Federal Reserve Bank of Minneapolis
Research Department Working Paper

TEMPORAL AGGREGATION AND THE STOCK ADJUSTMENT MODEL OF INVENTORIES

Lawrence J. Christiano and Martin Eichenbaum*

Working Paper 357

May 1987

*Christiano, Federal Reserve Bank of Minneapolis; Eichenbaum, Carnegie-Mellon University. This is a revised version of section 3 of our paper, "Temporal Aggregation and Structural Inference in Macroeconomics," in Bubbles and Other Essays, Brunner and Meltzer, Carnegie-Rochester Conference Series on Public Policy, vol. 26, Spring 1987.

The views expressed herein are those of the authors and not necessarily those of the Federal Reserve Bank of Minneapolis or the Federal Reserve System. The material contained is of a preliminary nature, is circulated to stimulate discussion, and is not to be quoted without permission of the authors.

1. Introduction

Application of the stock adjustment model to the study of inventory behavior frequently produces implausibly low estimates of the speed of adjustment of actual to target inventories. For example, the parameter estimates reported by Feldstein and Auerbach [1976] imply that firms take almost 19 years to close 95 percent of the gap between actual and desired inventories. Application of the stock adjustment model to other problems such as the demand for money also yields implausibly low speeds of adjustment.

A variety of interesting explanations for these anomalous results exist. Blinder [1986], Eichenbaum [1984], Maccini and Rossana [1984] explore different explanations for the slow estimated speed of adjustment of inventories. Goodfriend [1985] discusses this problem with respect to the demand for money. In this paper we explore the possibility that estimated slow speeds of adjustment reflect temporal aggregation bias. Mundlak [1961] and Zellner [1968] showed theoretically that, if agents make decisions at intervals of time that are finer than the data sampling interval, then the econometrician could be led to underestimate speeds of adjustment. This is consistent with findings reported in Bryan [1967] who applied the stock adjustment model to bank demand for excess reserves. Bryan found that when the model was applied to weekly data, the estimated time to close 95 percent of the gap between desired and actual excess reserves was 5.2 weeks. When the model was applied to monthly aggregated data, the 95 percent closure time was estimated to be 28.7 months.

Our strategy for investigating the role played by time aggregation on the empirical estimate of speed of adjustment is as follows. First, we construct a continuous time equilibrium rational expectations model of inventories and sales. The model rationalizes a continuous time inventory stock adjustment equation. Using techniques developed by Hansen and Sargent [1980a, 1981] we estimate the model using monthly data on inventories and sales in the nondurable manufacturing sector. The parameter estimates from the continuous time model imply that firms close 95 percent of the gap between actual and "desired" inventories in 17 days. We then estimate an analogous discrete time model using monthly, quarterly and annual data. The parameter estimates obtained using monthly data imply that it takes firms 46 days to close 95 percent of the gap between actual and "desired" inventories. The analogous figure obtained with annual data imply that it takes firms 1,980 days to close 95 percent of the gap between actual and "desired" inventories. These results indicate that estimates of speed of adjustment are very sensitive to the effects of time aggregation.

Unfortunately, we cannot claim that temporal aggregation effects account for the statistical shortcomings of existing stock adjustment models. Both the discrete and continuous time versions of our equilibrium stock adjustment model impose strong over identifying restrictions on the data. Using a variety of tests and diagnostic devices, we find substantial evidence against these restrictions. In addition, we find no evidence that the overall fit for the continuous time model is superior to that of the discrete time model.

In section 2 we formulate a continuous time equilibrium model of employment, inventories of finished goods and output. In section 3 we discuss an estimation strategy which explicitly takes the temporal aggregation problem into account. Section 4 presents the empirical results.

2. A Continuous Time Model of Inventories, Output and Sales

In this section we discuss a modified continuous time version of the model in Eichenbaum [1984]. Our model is designed to nest, as a special case, the model considered by Blinder [1981, 1986] and Blinder and Holtz-Eakin [1984]. We take model to be representative of an interesting class of inventory models. An important virtue of our model is that it provides an explicit equilibrium rationale for a continuous time version of the stock adjustment equation for inventories. An additional advantage of proceeding in terms of an equilibrium model is that we are able to make clear both the theoretical underpinnings and the weaknesses of an important class of inventory models which has appeared in the literature.

Consider a competitive representative household that ranks alternative streams of consumption and leisure using the utility function:[1]

$$(1) \qquad E_t \int_0^\infty e^{-r\tau}\{u(t+\tau)s(t+\tau)-.5A[s(t+\tau)]^2-N(t+\tau)\}d\tau.$$

In (1),

t = the time unit, measured in months,

E_t = the linear least squares projection operator, conditional on the time t information set,

$s(t)$ = time t consumption of the single nondurable consumption good,

$N(t)$ = total work effect at time t,

$u(t)$ = a stochastic disturbance to the marginal utility of consumption at time t, and,

A,r = positive constants.

We now specify the technology for the production of new consumption goods and storing inventories of finished goods. Let $Q(t)$ denote the total output of new consumption goods at time t. The production function for $Q(t)$ is given by:

$$(2) \qquad Q(t) = [(2/a)N(t)]^{1/2},$$

where a is a positive scalar. In order to accommodate two different types of costs associated with inventories that have been considered in the literature we suppose that total inventory costs, measured in units of labor, are given by:

$$(3) \qquad C_I(t) = (b/2)[s^*(t)-cI(t)]^2 + v(t)I(t) + (e/2)I(t)^2,$$

where b, c, and e are positive scalars, v(t) is a stochastic shock to marginal inventory holding costs and s*(t) denotes time t sales of the good. The last two terms in (3) correspond to the inventory holding cost function adopted by Blinder [1981, 1986] and Blinder and Holtz-Eakin [1984], among others. This component of costs reflects the physical costs of storing inventories of finished goods. The first term in (3) reflects the idea that there are costs, denominated in units of labor, associated with allowing inventories to deviate from some fixed proportion of sales. Blanchard ([1983], p. 378) provides an extensive motivation of this component of inventory costs. Similar cost functions appear in Eichenbaum [1984], McCallum [1984] and Eckstein and Eichenbaum [1985].

The link between current production, inventories of finished goods and sales is given by,

$$(4) \qquad Q(t) = s^*(t) + DI(t),$$

where D is the derivative operator, $Dx(t) = dx(t)/dt$.

It is well known that, in the absence of externalities or similar types of distortions, rational expectations competitive equilibria are Pareto optimal. Since our representative consumer economy has a unique Pareto optimal allocation, we could solve directly for the competitive equilibrium by considering the relevant social planning problem (see Lucas and Prescott

[1971], Hansen and Sargent [1980b] and Eichenbaum, Hansen and Richard [1985]). On the other hand there are a variety of market structures which will support the Pareto optimal allocation. In the interest of preserving comparability with other papers in the inventory literature, we find it convenient to work with a particularly simple market structure that supports this allocation. As in Sargent [1979] we require only competitive spot markets for labor and the consumption good to support the Pareto optimal allocation.[2]

Suppose that the representative consumer chooses contingency plans for $s(t+\tau)$, $\tau \geq 0$, to maximize (1) subject to the sequence of budget constraints,

$$(5) \qquad P(t+\tau)s(t+\tau) = N(t+\tau) + \pi(t+\tau).$$

In (5),

$P(t)$ = the price of the consumption good, denominated in labor units, and
$\pi(t)$ =lump sum dividend earnings of the household, denominated in labor units.

Solving the representative consumer's problem we obtain the following inverse demand function,

$$(6) \qquad P(t) = -As(t) + u(t).$$

Given the very simple structure of relation (6) it is important to contrast our specification of the demand function with different specifications that have been adopted in the literature. In constructing empirical stock adjustment models, most analysts abstract from modelling demand. Instead, the analysis is conducted assuming a particular time series representation for an exogenous sales process (see for example Feldstein and Auerbach [1976] or Blanchard [1983]). Our model is consistent with this practice when A is very large. To see this, rewrite (6) as, (6)' $\quad s(t) = -(1/A)P(t) + \eta(t),$

where $\eta(t) = -(1/A)u(t)$. The assumptions we place on $u(t)$ below guarantee that $\eta(t)$ has a time series representation of the form $\gamma(D)\eta(t) = \nu(t)$, where $\nu(t)$ is continuous time white noise, uncorrelated with past values of $s(t)$ ad $I(t)$. Also, $\gamma(t)$ is a finite ordered polynomial satisfying the root condition required for covariance stationarity. If A is very large ("infinite") then sales have the reduced form time series representation $\gamma(D)s(t) = \nu(t)$. This is the continuous time analogue of the assumption, made in many stock adjustment models, that sales are exogenous stochastic process in the sense of not being Granger caused by the actions of the group of agents who make inventory decisions. (Our empirical results indicate that the assumption of one way Granger causality from sales to inventory stocks is reasonably consistent with the data.)

