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HOW SHOULD TAXES BE SET?
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## I. Introduction

Are current deficits too high? Is a balanced budget amendment to the constitution a good idea? After all, most state governments are required to balance their budgets over a two year period. Why not insist that the federal budget also be balanced over some suitably short period of time, if not year by year? ${ }^{1}$ Should exceptions be made in the event of a war? For instance, would it have been better to have raised taxes immediately to pay for the Vietnam war instead of financing it by (as was done) running up the federal debt? How should tax policy respond if the government anticipates entering a medium term war in the near future? All of these questions have one thing in common. They force one to confront the issue of how taxes should be set given the usually fluctuating and, of ten unpredictable, requirements for government expenditures. ${ }^{2}$ The purpose of this article is to explain the principles of tax setting and budget management and use these principles to throw some light on the above questions. The point of view that will be adopted in this explanation is that of a benevolent federal government which has the welfare of its citizens as its prime consideration and properly takes account of the impact of taxes on incentives and welfare.

To give the reader an advance flavor of some of the conclusions, I will show that, roughly speaking, tax policy should attempt to maintain a smooth pattern of tax rates over time while the average level of tax rates would have to be such as to generate tax revenues which match the average level of government expenditures plus interest payments on government debt. ${ }^{3}$ It is,
therefore, entirely appropriate to finance unusually high expenditures (i.e., higher than average) by issuing debt rather than raising tax rates and to use the surpluses in periods of below average expenditures to retire some of the debt. In this way, high expenditures are allowed to result in deficits and debt accumulation which are offset by surpluses and debt retirement in periods of low expenditures so that tax rates can be held steady. It follows that a constitutional amendment to balance the budget over short horizons is probably not a good idea. Whether current deficits are "too high" depends on whether current expenditures are viewed as unusually higher than average or not. In the former case current deficits are not too high and will be offset by future surpluses when expenditures dip below average. In the latter case current deficits are too high and present tax policy inappropriate--it would be beneficial to let the tax rate rise gradually. If one associates higher than average levels of expenditures with wars and similar events then one might be led to judge current deficits as being too high.

The above conclusions are based on a relatively simple model of tax determination and debt policy first developed by Robert J. Barro in 1979. ${ }^{4}$ In order to provide some background to this model I first discuss, in Section II, the variety of effects that different types of taxes generally have and what might be appropriate ways of analyzing these. These considerations will lead to the Barro (1979) model as an appropriate tool with which to address the questions posed in the beginning. In Section III, I then give an exposition of the Barro (1979) model. In Section

IV, I explain how and why the model leads to the conclusions outlined above. In Section V, I note some limitations of the simple model and the resulting qualifications to the conclusions. Finally in Section VI, I summarize the lessons to be learnt regarding tax policy and debt management.

## II. Effects of Taxes

The variety of effects that different types of taxes can have can be classified into the following:
a. intragenerational wealth redistribution effects,
b. intergenerational wealth redistribution effects, and
c. incentive effects due to nonlump sum nature of taxes.

In general, taxes can and do impinge differently on different people thereby creating some wealth redistribution effects. Progressive income taxes are designed to take more from the high income than from the low income people. Sales taxes ${ }^{6}$ are generally regressive because low income people tend to spend a higher percent of their income than high income people. Specific taxes on various goods, the best examples of which are probably the "sin" taxes on tobacco and liquor, also affect different people differently. These distribution effects may occur either among people of a given generation as in (a) or among people of different generations as in (b). The most interesting case illustrating (b) is when the government chooses to finance current expenditures by borrowing (instead of taxing the currently alive generation), rolls over the debt for many years and then pays off the debt by taxing later generations. Indeed, many critics of
current deficits have argued exactly this point, that by failing to raise enough taxes presently the burden of financing current government expenditures is being passed on to future generations.

The incentive effects of taxes noted in (c) arise only because taxes are not lump sum but are generally related to the level at which people choose to undertake various economic activities. ${ }^{7}$ An income tax will generally affect people's incentives to work (the after-tax income from taking up a second job may not be worth the loss in leisure) and to save (since interest income is also taxed). There can also be more subtle intertemporal effects. If it is known that the tax rate next year will be much higher than this year then there will be a great incentive to increase work this year (and postpone the unpaid vacation to next year!) and to reduce saving. The converse will be the case if it becomes known that the tax rate next year will be lower.

Which of these effects are interesting and important from the point of view of determining the timing pattern of taxes? For the moment suppose that taxes are lump sum. Then they can at most have effects of the types (a) and (b). Clearly, whose hide is being gored by taxes is important to the persons involved and to society as a whole from the point of view of the distribution of income and wealth. Essentially, this involves value judgments but has little to do with the determination of the time pattern of taxes. The reason is that with lump sum taxes the only thing that matters to an individual is the present value of taxes that he/she is responsible for paying. The time pattern of taxes over his/her life is irrelevant. Thus, while considerations of
intergenerational equity may limit the extent to which taxes can vary over time (since some taxes would in general have to be levied on each generation) they cannot uniquely determine the time path of taxes and hence the time path of deficits and government debt.

Another consideration that arises in this context is the following. Not only may the time pattern of taxes over the life of a single generation be irrelevant but also irrelevant may be the pattern of taxes across different generations. This view dates back to the English economist, David Ricardo (1772-1823) and is known as the Ricardian doctrine and was resurrected by Robert J. Barro in $1974 .^{8}$ In a previous Quarterly Review article I explained the economics behind this position. ${ }^{9}$ Briefly, the idea is that members of a family (who typically belong to different generations) care for each others' welfare and that while individual members of a family come and go (that is, some die and some new ones are born) the family itself effectively lives forever by constantly replacing its members. The altruistic linkages across members of different generations may render the intergenerational distribution of taxes irrelevant, at least within some range. For instance, the potential distribution effects of a cut in taxes for the present generation and a rise in taxes for a future generation may be neutralized by the present generation saving the tax cut and passing it on to its descendent generation as a larger bequest with which it can meet its larger tax bill. Thus, the intergenerational distribution of taxes and hence the time pattern of taxes generally, may be largely irrelevant in the presence of such altruistic motives. ${ }^{10}$

From the above discussion we can conclude that the most fruitful and interesting avenue towards understanding the determination of the timing pattern of taxes is the analysis of the incentive effects of non lump sum taxes. An intuitive feel for the importance of this consideration can be given as follows. Suppose that government expenditures are fluctuating in a regular and predictable manner. What would be the effects of raising and lowering the income tax rate in step with expenditures in order to maintain a balanced budget? From previous discussion it should be clear that this creates incentives for people to work less and therefore produce less in periods when expenditures are high. We will see that, on average, this leads to a lower level of output and hence private consumption (total output less government expenditures less investment). A policy of maintaining the tax rate roughly constant would not create similar incentives to shift work intertemporally and would lead to a higher average level of private consumption. Thus, the intertemporal incentive effects of fluctuating tax rates can dictate a unique time pattern of tax rates that is best from a social point of view.

