of equilibria. Second, there have been a number of extensions of these models (essentially intact) to applications in new areas. These efforts largely have ignored the effects of altering the underlying economic environments in these settings, however.

Recent research has begun to examine the results of fairly basic alterations in economic environments in these contexts. Riley [1985], for instance, and Judd [1984] examine the implications for the nature of an equilibrium (when one exists), for existence of equilibrium, and for optimality of equilibrium when agents are given the option of nonparticipation in certain markets (i.e., when they have an alternative opportunity). This simple modification of a standard adverse selection environment turns out to have important implications for such questions. Given this fact, it would seem to be important to investigate the effects of other kinds of straightforward alterations of standard adverse selection-signaling environments.[1]

This paper undertakes an investigation of one such alteration. In particular, a standard variation of a Rothschild-Stiglitz [1976] adverse selection environment is considered, in this case in the context of a model of borrowing and lending. The analysis also employs the game and equilibrium concept advanced by Rothschild and Stiglitz. The single variation on their environment is as follows. Borrowers are of several types, indexed by i. A

borrower of type i has a probability of repaying his loan $p_i(s)$, where s is an aggregate shock. There are also a large number of borrowers of each type, so that contingent on the realization of s, a fraction $p_i(s)$ of type i agents repay their loans. In general $p_i(s_1) \neq p_i(s_2)$ ∀ i. Moreover, the realization of s occurs after loan contracts are set. Lenders in the model are risk neutral. Hence, as in Rothschild and Stiglitz, competition among lenders will dictate that loan contracts earn zero expected profits. However, in contrast to the Rothschild-Stiglitz setting, ex post profits of lenders will be random variables.

While this alteration of the Rothschild-Stiglitz environment seems fairly innocuous (especially since it will occur in a context in which all agents are risk neutral), it turns out to have important implications for the nature of the Nash equilibrium studied by Rothschild and Stiglitz. Specifically, the loan market will, under a simple nondegeneracy condition on the values $p_i(s)$, consist of two types of lenders. One type, which will service "high-risk" borrowers, will be organized in exactly the same way as Rothschild-Stiglitz firms, and will offer the same types of contracts. The second type of lender is organized as a "mutual" enterprise. In particular, this type of lender (who services "low-risk" borrowers) functions not only by offering standard Rothschild-Stiglitz like interest rate-loan quantity contracts, but also by paying dividends to borrowers. Or, to place a somewhat different interpretation on the result, lenders who service "high-risk" borrowers purchase debt that specifies repayments which are not contingent on the prevailing aggregate state of nature. Lenders who service "low-risk" borrowers purchase debt that specifies (nontrivially) state contingent repayment, in the form of dividends which depend on an aggregate shock. Thus, the simple change in the specification of the environment described above fundamentally

changes the nature of equilibrium "contracts" (or industrial organization). This change also has implications for results on the existence of equilibria, and for the nature of optimal resource allocations, which are discussed below.

The basic result of interest, however, would seem to be the prediction that "mutual" forms of organization should coexist in the same markets with "nonmutual" forms of organization, or that both contingent and uncontingent debt should coexist, since this is in fact what is observed in many intermediation contexts. For example, farm credit cooperatives coexist with commercial farm banks, while mutual savings banks coexist with stockholder-owned savings and loan associations, and mutual insurance firms coexist with insurance firms organized in other ways.

The main part of the analysis proceeds in the context of a loan market. This is for simplicity, since a loan market setting permits a specification where all agents have linear preferences (in contrast with an insurance setting). While equilibrium contracts are not readily computed in an insurance context, i.e., when some agents have nonlinear preferences, it will be shown that our results can be extended to it. Hence the analysis also helps explain the coexistence of mutual insurance firms with investor-owned insurance firms.

Finally, the modification discussed above does not just have implications for the nature of the Nash equilibrium that emerges in these markets, but also has more fundamental implications. An example of one such implication is related to the set of Pareto optimal resource allocations for these environments. Specifically, Prescott and Townsend [1984] discuss the role of allocations which are contingent on the outcome of lotteries. They show that in the original Rothschild-Stiglitz environment, Pareto optimal allocations can depend only trivially on lotteries. Here, in an insurance context, risk

averse agents must bear some risk in any Pareto optimum, even when there is a risk neutral agent who is able to absorb all such risk. Hence the introduction of aggregate shocks has implications that are independent of the equilibrium concept employed.

The scheme of the paper is as follows. Section I describes the (loan market) environment, and examines its behavior when lenders are restricted to offer only Rothschild-Stiglitz like "loan quantity-interest rate" contracts. Section II discusses equilibrium contracts when this restriction is relaxed. Section III discusses how the results obtained can be extended to an insurance market context. Section IV concludes.

I. A Loan Market Model

A. The Environment

The economy considered is one in which there are two dates ($t = 1, 2$). At $t = 1$ there are three types of economic actors. The first type is a set of agents (lenders) who have a positive endowment of a single good when young, and no endowment when old. These agents are assumed to have a sufficiently large first period endowment so that any one of them could service all of the borrowers in the model. (This amounts to assuming that lenders are able to raise all the funds they require without affecting the price of these funds to themselves.) The number of lenders is greater than one, but finite. Let c_t denote date t consumption. Then lenders have utility functions $V(c_1, c_2)$ defined on R_+^2 of the form.

$$V(c_1, c_2) = c_1 + c_2.$$

In addition there are a large number of borrowers, who are divided into two "types." Type is indexed by $i = 1, 2$. The fraction who are of type 1 is denoted by θ. Borrowers have no endowment of the good when young. When

old their endowment, denoted w, is a random variable; $w \in \{y,0\}$, $y > 0$. Thus borrowers either have a positive endowment when old, or not. Receipt of the old age endowment is observable, so loans must be repaid if $w = y$. Obviously, any borrower with $w = 0$ defaults on his loan.

