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MONETARY POLICY TRANSMISSION
WHEN THERE ARE NONTRADED GOODS

Preston J. Miller and Richard M. Todd*

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ABSTRACT

This paper investigates the effects of changes in a country's monetary policies on its economy and the welfare of its citizens and those of other countries. Each country is populated by two-period lived overlapping agents who reside in either a home service sector or a world-traded good sector. Policy effects are transmitted through changes in the real interest rate, relative prices, and price levels. Welfare effects are sometimes dominated by relative price movements and can thus be opposite of those found in one-good models. Simulation of dynamic paths also reveals that welfare effects for some types of agents reverse between those born in immediate post-shock periods and those born later.

Events following the Federal Open Market Committee's policy change of 1979 influenced economists' thinking about the transmission of monetary policy. Historical accounts and statistical evidence suggest that U.S. monetary policy shifted to a significantly more restrictive stance in the fall of 1979 (see, for example, Blanchard 1984 and Miller-Roberds 1991). The change seemed to result in real and distributional effects which could not easily be explained by general equilibrium models then in use.

One observation which seemed in conflict with those models' predictions was the behavior of real interest rates. After the policy change real interest rates rose in the U.S. and stayed high (see Figure 1). Standard monetary models, however, implied that only surprises in policy would have real effects (see, for example, Barro (1976), and Sargent-Wallace (1975)). Clearly, the rise in real interest rates was sustained too long to be attributable to a surprise.

The policy change also seemed to have distributional effects across different types of agents. Farmers, home builders, and other borrowers regularly complained about how they were being hurt by the tighter monetary policy. Monetary models with only one type of agent could not even confront this type of observation.

But, perhaps the most surprising development following the monetary policy change was the behavior of exchange rates. The value of the dollar rose, and most of the gain was in real terms (see Figure 2). The apparent response of the real exchange rate to the change in monetary policy was not predicted by general equilibrium models then in use. Those models included only one good, while real exchange movements necessarily involve changes in relative prices. However, some economists argued that the rise in real exchange rates was best explained as a consequence of sticky prices, and that general equilibrium models not only had not offered (Dornbusch, 1988, p.21) but could not even offer (Krugman, 1989) any hope of explaining this phenomenon.

In this paper we explore how adding nontraded goods to Miller and Wallace's (1985) simple general equilibrium model affects the transmission of monetary policy to real interest rates, relative prices and real exchange rates, and the welfare of agents in the traded- and nontraded-goods sectors. As in Miller-Wallace and many other general equilibrium models of money, monetary policy in our model affects real variables by changing the wealth of agents with nominal assets and debts. However, the presence of nontraded goods in our model provides a relative price and real exchange rate channel for transmission of the wealth effects of monetary policy. Our model thus provides an explicit intertemporal equilibrium framework that links changes in monetary policy to movements in relative prices and real exchange rates, with no stickiness of prices.

Adding nontraded goods to the Miller-Wallace model also affects the welfare implications of monetary policy. In Miller-Wallace, agents who borrow and whose good is traded in world markets, such as farmers, are clearly made worse off by a tightening in monetary policy due to the resulting rise in the real interest rate. In our model, the effects on borrowers of higher real interest rates eventually can be mitigated or even reversed by a rise in the relative price of traded goods to nontraded goods.

The channels of monetary policy transmission in our model are shaped by a few key features of the model. These channels and the features of the model that shape them do not seem inconsistent with casual observations.

In our model there is a difference between policy effects in the short-run (the initial period) and the long-run (the steady state). This is driven primarily by the model's monetary structure. A combination of monetarism and unpleasant monetarist arithmetic produces distinct price level and inflation rate effects. An open-market sale, for instance, reduces the initial price level, because it lowers the current stock of money. Subsequently, however, it raises the monetary growth rate, and hence the rate of inflation, because higher interest payments increase the government's gross budget

deficits and require an increased amount of seignorage. Eventually in our model, the effects of a higher inflation rate overtake the effects of a lower price level.

In our model monetary policy has real effects due to a friction that we impose. That friction is a reserve requirement in each country that specifies that a fraction of savings must be held in that country's currency. In our model this friction provides the only motive for holding money. As Wallace (1984) points out, this assumption corresponds to all borrowing and lending being intermediated through banks having reserve requirements on deposits.

Reserve requirements explain why changes in monetary policy affect the real interest rate. The reserve requirement generates separate demands for fiat money and fiat bonds. We assume that bonds are traded in a world capital market and compete with private loans in investors' portfolios, so that they bear the common world real interest rate. The assumption of a world capital market seems consistent with observed interest rate differentials across countries (see Frankel 1989). An open-market operation, then, results in a change in the relative supplies of money and bonds, which in our model requires a change in the real interest rate to equate the demands to the new supplies.