We will, therefore, concentrate on analyzing the incentive effects of taxes and how they might help determine the time pattern of taxes. Consequently, we will ignore the intergenerational distributional effects of taxes by pretending that the different generations are altruistically linked and hence can be thought of as a single infinitely lived family. That is, we will be focusing on the incentives to allocate family work over time in response to taxes and ignore the wealth distribution among members
of the family. Effectively, a family may be thought of as consisting of a single representative individual who controls total family wealth and decides the allocation of total family work across time. Thus, the entire economy consists of a large number of such infinitely lived agents. Under some conditions it is appropriate to ignore distributional effects among these agents and represent the entire economy as consisting of a single infinitely lived agent. ${ }^{11}$ In the next section I will describe and analyze the Barro (1979) model of tax determination which is based on the incentive effects of taxes on the behavior of a single representative infinitely lived agent.

## III. A Simple Model of Tax Determination

The model has the following features:
a. There is a single infinitely lived agent who works and produces, thereby generating labor income. Work (measured in, say hours/week) involves an opportunity cost measured in units of foregone consumption and the agent cares for net consumption, that is, consumption net of the opportunity cost of working. He/she also has access to a simple storage technology yielding a constant net return,
b. The path of government expenditures is given exogenously and taxes are levied as a percent of labor income. ${ }^{12}$ The government may also issue debt to meet expenditures,
c. The individual maximizes welfare given by the discounted sum of the utility of net consumption by choosing the allocation of work, net consumption and saving over time and taking the time pattern of tax rates as given,
d. The government chooses the time pattern of tax rates to maximize the agent's welfare subject to its own budget constraint and taking account of the effects of changing tax rates on the agent's behavior. The government also treats the time path of expenditures exogenously.

Let $C(t), l(t)$ be consumption and work, respectively, in period $t$ where $t$ takes values $0,1,2, \ldots$, etc. Let $H(1)$ denote the opportunity cost of work in units of foregone consumption and let

$$
\begin{equation*}
c(t)=C(t)-H(1(t)) \tag{1}
\end{equation*}
$$

denote net consumption. A fairly typical opportunity cost function $\mathrm{H}($.$) is shown in Figure 1. The marginal opportunity cost of$ work is defined as the increase in opportunity cost resulting from a unit increase in work, and corresponds to the slope of the curve $\mathrm{H}(\cdot)$ in Figure 1. We assume that both the opportunity cost as well as the marginal opportunity cost are zero at zero work and that both are increasing as the level of work increases. This means that both the level of work as well as the marginal unit of work are costless at zero work level and both become increasingly unpleasant as work increases. Next we assume that each unit of work results in w units of output, ${ }^{13}$ and denote by $y(t)$ net labor income, that is, labor income net of the opportunity cost of work as well as taxes. This is given by the following where $\theta(t)$ is the tax rate on labor income in period $t$

$$
\begin{equation*}
y(t)=(1-\theta(t)) w l(t)-H(l(t)) . \tag{2}
\end{equation*}
$$

Tax revenues are denoted by $T(t)$ and are obviously given by,

$$
\begin{equation*}
T(t)=\theta(t) W l(t) . \tag{3}
\end{equation*}
$$

Let $W(t)$ be the total wealth of the individual measured in units of consumption at the beginning of period $t$, consisting of $K(t)$ units of capital and $D(t)$ units of government debt. The agent earns interest on both of these at the constant rate $r$. We can now write the individual's intertemporal budget constraint in the following way

$$
\begin{equation*}
c(t)+W(t+1) /(1+r)=y(t)+W(t) . \tag{4}
\end{equation*}
$$

Equation (4) says that net labor income plus wealth is either spent on net consumption or accumulated as future wealth. Note that since $W(t+1)$ is wealth at the beginning of period ( $t+1$ ) in units of $(t+1)$ consumption, its value in units of period $t$ consumption is only $W(t+1) /(1+r)$. Now assume that the agent maximizes welfare as of period zero, denoted by $V(0)$ which is given by the following

$$
\begin{equation*}
V(0)=\sum_{t=0}^{\infty} \beta^{t} U(c(t)) \tag{5}
\end{equation*}
$$

In the above expression, $U(\cdot)$ measures the utility derived in period $t$ and depends on period $t$ net consumption, ${ }^{14}$ and $B$ is the discount factor assumed to be positive but less than unity. This implies that a unit of utility derived tomorrow is less valuable (by the factor $\beta$ ) than a unit of utility derived today, that is, the consumer exhibits impatience with regard to the future.

In order to analyze the consumer's welfare maximization problem it will be convenient to rewrite the budget constraint (4) in present value form. To do this we will assume that $r$ is positive and that wealth is bounded below, i.e., wealth is always greater than some (possibly negative) number. Under these conditions it can be shown that the present value budget constraint can be written as follows ${ }^{15}$

$$
\begin{equation*}
\sum_{t=0}^{\infty} c(t) /(1+r)^{t}=\sum_{t=0}^{\infty} y(t) /(1+r)^{t}+W(0) . \tag{6}
\end{equation*}
$$

The consumer maximizes $\mathrm{V}(0)$ given by (5) subject to (6) by choosing the time paths of net consumption and net income. Note that from equation (2) net income is determined by the choice of work and will depend on the tax rate, $\theta(t)$. These choices may be analyzed as follows.

First, the consumer will equate the marginal rate of substitution between $c(t)$ and $c(t+1)$ to the corresponding opportunity cost of $c(t)$ in terms of $c(t+1)$. The marginal rate of substitution is the ratio of the marginal welfare of $c(t)$ to that of $c(t+1)$. It measures the number of units of $c(t+1)$ that the consumer is willing to sacrifice in order to obtain an additional unit of $c(t)$. The marginal welfare of $c(t)$ is the extra welfare obtained by an additional unit of $c(t)$ and is given by $B^{t} U^{\prime}(c(t))$, where $U^{\prime}(\cdot)$ is the derivative of $U(\cdot)$. Therefore, the marginal rate of substitution is given by $U^{\prime}(c(t)) /\left[\beta U^{\prime}(c(t+1))\right]$. The opportunity cost is (1+r) because if the consumer were to increase $c(t)$ by one unit, financing it by borrowing, then $c(t+1)$ would have to be reduced by ( $1+r$ ) units to pay back the loan. Therefore, we obtain the following equation

$$
\begin{equation*}
U^{\prime}(c(t)) /\left[\beta U^{\prime}(c(t+1))\right]=1+r . \tag{7}
\end{equation*}
$$