For a borrower of type i, define

$$p_i(s) \equiv prob\ [w=y|s]$$

where s is a random aggregate shock; $s \in \{1,2\}$. Let $\pi(s)$ denote the probability of state s, so that $1 \geq \pi(s) \geq 0$ ∀ s, and $\sum \pi(s) = 1$. The actual value of s is realized at the beginning of period 2, so it is not known when borrowing occurs. Given the realization of s in period 2, then, since there are a "large number" of agents of each type, a fraction $p_i(s)$ of type i agents receive a positive endowment.[2] It is assumed that $p_2(s) > p_1(s)$, $s = 1, 2$, so type 2 agents are "low-risk" borrowers, independently of s. It is also assumed that $p_i(2) > p_i(1)$, $i = 1, 2$, so that state $s = 2$ is a "good" state with respect to aggregate endowments. Borrowers of type i have utility functions $U_i(c_1,c_2)$ defined on R^2_+ of the form

$$U_i(c_1,c_2) = \beta_i c_1 + c_2,$$

with $\beta_i > 1$; $i = 1, 2$. Finally, it is assumed that

$$\beta_1/\beta_2 > \sum_s \pi(s)p_1(s)/\sum_s \pi(s)p_2(s).$$

B. A Simple Borrowing/Lending Game

As a point of reference, this section develops a game played by lenders such that the equilibrium which emerges is the Rothschild-Stiglitz equilibrium (when this exists). In this game the strategies of lenders are interest rate-loan (quantity) pairs. Let x_i denote the quantity borrowed by a

representative type i borrower, and R_i the (gross) interest rate offered to type i agents. Then lenders announce (R_i, x_i) pairs, i = 1, 2. These announcements are made taking the announced interest rate-loan pairs of all other lenders as given. Finally, an analogue of the Rothschild-Stiglitz-Wilson assumption that rules out cross-subsidization is imposed. In particular, each lender is restricted to offer only a single (R,x) pair. Then, each borrower selects his most preferred pair from the entire set of announced pairs (R_i, x_i).

The equilibrium concept imposed on this game is the Rothschild-Stiglitz Nash equilibrium concept.

<u>Definition</u>. A set of announced interest rate-loan pairs (R_i^*, x_i^*), i = 1, 2, is a Nash equilibrium if, given these announcements, no lender has an incentive to announce (earns an expected profit by announcing) an alternate pair.

Notice that (R,x) pairs are set prior to the realizations of the aggregate state and the endowments of individuals, so that these random variables are realized after loans are made. Then the ex post profits of a lender offering the interest rate-loan pair (R,x) are:

(1) $\psi_i(s) \equiv [p_i(s)R-1]x$

if his offer attracts only type i agents, and

(2) $\psi(s) \equiv \{[\theta p_1(s)+(1-\theta)p_2(s)]R-1\}x$

if his offer attracts both types of agents in their population proportions. Clearly, given the preferences of lenders, each lender's objective when announcing (R,x) pairs is to maximize the quantity $\sum_s \pi(s)\psi(s)$, given the announcements of other lenders.[3]

Finally, announced (R,x) pairs are required to be incentive compatible. In particular, if type 1 agents are meant to accept the loan contract (R_1, x_1) and type 2 agents the loan contract (R_2, x_2), then these contracts must satisfy the following self-selection conditions:

(3) $\qquad \beta_1 x_1 + \sum_s \pi(s) p_1(s) [y - R_1 x_1] \geq \beta_1 x_2 + \sum_s \pi(s) p_1(s) [y - R_2 x_2]$

(4) $\qquad \beta_2 x_2 + \sum_s \pi(s) p_2(s) [y - R_2 x_2] \geq \beta_2 x_1 + \sum_s \pi(s) p_2(s) [y - R_1 x_1].$

Also, given the nature of these contracts, feasibility of an (R,x) pair requires that

(5) $\qquad y \geq Rx.$

C. Equilibrium

The features of equilibrium (R,x) pairs (if an equilibrium exists) are exactly as in Rothschild and Stiglitz. In particular, self-selection of borrowers by contract accepted occurs in equilibrium, or in other words $(R_1^*, x_1^*) \neq (R_2^*, x_2^*)$. In addition, each contract (R_i^*, x_i^*) earns zero expected profits.[4] Finally, each (R_i^*, x_i^*) pair must be maximal for type i agents among the set of contracts that earns nonnegative expected profits when offered and that are consistent with self-selection. Thus, equilibrium contracts are readily derived here.

To begin, expected (per capita) profits for a lender offering the pair (R_i, x_i) are

(6) $\qquad \left[\sum_s \pi(s) p_i(s) R_i - 1 \right] x_i = \sum_s \pi(s) \psi_i(s).$

In equilibrium $\sum_s \pi(s) \psi_i(s) = 0$; $i = 1, 2$. Thus, in equilibrium,

(7) $\qquad R_i^* = \left[\sum_s \pi(s) p_i(s) \right]^{-1} \equiv \bar{R}_i$; $i = 1, 2.$

Second, again as in Rothschild and Stiglitz, only one of the self-selection conditions (3) and (4) holds with equality in equilibrium. In particular, low-risk (i.e., type 2) borrowers have no incentive to claim to be high-risk borrowers. Hence (R_1^*, x_1^*) (if an equilibrium exists) is unconstrained by (4). Thus, since (R_1^*, x_1^*) must be maximal for type 1 agents, x_1^* is the solution to the problem

$$\max \; \beta_1 x_1 + \sum_s \pi(s) p_1(s)[y - R_1^* x_1]$$

subject to

(8) $y \geq R_1^* x_1.$

From (7), $R_1^* = \left[\sum_s \pi(s) p_1(s)\right]^{-1}$, so this problem is equivalent to choosing x_1 to maximize $(\beta_1 - 1) x_1$ subject to (8). As $\beta_1 > 1$, the solution to this problem is:

(9) $x_1^* = y/R_1^*$

with second period consumption equaling zero. For future reference, the expected value of utility for type 1 agents under this contract is

$$(\beta_1 - 1)(y/R_1^*) + \sum_s \pi(s) p_1(s) y = \beta_1 (y/R_1^*),$$

where we have used $R_1^* = \left[\sum_s \pi(s) p_1(s)\right]^{-1}$.

As with type 1 contracts, type 2 contracts have $R_2^* = \bar{R}_2$ [with $\bar{R}_2$ defined by (7)]. Also, as before, type 2 contracts must be maximal for type 2 agents given $R_2^* = \bar{R}_2$, and given that (R_2^*, x_2^*) must satisfy (3) [given the announcements (R_1^*, x_1^*)]. It is straightforward to show that (3) must always bind on the determination of (R_2, x_2). Thus x_2 is the solution to

$$\max \; \beta_2 x_2 + \sum_s \pi(s) p_2(s)[y - R_2^* x_2]$$

subject to

(10) $\qquad (\beta_1-1)x_1^* + \sum_s \pi(s)p_1(s)y = \beta_1 x_2 + \sum_s \pi(s)p_1(s)[y-R_2^*x_2]$

(11) $\qquad y \geq R_2^*x_2.$

It will be convenient to rewrite (10) as

(10') $\qquad (\beta_1-1)x_1^* = \left[\beta_1-(\bar{R}_2/\bar{R}_1)\right]x_2$

where we have used $\sum_s \pi(s)p_1(s) = 1/\bar{R}_1$ from (7). Clearly, then,

$$x_2^* = \frac{(\beta_1-1)x_1^*}{\beta_1 - (\bar{R}_2/\bar{R}_1)}.$$

Since $p_2(s) > p_1(s)$ ∀ s, (7) implies that $\bar{R}_2 = R_2^* < R_1^* = \bar{R}_1$. Then $x_2^* < x_1^*$, so that the contract (R_2^*, x_2^*) clearly satisfies the feasibility requirement (11).