Other authors like Blinder [1986] and Eichenbaum [1984] begin their analysis by postulating the industry demand curve (6). Our analysis provides an equilibrium interpretation of this demand specification. In so doing we are forced to confront the strong assumptions implicit in (6). For example, we implement our model on nondurable manufacturing shipment and inventory data. This choice of data was dictated by the desire for our results to be comparable with those appearing in the relevant literature. Notice however that manufacturer' shipments do not enter directly as arguments into consumers' utility functions. Rather they represent sales from manufacturers to wholesalers and retailers who in turn sell them to households. Consequently, objective function (1) consolidates the wholesale, retail and household sectors. We know of no empirical justification for this assumption. By focusing on nondurable manufacturers, we place more faith than we care to on the stability of their relation to wholesalers and retailers. For example, shifts through time in the pattern of inventory holdings between manufacturers and retailers and wholesalers would have effects on our empirical results that are hard to predict. At the same time they do not represent phenomena that we wish to model in this paper. In future research we plan to avoid this type of problem by consolidating data from the wholesale, retail and manufacturing sectors.

We assume that the representative firm seeks to maximize its expected real present value. The firm distributes all profits in the form of lump sum dividends to consumers. The firm's time t profits are equal to

$$(7) \qquad \pi(t) = P(t)s^*(t) - N(t) - C_I(t).$$

Substituting (2), (3), and (4) into (7) we obtain,

$$(8) \qquad \pi(t) = P(t)s^*(t) - (a/2)[s^*(t)+DI(t)]^2 - (b/2)[s^*(t)-cI(t)]^2$$
$$- v(t)I(t) - (e/2)I(t)^2.$$

The firm chooses contingency plans for $s^*(t+\tau)$ and $DI(t+\tau)$, $\tau \geq 0$, to maximize,

$$(9) \qquad E_t \int_0^\infty e^{-r\tau}\pi(t+\tau)d\tau$$

given $I(t)$, the laws of motion of $v(t)$ and $u(t)$, (1) and beliefs about the law of motion of industry wide sales, $s^*(t)$.[3] In a rational expectations equilibrium these beliefs are self-fulfilling. Sargent ([1979], p. 375) describes a simple procedure for finding rational expectations equilibria in linear quadratic, discrete time models. The discussion in Hansen and Sargent [1980a] shows how to modify Sargent's solution procedure to accommodate our continuous time setup. Briefly, the procedure is as follows. Write,

(10) $F[I(t),DI(t),s^*(t),v(t),P(t),t] = e^{-rt}\pi(t)$,

so that (9) can be written as,

(11) $E_t \int_0^\infty F[I(t+\tau),DI(t+\tau),s^*(t+\tau),v(t+\tau),P(t+\tau),\tau]d\tau$,

by choice of $DI(t+\tau)$, $s^*(t+\tau)$, $\tau \geq 0$, subject to $I(t)$ and the laws of motion of $v(t)$ and $P(t)$. Notice that the principle of certainty equivalence applies to this problem. Accordingly, we first solve a version of (11) in which future random variables are equated to their time t conditional expectation. Then we use a continuous time version of the Weiner-Kolmogorov forecasting formula to express the time t conditional expectation of time $t + \tau$ variables in terms of elements of agents' time t information set.

The variational methods discussed by Luenberger [1969] imply that firm's Euler equations for $s(t)$ and $I(t)$ are:

(12a) $\partial F/\partial s^*(t) = 0$

and

(12b) $\partial F/\partial I(t) = D\{\partial F/\partial DI(t)\}$.

These imply respectively:

(13a) $P(t) - (a+b)s^*(t) - aDI(t) + bcI(t) = 0$,

and,

(13b) $aD^2I(t) - raDI(t) - (c^2b+e)I(t) + asDs^*(t) + (cb-ra)s^*(t) = v(t)$.

In a rational expectations competitive equilibrium, P(t) must satisfy (6), with $s(t) = s^*(t)$. Substituting (6) into (13a) and replacing $s^*(t)$ by s(t) we obtain,

$$(14) \qquad s(t) = -[a/(a+b+A)]DI(t) + \{bc/(a+b+A)\}I(t) + [1/(a+b+A)]u(t).$$

It is convenient to collapse (13b) and (14) into one differential equation in $\dot{I}(t)$. Substituting s(t) and Ds(t) from (14) into (13b) we obtain,

$$(15a) \qquad (D-\lambda)[D-(r-\lambda)]I(t) = \frac{(a+b+A)}{a(b+A)} v(t) - \frac{1}{(b+A)}[(bc-ra)/a+D]u(t)$$

where,

$$(15b) \qquad \lambda = .5r + (k+.25r^2)^{1/2}$$

and,

$$(15c) \qquad k = [(a+b+A)/a(b+A)]\{(bc[c(a+A)+ra]/(a+b+A))+e\}.$$

Since $k > 0$, it follows from (15b) that $\lambda > 0$ is real. Moreover, it is easy to verify that $r - \lambda = .5r-[k+.25r^2]^{1/2}$.[4] Solving the stable root $(r-\lambda)$ backward and the unstable root λ forward in (15a) we obtain,

$$(16) \qquad DI(t) = (r-\lambda)I(t) \frac{a+b+A}{a(b+A)} \int_0^\infty e^{-\lambda\tau}v(t+\tau)d\tau$$

$$+ \frac{1}{b+A} \int_0^\infty e^{-\lambda\tau}[(cb-ra)/a+D]u(t+\tau)d\tau$$

$$= (r-\lambda)I(t) - \frac{a+b+A}{a(b+A)} \int_0^\infty e^{-\lambda\tau}v(t+\tau)d\tau - \frac{1}{b+a} u(t)$$

$$+ \frac{1}{b+a} [\frac{bc}{a} -(r-\lambda)]\int_0^\infty e^{-\lambda\tau}u(t+\tau)d\tau,$$

where the second equality is obtained using integration by parts. Substituting (16) ad (14), we obtain,

$$(17) \qquad s(t) = \frac{bc - a(r-\lambda)}{a + b + A} I(t) + \frac{1}{b + A}\int_0^\infty e^{-\lambda\tau}v(t+\tau)d\tau + \frac{1}{b + A} u(t)$$

$$- \frac{1}{(b+A)(a+b+A)}[(bc/a)-(r-\lambda)]\int_0^\infty e^{-\lambda\tau}u(t+\tau)d\tau.$$

Equations (16) and (17) are the equilibrium laws of motion for inventory investment and consumption in the perfect foresight version of our model. Before allowing for uncertainty we discuss some qualitative features of this equilibrium.

First, suppose that the parameter b is equal to zero and there are no technology shocks. This is the model considered by Blinder [1981, 1986] and Blinder and Holtz-Eakin [1984]. The role of inventories in this version of the model is to smooth production in the sense that inventory investment is negatively related to current demand shocks and positively related to expected future demand shocks (see (16) and recall that $r-\lambda<0$). As Blinder [1986] points out, production smoothing, as defined here, does not necessarily imply that the variance of sales will exceed that of production. For example, if the serial correlation structure of u(t) were such that a jump in u(t) typically implies a large increase in u(t) in the future, then the current jump in u(t) could lead to an increase in inventory investment, as well as sales. We rule out these types of u(t) processes below. Consequently, production smoothing in our model implies that the variance of production is lower than the variance of sales when $b = v(t) = 0$.

Second, suppose that there are no preference shocks. Then, the role of inventories is to smooth sales. To see this, notice that inventory investment depends negatively on current and future shocks to the inventory holding cost function. The firm holds less inventories when the marginal cost of holding inventories increases. Suppose that inventory holding costs are viewed as general shocks to production costs. Firms will use inventories to smooth production costs, as opposed to production levels, over time in the face of stable demand for their product. For the kinds of production cost shocks that we consider in this paper, this implies that the variance of sales will be smaller than the variance of production.