The reserve requirement friction also explains how monetary policy affects the relative prices of nontraded and traded goods. The effect can be thought to arise from a transfer of wealth between the home country and rest of world. Consider, for example, an open-market sale. In the initial period the home country price level must fall, since the reserve requirement makes prices proportional to the money stock. The higher real interest rate that results, however, causes the price level in the rest of the world to rise, because with a constant monetary policy abroad, money there rises as more fiat debt is issued to finance budget deficits inclusive of interest. Thus, real wealth is transferred from foreign owners of nominal debt to domestic owners. This causes more of the traded good to be consumed in the home country, worsening its trade balance, and less to be consumed abroad. Since the home country consumes more of the traded good and the same amount

of the nontraded good, the relative price of traded to nontraded goods must fall. The opposite occurs in the rest of the world.[1]

The relative price effects are reversed in later periods as the inflation effects overtake the price level effects. The rise in the world real interest rate affects inflation equally in all countries. However, because the open-market sale shrinks the base for the inflation tax in the home country, its inflation rate rises more than in the rest of the world. This causes the real rate of return on savings in the home country to fall relative to that in the rest of the world, since the reserve requirements imply that those rates of return are weighted averages of the rates of return on money and bonds. The inflation effects then result in a transfer of wealth from home savers to foreign savers and thus cause relative price changes in the opposite direction from their initial changes.

The distributional effects in our model derive from our assumptions about endowments and markets. We assume two different types of agents. Agents of the first type are endowed with the world traded good in a pattern which makes them want to borrow in equilibrium. Agents of the second type are endowed with the nontraded good in a pattern which makes them want to lend in equilibrium. Because the two types are on the opposite sides of the loan and goods markets, their welfare is affected in opposite directions by changes in the real interest rate and relative prices.

Qualitatively, the model's implications do not seem inconsistent with developments following the 1979 tightening in U.S. monetary policy. We interpret those developments as initial period effects from a sequence of monetary policy restrictive actions, which became apparent in 1980 (see Figure 3). The model predicts, initially, rises in U.S. nominal and real interest rates (see Figure 1), a rise in the nominal exchange rate of the dollar (see Figure 2), a rise in the relative price of nontraded goods in the U.S. and a consequent rise in the real exchange rate of the dollar (see Figure 2), and a deterioration in the U.S. balance of trade (see Figure 4). It also predicts a lower U.S.

price level than otherwise would have occurred, and this also seems born out, as the rate of growth of the consumer price index fell from 13.3 percent in 1979 to 3.9 percent in 1984.

Our model thus suggests that allowing the wealth effects of monetary policy to affect relative prices can help general equilibrium models account for the real exchange rate movements of the 1980s. Quantitatively, however, the contribution in our model is small. In particular, we set the model's parameters so as to roughly match the tightening of U.S. monetary policy and the consequent increases in the real interest rate and nominal exchange value of the U.S. dollar which occurred in the early 1980s. However, we found that the corresponding paths of the model's home country relative prices and the real exchange rate all increase by only small fractions of the observed changes of the 1980s. The failure of our model to match the data in this dimension suggests that its demands for goods are too price elastic. To make our model consistent with the data, small changes in demand resulting from small changes in wealth would have to lead to big changes in relative prices.

Since the structure of our model, which is in the tradition of the Wallace (1984) and Miller-Wallace (1985) models, differs from standard trade models, we offer a defense for two of its nonstandard features. We chose an overlapping generations structure because it provides a tractable way to examine the distributional effects of alternative policies. For monetary policy we believe it is important to distinguish between the effects on borrowers and the effects on lenders. We used reserve requirements, a form of Wallace's legal restrictions, as a way of generating separate money and bond demands in order to allow a distinction between monetary and budget policies. In our model a monetary policy action, be it an open-market operation or a change in reserve requirements, involves no change in the total outside debt, that is, no change in budget policy. In contrast, in most monetary models, where monetary policy consists of helicopter drops of money, total outside debt

changes when the quantity of money changes, so that there is not a clear distinction between monetary and budget policies.

In the text that follows we describe the model, conditions for equilibrium, and both qualitative and quantitative comparisons of equilibria for alternative monetary policies.

The Model

A. Structure

Our model is a two-country, traded-and-nontraded-goods version of the multi-country, one-good model developed in Miller-Wallace (1985). In this section we describe the structure of the model and its equilibrium.