D. Existence of Equilibrium

The existence issue here is again essentially identical to that in Rothschild and Stiglitz. In particular, the contracts derived above were the maximal contracts for type 1 and 2 borrowers that were consistent with the occurrence of self-selection, with lenders earning nonnegative profits, and with each lender restricted to the announcement of a single contract. Therefore it is not possible for any lender to offer a single contract which attracts agents of only one type [given the presence of the announced contracts (R_i^*, x_i^*)] and earns a nonnegative profit. Thus if the contracts (R_i^*, x_i^*) are not equilibrium contracts, this must be because some lender has an incentive to announce a contract (R,x) which is designed to attract borrowers of all types. In other words, the contracts (R_i^*, x_i^*) are not equilibrium contracts if and only if there exists a contract (R,x) such that

(12) $\qquad \beta_i x + \sum_s \pi(s) p_i(s)[y-Rx] \geq \beta_i x_i^*$

$\qquad\qquad + \sum_s \pi(s) p_i(s)[y-R_i^* x_i^*]; \quad i = 1, 2,$

with strict inequality for some i, and such that

(13) $\qquad \sum_s \pi(s)[\theta p_1(s)+(1-\theta)p_2(s)]Rx - x \geq 0.$

Define $\bar{R} = [\sum_s \pi(s)[\theta p_1(s)+(1-\theta)p_2(s)]]^{-1}$. Then we may rewrite (12) and (13) as

(12') $\qquad (\beta_i - R/\bar{R}_i) x \geq (\beta_i - 1)x_i^*; \quad i = 1, 2,$

and

(13') $\qquad R \geq \bar{R}.$

It is now easy to derive conditions sufficient for (R_i^*, x_i^*) to be equilibrium loan contracts. In particular (12') cannot be satisfied if there is no value x such that

(14) $\qquad [\beta_i - (\bar{R}/\bar{R}_i)]x \geq (\beta_i - 1)x_i^*$

for some i. There will be no such x, for instance, if $\beta_i < \bar{R}/\bar{R}_i$ for some i. Since $\beta_i > 1 \; \forall \; i$ and $\bar{R}_1 > \bar{R} > \bar{R}_2$, a sufficient condition for ruling out (14) is

(15) $\qquad \beta_2 < \bar{R}/\bar{R}_2.$

(15) rules out a pooling contract which earns nonnegative profits, and which attracts type 2 agents in the presence of the announced contract (R_2^*, x_2^*). Hence (15) is sufficient for the contracts (R_i^*, x_i^*) derived above to be equilibrium contracts.

In the remainder of the paper contracts more complicated than (R_i^*, x_i^*) will be considered. When the set of contracts that lenders can offer is augmented, as below, then a stronger condition than (15) can be used to rule out the possibility that any lender will have an incentive to offer a pooling contract. In particular, it is henceforth assumed that

$$(15') \qquad p_2(s) \geq \beta_2 \left[\theta p_1(s) + (1-\theta) p_2(s) \right]; \quad s = 1, 2.$$

This condition implies (15), and in the sequel is a sufficient condition for the existence of an equilibrium.

II. Sorting by the Nature of Firm Organization

A. The Game

In this section the strategies (or contract offers) of firms are allowed to be more complex than in Section I. The strategies of firms now consist of contract offers that specify a (gross) interest rate R, as before, a loan quantity x, as before, and a state-contingent fraction of ex post profits to be rebated (paid out as dividends) to borrowers $\alpha(s)$.[5] As before, each lender is permitted to offer only a single contract. Thus, under assumption (15') any equilibrium contracts must induce self-selection by contract accepted and an equilibrium exists [as profitable pooling contracts will be ruled out by (15')].

To be more specific about the nature of these contracts, any announced contract which attracts type i agents specifies a loan quantity x_i, and a (noncontingent) repayment $R_i x_i$. The ex post (per capita) profits received by a lender offering this contract are (as before)

$$\psi_i(s) = \left[p_i(s) R_i - 1 \right] x_i.$$

Moreover, the contract specifies that each type i agent receives a dividend (possibly negative) of $\alpha_i(s)\psi_i(s)$ if (and only if) his loan has been repaid. (Defaulters do not share in dividends for simplicity.) Then total (per capita) dividend payments by this lender in state s are $p_i(s)\alpha_i(s)\psi_i(s)$, since fraction $p_i(s)$ of type i agents do not default in state s. $\alpha_i(s)$ is restricted to be nonnegative, and since lenders have no endowment when old, $\alpha_i(s) \leq 1/p_i(s)$ if $\psi_i(s) > 0$. Finally, given the risk neutrality of lenders, they care only about expected profits net of dividends when they announce contracts. Hence their objective is to maximize

$$(16) \qquad \sum_s \pi(s)\left[1-p_i(s)\alpha_i(s)\right]\psi_i(s) \equiv \tilde{\psi}_i(s)$$

by choice of contract, subject to the announcements of other lenders and considerations of self-selection.

In order to discuss incentive compatibility, it is useful to introduce some additional notation. Therefore, let c_1^i denote the consumption of type i borrowers in period 1, and $c_2^i(s)$ the consumption of type i agents in period 2 given the realization s. Then a contract specifying R_i, x_i, $\alpha_i(1)$, and $\alpha_i(2)$ implies consumption values

$$(17) \qquad c_1^i = x_i$$

$$(18) \qquad c_2^i(s) = y - R_i x_i + \alpha_i(s)\psi_i(s), \text{ if } w = y$$

$$c_2^i(s) = 0 \text{ otherwise.}$$

A set of announced contracts is incentive compatible, then, if they are such that

$$(19) \qquad \beta_1 c_1^1 + \sum_s \pi(s)p_1(s)c_2^1(s) \geq \beta_1 c_1^2 + \sum_s \pi(s)p_1(s)c_2^2(s)$$

and

(20) $\qquad \beta_2 c_1^2 + \sum_s \pi(s)p_2(s)c_2^2(s) \geq \beta_2 c_1^1 + \sum_s \pi(s)p_2(s)c_2^1(s).$

where consumption values are obtained from the contract specifications by (17) and (18). Conditions (19) and (20) may be rewritten in terms of the original contracts:

(19') $\qquad \beta_1 x_1 + \sum_s \pi(s)p_1(s)[y-R_1x_1] + \sum_s \pi(s)p_1(s)\alpha_1(s)\psi_1(s) \geq \beta_1 x_2$