A slightly different way of seeing these points is to remember that the competitive equilibrium solves the problem of a fictitious social planner/representative consumer. The representative consumer has a utility function which is locally concave in consumption so that, other things equal, he prefers a smooth consumption path. If preference shocks predominate we would expect sales/consumption to be volatile relative to production. On the other hand if technology shocks predominate, we would expect sales/consumption to be smooth relative to production. Blinder [1981, 1986] and West [1986] document the fact that, at least for post World War II data, the variance of production exceeds the variance of sales/consumption. This suggests that the primary role of inventories is to smooth sales rather than production levels.

We now consider the equilibrium of the system in the uncertainty case. In order to derive explicit expressions for the equilibrium laws of motion of the system we parameterize the stochastic laws of motion of the shocks to preferences and technology. To this end we assume that u(t) and v(t) have the joint AR(1) structure,

$$(18a)\qquad u(t) = \epsilon_1(t)/(\beta+D) = \int_0^\infty e^{-\beta\tau}\epsilon_1(t-\tau)d\tau,$$

and

(18b) $$v(t) = \epsilon_2(t)/(\alpha+D) = \int_0^\infty e^{-\alpha\tau}\epsilon_2(t-\tau)d\tau,$$

where α and β are positive scalars. The vector $\epsilon(t) = [\epsilon_1(t)\epsilon_2(t)]'$ is the continuous time linear least squares innovation in $[u(t)v(t)]'$ $E\epsilon(t)\epsilon(t-\tau)' = \delta(\tau)\tilde{V}$, where $\tilde{V}$ is a positive definite 2 x 2 symmetric matrix and $\delta(\tau)$ is the Dirac delta generalized function.

Given the above specification for the shocks it is obvious that, for $\tau \geq 0$,

(19a) $$E_t u(t+\tau) = \int_0^\infty e^{-\beta s}\epsilon_1(t+\tau+s)ds = e^{-\beta\tau} \int_0^\infty e^{-\beta s}\epsilon_1(t-s)ds = e^{-\beta\tau}u(t).$$

Similarly,

(19b) $$E_t v(t+\tau) = e^{-\alpha\tau}v(t).$$

Simple substitution from (19) yields,

$$E_t \int_0^\infty e^{-\lambda\tau}u(t+\tau)d\tau = u(t)/(\beta+\lambda)$$

and

$$E_t \int_0^\infty e^{-\lambda\tau}v(t+\tau)d\tau = v(t)/(\alpha+\lambda).$$

Substituting these expressions into (16) and (17) we obtain the equilibrium laws of motion for $s(t)$ and $DI(t)$,

(20a) $$DI(t) = (r-\lambda)I(t) - \frac{a+b+A}{a(b+A)(\alpha+\lambda)}v(t) + \frac{(cb-ra) - a\beta}{a(b+A)(\beta+\lambda)}u(t)$$

(20b) $$s(t) = \frac{bc - a(r-\lambda)}{a+b+A}I(t) + \frac{v(t)}{(b+A)(\alpha+\lambda)} + \frac{a}{(b+A)(a+b+A)}\frac{[(bc-ra)-\beta a]}{a(\beta+\lambda)}$$

$$+ [\frac{1}{a+b+A}]u(t).$$

It is convenient to write the equilibrium laws of motion for I(t) an s(t) in the form of a continuous time moving average of $\epsilon_1(t)$ and $\epsilon_2(t)$. Substituting (18)into (20) and rearranging we obtain, in operator notation,

$$(21) \qquad \begin{matrix} I(t) \\ s(t) \end{matrix} = \theta(D)^{-1}\tilde{C}(D)\epsilon(t)$$

where

$$(22) \qquad \theta(D) = (\alpha+D)(\beta+D)\left[D-(r-\lambda)\right],$$

$$(23) \qquad \tilde{C}(D) = \tilde{C}_0 + \tilde{C}_1 D + \tilde{C}_2 D^2,$$

$$\tilde{C}_0 = \begin{matrix} q_1\alpha & q_2\beta \\ \frac{\alpha\left(q_1 bc-(r-\lambda)\right)}{a+b+A} & \frac{q_2 bc\beta}{a+b+A} \end{matrix}$$

$$\tilde{C}_1 = \begin{matrix} q_1 & q_2 \\ \frac{-aq_1(\alpha-\frac{bc}{a})+\alpha-(r-\lambda)}{a+b+A} & \frac{-aq_2(\beta-bc/a)}{a+b+A} \end{matrix}$$

$$\tilde{C}_2 = \begin{matrix} 0 & 0 \\ \frac{1-aq_1}{a+b+A} & \frac{-aq_2}{a+b+A} \end{matrix}$$

$$(24) \qquad q_1 = \frac{(cb-ra)-a\beta}{a(\lambda+\beta)(b+A)}$$

and

$$q_2 = \frac{'(a+b+A)}{a(b+A)(\lambda+\alpha)}.$$

We find it useful to write (21)' as,

$$(21) \qquad \begin{matrix} I(t) \\ s(t) \end{matrix} = \theta(D)^{-1}C(D)e(t)$$

where $e(t) = \tilde{C}_0\epsilon(t)$, $C(D) = \tilde{C}(D)\tilde{C}_0^{-1}$, and $Ee(t)e(t)' = \delta(\tau)V = \delta(t)\tilde{C}_0\tilde{V}\tilde{C}_0'$. With this definition of C(D) and e(t), equations (21)-(24) summarize all of the restrictions that our model imposes on the continuous time Would MAR of I(t) ans s(t).

We conclude this section by showing that our model is consistent with a stock adjustment equation for inventories. Let I(t)* denote the aggregate level of inventories such that if I(t) = I(t)*, then actual inventory investment, DI(t), is equal to zero. I(t)* is taken to be the level of "desired" or "target" inventories. Relation (20a) implies that,

$$(25) \qquad I(t)^* = + \frac{a + b + A}{(r-\lambda)a(b+A)(\alpha+\lambda)} v(t) - \frac{(bc-ra) - a\beta}{a(r-\lambda)(b+A)(\beta+\lambda)} u(t).$$

Substituting (25) into (20a) we obtain a stock adjustment equation for inventory investment,

$$(26) \qquad DI(t) = (\lambda-r)\left[I(t)^*-I(t)\right].$$

We require a measure of the "speed of adjustment" which can be compared with similar measures reported in the literature. In order to make this concept precise we imagine, counterfactually, that movements in I(t)* can be ignored over an interval $\tau \in (t,t+1)$, so that $I(\tau)^* = I(t)^*$ for $\tau \in (t,t+1)$. Then the solution to (26) is

$$(27) \qquad I(t+\tau) - I(t)^* = e^{-(\lambda-r)\tau}\left[I(t)-I(t)^*\right].$$

Relation (27) gives rise to an interesting summary statistic regarding the speed of adjustment of actual to target inventories. In particular, the number of days required to close 95 percent of the gap between actual and target inventories is,

$$(28) \qquad T^c = -30\ [\log\ (1-.95)]/(\lambda-r),$$

where 30 is approximately the number of days in a month.

Given the estimates of the structural parameters it is straightforward to calculate this statistic. In the next section we discuss a strategy for estimating the parameters of our model from discrete data. In addition we formulate a discrete time version of the model which is useful for estimating speeds of adjustment under the assumption that agents' decision intervals coincide with the data sampling interval.

3. Estimation Issues

In this section we discuss a strategy for estimating the continuous time model of section 2 from discrete observations on inventories and sales. Since our estimator corresponds to the one discussed in Hansen and Sargent [1980a] we refer the reader to that paper for technical details. Christiano and Eichenbaum [1985] provide additional details for the model considered here. In this section we also display a discrete time version of our basic model and describe a method for estimating its parameters. By estimating both models we are able to derive an empirical measure of the effects of temporal aggregation on speed of adjustment estimates.

We now describe the procedure used to estimate the parameters of the continuous time model described in section 2. This procedure takes into account the fact that the inventory data are point-in-time, and measured at the beginning of the sampling interval, while sales are averages over the month.

Our estimation strategy involves maximizing an approximation of the Gaussian likelihood function of the data with respect to the unknown parameters, ζ, which we list explicitly in section 4. The approximation we use is the frequency domain approximation studied extensively in Hannan [1970]. Hansen and Sargent [1980a] show how to use this approximation to estimate continuous time linear rational expectations models from discrete data records.