By assuming a special form for the utility function (known as the constant elasticity form) ${ }^{16}$ it is possible to give an explicit solution to the consumer's choice of the time path of net consumption. This solution is described by the following two equations

$$
\begin{equation*}
c(t+1)=(1+n) c(t) \tag{8a}
\end{equation*}
$$

$c(0)=\alpha \sum_{t=0}^{\infty} y(t) /(1+r)^{t}+W(0)$.
In this solution net consumption grows at a constant geometric rate $n$ that depends only on $\beta,(1+r)$ and the form of the utility function, and net consumption at the starting date, $c(0)$ is proportional to total wealth which is $W(0)$ plus the present value of net income. ${ }^{17}$

The solution to the problem of choosing the time path of work simply amounts to choosing $l(t)$ in each period to maximize net income $y(t)$. As is clear from equation ( 8 b ) this results in the maximum possible value of $c(0)$ and hence from ( 8 a ), the best possible value of $c(t)$ and hence also of welfare $V(0)$. The choice of $l(t)$ is illustrated in Figure 1. In this figure the straight line $O A$ with slope w represents the relationship between before tax income and work, the straight line $O B$ with slope $w(1-\theta(t))$ represents the relationship between after tax income and work, and finally the curve marked $H(1)$ represents the opportunity cost of work. The marginal after tax income is defined as the extra income after taxes obtained by working an extra unit, and corre-
sponds to the slope of $O B$. For a given value of $l(t)$ the vertical distance between $O A$ and $O B$ gives the tax revenues and the vertical distance between OB and $\mathrm{H}(1)$ gives net income (see equations (2) and (3)). The maximum value of net income occurs when the marginal after tax income which is $w(1-\theta(t))$ equals the marginal opportunity cost of work which is the slope of the curve marked $H(1)$. In Figure 1, maximum net income occurs at a value of $l(t)$ given by OL. Tax revenues and net income are as shown in the figure. This completes the description of the consumer's behavior.

I will now describe the government's budget constraint and maximization problem. Let $g(t)$ be government expenditures in period $t$, and let $D(t)$ be the face value of government debt outstanding at the beginning of period $t$. Then the period $t$ government budget constraint is given by the following

$$
\begin{equation*}
g(t)+D(t)=T(t)+D(t+1) /(1+r) \tag{9}
\end{equation*}
$$

Equation (9) says that expenditures plus debt must be paid off by tax revenues and additional borrowing. Note that since $D(t+1)$ is the face value of debt at the beginning of period $(t+1)$, its market value as of period $t$ is only $D(t+1) /(1+r)$. We can now develop the resource constraint for this economy as follows. In equation (4), substitute for $c(t)$ from (1), for $y(t)$ from (2) and for $\theta(t) w l(t)$ from (3). Next substitute for $T(t)$ from (9) and use the fact that $W(t)$ equals $K(t)$ plus $D(t)$. This yields the following resource constraint

$$
\begin{equation*}
C(t)+g(t)+K(t+1) /(1+r)=w l(t)+K(t) . \tag{10}
\end{equation*}
$$

Just as we developed the consumer's budget constraint in present value form (equation (6)), we can use (9) to develop the government budget constraint also in present value form. We will assume that there is some upper bound to $D(t)$ so that the government is also prohibited from running a ponzi game by perpetually rolling over debt (see footnote 15). With this assumption we can rewrite equation (9) as follows by repeatedly substituting for $D(t+1)$ in terms of $D(t+2)$ and so on

$$
\begin{equation*}
\sum_{t=0}^{\infty} g(t) /(1+r)^{t}+D(0)=\sum_{t=0}^{\infty} T(t) /(1+t)^{t} . \tag{11}
\end{equation*}
$$

Equation (11) says that the present value of tax revenues must be sufficient to pay for the present value of expenditures plus the debt outstanding at date zero. An equivalent way to write this equation is the following

$$
\begin{equation*}
\sum_{t=0}^{\infty}[g(t)+r D(0) /(1+r)-T(t)] /(1+r)^{t}=0 . \tag{12}
\end{equation*}
$$

The term $r D(0) /(1+r)$ represents interest payments on debt. Therefore, equation (12) says that in present value terms the government's budget is always balanced. The present value of deficits (total expenditures plus interest on debt minus tax revenues) must be zero. Therefore, it is impossible to run deficits forever. Indeed, it is not possible to run surpluses forever either. Thus periods of deficits must be followed by periods of surpluses and in a present value sense they must cancel each other out. In fact, if we define the average levels of tax revenues and government expenditures as follows, then it must be that on average the deficit is zero

$$
\begin{equation*}
\bar{T}=[r /(1+r)] \sum_{t=0}^{\infty} T(t) /(1+r)^{t} \tag{13a}
\end{equation*}
$$

$$
\begin{equation*}
\bar{g}=[r /(1+r)] \sum_{t=0}^{\infty} g(t) /(1+r)^{t} \tag{13b}
\end{equation*}
$$

$$
\begin{equation*}
\overline{\mathrm{T}}=\overline{\mathrm{g}}+\mathrm{rD}(0) /(1+\mathrm{r}) . \tag{13c}
\end{equation*}
$$

$\overline{\mathrm{T}}$ and $\overline{\mathrm{g}}$ are the geometrically weighted averages of the time paths of tax revenues and government expenditures, respectively. The result that the average deficit is zero is a simple consequence of the government budget constraint (12). $\overline{\mathrm{T}}$ and $\overline{\mathrm{g}}$ may also be interpreted as the permanent levels of tax revenues and expenditures, analogously to the concept of permanent income. ${ }^{18}$ That is, $\overline{\mathrm{T}}$ and $\overline{\mathrm{g}}$ are the constant levels of tax revenues and expenditures that have the same present value as the actual time paths of taxes and expenditures, respectively. Thus, in this (geometrically) weighted average sense the budget is always in balance, even though during any period $t$, tax revenues $T(t)$ may exceed or fall short of expenditures plus interest on debt. The average level of tax revenues is thus determined by the average level of government expenditures and the initial level of debt. The question that remains to be answered is what is the best time pattern of tax revenues given the average value. Should tax revenues be fairly smooth over time, staying close to their average value or should they vary highly up and down, perhaps in step with government expenditures? We will now proceed to answer this question.