$\qquad\qquad + \sum_s \pi(s)p_1(s)[y-R_2x_2] + \sum_s \pi(s)p_1(s)\alpha_2(s)\psi_2(s)$

(20') $\qquad \beta_2 x_2 + \sum_s \pi(s)p_2(s)[y-R_2x_2] + \sum_s \pi(s)p_2(s)\alpha_2(s)\psi_2(s) \geq \beta_2 x_1$

$\qquad\qquad + \sum_s \pi(s)p_2(s)[y-R_1x_1] + \sum_s \pi(s)p_2(s)\alpha_1(s)\psi_1(s).$

Also, realized (per capita) profits net of dividends may be written in terms of consumption values as

$$\tilde{\psi}_i(s) = p_i(s)[y-c_2^i(s)] - c_1^i$$

and (ex ante) expected profits, in terms of consumption values, are

$$\sum_s \pi(s)\tilde{\psi}_i(s) = \sum_s \pi(s)p_i(s)[y-c_2^i(s)] - c_1^i.$$

As before, suppose type i contracts are constructed so as to be maximal for type i agents among the set of contracts that earn nonnegative expected profits (when offered singly), and that are consistent with self-selection. These will be equilibrium contracts, since such a construction rules out the possibility of profitable alternative offers to agents of one type, and since (15') rules out any incentive for lenders to announce pooling contracts. Attention is now directed to the construction of these contracts.

B. Equilibrium

The equilibrium contracts derived here will be denoted $[\hat{R}_i, \hat{x}_i, \hat{\alpha}_i(1), \hat{\alpha}_i(2)]$. The primary result of this section is:

Proposition 1. If the values $p_i(s)$ satisfy

$$(21) \qquad \frac{p_1(1)}{p_1(2)} \ne \frac{p_2(1)}{p_2(2)},$$

then Nash equilibrium contracts have $\hat{R}_1 = \bar{R}_1 = R_1^*$, $\hat{x}_1 = x_1^*$, $\hat{\alpha}_1(s) = 0$, $s = 1, 2$, and $\hat{\alpha}_2(s) \ne 0$ for some s.

Thus any Nash equilibrium involves some payment of dividends by lenders to borrowers (i.e., some mutual aspect to the contractual arrangement), since $\hat{\alpha}_2(s) \ne 0$. In addition, in equilibrium some lenders (those serving type 2 borrowers) will offer contracts with this mutual aspect, and some others will pay no dividends ($\hat{\alpha}_1(s) = 0$; $s = 1, 2$). Thus, mutual-type contracts will coexist with other types of contracts in equilibrium.

The proof of proposition 1 will simply be a construction of equilibrium contracts. For the purposes of this construction, it will be convenient to work with consumption values c_1^i, $c_2^i(1)$, and $c_2^i(2)$. As before, it will be the case that the contracts offered to type 1 borrowers in equilibrium will be unconstrained by (20), the self-selection constraint for type 2 borrowers. Thus, the consumption values implied by these contracts must solve the following problem:

(A) $\max \beta_1 c_1^1 + \sum_s \pi(s)p_1(s)c_2^1(s)$

subject to

(22) $\sum_s \pi(s)p_1(s)[y-c_2^1(s)] - c_1^1 = 0$

(23) $c_2^1(s) \geq 0,\ c_1^1 \geq 0,$

where equation (22), of course, is the condition that the contracts offered to type 1 agents must earn zero expected profits net of dividend payments.

The solution to problem (A) has $c_2^1(1) = c_2^1(2) = 0$, and $c_1^1 = \sum_s \pi(s)p_1(s)y \equiv y/\overline{R}_1 = y/R_1^*$, since $\beta_1 > 1$. Given this solution, it is possible to reconstruct equilibrium contracts from (17) and (18). In particular, $c_1^1 = \hat{x}_1 = y/\overline{R}_1 = x_1^*$, i.e., the quantity loaned to type 1 agents is the same as for the game with a simpler set of contracts discussed in Section I. From (18),

$$c_2^1(1) = 0 = y - \hat{R}_1\hat{x}_1 + \hat{\alpha}_1(1)\psi_1(1)$$

$$c_2^1(2) = 0 = y - \hat{R}_1\hat{x}_1 + \hat{\alpha}_1(2)\psi_1(2).$$

Thus

(24) $\hat{\alpha}_1(1)\psi_1(1) = \hat{\alpha}_1(2)\psi_1(2),$

or

(24') $\hat{\alpha}_1(1)[p_1(1)\hat{R}_1-1]\hat{x}_1 = \hat{\alpha}_1(2)[p_1(2)\hat{R}_1-1]\hat{x}_1.$

We will prove by contradiction that $\hat{\alpha}_1(1) = \hat{\alpha}_1(2) = 0$. First, suppose that $\hat{\alpha}_1(1) \neq 0$ while $\hat{\alpha}_1(2) = 0$. Then, because $\hat{x}_1 = y/\overline{R}_1 \neq 0$, (24') implies that $\hat{R}_1 = 1/p(1)$. Substituting $\hat{x}_1$, $\hat{R}_1 = 1/p_1(1)$, $\overline{R}_1$ (from 7) and $\psi_1(1) = 0$ into the expression for $c_2^1(1)$ above yields

$$y\left[1- \frac{\pi(1)p_1(1)+\left[1-\pi(1)\right]p_1(2)}{p_1(1)}\right] = 0.$$

But this implies $p_1(2)/p_1(1) = 1$, contrary to assumption. Hence this is impossible. Second, the same contradiction arises if $\hat{\alpha}_1(2) \neq 0$ while $\hat{\alpha}_1(1) = 0$, because (24') then implies $\hat{R}_1 = 1/p_1(2)$, etc. Then it must be the case that $\hat{\alpha}_1(1)$ and $\hat{\alpha}_1(2)$ are both positive. Note that (24) and $\hat{x}_1 \neq 0$ imply that $\psi_1(1) \neq 0$ and $\psi_1(2) \neq 0$ (since if $\psi_1(1) = \psi_1(2) = 0$ held, $\hat{R}_1 = 1/p_1(1)$ and $\hat{R}_1 = 1/p_1(2)$ would also hold, yielding the same contradiction as above). But then $\psi_1(1)$ and $\psi_1(2)$ must have the same sign. Since in equilibrium expected profits net of dividends must be zero, it must then be the case that $\hat{\alpha}_1(s) = 1/p_1(s)$, $s = 1, 2$. However, substitution of $\hat{\alpha}_1(s) = 1/p_1(s)$ into (24') again implies $p_1(1) = p_1(2)$, contrary to assumption. Therefore, $\hat{\alpha}_1(1) = \hat{\alpha}_1(2) = 0$, i.e., lenders who service high-risk borrowers do not pay dividends. Finally, to complete the characterization of equilibrium contracts, we have seen that $\hat{x}_1 = x_1^*$. Then, since expected profits must be zero, $\hat{R}_1 = \bar{R}_1 = R_1^*$.