One way to describe our estimation strategy exploits the observation that estimation of a continuous time model actually is a special case of estimating a constrained discrete time model. Recall from the discussion of section 2 that ζ implies a continuous time ARMA model, characterized by the polynomials $\theta(D)$ and $C(D)$ and a symmeric matrix, V (see (21)-(24)). This continuous time series representation implies a particular discrete time series representation for the sampled, averaged data. In Christiano and Eichenbaum ([1987], Theorem 2) we characterized this discrete time representation by a scalar third order polynomial $\theta^c(L)$, a third order 2 x 2 matrix polynomial $\overline{C}^c(L)$, and an innovation variance matrix, V^c. Here, L is the lag operator where $L^j x(t) = x(t-j)$. The polynomial θ^c satisfies $\theta^c(e^{-\alpha}) = \theta^c(e^{-\beta}) = \theta^c(e^{r-\lambda}) = 0$ and $\theta^c(0) = 1$. Also, det $\overline{C}^c(z) = 0$ implies $|z| \geq 1$. Letting $Y(t)$ denote the measured data on inventories and sales ($Y(t)$ is defined precisely below), then the time series representation of ($Y(t)$, integer t) is

$$[1+\theta_1^c L+\theta_2^c L^2+\theta_3^c L^3]Y(t) = [I+\overline{C}_1^c L+\overline{C}_2^c L^2+\overline{C}_3^c L^3]u_t,$$

where u_t the white noise innovation in $Y(t)$ with variance V^c.

Given θ^c, $\overline{C}^c$, V^c it is possible to compute the spectral density of the data, $S_y(z;\zeta)$, which is one of the two ingredients of the spectral approximation to the likelihood function. It can be shown that $S_y(z;\zeta)$ is given by,

$$S_y(z;\zeta) = \overline{C}^c(z)V^c\overline{C}^c(z^{-1})'/\theta^c(z)\theta^c(z^{-1}),$$

for $z = e^{-iw}$, $w\epsilon(-\pi,\pi)$.

The other ingredient of the spectral approximation to the likelihood function is the periodogram of the data. We denote the available data by $\{Y(t),t=1,2,..,T\}$. Here, $Y(t) \equiv [I(t),\bar{s}(t)]'$, where $\bar{s}(t)$ denotes average sales:

$$(29) \qquad \bar{s}(t) = \int_0^1 s(t+\tau)d\tau.$$

The periodogram of the data at frequency w_j, $I(w_j)$, is

$$I(w_j) = (1/T)Y(w_j)Y(w_j)^H,$$

where H denotes the Hermetian transpose and,

$$Y(w_j) = \sum_{t=1}^{T} Y(t)e^{-iw_jt}.$$

Here, $w_j = 2\pi j/T$, $j = 1, 2, ..., T$. Given these expressions for $S_y(z;\zeta)$ and $I(w_j)$ we can compute the spectral approximation to the likelihood function,

$$(30) \qquad L_T(\zeta) = -T \log 2\pi - .5 \sum_{j=1}^{T} \log \det [S(e^{-iw_j};\zeta)]$$

$$- .5 \sum_{j=1}^{T} \text{trace } [S(e^{-iw_j};\zeta)^{-1}I(w_j)].$$

Since the likelihood function (30) is a known function of the data and the parameters of the model it can be maximized with respect to those parameters. We obtain an estimate of the variance-covariance matrix of the estimated coefficients by computing the negative of the inverse of the second derivative of L_T with respect to ζ, evaluated at the estimated values of ζ

We now consider the problem of estimating a discrete time version of the model. Accordingly, we suppose that the representative consumer maximizers,

$$(31) \qquad E_t \sum_{j=0}^{\infty} \phi^j \{u(t+j)s(t+j)-.5As(t+j)^2-N(t+j)\},$$

subject to (5) by choice of linear contingency plans for s(t) and N(t). The parameter ϕ is a subjective discount rate that is between zero and one. As before the solution to the consumer's problem is given by the inverse demand function (6).

The representative competitive firm chooses linear contingency plans for s*(t) and I(t) to maximize,

$$(32) \qquad E_t \sum_{j=0}^{\infty} \phi^j \{P(t+j)s^*(t+j)-(a/2)[s^*(t+j)+I(t+j)-I(t+j-1)]^2$$

$$-(b/2)[s^*(t+j)-cI(t+j)]^2-v(t+j)I(t+j)-(e/2)I(t+j)^2\},$$

subject to I(t) given and the laws of motion of v(t) and P(t). We suppose that the shocks to technology and preferences have a discrete time AR(1) representation:

$$(33a) \qquad u(t) = \mu u(t-1) + \epsilon_1(t),$$

and

$$(33b) \qquad v(t) = \rho v(t-1) + \epsilon_2(t),$$

where $|\mu| < 1$ and $|\rho| < 1$. Also $\epsilon(t) = [\epsilon_1(t)\epsilon_2(t)]'$ is a vector white noise that satisfies,

(34) $$E\epsilon(t)\epsilon(t-\tau)' = \Omega \quad \tau \text{ equal to zero,}$$

$$= 0 \quad \tau \text{ equal to zero.}$$

The model summarized by (31)-(34) is the discrete time version of our continuous time model in that, essentially, it has been obtained by replacing the D operator by its "approximation," $1 - L$. An alternative would have been to specify the discrete time model so that the implied reduced form time series representation for inventories and sales would be an ARMA of the same order as that predicted by the continuous time model. In order to do this we would have to abandon the assumption that $u(t)$ and $v(t)$ have first order autoregressive representations or change other basic features of the discrete time model. This is an important point which we will return to in section 4.

Eichenbaum and Christiano [1985] show that the equilibrium laws of motion for inventories and sales are given by,

(35a) $$I(t) = \psi I(t-1) + hu(t) + gv(t)$$

(35b) $$s(t) = -(a-bc)/(a+b+A)I(t) + a/(a+b+A)I(t-1) + \left[1/(a+b+A)\right]u(t)$$

where

(35c) $$h = \frac{-1}{a\left[b(c+1)+A\right]}\left\{\psi(a-bc)+\left[(a-bc)\psi-a\right]\psi\phi\mu/(1-\psi\phi\mu)\right\},$$

$$g = \frac{-(a+b+A)\psi}{a\left[b(c+1)+A\right](1-\psi\phi\rho)},$$

$$\psi + 1/(\psi\phi) = \frac{-(a+b+A)}{\phi a\left[b(c+1)+A\right]}\left[\frac{\phi a^2+(a-bc)^2}{a+b+A} - (a+bc^2+e+\phi a)\right],$$

and $|\psi| < 1$.

The relevant measure of the speed of adjustment of inventories which can be compared to the measure which emerges from the continuous time model is,

$$(28) \qquad T^d = X[\log(.05)]/\log\psi,$$

where X is the number of days in the data sampling interval.

It is convenient to write the equilibrium law of motion for s(t) and I(t) in the form of a moving average representation of the discrete time innovations to agents' information sets. Substituting (33) into (35) and rearranging we obtain,

$$(36) \qquad \begin{pmatrix} I(t) \\ s(t) \end{pmatrix} = \theta^d(L)^{-1}\overline{C}^d(L)\epsilon(t)$$

where,

$$(37) \qquad \theta^d(L) = (1-\rho)(1-\mu L)(1-\psi L),$$

$$(38) \qquad \overline{C}^d(L) = \overline{C}^d_0 + \overline{C}^d_1 + \overline{C}^d_2 L^2,$$

$$\overline{C}^d_0 = \begin{pmatrix} h & g \\ \frac{1-(a-bc)h}{a+b+A} & \frac{g(bc-a)}{a+b+A} \end{pmatrix}$$

$$\overline{C}^d_1 = \begin{pmatrix} -h\rho & -g\mu \\ \frac{(ah-\psi)-\rho[1-(a-bc)h]}{a+b+A} & \frac{g[a-\mu(bc-a)]}{a+b+A} \end{pmatrix}$$

$$\overline{C}^d_2 = \begin{pmatrix} 0 & 0 \\ \frac{-\rho(ah-\psi)}{a+b+A} & \frac{-g\mu a}{a+b+A} \end{pmatrix}.$$

Given these relations the free parameters of the discrete time model can be estimated by maximizing Hannan's spectral approximation to the likelihood function.