As mentioned in the introduction and as outlined at the beginning of this section, we will assume that the government
chooses the time path of $\operatorname{tax}$ rates $\theta(t)$ subject to its budget constraint and taking account of its effect on the consumer's behavior in such a way as to maximize the consumer's welfare $\mathrm{V}(0)$. From previous discussion this is equivalent to maximizing $c(0)$ and hence to maximizing the present value of net income (see $8(b)) .{ }^{19}$

To analyze this problem it is convenient to let the government choose tax revenues $T(t)$ directly and the tax rate $\theta(t)$ indirectly, since the government's budget constraint is written in terms of $T(t)$. That is, the government picks $T(t)$ and then lets the tax rate $\theta(t)$ be whatever it must be to raise the required tax revenues. For this purpose we need to determine the relationship between tax revenues $T(t)$, the tax rate $\theta(t)$ and net income $y(t)$. This can be obtained from Figure 1 by varying the tax rate $\theta(t)$ between zero and unity, calculating the consumer's choice of work at each different tax rate and thereby the amount of tax revenues and the net income. The result of this exercise is shown in Figure 2. The relationship between the $\operatorname{tax}$ rate $\theta(t)$ and tax revenues $T(t)$ has an inverted $U$-shape because when the tax rate is zero, tax revenues will also be zero and when the tax rate is unity, the consumer will choose zero work and hence tax revenues will again be zero. ${ }^{20}$ Net income $\mathrm{y}(\mathrm{t})$ on the other hand will always be decreasing with the tax rate. We may as well restrict attention to tax rates below $\theta^{*}$ because any tax revenue that can be raised by a tax rate above $\theta^{*}$ can also be raised by a tax rate below $\theta^{*}$ and net income will be higher. Therefore, the government will never choose a tax rate above $\theta^{*}$. Given this restriction we
see that there is a one-to-one and increasing relationship between tax revenues and the tax rate, and a one-to-one and decreasing relationship between tax revenues and net income. Therefore, we may as well let the government specify tax revenues $T(t)$ and we can calculate the corresponding value of net income $y(t)$ from Figure 2. This relationship is given by the following equation and is illustrated in Figure 3

$$
\begin{equation*}
y(t)=y(T(t)) . \tag{14}
\end{equation*}
$$

It can be seen that net income is decreasing in tax revenues. It will be useful to define the concept of marginal net income loss as the loss in net income due to an increase in tax revenues by one unit. This corresponds to the slope of the curve in Figure 3. Under reasonable assumptions it is possible to show that the marginal net income loss is always increasing in tax revenues, is unity at zero and infinitely high at the value $T^{*}$ which is the maximum possible tax revenue. Intuitively, the loss in net income from raising the tax revenue marginally is higher, the higher is the level of taxes. It always exceeds unity since an increase in the tax revenues by one unit will decrease net income by at least one unit. The shape of the curve in Figure 3 reflects this assumption.

The government's problem may now be written as choosing the time path of tax revenues $T(t)$ to maximize
(15a) $\sum_{t=0}^{\infty} y(T(t)) /(1+r)^{t}$
subject to the constraint

$$
\begin{equation*}
\sum_{t=0}^{\infty} T(t) /(1+r)^{t}=\bar{g}(1+r) / r+D(0) . \tag{15b}
\end{equation*}
$$

Equation (15b) is simply a rewritten form of (13c) using (13a). Since we are taking the time path of expenditures $g(t)$ as given and the amount of government debt outstanding in period zero is also given by past budget policies, the right side of (15b) is independent of tax rates.

The solution to problem (15) is quite simple. Keep tax revenues constant at the level $\overline{\mathrm{T}}$ forever! The explanation for this remarkable conclusion is as follows. We will first argue that the marginal net income loss in any two successive periods must be the same. For, suppose to the contrary that the marginal net income loss in period $t$ (say, 2 units) is greater than that in period ( $t+1$ ) (say, 1 unit). If the government reduces tax revenues in period $t$ by one unit and increases them by ( $1+r$ ) units in period ( $t+1$ ), then the government budget constraint (15b) will still be satisfied. Net income in period $t$ will go up by 2 units and will go down in period ( $t+1$ ) by ( $1+r$ ) units. Hence, the present value of net income will go up by 1 unit. A similar argument can be made if the marginal net income loss in period $t$ is less than in period $(t+1)$. This proves that unless the marginal net income loss in every period is the same, the present value of net income cannot be at its maximum possible value. The conclusion that tax revenues $T(t)$ must be the same in every period follows because the slope of the curve in Figure 3 (which measures the marginal net income loss) is, by assumption, always increasing in tax revenues. Therefore, if tax revenues differ in any two periods the marginal net income loss cannot be the same in those
two periods. It also follows that the tax rate must then be the same in every period.

It should be emphasized here that the conclusion that tax revenues must be the same in every period depends on our earlier assumption that labor productivity $w$ is constant over time. If $w$ is changing over time then the relationship between $y(t)$ and $T(t)$ shown in Figure 3 will be changing over time. Consequently, a constant path of the marginal net income loss will lead to a fluctuating path of tax revenues over time. In general, the tax rate will also be changing over time. However, if the elasticity of work with respect to after tax productivity (percent change in work due to a one percent change in the after tax productivity) is constant ${ }^{21}$ then the tax rate will be constant even if productivity varies over time. Tax revenues $T(t)$ will however be varying over time because higher productivity will lead to a higher level of work and hence to a higher level of before tax income. In general, the tax rate depends inversely on the elasticity of work. This result is an instance of the Ramsey principle of taxation which states that tax rates on different goods should be set in inverse relation to their demand/supply elasticities. ${ }^{22}$

As noted in the introduction, the result that the tax rate should be constant over time is a simple application of the principle of uniform taxation from public finance theory. The crucial assumptions in our context are: (1) the supply function of work in period $t$ depends only on the period $t$ productivity (i.e., all cross price elasticities for dated work are zero) and
is the same in each period, (2) pre-tax productivity is constant over time. However, these are not the only assumptions that give rise to the constant tax rate result. The literature on uniform taxation (Sandmo 1974, Sadka 1977) has identified other assumptions on utility functions that yield the same result, even if the pre-tax productivity is varying over time. Our choice of the utility function was made for ease of exposition. ${ }^{23}$

It is possible to give a slightly different and somewhat interesting interpretation of the government's problem of choosing tax rates over time. This interpretation is in terms of minimizing the present value of "excess burden." The concept of excess burden may be explained as follows. ${ }^{24}$ Suppose the government has to raise tax revenues in the amount $T(t)$. Then from Figure 2 we can calculate the tax rate $\theta(t)$ that it has to set (always below $\theta^{*}$ ) and also what the net income would be. If, however, the government had the option of raising the same amount of tax revenues by levying a lump sum tax then net income would be higher. This difference in net incomes when the tax is proportional and when it is lump sum (given that both yield the same tax revenues) is said to be the "excess burden" on the consumer. The idea is that for any given level of tax revenues being raised by a proportional tax, a lump sum tax raising the same amount would leave the consumer with a larger net income. ${ }^{25}$ Hence, the burden of a proportional tax is in "excess" of what it need be. Equivalently, for a given level of net income for the consumer, a lump sum tax would raise more revenues than a proportional tax and the difference is the excess burden.