It remains to derive equilibrium contracts for type 2 agents. As above, these contracts must be maximal for type 2 agents among the set of contracts earning nonnegative profits (when accepted by type 2 borrowers) and which are consistent with the self-selection constraint (19). Thus, consumption values resulting from these contracts must solve the following problem:

(B) $\max \beta_2 c_1^2 + \sum_s \pi(s)p_2(s)c_2^2(s)$

subject to

(25) $\beta_1 c_1^1 = \beta_1 c_1^2 + \sum_s \pi(s)p_1(s)c_2^2(s)$

(26) $\sum_s \pi(s)p_2(s)\left[y-c_2^2(s)\right] - c_1^2 = 0$

(27) $c_1^2, \; c_2^2(s) \geq 0.$

(25) results from substituting the solution of problem A into (19), and (26) is the zero expected profit condition.

Solving (26) for c_1^2 and substituting it into the objective function (B) and the constraint (25) reduces (B) to a linear programming problem in the two controls $c_2^2(1)$ and $c_2^2(2)$, with one constraint, (25). Thus, there is always an optimal solution with exactly one of the values $c_2^2(s) = 0$. Which state has zero consumption depends on the sign of $p_1(1)/p_1(2) - p_2(1)/p_2(2)$. When $p_1(1)/p_1(2) < p_2(1)/p_2(2)$ holds, then $c_2^2(2) = 0$, while $c_2^2(1) = 0$ in the other case. The entire solution when $p_1(1)/p_1(2) > p_2(1)/p_2(2)$, for example, is:

(28) $c_1^2 = y/\overline{R}_2 - \pi(2)p_2(2)\gamma$

$c_2^2(1) = 0$

$c_2^2(2) = \gamma$

where

$$\gamma = \beta_1 y [\overline{R}_2^{-1} - \overline{R}_1^{-1}]/\pi(2)[\beta_1 p_2(2) - p_1(2)] > 0,$$

and the inequality follows from the fact that $\overline{R}_1 > \overline{R}_2$, $\beta_1 > 1$, and $p_2(2) > p_1(2)$.[6]

If $p_1(1)/p_1(2) > p_2(1)/p_2(2)$, for instance, any equilibrium contract for type 2 agents must induce the consumption values given in (28). Then, using (17) and (18) to reconstruct the underlying contract, any Nash equilibrium contract $[\hat{R}_2, \hat{x}_2, \hat{\alpha}_2(1), \hat{\alpha}_2(2)]$ must satisfy

(29) $c_1^2 = \hat{x}_2$

(30) $c_2^2(1) = 0 = y - \hat{R}_2\hat{x}_2 + \hat{\alpha}_2(1)[p_2(1)\hat{R}_2 - 1]\hat{x}_2$

(31) $\qquad c_2^2(2) = \gamma = y - \hat{R}_2\hat{x}_2 + \hat{\alpha}_2(2)[p_2(2)\hat{R}_2-1]\hat{x}_2.$

Clearly (29) implies a determinate equilibrium loan quantity. Equations (30) and (31) constitute two equations in the three unknowns $\hat{R}_2$, $\hat{\alpha}_2(1)$, and $\hat{\alpha}_2(2)$. Many resolutions of the resulting indeterminacy are possible. We suggest two "natural" resolutions: (i) One would require interest rates to be set in an "actuarially fair" manner, so that $\hat{R}_2 = \bar{R}_2$, and then (30) and (31) determine $\hat{\alpha}_2(1)$ and $\hat{\alpha}_2(2)$. This implies $\psi_1(1) < 0$, however, while $\hat{\alpha}_2(1) > 0$, which requires type 1 borrowers to receive negative dividend payments when s = 1. In practice mutual forms of organization do not impose negative dividend payments. Thus a more desirable resolution of the indeterminacy is: (ii) to set $\hat{\alpha}_2(1) = 0$, in which case $\hat{R}_2 = y/\hat{x}_2$ and (31) determines the desired positive value for $\hat{\alpha}_2(2)$. Then, the equilibrium type 2 contract has $\hat{R}_2 = y/\hat{x}_2$, $\hat{x}_2 = c_1^2$ [given by (28)], $\hat{\alpha}_2(1) = 0$, and $\hat{\alpha}_2(2) > 0$ (where $\hat{\alpha}_2(2)$ solves (31) given $\hat{R}_2$ and $\hat{x}_2$).[7] In the other case, i.e., when $p_1(1)/p_1(2) < p_2(1)/p_2(2)$, a similar construction results in $c_2^2(2) = 0$ and $\hat{\alpha}_2(1) > 0$. Proposition 1 is thus proven.

C. State Contingent Interest Rates

As noted previously, there are alternative possible specifications for the choices of contracts that can be employed by lenders. One such possibility is to have lenders specify contracts consisting of loan quantities offered to type i agents, x_i, and state contingent (gross) interest payments $R_i(s)$ for type i agents, i.e., for lenders to make variable rate loans. Under this specification, our analysis proceeds as above, with R_i replaced by $R_i(s)$ and with $\hat{\alpha}_1(s) = \hat{\alpha}_2(s) = 0$; s = 1, 2.

As before, equilibrium consumption values continue to be given by the solutions to problems (A) and (B). Then, implied equilibrium contracts

would have $x_i = \hat{x}_i$, $i = 1, 2$. In addition, since $c_2^1(1) = c_2^1(2)$, it would necessarily be the case that $R_1(1) = R_1(2)$, i.e., that type 1 agents would face non-state-contingent interest rates. On the other hand, since $c_2^2(1) \neq c_2^2(2)$, $R_2(1) \neq R_2(2)$ in equilibrium. Thus some borrowers would face fixed interest rates in equilibrium (type 1 borrowers), while other borrowers would face state contingent repayments (type 2 borrowers). Or, in other words, under this respecification of the set of possible contract offers, the coexistence of state contingent and uncontingent contracts is predicted.

D. Implications

The nature of the equilibrium contracts just derived has several implications. First, if $p_i(1) = p_i(2)$; $i = 1, 2$, were to hold, then equilibrium contracts would be simple interest rate-loan quantity contracts, as in Section I. Thus, even though the environment was specified in such a way that the presence of nontrivial aggregate uncertainty might appear to be innocuous (in particular, all agents were assumed to be risk neutral), the nature of equilibrium contracts is quite different in the presence of aggregate disturbances. The differences in these contracts also have further implications, which will now be discussed briefly.