We are now in a position to demonstrate some of the possible sources of temporal aggregation bias in estimates of the speed of adjustment. Relations (21)-(24) and (36)-(38) summarize the restrictions on the continuous and discrete time Wold representation imposed by the continuous and discrete versions of the model, respectively. It can be shown that the continuous and discrete time models imply that I(t) and s(t) have continuous and discrete time VAR(2) representations, respectively. For example, to see this for the continuous time model, notice that (21)-(24) imply

(39) $$\det C(D) = (\alpha+D)(\beta+D)\left[D-(r-\lambda)\right]/(\lambda-r)a(b+A).$$

Premultiplying (3.22) by $C(D)^{-1} = C(D)^{a}/\det C(D)$ we obtain,

(40) $$(\lambda-\alpha)a(b+A)C(D)^{a}Y(t) = e(t).$$

Here $C(D)^{a}$ denotes the adjoint matrix of C(D). Thus $\{Y(t)\}$ is a pure VAR(2) in continuous time. However, Theorem 1 of Christiano and Eichenbaum [1987] implies that sampled and averaged $\{Y(t)\}$ is a discrete time ARMA(2,2) process. One moving average term is due to sampling and the other is due to averaging. We choose not to focus upon this representation of the discrete data because its AR part requires stronger than usual restrictions to ensure identification (see Christiano and Eichenbaum [1985], pp. 29-31). Instead we focus on an alternative reduced form representation for the data which emerges from the continuous time model,

(41) $$\theta^{c}(L)Y(t) = \left[I+C_1^{c}+C_2^{c}+C_3^{c}L^{3}\right]e^{c}(t),$$

where $e^c(t)$ is the innovation in Y(t) which has covariance matrix V^c. Here dec $C^c(L) = \theta^c(L)\kappa(L)$, where $\kappa(L)$ is a second order polynomial in the lag operator L. The presence of $\kappa(L)$ is a symptom of the effects of sampling and of averaging s(t). Since det $C^c(L)$ is not proportional of $\theta^c(L)$, the sampled representation is not VAR(2). As we indicated it is vector ARMA (2,2). Christiano and Eichenbaum [1985] discuss the mapping between the representations (40) ad (41).

Of course the discrete time model remains a VAR(2). It is useful to write the reduced form of the discrete model in a manner that is analogous to (43). Define $e^d(t) = \overline{C}_0\epsilon(t)$ and $C^d(L) = \overline{C}^d(L)(\overline{C}^d_0)^{-1}$. Then (36) implies that the reduced form representation for Y(T) emerging from the discrete time model is

$$\theta^d(L)Y(t) = [I+C^d_1L+C^d_2L^2]e^d(t), \tag{42}$$

where the first row of C^d_2 is composed of zeros. We denote the covariance matrix of $e^d(t)$ by V^d.

Comparing (41) and (42) we see that the moving average component of the reduced form for the discrete model is of smaller order than that of the continuous time model. Again, this reflects the fact that the continuous time and discrete time models have different implications for measured data. Not surprisingly, estimation of the two models will yield different estimates of the underlying structural parameters and speeds of adjustment of actual to target inventories.

4. Empirical Results

In this subsection we report empirical results obtained from estimating four different models. The continuous time model was estimated using monthly data. Three discrete models were estimated, one each using monthly, quarterly, and annual data. Our main results can be summarized as follows. First, the parameter estimates from the different models that we estimated are consistent with the Mundlak-Zellner hypothesis that temporal aggregation can account for slow speeds of adjustment in stock adjustment models. Secondly, we find that while the effects of temporal aggregation are substantial as we move from annual to quarterly to monthly specifications of the model, they are rather small when we move from the monthly to the continuous time specification. This second result is consistent with findings in Christiano [1986b] where the length of the timing interval in a rational expectations model is treated as a free parameter. Christiano [1986b] plots the maximized value of the likelihood function of an annual data record against various values of the model timing interval. As the interval is reduced from an annual to a quarterly specification the value of the likelihood function rises substantially. However, further decreases in the model timing interval result in smaller increases in the value of the likelihood function. This result is also consistent with findings in Christiano [1986a] in which a continuous time model of hyperinflation is estimated using monthly data. When an analogous discrete time model is fit to the same data, the results are virtually indistinguishable from the continuous time results.

The 11 free parameters of our continuous time model are:

$$\Lambda^{c} = (r,a,b,c,e,A,\alpha,\beta,V_{11},V_{22},V_{12}).$$

Our discrete time model also hass 11 free parameters:

$$\Lambda^{d} = (\phi,a,b,c,e,A,\rho,\mu,V^{d}_{11},V^{d}_{22},V^{d}_{12}).$$

Equation (40) implies that no more than 9 parameters of the continuous time can be identified. The same is true for the discrete time model. Consequently, we searched for a lower dimensional parameter set that was identified. We restricted our attention to sets that included $(\lambda - r)$ and ψ for the continuous and discrete time models respectively. For present purposes, it does not concern us that we cannot identify all the elements of Λ^c and Λ^d, since our principle motivation is to identify the adjustment speeds implied by the two models. These are controlled by $(\lambda - r)$ and ψ in the continuous and discrete time cases, respectively. The parameter sets that we estimated are the following:

$$\zeta = \left(r, \alpha, \beta, \lambda - r, bc/a, (a+b+A)/a, V_{11}^c, V_{22}^c, V_{12}^c\right),$$

and

$$\xi = \left(\phi, \rho, \mu, \psi, bc/a, (a+b+A)/a, V_{11}^d, V_{22}^d, V_{12}^d\right).$$

Christiano and Eichenbaum [1985] establish that ζ and ξ are identified.[6] In practice we fixed the discount rates r and ϕ, a priori, at values which imply a monthly discount rate of .997.[7]

Both models were estimated using seasonally adjusted monthly data on nondurable manufacturing shipments and finished goods inventories. The data correspond to those used by Blinder [1986]. This data is published by the Bureau of Economic Analysis (BEA) except that Blinder has converted BEA's end-of-month inventory stocks to beginning-of-month figures. We constructed quarterly and annual data by taking arithmetic averages of the monthly data. The data cover the period February 1959 to April 1982 and are measured in millions of 1972 dollars. Shipments data are averages over the month. All data were demeaned and detrended using a second order polynomial function of time and seasonal dummies.[8]

Table 1 reports the results of estimating the continuous time model using monthly data.[9] We are particularly interested in the implications of these estimates for the speed of adjustment statistics. The point estimate for $\lambda - r$ is 5.29 with 90 percent confidence interval given by (1.83, 8.75). This implies that,

$$T^c = 17\ (10,49).$$

The ninety percent confidence interval is reported in parentheses. Thus the continuous time model implies that it takes 17 days to eliminate 95 percent of the gap between actual and desired inventories. This speed of adjustment seems plausible, especially in light of Feldstein and Auerbach's [1976] observation that even the largest swings in inventory stocks involve only a few days' worth of production.

We now turn to the results obtained with the discrete time models. Table 2, 3, and 4 report results obtained with monthly, quarterly and annual data, respectively. The point estimates of ψ obtained with monthly, quarterly and annual data are .14 (.036,.244), .28 (.070,.490) and .58 (.150,1.01), respectively. Ninety percent confidence intervals are reported in parentheses. The standard errors of the estimates of ψ increase with the degree to which the data are temporally aggregated. Presumably this reflects the smaller number of data points that are available for the more temporally aggregated data.

The implied speed of adjustment statistics are given by,

	Continuous	Monthly	Quarterly	Annual
Days to Close 95% of the Gap	17	46	212	1980
Confidence Interval	(10,49)	(27,63)	(101,378)	$(577,\infty)$[10]

The continuous time figures are repeated here for ease of comparison. The numbers in the last three columns of the first row correspond to T^d in (28)'. The number in the first column of row one corresponds to T^c in (28). Numbers in parentheses in the second row are 90 percent confidence intervals.

Notice that the number of days required to close 95 percent of the gap between actual and desired inventories (T^d) is more than twice as large with monthly data, more than twelve times as large with quarterly data, and more than one hundred and fifteen times as large with annual data, than the estimate obtained using the continuous time model. Evidently, the estimated speeds of adjustment are a monotonically decreasing function of the degree to which the data are temporally aggregated. We take this result to be supportive of the Mundlak-Zellner conjecture that temporal aggregation can account for slow speeds of adjustment in stock adjustment models. The estimated adjustment speeds are plausible for the continuous time and monthly models, but implausibly slow--in our view--in the quarterly and annual models.