First let us show that net income would indeed be higher under lump sum taxes for a given amount of tax revenues to be raised. In terms of Figure 1, the consumer equates the marginal after tax income to the marginal opportunity cost of work. Therefore, under proportional taxes the marginal opportunity cost of work equals $w(1-\theta(t))$. Under lump sum taxes the marginal after tax income is $w$ (since the amount of the tax is independent of work) which exceeds $w(1-\theta(t))$. Therefore, with a lump sum tax, maximizing net income will lead the consumer to choose a larger value for $l(t)$ since the marginal opportunity cost of work is increasing. That is, if the tax were lump sum the consumer would be able to increase net income by increasing work. It follows that net income under the lump sum tax is higher. In Figure 1, the choice of $l(t)$ under a lump sum tax is shown as $O L^{\prime}$ and the corresponding net income is also indicated. Let $l^{\prime}(t)$ be the choice of work, and $y^{\prime}(t)$ be the net income under a lump sum tax. Note that the choice of $l^{\prime}(t)$ is in fact independent of tax revenues (it depends only on labor productivity $w$ and the opportunity cost function $H(\cdot))$. Let $E B(t)$ be the excess burden in period $t$. We then have,

$$
\begin{equation*}
E B(t)=y^{\prime}(t)-y(t) \tag{16}
\end{equation*}
$$

$$
=\left[w l^{\prime}(t)-H\left(l^{\prime}(t)\right)-T(t)\right]-y(t) .
$$

The above relationship is illustrated in Figure 3. Since $l^{\prime}(t)$ is independent of tax revenues, $y^{\prime}(t)$ decreases one-to-one with $T(t)$. When tax revenues are zero $y^{\prime}(t)$ equals $y(t)$ because the tax rate is also zero. Therefore, $y^{\prime}(t)$ always lies
above $y(t)$ and the vertical distance between the two at a given level of tax revenues measures the excess burden. We now rewrite equation (16) in the following present value terms

$$
\begin{align*}
\sum_{t=0}^{\infty} y(t) /(1+r)^{t}= & \sum_{t=0}^{\infty}\left[w l^{\prime}(t)-H\left(l^{\prime}(t)\right)\right] /(1+r)^{t}  \tag{17}\\
& -\sum_{t=0}^{\infty} T(t) /(1+r)^{t}-\sum_{t=0}^{\infty} E B(t) /(1+r)^{t} .
\end{align*}
$$

Since the present value of tax revenues is also independent of the time path of tax revenues (from the government budget constraint (11)) maximizing net income by choice of the time path of tax rates is equivalent to minimizing the present value of the excess burden. Thus, the government's problem can be stated as one of minimizing the present value of the stream of losses in the consumer's net income due to the fact that taxes are proportional rather than lump sum.

In the next Section I will describe the implications of the above analysis for the questions posed in the introduction.

## IV. The Time Path of Optimal Taxes Under Various Cases

Is a balanced budget amendment a good idea? The above model clearly says no. Even if expenditures are fluctuating up and down tax rates should be maintained at a constant level sufficient to match the permanent level of expenditures plus interest. It would not be a good idea to raise and lower the tax rate in step with expenditures so as to maintain a balanced budget. The reason, as explained before is that the incentive on the part of consumers to vary work over time in response to changing tax rates generates losses in the present value of net income (equiva-
lently raises the present value of the excess burden) and hence in welfare. Therefore, it is better to let the deficit rise in periods of unusually high expenditures and issue more government debt and make up for it by surpluses in periods of below average expenditures.

It follows from this discussion that the issue of whether current deficits are "too high" hinges on what we believe to be the permanent level of government expenditures. If we associate high levels of expenditures with war times and other periods of unusually high demands on the government (say, disaster relief at home or abroad) then one may judge current expenditures not to be above average. In that case current deficits are too high and tax revenues too low.

It is also interesting to consider the response of tax rates to a variety of anticipated and unanticipated changes in the time path of government expenditures. The relationship of this discussion to the question posed in the introduction regarding the appropriateness of tax policy before and during the Vietnam war will be obvious and we will not comment on it further.

To simplify the discussion I will assume that initially the level of government expenditures is in fact constant and that tax rates are in fact chosen as described in the previous section. Therefore, expenditures plus interest on debt will also be constant over time and equal to tax revenues. Consequently, the budget will be in balance in every period, initially. I will consider the following examples.

## A. An Anticipated Temporary Rise in Expenditures

Suppose that starting in some future period expenditures are expected to go up for a certain number of periods before coming back down to the previous level. How should tax policy respond? A balanced budget rule says to do nothing until the periods in which expenditures actually go up and then to raise tax rates by the required amount to keep the budget in balance. Our model recommends a different policy. This is to raise tax rates immediately to a higher constant level to match the increase in permanent expenditures thereby running budget surpluses until the period in which expenditures actually go up. The surpluses are used to make loans to the consumer and credit is built up. Then the interest income from these loans plus tax revenues is used to partly offset the higher level of expenditures, the rest being made up by issuing more debt. When expenditures return to normal, the level of debt will be higher than initially but the budget will be in balance and stay that way. This happens because tax revenues go up uniformly in all periods and hence will be higher than the original level of expenditures plus interest.

The pattern of response is pretty much the same regardless of the magnitude of the anticipated rise in expenditures or the number of periods for which the rise is expected to last, or how soon the rise occurs. What differs is the magnitude of the immediate (and permanent) rise in tax revenues. This happens simply because the magnitude of the rise in permanent expenditures increases with the magnitude of the rise in expenditures and the number of periods for which it lasts and the proximity of the rise
in expenditures. This is a simple consequence of the definition of permanent expenditures given in (13b). However, it should be clear that the magnitude of the rise in tax revenues is always less than the anticipated rise in expenditures. Figure 4 illustrates this case.

## B. An Immediate Permanent Rise in Expenditures

Suppose that starting in period zero, the level of expenditures goes up permanently and uniformly by, say one unit. From the definition of permanent expenditures in (13b) we see at once that permanent expenditures also goes up by one unit. Therefore, tax revenues should be raised immediately and permanently by one unit. This case is illustrated in Figure 5. Note that this conclusion is independent of the assumption that the initial path of expenditures was constant and that, initially, the budget was always in balance. Therefore, what remains unaffected under such a tax response is the time path of the deficit.
C. An Immediate Temporary Rise in Expenditures

Suppose that starting in period zero the level of expenditures goes up temporarily for a certain number of periods by, say one unit. From (13b) it is clear that permanent expenditures go up by less than one unit and hence tax revenues should go up immediately and permanently by the amount of the rise in permanent expenditures. This implies that the government will be running deficits during the time that expenditures are high and hence more and more debt will be issued. Once expenditures return to normal, tax revenues will exceed expenditures by just enough to meet
interest payments on the higher level of debt. That is, the budget will be in balance and stay that way. This case is shown in Figure 6.

## V. Limitations and Qualifications

We have used a very simple model of tax determination and debt management and obtained some interesting and sharp conclusions. It is, therefore, appropriate at this stage to highlight the simplifying assumptions we have made and consider how our results may be affected if these simplifications are relaxed. I discuss some of these in the following.