One of these is that the model predicts the coexistence of contracts that specify dividend (Section B) or state contingent (Section C) payments with contracts that specify only noncontingent payments. This is in accordance with observation in the banking, insurance, and other intermediation industries.

Second, the change in the specification of the environment that has been introduced here has implications for the existence of an equilibrium. In Section I, condition (15) was derived as sufficient for the existence of a Nash equilibrium (in pure strategies). On the other hand, suppose that

(32) $\beta_i y/\bar{R} > (\beta_i-1)x_i^* + y/\bar{R}_i$; $i = 1, 2$

holds, where x_i^* is the candidate Nash equilibrium loan quantity for type i
borrowers derived in Section I. Under (32) a Nash equilibrium in pure strate-
gies fails to exist in Section I, since it implies the existence of a pooling
contract which earns zero expected profits, and which is preferred by all
agents to (R_i^*, x_i^*).

However, when firms are permitted to announce the richer contracts
$[R_i, x_i, \alpha_i(1), \alpha_i(2)]$ a Nash equilibrium may exist even when (32) holds. In
particular, if $\beta_i[\theta p_1(s)+(1-\theta)p_2(s)] \geq p_i(s)$; i, s = 1, 2, then $R = \bar{R}$,
$x = y/\bar{R}$, and $\alpha(1) = \alpha(2) = 0$ is the maximal pooling contract for agents of
both types. Then the left-hand side of (32) continues to represent the ex-
pected utility obtained by type i agents under this contract. Also, it is
feasible in the problem (B) to set $c_1^2 = x_2^*$, and $c_2^2(s) = y - R_2^* x_2^*$. Since this
is not the solution, the expected utility of type 2 borrowers exceeds
$(\beta_2-1)x_2^* + y/\bar{R}_2$. Moreover, for sufficiently large values of $\bar{R}/\bar{R}_2$, their
expected utility will also exceed the left-hand side of (32). Hence, when
$\bar{R}/\bar{R}_2$ is chosen appropriately, an equilibrium fails to exist when contracts
must be interest rate-loan pairs, while an equilibrium does exist when divi-
dend payments can be made. Thus, as in Riley [1985], minor modifications of
the standard adverse selection environment have implications for the existence
of Nash equilibria.

III. A Rothschild-Stiglitz Insurance Environment

The results of the previous section do not depend in any way on the
assumed linearity of agents' preferences. In this section a standard
Rothschild-Stiglitz insurance environment with aggregate uncertainty is ana-
lyzed. In particular, it will be assumed that the probability of suffering a

loss is contingent on the realization of some aggregate state variable, which is realized after insurance contracts are written. The analysis of the section then demonstrates that, as before, any equilibrium must involve the coexistence of "mutual" and other forms of organization in insurance markets. It will also be demonstrated that this modification alters the set of Pareto optimal allocations for some economies in the class considered. This result, in turn, indicates that the implications of altering the environment are not induced solely by the choice of an equilibrium concept.

A. The Environment

Consider the insurance environment discussed by Rothschild and Stiglitz. In particular, there are three groups of agents. One group is a set (with a fixed number of members that exceeds one) of sellers of insurance. These will be called insurance companies. Insurance companies are risk neutral, and have some endowment of the single consumption good (which is assumed to be sufficiently large for the purposes of subsequent discussion).

In addition to these insurance companies, there are some risk averse economic agents, who may be divided into two types. All of these agents have identical utility functions $U(c)$ with $U' > 0$, $U'' < 0$ ∀ $c \in R_+$. Each agent receives a random endowment e of the consumption good, drawn from the two element set $\{e_1, e_2\}$; $e_1 > e_2 > 0$. An agent whose endowment is e_1 has not suffered a loss, while an agent with endowment e_2 is in a "loss" state.

Given the realization of some aggregate shock s, a type i agent, $i = 1, 2$, has probability $p_i(s)$ of receiving the endowment e_1, or probability $1 - p_i(s)$ of suffering a loss. Assume that $p_2(s) > p_1(s)$ ∀ s, so that type 2 agents are the "low-risk" group. As before, $\pi(s)$, $s \in \{1,2\}$, is the probability of state s, and $p_i(2) > p_i(1)$ ∀ i. A fraction θ of all agents are of type 1. Endowment realizations are independent across a large number of agents, so

the fraction of type i agents receiving e_1 is $p_i(s)$.[8] Finally, it is assumed that

$$\frac{p_1(1)}{p_2(1)} \neq \frac{p_1(2)}{p_2(2)}.$$

B. **The Game**

Let $c_j^i(s)$ denote the consumption of a type i agent who receives endowment e_j in state s.   Then insurance firms offer contracts which consist of consumption schedules $c_j^i(s)$; i, j, s = 1, 2.   As before, and as in Rothschild and Stiglitz, each insurance firm is restricted to the offer of a single consumption schedule.

Given an offered consumption schedule $c_j^i(s)$, an insurance firm attracting type i agents earns the ex post profit

$$(33) \qquad \psi_i(s) = p_i(s)\left[e_1 - c_1^i(s)\right] + \left[1 - p_i(s)\right]\left[e_2 - c_2^i(s)\right]; \ i = 1, 2.$$

As before, the objective of any firm is to maximize expected profits, given the offers of other firms.   Expected profits associated with any contract offer, of course, are $\sum_s \pi(s)\psi_i(s)$.   Finally, contract offers must be incentive compatible in the presence of other announced contracts.   Incentive compatibility here requires that

$$(34) \qquad \sum_s \pi(s)p_1(s)U\left[c_1^1(s)\right] + \sum_s \pi(s)\left[1-p_1(s)\right]U\left[c_2^1(s)\right]$$

$$\geq \sum_s \pi(s)p_1(s)U\left[c_1^2(s)\right] + \sum_s \pi(s)\left[1-p_1(s)\right]U\left[c_2^2(s)\right]$$

$$(35) \qquad \sum_s \pi(s)p_2(s)U\left[c_1^2(s)\right] + \sum_s \pi(s)\left[1-p_2(s)\right]U\left[c_2^2(s)\right]$$

$$\geq \sum_s \pi(s)p_2(s)U\left[c_1^1(s)\right] + \sum_s \pi(s)\left[1-p_2(s)\right]U\left[c_2^1(s)\right].$$

As before, a Nash equilibrium is a set of announced insurance contracts such that, given these announcements, no firm has an incentive to offer an alternate insurance contract.

C. The Rothschild-Stiglitz Equilibrium

As a point of reference, this section constructs the analogue of the Rothschild-Stiglitz equilibrium for this economy. Specifically, for the purposes of this section, it is further assumed that firms are precluded from offering insurance contracts contingent on s or, in other words, announced contracts must obey $c_j^i(1) = c_j^i(2)$ ∀ i, j. This restriction will essentially reproduce the Rothschild-Stiglitz equilibrium allocation of resources.