An interesting feature of our results is that the estimated speed of adjustment increases in diminishing increments as the model timing interval is reduced. The increase is very large going from annual to quarterly data, but appears to have approximately converged at the monthly level. To see this, notice that the adjustment speed confidence intervals for the monthly and continuous time models overlap considerably. To investigate the conjecture that convergence has occurred with the monthly specification, we compared the discrete time reduced forms of the monthly and continuous time models.

The reduced forms of the continuous and discrete time models are reported in the second columns of Tables 1 and 2 respectively. These are similar along a number of interesting dimensions. First, C_3^c is close to zero, while the third order term in $C^d(L)$ is exactly zero. Also, the 2,1 elements of C_1^c and C_2^c are small, and so compare well with the implication that sales fail to be Granger-caused by inventories.[11] One dimension along which the reduced forms differ concerns the first row of C_2^c, which does not appear to be close zero. In contrast, the first row of C_2^d is identically equal to zero. Also, the variance of the second innovation error is three times larger in the continuous time model than in the discrete time model. Unfortunately, the importance of these differences and similarities is hard to judge, since we do not have the relevant distribution theory. Moreover, it is not clear that a direct comparison of the reduced form parameters is the most revealing one.

In our view, it is more interesting to compare the implications of the two reduced forms for both sets of structural parameters. We are particularly interested in the implications of the reduced form representation of the data emerging from the continuous (discrete) time model for the structural parameters of the discrete (continuous) time model. Consider first the implications of the reported reduced forms for the structural parameters of the continuous time model. Since the continuous time model is identified the reduced form parameters in column 2 of Table 1 map uniquely into the parameter values reported in the first column of Table 1. It is less obvious how to deduce the implications of the reduced form emerging from the discrete time model for the structural parameters of the continuous time model. Since the reduced form of the discrete time model does not satisfy the cross equation restrictions implied by the continuous time model, there is in fact no set of continuous time structural parameters consistent with the discrete time model reduced form. In view of this, we decided that the most sensible thing to do was to compute the set of continuous time parameters that comes "closest" to reproducing the discrete time reduced form in Table 2.

A natural candidate for this set of parameters is the probability limit of the maximum likelihood estimator of the continuous time structural parameters calculated under he assumption that the data are generated by the estimated reduced form corresponding to the discrete time model.[12] If the discrete time model is true then the estimates of the continuous time model obtained using monthly data ought to be close to this probability limit. These probability limits are reported in the second of the two columns labeled "Plim" in Table 5. Numbers in parentheses are the estimated parameter values taken from columns one of Table 2. We find some discrepancies. For example, the plim of α is .035, while its estimated value is .081. Other discrepancies which stand out are the results for bc/a, V_{22}, and V_{12}. Unfortunately, we cannot draw any definitive conclusions regarding the magnitude of these differences in the absence of the relevant distribution theory. Nevertheless it is interesting to note the similarity between the estimated value of $\lambda - r$ and its reported probability limit. As noted earlier, the estimated value of $\lambda - r$ implies that firms close 95 percent of the gap between actual and desired inventories in 17 days. The estimated probability limit of this number under the assumption that the data are generated by the discrete time monthly model is 19.5 days.

We now consider the implications of the two reduced form representations for the structural parameters of the discrete time model. In column 1 of Table 5 we report the probability limits of the structural parameters of the discrete time monthly model. These were calculated under the assumption that the data are generated by the continuous time model. If the continuous time model is true then the estimates of the structural parameters of the discrete time model obtained using the monthly data ought to be close to the corresponding probability limits reported in Table 5. In fact these appear to be quite close to each other. The principal discrepancy is that bc/a is larger than the value reported in Table 2. In addition V_{22}^{d} and V_{12}^{d} are somewhat different from the values reported in Table 2. As before we cannot draw any definitive conclusions from this exercise without the relevant distribution theory. Nevertheless, it is interesting to note how similar the estimate of ψ reported in Table 2 is to its plim in Table 5. In particular, inferences about the speed of adjustment of actual to target inventories are basically the same for the two values of ψ.

We conclude from the results in Table 5 that, when viewed from the point of view of their implications for the discrete time parameters, the reduced forms in Tables 1 and 2 are fairly similar. Some differences are apparent when examined from the point of view of certain structural parameters of the continuous time model.

A third way to compare the two reduced form representations is to compare their log likelihood values. The difference between the log likelihood vale of the discrete time monthly and continuous time models is equal to 25.36. In this sense the discrete time monthly model "fits" the data better than the continuous time model. On the other hand, the likelihood ratio statistic obtained when either of the two models is compared with an unrestricted reduced form ARMA(3,3) model indicates rejection of both structural models at essentially the same level. The log likelihood value of the unrestricted ARMA(3,3) model is 3307.5 which is significantly greater than the log likelihood values associated with both the continuous ad discrete time monthly models (see Tables 1 and 2).

Overall, we conclude that the monthly discrete time and continuous time models appear to be fairly similar when examined from the perspective of the reduced form time series representations that they imply for the monthly data. Next, we report some diagnostic tests on the underlying statistical adequacy of the two structural models.

The validity of the formulas used to compute the confidence intervals around our speed of adjustment estimates requires that the underlying models be correctly specified. Unfortunately, we found evidence against this hypthesis. As we indicated, a likelihood ratio test rejects both models multivariate Box-Pierce statistics proposed by Li and McLeod [1981] to test for serial correlation in the fitted residuals from the continuous time and monthly discrete time models. These statistics were computed at lags 12 and 24 and are denoted by BP(12) and BP(24), respectively. Under the null hypothesis that the underlying disturbances are white noise, BP(k) is drawn from a chi-square distribution with $4 \times k - n$ degrees of freedom, where n is the number of free parameters.[13] In our case, $n = 9$. The Box Pierce statistics for the continuous time model are BP(12) = 162 and BP(24) = 278. For the discrete time model, they are BP(12) = 386 and BP(24) = 602. These statistics indicate a substantial departure from white noise in the fitted residuals. Because the likelihood ratio statistic and Box-Pierce statistics supply evidence against our models the speed of adjustment confidence intervals that we reported above must be interpreted with caution.

To what extent are our results sensitive to the way in which we specified our discrete time model? As we indicated in section 3 there are at least two ways to choose a discrete time analogue to the continuous time model of section 2. Our procedure was to specify the shocks in the discrete time model to have the same representation as the point-in-time sampled representations the continuous time shocks. Since our continuous time shocks are AR(1), this implies an AR(1) representation for the shocks in the discrete time model. We adopted this specification of the discrete time model because it matches well with what is commonly done in the literature.[14] An alternative would have been to specify the shocks in the discrete time model so as to produce a reduced form for that model with AR and MA orders identical to those implied by the continuous time model. This can be accomplished by adding a first order moving average term to the shocks in the discrete time model. We conjecture that the effect of these moving average terms would be to raise the estimated speed of adjustment implied by the discrete time model. This conjecture is based on the belief that the additional MA terms would take over some of the burden borne by the AR parameters--one of which controls the speed of adjustment--for accommodating the serial correlation in the data. This would be consistent with results in Telser [1967]. As yet, we have not formally investigated this conjecture. However, it is important to note that these comments illustrate the observations made in Christiano and Eichenbaum ([1987], section 2B) where we argued that the temporal aggregation effects of shrinking the model timing interval can have the same effect on the reduced form implications of a model as allowing for more serial correlation in the unobserved shock terms.

Table 1

Continuous Time Model

Monthly Data

Structural Parameters*		Reduced Form Parameters
α	.081 (.021)	$\theta_1^c = -1.85$
β	.082 (0.21)	$\theta_2^c = .851$
$\lambda - r$	5.29 (2.10)	$\theta_3^c = -.004$
bc/a	610.9 (9120.5)	$C_1^c = \begin{matrix} -.772 & -.035 \\ -.032 & -.698 \end{matrix}$
a/(a+b+A)	0.00 (.001)	
		$C_2^c = \begin{matrix} -.104 & .009 \\ .088 & -.243 \end{matrix}$
V	$\begin{matrix} 13244.5 & -507.3 \\ (12046.2) & (4854.8) \\ & 28310.7 \\ & (25150.5) \end{matrix}$	
		$V^c = \begin{matrix} 24852.1 & 12459.9 \\ & 187924.0 \end{matrix}$
$\pounds$** = -3352.33		

*Standard errors are displayed in parentheses.