## A. Robustness of Result

Our principal conclusion is that tax rates should be constant over time at a level sufficient to generate revenues equal to permanent government expenditures plus interest on debt. How robust is this result? We have already noted one qualification that arises if labor productivity is not constant but varying over time. In general, neither tax revenues nor tax rates will be constant. However, it will still be true that the marginal net income loss (equivalently, the marginal excess burden) should be constant over time. Even this result goes away if the utility function has a more complicated form. In our model, utility depends only on the difference between consumption and the opportunity cost of work rather than separately on each. In the more general case maximizing consumer welfare is not equivalent to maximizing the present value of net income (see footnote 25). Therefore, the solution for the time path of tax rates will be
more complicated. Another assumption that simplifies matters considerably for us is the assumption that the rate of return on capital is fixed independently of the consumer's choice of work. ${ }^{26}$ If this is not so then the problem becomes more complicated and so will be the time path of tax rates. However, to the extent that our specification is a reasonably good approximation to whatever might be more realistic forms for the utility and production functions our result is also likely to closely approximate the best choice of tax rates over time.

## B. Problem of Commitment and Time Consistency

Our problem has been formulated as one in which the government chooses and announces at date zero the entire infinite sequence of $\operatorname{tax}$ rates $\{\theta(t), t \geq 0\}$. But once period zero passes and period one arrives, is the government required to remain committed to the remaining time path of tax rates that was previously announced or is it allowed to choose a possibly different time path of $\operatorname{tax}$ rates $\left\{\theta^{\prime}(t), t \geq 1\right\}$ from period one onwards? Would it make any difference whether or not the government's hands were tied in period zero? If the government is not committed to follow through with whatever tax rates it announces for future periods, how exactly is the consumer supposed to form beliefs about future tax rates?

First, it might be useful to point out that it is indeed necessary to have the government announce current and all future tax rates so that the consumer's welfare maximization problem is well posed. From (5) and (6) we see that the consumer's choice of the time path of consumption depends on the time path of net
income, which in turn depends on the time path of tax rates (from (2)). In solving the government's problem of choosing tax rates we have implicitly adopted the view that it is fully committed to implementing whatever time path of tax rates it chooses at time zero and is not allowed to change its mind as time passes. For our simple model, it turns out that it makes no difference even if we assume that the government is not committed and is allowed to choose a different time path of tax rates after period zero has passed. It will in fact choose the same time path of taxes from period one onwards whether the choice is made in period zero or in period one. It is not difficult to verify this. We know that $T(0)$ equals $\bar{T}$ where $\bar{T}$ is given by (13c). Substitute this value for $T(0)$ in the period zero version of (9) and calculate the resulting value of $\mathrm{D}(1)$, the amount of government debt at the beginning of period one. Now use the following updated version of (13b) to calculate the permanent level of government expenditures as of period one. This value, denoted by $\mathrm{g}^{\prime}$ is given by the following

$$
\begin{equation*}
g^{\prime}=[r /(1+r)] \sum_{t=1}^{\infty} g(t) /(1+r)^{t-1} . \tag{18}
\end{equation*}
$$

By the same argument that was used before, the best choice of tax revenues for the government from period one onwards will be constant and equal to $\mathrm{T}^{\prime}$ where T ' is given by the following

$$
\begin{equation*}
T^{\prime}=g^{\prime}+r D(1) /(1+r) . \tag{19}
\end{equation*}
$$

If we substitute for $\mathrm{g}^{\prime}$ and $\mathrm{D}(1)$ in (19) and use (13b) it is easy to see that $T^{\prime}$ equals $\overline{\mathrm{T}}$. That is, the government will indeed find it in its best interest to continue implementing the time path of tax rates that it found best as of period zero, even as time passes. Thus, the lack of commitment on the part of the government poses no problem and the consumer will be entirely justified in believing the government's announcement of the time path of tax rates made at time zero.

As already noted, this conclusion is very special to our simple model and does not hold in more general models. ${ }^{27}$ Whenever it turns out that the best choice of tax rates made in period $(t+1)$ for periods ( $t+1$ ) and beyond differs from the best choice of tax rates made in period $t$ for periods $(t+1)$ and beyond, there is said to be a problem of time consistency. ${ }^{28}$ One way around this is to simply assume (as we did) that the government is committed to implementing whatever time path of tax rates it announces at time zero, and cannot change its mind later on. Alternatively, we could assume that such commitment possibility does not exist (which seems realistic) and that the government is free to depart from the time path of tax rates it has announced previously, if it wishes. Whenever there is a problem of time consistency, the two assumptions regarding commitment will yield different choices for the time path of tax rates.

In some instances (our model is one example) it is possible to show that the two choices will be the same. Lucas and Stokey (1983) show this in a somewhat different model than ours. What is interesting in their model is that it is necessary to
specify the entire maturity structure of government debt. The government's problem involves choosing not just tax rates and the total value of debt issue but also managing the maturity structure of debt. In our simple model the maturity structure of debt is irrelevant. Interesting analyses of taxation along these lines are contained in Chari and Kehoe (1988a,b).

## C. Debt Default

Implicit in our analysis is the assumption that the government is not allowed to default on its debt. ${ }^{30}$ For instance, suppose that $D(0)$ is positive so that the government is initially a debtor. If it could default on its debt then we can see from (13c) that tax revenues would be lower permanently. It then follows that excess burden will be lower and consumer welfare higher. It is perhaps worth emphasizing that the government's incentive to default arises not because its objective differs from that of the consumer but in spite of the fact that its objective is the same as that of the consumer. That is, an unanticipated default by the government ${ }^{31}$ increases consumer welfare. The reason for this is that a debt default is equivalent to a lump sum tax. The loss to the consumer on government bonds is independent of current and future work. From our discussion of excess burden we know that a lump sum tax that extracts the same tax revenues as a proportional tax leaves the consumer with higher net income and hence leads to higher welfare. If the government could promise never to default and do so always, it would in effect have access to lump sum taxes and consumer welfare would be higher. The obvious hitch in this game plan is that the consumer would have to be incredibly stupid!

If the government is assumed unable to commit in advance to the time path of tax rates and in addition unable to commit never to default, the problem of choosing tax rates and debt issues becomes quite complicated. A very interesting analysis of this problem can be found in Chari and Kehoe (1988b).

## D. Capital Taxation

This problem is very similar to that of defaulting on debt. In our model we have assumed that only labor income is taxed and there is no tax on capital. If the government is assumed to be committed once and for all to whatever time path of tax rates it chooses at time zero, then allowing for capital taxation does not create too many complications [see, for example, Chamley (1984), Judd (1987)]. However, if such commitment is not feasible then difficulties arise. Essentially, the government would have an incentive to promise that it would only levy a very small (may be zero) tax on capital thereby encouraging saving and capital accumulation and later on renege on its promise and raise the tax on capital. An unanticipated tax on capital is like a lump sum tax and is similar to a debt default. Again, it should be emphasized that the difficulty arises in spite of the fact that the government is maximizing the consumer's welfare. The paper by Fischer (1980) contains an early exposition of this problem. Chari and Kehoe (1988a) is an interesting extension of Fischer's work using the modern tools of game theory.