Under this restriction on announced contracts, any equilibrium contract offer must have the properties derived by Rothschild and Stiglitz: (i) self-selection of types by contract selected must occur, (ii) all offered contracts must earn zero expected profits, and (iii) all contracts must be maximal for the agents selecting them among the set of contracts that earn nonnegative profits and that are consistent with self-selection. The arguments to this effect are identical to those given by Rothschild and Stiglitz, and are therefore omitted here.

Equilibrium contracts are easy to construct here, and in fact are constructed in essentially the same way as in Section I. To begin, define

$$\bar{p}_i = \sum_s \pi(s)p_i(s); \quad i = 1, 2.$$

In addition, noting that (as in Rothschild and Stiglitz) incentive constraints do not bind on the determination of (c_1^1, c_2^1), this contract must be maximal for type 1 (high-risk) agents among the set of contracts that earn nonnegative expected profits. Hence, in equilibrium (c_1^1, c_2^1) must solve the problem

$$\max \bar{p}_1 U(c_1^1) + (1-\bar{p}_1)U(c_2^1)$$

subject to

(36) $\qquad \bar{p}_1(e_1-c_1^1) + (1-\bar{p}_1)(e_2-c_2^1) = 0.$

The solution to this problem is characterized by complete insurance, i.e.,

$$c_1^1 = c_2^1 = \bar{p}_1 e_1 + (1-\bar{p}_1)e_2 \equiv c^*.$$

Determination of (c_1^2, c_2^2) involves finding the maximal contract for type 2 agents that earns nonnegative expected profits, and that is incentive compatible in the presence of the contract offer $(c_1^1, c_2^1) = (c^*, c^*)$. In equilibrium, (c_1^2, c_2^2) must solve the problem

$$\max \bar{p}_2 U(c_1^2) + (1-\bar{p}_2)U(c_2^2)$$

subject to

(37) $\qquad \bar{p}_1 U(c_1^2) + (1-\bar{p}_1)U(c_2^2) \leq U(c^*)$

(38) $\qquad \bar{p}_2(e_1-c_1^2) + (1-\bar{p}_2)(e_2-c_2^2) = 0.$

As before, (37) can be shown to be binding in equilibrium. This equation and (38) in fact determine a unique consumption pair $(\hat{c}_1^2, \hat{c}_2^2)$.

The contract offers $(c_1^1, c_2^1) = (c^*, c^*)$ and $(c_1^2, c_2^2) = (\hat{c}_1^2, \hat{c}_2^2)$ are exactly the Rothschild-Stiglitz contracts with the expected loss probabilities $\bar{p}_i = \sum_s \pi(s)p_i(s)$ playing the role of the Rothschild-Stiglitz probabilities. Finally, existence issues are exactly as in Rothschild and Stiglitz, so that existence of equilibrium can be guaranteed by appropriate choice of the fraction θ of type 1 agents.

D. An Equilibrium With Mutual Insurance Firms

It is now possible to investigate the properties of equilibrium contracts with the restrictions $c_j^i(1) = c_j^i(2)$ relaxed. Equilibrium contracts are not readily derived in this section, as they were above. However, it will nonetheless be possible to show that in equilibrium "mutual insurance firms" must coexist with firms organized in other ways.

In order to show this, it is natural to begin by considering equilibrium contracts $c_j^1(s)$; j, $s = 1, 2$. As previously, these contracts must be maximal for type 1 agents among the set of all such contracts earning nonnegative expected profits, since incentive constraints cannot bind on type 1 contracts in equilibrium. Then $c_j^1(s)$; j, $s = 1, 2$, solves the problem

$$\max_{s} \sum_s \pi(s)p_1(s)U\left[c_1^1(s)\right] + \sum_s \pi(s)\left[1-p_1(s)\right]U\left[c_2^1(s)\right]$$

subject to

$$(39) \qquad \sum_s \pi(s)p_1(s)\left[e_1-c_1^1(s)\right] + \sum_s \pi(s)\left[1-p_1(s)\right]\left[e_2-c_2^1(s)\right] = 0.$$

The first order conditions for this problem are

$$(40) \qquad \pi(s)p_1(s)U'\left[c_1^1(s)\right] = \lambda\pi(s)p_1(s); \quad s = 1, 2$$

$$(41) \qquad \pi(s)\left[1-p_1(s)\right]U'\left[c_2^1(s)\right] = \lambda\pi(s)\left[1-p_1(s)\right]; \quad s = 1, 2,$$

where λ is the Lagrange multiplier associated with the constraint (39). As is clear from (40) and (41), $c_j^1(s)$ must be constant across s and j, so from (39), $c_j^1(s) = c^*$. As before, type 1 agents do not share aggregate risk with their insurers, i.e., type 1 agents purchase complete insurance contracts from firms which are <u>not</u> mutual firms.

If the same were true for type 2 agents, i.e., if $c_j^2(1) = c_j^2(2)$; $j = 1, 2$, held in equilibrium, then there would be no mutual insurance firms,

and the Nash equilibrium contracts here would have $c_j^2(1) = c_j^2(2) = \hat{c}_j^2$, as above. It is now shown that $c_j^2(1) \neq c_j^2(2)$ in equilibrium for some j, so that type 2 agents do share aggregate risk with their insurers, or in other words, they purchase insurance from firms that are organized as mutuals. In order to see this, note again that $c_j^2(s)$; j, s = 1, 2, must be maximal for type 2 agents among the set of contracts that earn nonnegative expected profits, and that are incentive compatible in the presence of the contract offers $c_j^1(s) = c^*$; j, s = 1, 2. Then, in equilibrium, $c_j^2(s)$ must solve the problem

$$\max_{s} \sum_s \pi(s)p_2(s)U[c_1^2(s)] + \sum_s \pi(s)[1-p_2(s)]U[c_2^2(s)]$$

subject to

(42) $\qquad U(c^*) = \sum_s \pi(s)p_1(s)U[c_1^2(s)] + \sum_s \pi(s)[1-p_1(s)]U[c_2^2(s)]$

(43) $\qquad \sum_s \pi(s)p_2(s)[e_1-c_1^2(s)] + \sum_s \pi(s)[1-p_2(s)][e_2-c_2^2(s)] = 0.$

The first order conditions for this problem are

(44) $\qquad [p_2(s)-\mu p_1(s)]U'[c_1^2(s)] - \eta p_2(s) = 0; \quad s = 1, 2$

(45) $\qquad \{1-p_2(s)-\mu[1-p_1(s)]\}U'[c_2^2(s)] - \eta[1-p_2(s)] = 0; \quad s = 1, 2.$

where μ and η are the Lagrange multipliers associated with (42) and (43) respectively. To obtain the result that $c_j^2(1) \neq c_j^2(2)$ for at least one j, note the following. If the solution to the problem above had $c_1^2(1) = c_1^2(2)$, then from (44)

(46) $\qquad \dfrac{p_2(1) - \mu p_1(1)}{p_2(2) - \mu p_1(2)} = \dfrac{p_2(1)}{p_2(2)}$

would hold. However, since $\mu > 0$, (46) requires that $p_1(1)/p_2(1) = p_1(2)/p_2(2)$, contrary to assumption. Hence $p_1(1)/p_2(1) \neq p_1(2)/p_2(2)$ implies that $c_1^2(1) \neq c_1^2(2)$, as asserted. Thus, in equilibrium, type 2 agents must share in aggregate risks with insurance firms.