**Value of the log likelihood function.

Table 2

Discrete Time Model

Monthly Data

Structural Parameters*		Reduced Form Parameters
μ	.910 (.027)	$\theta_1^d = -2.01$
ρ	.960 (.021)	$\theta_2^d = 1.14$
ψ	.140 (.063)	$\theta_3^d = -.12$
bc/a	1.00 (1.17)	$C_1^d = \begin{matrix} -.910 & .008 \\ 0.00 & -1.10 \end{matrix}$
a/(a+b+A)	0.00 (.001)	
		$C_2^d = \begin{matrix} 0.00 & 0.00 \\ 0.00 & .130 \end{matrix}$
$V^d =$	$\begin{matrix} 24808.7 & 7594.0 \\ (2110.4) & (3781.3) \\ & 54792.8 \\ & (13156.5) \end{matrix}$	
$\pounds = -3326.97$**		

*Standard errors are displayed in parentheses.

**Value of log likelihood function.

Table 3

Discrete Time Model

Quarterly Data

Structural Parameters*		Reduced Form Parameters
μ	.824 (.077)	$\theta_1^d = -1.96$
ρ	.854 (.063)	$\theta_2^d = 1.18$
ψ	.283 (.132)	$\theta_3^d = -.20$
bc/a	.078 (.602)	$C_1^d = \begin{matrix} -.824 & -.007 \\ 0 & -1.14 \end{matrix}$
a/(a+b+A)	0.00 (.001)	
$V^d = \begin{matrix} 65530.8 & 8337.6 \\ (9731.9) & (14396.0) \\ & 276318.4 \\ & 41021.8 \end{matrix}$		$C_2^d = \begin{matrix} 0.00 & 0.00 \\ 0.00 & .242 \end{matrix}$
$\mathcal{L}$** = -1161.52		

*Standard errors are displayed in parentheses.

**Value of the log likelihood function.

Table 4

Discrete Time Model

Annual Data

Structural Parameters*		Reduced Form Parameters
μ	.139 (.224)	$\theta_1^d = -1.31$
ρ	.584 (.256)	$\theta_2^d = .500$
ψ	.584 (.525)	$\theta_3^d = -.050$
bc/a	.998 (.525)	$C_1^d = \begin{matrix} -.139 & -.038 \\ .206 & -1.17 \end{matrix}$
a/(a+b+A)	.021 (.396)	
$V^d =$	$\begin{matrix} 133765.1 & -42203.5 \\ (42042.6) & (60428.8) \\ & 468030.5 \\ & (146721.4) \end{matrix}$	$C_2^d = \begin{matrix} 0.00 & 0.00 \\ -.029 & .333 \end{matrix}$
£**		

*Standard errors are in parentheses.

**Value of log likelihood function.

Table 5

Probability Limits

Discrete[1] Parameter	Plim[3]	Continuous[2] Parameter	Plim[3]
ρ	.940 (.960)	α	.035 (.081)
μ	.938 (.910)	β	.164 (.082)
ψ	.116 (.140)	$\lambda - r$	4.60 (5.30)
bc/a	51.45 (1.00)	bc/a	.879 (611.1)
a/(a+b+A)	0.00 (0.00)	a/(a+b+A)	0.00 (0.00)
V_{11}^d	24951.7 (24808.7)	V_{11}	19013.6 (13244.5)
V_{22}^d	200570.0 (54792.8)	V_{22}	8276.7 (28310.7)
V_{12}^d	11701.0 (7594.0)	V_{12}	2661.6 (-507.3)

[1]Probability limit of parameters of monthly discrete time model, assuming data are generated by reduced form in column 2, Table 1.

[2]Probability limit of parameters of continuous time model, assuming data are generated by reduced form in column 2, Table 2.

[3]Numbers in parentheses are parameter estimates obtained from the data.

Footnotes

[1]The fact that we specify utility to be linear in leisure warrants some discussion because it appears to be inconsistent with findings in two recent studies. Our specification implies that leisure in different periods are perfect substitutes from the point of view of the representative consumer. MaCurdy [1981] and Altonji [1986] argue, on the basis of panel data, that leisure in different periods are imperfect substitutes from the point of view of private agents. Rogerson [1984] and Hansen [1985] describe conditions under which the assumption that the representative consumer's utility function is linear in leisure is consistent with any degree of intertemporal substitutability at the level of private agents.

[2]It is of interest to contrast our model with the equilibrium model in Sargent ([1979], chapter XV). In that model, the representative agent's utility function is linear in consumption and quadratic in leisure. As a result, the interest rate on risk free securities, denominated in units of the consumption good, is constant. In our model, the representative agent's utility function is quadratic in consumption, with the result that the interest rate on risk free securities, denominated in units of the consumption good, is time varying and stochastic. This feature of our model is attractive in view of the apparent nonconstancy of real interest rates in the U.S. In order to remain within the linear-quadratic framework, we specify utility to be linear in leisure. This implies that the interest rate on risk free securities, denominated in units of leisure, is constant.

[3]To avoid proliferating notation we do not formally distinguish between variables chosen by individual households and firms and their economy wide counterparts. Nevertheless the distinction between them plays an important role in the model. By assumption agents are perfectly competitive and

view economy wide variables, such as P(t) and economy wide sales and inventories, parameterically.

[4]To see that $\lambda > 0$ consider $f(k) = .5r - [k+2.5r^2]^{1/2}$ and note that $f(0) = 0$ and $f'(k) < 0$ for $k \geq 0$.

[5]See Hansen and Sargent [1980a] who show that this procedure yields the unique optimal solution to the social planning problem which the competitive equilibrium solves.

[6]Specifically, Christiano and Eichenbaum [1985] show that ζ and ξ are <u>locally</u> identified. In addition, we show that, given any admissable ζ, then there are at least 5 other values of ζ which are observationally equivalent, i.e., yield an identical value for the likelihood function. We constructed an algorithm to find these ζ's in order to determine whether any of them is admissable in the sense of satisfying the nonnegativity conditions imposed by the model. Generally, we find that one other ζ is admissable in this sense. This value of ζ is obtained by exchanging the values of α and $(\lambda-r)$ and suitably adjusting r. As we point out later, our continuous time parameter estimates imply $\alpha = .082$ and $(\lambda-r) = 5.29$ with $r = .003$. This parameterization implies a relatively rapid speed of adjustment of actual to desired inventories. An alternative parameterization which yields the same value of the likelihood function is one in which $\alpha = 5.29$ and $(\lambda-r) = .082$. This implies that the speed of adjustment is very slow and relatively little serial correlation in the inventory holding cost shock. This parameterization can be ruled out as being implausible since it requires the discount rate to be r x 100 = 62,112 percent. We experimented with numerous parameterizations, and always found that if we placed a reasonable upper bound on r, then global identification obtained. We found the same result regarding ξ.

[7]Our results were insensitive to the different values of r and β that we considered.

[8]This time trend can be rationalized as follows. Suppose that u(t) and v(t) are the sum of a covariance stationary component, as given by equation (18) and a linear function of time and seasonal dummies. Then the equilibrium laws of motion will have two components. The first component will be the law of motion given in the text. The second component will be a deterministic function of time and seasonal dummies. There are no restrictions across the two components. These claims are established in Christiano and Eichenbaum [1985]. There are alternative ways to generate trend growth in inventories and sales. For example, the equilibrium laws of motion for s(t) and I(t) will inherit any unit roots in the VAR for u(t) and v(t). The fact that we choose to work with deterministic time trends does not necessarily reflect the view that this is the only reasonable model of trend growth for our variables. Instead it reflects the fact that almost the entire empirical literature that we wish to address assumes the existence of deterministic time trends.

[9]In models where the timing interval is finer than the data sampling interval, estimates of the AR and MA parameters can be sensitive to the scale in which the data are measured. This contrasts with the case in which the timing interval coincides with the data sampling interval. In the latter case, multiplying the data by a constant scalar affects only the innovation variances but not the AR and MA parameters. To check that our continuous time speed of adjustment estimate is robust to a change of scale, we divided the data by 100 and reestimated the model parameters. The results were virtually unchanged.