## E. Money and the Inflation Tax

Our model does not contain any fiat currency and hence does not contain any nominally denominated debt or prices. In a monetary economy there are additional means of taxation arising from the government's ability to control the time path of the money supply thereby influencing the time path of the general price level and the nominal interest rate. The anticipated rate of inflation acts as a proportional tax on all nominally denominated assets held by the private sector. ${ }^{32}$ In this sense it is not much different from a tax on labor income. Therefore, an interesting question might be: what is the best inflation rate (tax rate on nominal assets)? However, the presence of nominally denominated assets raises the same problems as debt default and capital taxation. The amounts of such assets that people are willing to accumulate is higher the lower is the tax rate or, equivalently the lower is the inflation rate. Therefore, there is an incentive on the part of the government to promise to maintain a low inflation rate and after the private sector has accumulated nominal assets, to tax these away by creating unanticipated inflation. There is a fairly large literature on these issues. The interested reader may consult the following: Friedman (1969), Phelps (1973), Calvo (1978), Helpman and Sadka (1979), Lucas and Stokey (1983), Chamley (1985a).

## VI. Conclusion

In this article, I have considered a number of interesting questions regarding the time path of tax rates and tax revenues and hence of the time paths of deficits and government
debt. Some of these that I addressed are as follows. (a) Are current deficits too high? (b) Is a constitutional amendment to balance the budget a good idea? (c) How should tax and debt management policy respond to foreseen changes in the path of government expenditures? I have presented a simple model of tax determination which was first analyzed by Barro in 1979. This model suggests that the government should attempt to maintain tax rates at a constant level, designed to balance the budget in an average sense over the very long run. I have used this result to comment on each of the questions posed above. Briefly, the answers are: (a) Given the relative tranquility in domestic and international affairs, current deficits may be judged too high since current expenditures seem to be at or below their long term average level, (b) Not a good idea, (c) Taxes should respond immediately and permanently whenever the path of expenditures in the future is perceived to be different from what was previously expected.

There are many qualifications to our analysis noted in the previous section and many interesting issues that were unexplored. This article is only an introduction to the issues involved in tax policy and budget management and the interested reader should consult the references at the end for further work along these lines.

## Footnotes

${ }^{1}$ The states maintain separate accounts for current and capital spending and are required to balance the budget on current account only, but are permitted to borrow for capital spending. The federal government does not have separate current and capital accounts.
${ }^{2}$ By government expenditures we will always mean net of interest expenditures, i.e., government purchases of goods and services plus transfer payments.
${ }^{3}$ The government budget constraint which will be developed later requires that the discounted present value of tax revenues be sufficient to finance the discounted present value of expenditures plus interest payments on government debt. In this present value sense the government budget is always balanced. Further, the average level of tax revenues would have to be equal to the average level of expenditures plus interest on debt. The question of how tax rates should be set is about the appropriate time path of tax rates given the present value, or equivalently, the average level of tax revenues. Such questions are studied in the theory of optimal taxation in public finance. The conclusion that tax rates should be smooth over time is really a simple application of the principle of uniform taxation from the theory of optimal taxation. See, Sandmo (1974), Sadka (1977), Barro (1979, p. 944, note 7).
${ }^{4}$ See his paper, Barro (1979).
${ }^{5}$ The tax is levied on income and the tax rate (percent of income that is taxes) increases with the level of income.
${ }^{6}$ The tax is levied on market transactions either as a percent of the value of sales (an ad valorem tax) or as a percent of the volume of sales.
${ }^{7}$ By definition, a lump sum tax is a tax whose total amount is not related to any economic decision. It is a head tax that specifies the total amount of tax to be paid regardless of what the individual does. It may be levied in different amounts on different people if they can be distinguished on the basis of characteristics which are unalterable. An example might be different taxes on men than on women (however, the possibility of sex change operations complicates this!), or different taxes on right handed people than on left handed people.
${ }^{8}$ See his paper, Barro (1974).
${ }^{9}$ See my article, Aiyagari (1987a).
${ }^{10}$ In a very thought provoking and provocative paper, Bernheim and Bagwell (1988) argue that this result holds not just for lump sum taxes but for all types of taxes. That is, even nonlump sum taxes may be completely neutral and have neither distribution effects nor incentive effects. Their analysis is based on the fact that family chains are not isolated but are connected by marriage. The obvious fact that nonlump sum taxes do have incentive effects even though different generations appear to be altruistically linked and there are large voluntary transfers of wealth from one generation to the next must be regarded as a puzzle.