E. Pareto Optima

In order to show that the changes that arise here in the presence of aggregate uncertainty are not purely the result of the selection of an equilibrium concept, we show that the presence of aggregate uncertainty also has implications for the set of Pareto optimal resource allocations. In order to demonstrate this result, it is sufficient to argue as follows. Suppose that $p_i(1) = p_i(2)$; $i = 1, 2$, held so that there is no genuine aggregate uncertainty here. Then, as Prescott and Townsend [1984] demonstrate, any Pareto optimal allocation of resources has $c_j^i(1) = c_j^i(2)$ ∀ i, j. Moreover, as Rothschild and Stiglitz and Prescott and Townsend [1984] show, in this case the Rothschild-Stiglitz insurance contract derived in Section C is Pareto optimal so long as

$$(47) \qquad \left(\frac{\theta}{1-\theta}\right)\left[\frac{\bar{p}_2-\bar{p}_1}{\bar{p}_2(1-\bar{p}_2)}\right] \geq \frac{U'(c*)\left[U'(\hat{c}_2^2)-U'(\hat{c}_1^2)\right]}{U'(\hat{c}_2^2)U'(\hat{c}_1^2)}.$$

Condition (47) obviously does not contradict anything assumed here.

When $p_i(1) \neq p_i(2)$; $i = 1, 2$, however, and when $p_1(1)/p_2(1) \neq p_1(2)/p_2(2)$, it has been shown that the allocation derived in Section C can never be Pareto optimal. Thus the presence of aggregate uncertainty does, in fact, change the set of Pareto optimal allocations for certain sets of parameter values.

IV. Conclusions

There seem to be two main conclusions to be derived from the preceding analysis. First, things work in a substantially different way in adverse selection economies in the presence of aggregate uncertainty. Second, such economies hold out a promise of explaining why a number of types of trading arrangements coexist. In particular, in Section III it was shown that, in the presence of adverse selection and aggregate uncertainty, it is natural to expect the coexistence of mutual insurance firms with other types of insurance firms. In Section II it was demonstrated that it is natural to expect mutual arrangements in lending organizations or to find the coexistence of debt which specifies uncontingent repayment with debt that pays off amounts contingent on the realization of the aggregate state. In previous attempts to explain the existence of uncontingent debt, it has proven difficult to develop models in which debt of various types coexists.[9] Hence these private information models with aggregate uncertainty appear to present a promising area of research.

Footnotes

[1]See also Eichenbaum and Peled [1986] and Jaynes [1978] who examine the implications of changing assumptions about what is observable in these contexts.

[2]For a justification see Green [1984] or Judd [1985]. An alternative justification using nonstandard analysis is given by Stutzer [1986].

[3]Parenthetically, it might be noted that loan contracts are contingent contracts requiring repayment of Rx if $w = y$, and zero otherwise.

[4]The arguments establishing these facts are essentially identical to those in Rothschild and Stiglitz, and hence are omitted here.

[5]Other specifications of contracts are possible in this context. In particular, an alternative specification of contracts would make interest payments state contingent, i.e., specify a loan quantity and a state contingent repayment $R(s)x$. The effect of this contract specification is considered below.

[6]It is straightforward to verify that $c_1^2 > 0$. $y \geq \gamma$ holds if and only if $\pi(2)p_1(2)(\beta_1-1) \geq \beta_1\pi(1)[p_2(1)-p_1(1)]$. If this condition is violated, equation (28) must be modified in an obvious way.

[7]In fact, feasibility of strategy (ii) requires that the value $\hat{\alpha}_2(2)$ solving (31) with $\hat{R}_2\hat{x}_2 = y$ also satisfies $\hat{\alpha}_2(2) \leq 1/p_2(2)$. This is henceforth assumed.

[8]See footnote 2.

[9]See Ross [1977] for one such attempt. Ross proceeds by specifying ad hoc remuneration functions for firm managers which imply the desirability of signaling via debt versus equity issues.

References

Eichenbaum, Martin and Dan Peled, "Capital Accumulation and Annuities in an Adverse Selection Economy," forthcoming, Journal of Political Economy, 1986.

Green, Edward, "Lending and the Smoothing of Uninsurable Income," unpublished, 1984.

Jaynes, Gerald, "Equilibria in Monopolistically Competitive Insurance Markets," Journal of Economic Theory 19, 1978, pp. 394-422.

Judd, Kenneth, "Efficiency, Adverse Selection, and Production," unpublished, 1984.

_____, "The Law of Large Numbers With a Continuum of IID Random Variables," Journal of Economic Theory 35, 1985, pp. 19-25.

Prescott, Edward and Robert Townsend, "Pareto Optima and Competitive Equilibrium With Adverse Selection and Moral Hazard," Econometrica 52, 1984, pp. 21-45.

Riley, John, "Competition With Hidden Knowledge," Journal of Political Economy 93, 1985, pp. 958-76.

Ross, Stephen, "The Determination of Financial Structure: The Incentive-Signalling Approach," Bell Journal of Economics 8, 1977, pp. 23-40.

Rothschild, Michael and Joseph Stiglitz, "Equilibrium in Competitive Insurance Markets: An Essay on the Economics of Imperfect Information," Quarterly Journal of Economics 90, 1976, pp. 629-50.

Spence, Michael, "Competitive and Optimal Responses to Signals: An Analysis of Efficiency and Distribution," Journal of Economic Theory 7, 1974, pp. 296-332.

_____, "Job Market Signalling," Quarterly Journal of Economics 82, 1973, pp. 355-74.

Stutzer, Michael, "Individual Risk Without Aggregate Uncertainty: A Nonstan-
    dard Approach," Federal Reserve Bank of Minneapolis, Working paper no.
    300, 1986.

Wilson, Charles, "A Model of Insurance Markets With Incomplete Information,"
    Journal of Economic Theory 16, 1977, pp. 167-207.