[10]The upper bound of the ninety percent confidence interval for ψ in the annual model is 1.01. This implies that firms never reach their target inventory level. This is why the reported upper bound of the ninety percent confidence interval for T^d in the annual model is ∞.

[11]We noted in section 2 that this assumption is frequently made in the inventory literature.

[12]These were computed by maximizing the frequency domain approximization to Gaussian likelihood function in which the periodogram was replaced by the spectral density function implied by the reduced form parameters in Table 2. The justification for calling the resulting numbers probability limits is given in Christiano [1984] where this technique is applied in another context.

[13]Li and McLeod [1981] derive the distribution for their test statistic under the assumption that the model being estimated is an unconstrained vector ARMA with independent, identically distributed disturbances. They show that BP(k) has an asymptotic chi-square distribution on $m^2k-\ell$ degrees of freedom, where m is the number of equations in the vector ARMA model and ℓ is the number of AR and MA parameters. We assume that the appropriate modification regarding the number of degrees of freedom, in our problem, is obtained by replacing ℓ by n.

[14]See for example, Blinder [1986], Eichenbaum [1984], Maccini and Rossana [1984] and the references in McCallum [1984].

References

Altonji, J. G. 1986. "Intertemporal Substitution in Labor Supply: Evidence From Micro Data," manuscript.

Blanchard, O. J. 1983. "The Production and Inventory Behavior of the American Automobile Industry," Journal of Political Economy, 91: 365-400.

Blinder, A. S. 1981. "Inventories and the Structure of Macro Models," American Economic Review, 71: 11-16.

Blinder, A. S. 1986. "Can the Production Smoothing Behavior Model of Inventory Behavior Be Saved," manuscript.

Blinder, A. S. and Holtz-Eakin, D. 1984. "Inventory Fluctuations in the United States Since 1929," manuscript.

Box, G. E. P. and Jenkins, G. M. J. 1976. Time Series Analysis: Forecasting and Control, Holden-Day.

Bryan, W. R. 1967. "Bank Adjustments to Monetary Policy: Alternative Estimates of the Lag," American Economic Review, 57: 855-64

Christiano, L. J. 1984. "The Effects of Aggregation Over Time on Tests of the Representative Agent Model of Consumption," manuscript.

Christiano, L. J. 1985a. "A Critique of Conventional Treatments of the Model Timing Interval in Applied Econometrics," manuscript.

Christiano, L. J. 1985b. "Temporal Aggregation and Government Policy Evaluation," manuscript.

Christiano, L. J. 1986a. "A Continuous Time Model of Cagan's Model of Hyperinflation Under Rational Expectations," manuscript.

Christiano, L. J. 1986b. "A Method for Estimating the Timing Interval in a Linear Econometric Model, with an Application to Taylor's Model of Staggered Contracts," Journal of Economic Dynamics and Control.

Christiano, L. J. and Eichenbaum, M. S. 1985. "A Continuous Time, General Equilibrium, Inventory-Sales Model," manuscript.

Christiano, L. J. 1987. "Temporal Aggregation and Structural Inference in Macroeconomics," in Bubbles and Other Essays, Brunner and Meltzer, Carnegie-Rochester Conference Series on Public Policy, vol. 26, Spring 1987.

Cowles, A. 1960. "A Revision of Previous Conclusions Regarding Stock Price Behavior," Econometrica, 28: 909-15.

Eckstein, Z. and Eichenbaum, M. S. 1985. "Quantity-Constrained Equilibria in Regulated Markets: The U.S. Petroleum Industry, 1947-1972," pps. 41-69, T. J. Sargent, ed., Energy, Foresight and Strategy, Resources for the Future, The Johns Hopkins University Press.

Eichenbaum, M. S. 1984. "Rational Expectations and the Smoothing Properties of Inventories of Finished Goods," Journal of Monetary Economics, 14: 71-96.

Eichenbaum, M. S., Hansen, L. P., and Richard, S. "The Dynamic Equilibrium Pricing of Durable Goods," manuscript.

Engle, Robert F. and Ta-Chung Liu. 1972. "Effects of Aggregation Over Time on Dynamic Characteristics of an Econometric Model," in Econometric Models of Cyclical Behavior, vol. II, Proceedings of NBER Conference on Research in Income and Wealth, edited by B. Hickman, Columbia University Press, New York.

Feldstein, M. S. and Auerbach, A. 1976. "Inventory Behavior in Durable Manufacturing: The Target Adjustment Model," Brookings Papers on Economic Activity, 2: 351-408.

Geweke, J. B. 1978. "Temporal Aggregation in the Multivariate Regression Model," Econometrica, 46: 643-62.

Goodfriend, M. 1985. "Reinterpreting Money Demand Regressions," (eds.) K. Brunner and A. H. Meltzer, Understanding Monetary Regimes, Carnegie-Rochester Conference on Public Policy, Vol. 22: 207-42, Amsterdan, North Holland.

Hannan, E. J. 1970. Multiple Time Series Wiley Press.

Hansen, G. 1985. "Indivisible Labor and the Business Cycle," Journal of Monetary Economics, 16: 309-27.

Hansen, L. P. and Sargent, T. J. 1980a. "Methods for Estimating Continuous Time Rational Expectations Models from Discrete Data," Staff Report 59, Federal Reserve Bank of Minneapolis.

Hansen, L. P. and Sargent, T. J. 1980b. "Formulating and Estimating Dynamic Rational Expectations Models," Journal of Economic Dynamics and Control, 2: 351-408.

Hansen, L. P. and Sargent, L. P. 1981. "Formulating and Estimating Continuous Time Rational Expectations Models from Discrete Data," manuscript.

Hansen, L. P. and Sargent, T. J. 1984. "Two Difficulties in Interpreting Vector Autoregressions," Working Paper 227, Federal Reserve Bank of Minneapolis.

Li, W. K. and McLeod, A. I. 1981. "Distribution of the Residual Autocorrelations in Multivariate ARMA Time Series Models," Journal of the Royal Statistical Society, (Series B), 43: 231-39.

Lucas, R. E. Jr. and Prescott, E. C. 1971. "Investment Under Uncertainty," Econometrica, 39: 154-163.

Luenberger, D. G. 1969. Optimization by Vector Space Methods, Wiley, New York.

Maccini, L. J. and Rossana, R. J. 1984. "Joint Production, Quasi-Fixed Factors of Production, and Investment in Finished Goods Inventories," Journal of Money Credit and Banking.

MaCurdy, T. E. 1981. "An Empirical Model of Labor Supply in a Life-Cycle Setting," Journal of Political Economy, 89: 1059-85.

McCallum, B. T. 1984. "Inventory Fluctuations and Macroeconomic Analysis: A Comment," manuscript.

Mundlak, Y. 1961. "Aggregation Over Time in Distributed Lag Models," International Economic Review, 2: 154-63.

Rogerson, R. 1984. "Indivisible Labor, Lotteries and Equilibrium," manuscript.

Rozanov, Yu. A. 1963. Stationary Random Processes, Holden-Day.

Sargent, T. J. 1979. Macroeconomic Theory, Academic Press.

Sargent, T. J. 1982. "Preliminary Introduction to Continuous Time Stochastic Processes," manuscript.

Sims, C. A. 1971a. "Approximate Specification in Distributed Lag Models," manuscript.

Sims, C. A. 1971b. "Discrete Approximations to Continuous Time Lag Distributions in Econometrics," Econometrica, 38: 545-64.

Telser, L. G. 1967. "Discrete Samples and Moving Sums in Stationary Stochastic Processes," Journal of the American Statistical Association 62, 484-99.

Tiao, G. C. 1972. "Asymptotic Behavior of Temporal Aggregates of Time Series," Biometrika, 59: 525-30.

West, K. D. 1986. "A Variance Bounds Test of the Linear-Quadratic Inventory Model," Journal of Political Economy, 94: 374-401.

Working, H. 1960. "Note on the Correlation of First Differences of Averages in a Random Chain," Econometrica 28, 916-18.

Zellner, A. 1968. "Note on Effect of Temporal Aggregation on Estimation of A Stock Adjustment Equation," manuscript.

Zellner, A. and Montmarquette, C. 1971. "A Study of Some Aspects of Temporal Aggregation Problems in Econometric Analyses," Review of Economics and Statistics, 63: 335-42.