[^0]generation to the next converts a model of overlapping generations of heterogeneous people to one with a single representative, infinitely lived person. Under some further assumptions this representation can also be extended to cover the case of many infinitely lived persons. See, Eichenbaum Hansen and Richards (1984). This may be viewed as a justification for using such an abstract representation of the economy.
${ }^{12}$ The assumptions that taxes are proportional (rather than progressive or regressive) and levied only on labor income and not on capital income simplifies the exposition considerably. Permitting capital taxation leads to some interesting complications which we will touch on later. Note that it is implicit in our assumption that interest income on government debt is also not taxed.
${ }^{13}$ For now we take labor productivity to be constant over time in order to focus on the relationship between the time paths of government expenditures and the tax rates. Note that with labor productivity fixed the time path of total tax revenues and tax rates will be similar. This need not be true when labor productivity also fluctuates over time. We will comment later on the effect this may have on tax setting.
${ }^{14}$ The perceptive reader will notice that our formulation of preferences is equivalent to one in which utility depends both on consumption, $\mathrm{C}(\mathrm{t})$ and on work, $\mathrm{l}(\mathrm{t})$ in the following special way: Utility $=U[C(t)-H(\ell(t))]$. This specification implies that the income effect on work is zero, and simplifies the exposition considerably. We are also implicitly assuming that government
purchases of goods and services do not enter the the consumer's welfare. This is only a simplification and makes no difference to the subsequent analysis since the path of government purchases will be treated as exogenous.
${ }^{15}$ Since wealth consists of capital (which is nonnegative) plus government debt (which may be negative, i.e., the agent may be borrowing from the government), the restriction that wealth be bounded below amounts to prohibiting the consumer from engaging in Ponzi games in which he/she borrows to finance consumption and keeps borrowing more and more to pay off previous debt without ever redeeming any debt. The present value budget constraint may be obtained by solving equation (4) for $W(0)$ by repeatedly substituting for future values of wealth. We are implicitly assuming that transfer payments from the government are zero. Otherwise they would have to be entered on the right side of (4). However, this makes no difference to the subsequent analysis since transfer payments will be treated as exogenous.
${ }^{16}$ The constant elasticity form of the utility function is given by: $U(c)=c^{\gamma} / \gamma$, where $\gamma \leq 1$. If $\gamma=0$, then $U(c)=10$ g c. With this form, $U^{\prime}(c)=c^{(\gamma-1)}$.
${ }^{17}$ Substituting for $U$ (c) from footnote 16 in equation (7) and manipulating we obtain equation (8a) where $(1+n)=$ $[\beta(1+r)]^{1 /(1-\gamma)}$. Next we can use equation (8a) to express $c(t)$ in terms of $c(0)$ as: $c(t)=c(0)(1+n)^{t}$. Now substitute this expression for $c(t)$ in equation (6) to express the present value of consumption as a geometric series. The assumption that $\beta(1+r)^{\gamma}<1$, will guarantee that $r>n$ so that the sum of the
geometric series is finite and is given by $c(0)(1+r) /(r-n)$. This then yields equation (8b) where $\alpha=(r-n) /(1+r)$.
${ }^{18}$ Permanent income is defined as that constant level of income that has the same present value as the actual time path of income. The concept was originated by Milton Friedman in his seminal work on the theory of consumption. See, Friedman (1957). Barro (1984, chapter 4, p. 92) contains a simple exposition of the concept. Sahasakul (1986) uses this concept for an empirical study of U.S. taxation.
${ }^{19}$ This conclusion is in fact independent of the constant elasticity form of the utility function we chose. It depends only on the facts that the interest rate $r$ is given by the return on capital independently of tax policy and that the tax rate (or the level of tax revenues) does not enter the utility function $U(\cdot)$. What is critical in generating this latter feature is the fact that the consumer cares for consumption net of the opportunity cost of work, or equivalently, the income effect on work is zero. If consumption and the opportunity cost of work entered the utility function in some other fashion, this would not be true. Given our specification, a moment's reflection will show that any net consumption path that is feasible for the consumer for a particular value of total wealth ( $\mathrm{W}(0)$ plus the present value of net income) is also feasible for a higher value of total wealth. That is, the maximum welfare that the consumer can attain depends only on total wealth and is always increasing with it. Therefore, regardless of the particular form of the utility function maximizing consumer welfare is equivalent to maximizing the present value of net income.
${ }^{20}$ This is the famous Laffer curve relationship between the tax rate and tax revenues.
${ }^{21}$ If we let $w^{\prime}$ be the after tax productivity then as explained previously the consumer will choose work such that, $W^{\prime}=H^{\prime}(1)$, where $H^{\prime}(\cdot)$ is the derivative of $H(\cdot)$ and measures the marginal opportunity cost of work. Differentiating this equation implicitly with respect to $W^{\prime}$ we obtain, elasticity $\equiv\left(w^{\prime} / l\right) d l / d w^{\prime}$ $=H^{\prime} /\left(1 \mathrm{H}^{\prime \prime}\right)$, where $\mathrm{H}^{\prime \prime}(\cdot)$ is the second derivative of $\mathrm{H}(\cdot)$. Therefore, elasticity of work will be constant whenever the elasticity of the opportunity cost function $\mathrm{H}(\cdot)$ is constant. This will happen if the function $H(\cdot)$ is of the form: $H(l)=a l^{b}$, with $\mathrm{b}>1$ and $\mathrm{a}>0$.
${ }^{22}$ This principle is named after the brilliant mathematician, philosopher, and economist, Frank Ramsey (1903-30) who was the first to pose and solve the problem of choosing tax rates on many different goods. This principle may explain why goods like cigarettes and liquor are more heavily taxed than other goods. In the present context consumption and work at different dates may be regarded as different goods.
${ }^{23}$ For instance, it follows from the analysis of Sandmo (1974) and Sadka (1977) that if $U(C(t), l(t))=Z(C(t))-(1(t))^{\gamma}$, then again the tax rate should be constant over time. See also, Sandmo (1976), Atkinson and Stiglitz (1980, lecture 12) and Sahasakul (1986).
${ }^{24}$ Barro's (1979) analysis was couched in terms of this concept.
${ }^{25}$ Then why does not the government use lump sum taxes? The short answer is that we have assumed that it cannot (except conceptually!). In general this requires a deeper look at taxation that is beyond the scope of this article. It should be emphasized that that there is a more general concept of excess burden in terms of consumer welfare rather than net income loss, for more general utility functions than ours. This alternative concept is defined as the loss in consumer welfare due to a proportional tax in comparison to a lump sum tax yielding the same revenues. The loss in consumer welfare is measured as the equivalent loss in income that would result in the same level of welfare as under a proportional tax. In the present instance, the two concepts are equivalent. In general, this is not so and the welfare based concept is more appropriate. Excess burden is also often referred to as dead weight loss. See, Judd (1987).
${ }^{26}$ It does not really matter that this rate of return is constant over time. Our conclusion about constant taxes over time will still follow. The rate of return on capital may depend on work if output is produced by capital and labor via a production function in which the two inputs are not separable. See, Chamley (1985b) for an analysis of the problem of efficient wage taxation in a more general model than ours.
${ }^{27}$ See, Chari, Kehoe, and Prescott (1988) and the references therein.
${ }^{28}$ There is a large and increasing volume of literature on this topic and it would be impossible to reference them all. The paper by Chari, Kehoe and Prescott (1988) contains a good
discussion of this problem as well as most of the relevant references.
${ }^{29}$ This does not imply that the government is somehow fooling the consumer into thinking that the time path of taxes will be, say $\{\theta(t)\}$ when it will actually choose something different. The time path of tax rates will have the property that at each point in time the consumer believes in whatever time path the government announces from that date onwards and given this, at each point in time the government has no incentive to depart from the time path it announced at time zero. Such a solution is known as the time consistent solution to the time path of tax rates.
${ }^{30}$ That is, whenever it happens to be a debtor. If it happens to be a creditor we will always assume that it collects from the consumer, i.e., the consumer is never allowed to default.
${ }^{31}$ The default would have to be unanticipated because if it is anticipated the consumer will not lend to the government. He/she would rather hold wealth in the form of capital than government debt.
${ }^{32}$ Let $M(t)$ be the total quantity of nominally denominated assets held by the private sector as of the end of period $t$, and let $\mathrm{p}(\mathrm{t})$ be the general price level in period t . Then the real value of these assets changes from $M(t) / p(t)$ to $M(t) / p(t+1)$ from period $t$ to period $t+1$. Therefore, the difference $[M(t) / p(t)-M(t) / p(t+1)]$ is like a tax. It is easy to see that the tax can be written as $(1-p(t) / p(t+1)) M(t) / p(t)$. Thus, the tax is proportional to the real value of assets $M(t) / p(t)$ and the tax rate is given by $(1-p(t) / p(t+1))$. Clearly, the higher is the inflation rate the higher is the tax rate.

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[^0]:    ${ }^{11}$ In Aiyagari (1987b) I show how the presence of altruism across generations along with positive bequests from one