There are two countries. Both have a private sector and a public sector. Both are populated by overlapping generations of agents who live two periods. At each integer date t a new generation, generation t, appears. The members of generation t are present at date t, when they are young, and at date t + 1, when they are old.

Each generation is made up of two types: service providers and farmers. Over time the number of each type of individual in each country is constant, and individuals can't switch types or countries.

The two types of individuals in each country have identical preferences and differ only with respect to their endowments. The preferences are assumed to be represented by a discounted log-linear utility function, U, over service and food consumption in the two periods of an individual's life:

$$U(c_t) \equiv U\!\left(c_t^S(t), c_t^F(t), c_t^S(t+1), c_t^F(t+1)\right)$$
$$= \theta \log c_t^S(t) + (1-\theta) \log c_t^F(t) + \delta\!\left[\theta \log c_t^S(t+1) + (1-\theta) \log c_t^F(t+1)\right],$$

where

S is services, and F is food,

θ is a parameter reflecting relative preferences for services versus food,

δ is a parameter reflecting the rate of time preference,

a subscript t refers to a member of generation t, and

an integer in parentheses is a date.

The endowments of the two types of individuals differ with respect to both the type of goods and the pattern of goods over time. The endowment of a generation t service provider $w_{t:S}$ is assumed to be:

$$w_{t:S} \equiv \left[w_{t:S}^S(t), w_{t:S}^F(t), w_{t:S}^S(t+1), w_{t:S}^F(t+1) \right] = \left[w_t^S(t), 0, 0, 0 \right],$$

where

$$w_t^S(t) > 0.$$

Thus, service providers are endowed with all of their country's services in their first period of life and with nothing else. The endowment of a generation t farmer, $w_{t:F}$, is assumed to be:

$$w_{t:F} \equiv \left[w_{t:F}^S(t), w_{t:F}^F(t), w_{t:F}^S(t+1), w_{t:F}^F(t+1) \right] = \left[0, w_t^F(t), 0, w_t^F(t+1) \right],$$

where

$$w_t^F(t+1) > w_t^F(t) \geq 0.$$

Farmers are endowed with some positive amount of food in their second period of life and with a lesser amount, perhaps zero, in the first period.

The preferences and the time-pattern of endowments apply to both countries, but the amounts of endowments can differ between the two. To distinguish the countries, we use a superscript asterisk on foreign variables or functions. For example, foreign preferences are represented by $U(c_t^*)$.

The pattern of endowments is intended to proxy for production, which is not included in our model. We think of service providers as supplying labor-intensive goods with a relatively short production lag. Hence, their endowment is concentrated in the first period of life. We think of farmers as supplying a capital-intensive good subject to a relatively long production lag that requires high initial investment. Hence, their endowment is concentrated in the second period of life. There is no storage. Because of the different endowment patterns, in equilibrium service providers lend and farmers borrow. We assume all private debt is in the form of one-period discount bonds, B^P. We think of these as nominal (nonindexed) bonds denominated in the currency of the country of the issuer, but these notions are irrelevant so long as we stick to perfect foresight equilibria.

Farmers are distinguished by two characteristics in our model. One is that they are borrowers. A second is that only their good is world traded. While services and food can be purchased and sold within a country, only food can be traded between countries.

In addition to a private sector, both countries have a public sector. The public sector has a fiscal authority that consumes some amounts of its country's service and food and levies lump-sum real taxes, quoted in food units, on service providers and farmers. We assume that its total consumption always matches or exceeds its total lump-sum tax revenue. (This permanent deficit assumption implies that, in steady states, inflation must be positive to generate the seignorage needed to finance government deficits.) A country's budget policy is defined by the fiscal authority's real consumption of services and food, $g^S \geq 0$ and $g^F \geq 0$, respectively; by its lump-sum taxes; and by its issue of one-period discount bonds, B^G, to finance the deficit. However, since lump-sum taxes are fixed throughout the analysis, without loss of generality they can be set to zero.

Each public sector also has a monetary authority that engages in open-market operations and imposes a reserve requirement. Open-market operations are conducted by exchanging fiat money for government bonds, and the reserve requirement is specified by a fraction of nominal savings

which must be held in the form of money. A country's monetary policy is defined by the ratio of government bonds to money, β, and by the reserve requirement, λ. Government bonds are traded in a world capital market, but in equilibrium money is held only by lenders in its country of issue to satisfy domestic reserve requirements.

B. Market Equilibrium

A market equilibrium is defined as an allocation of goods and a set of prices such that at each date the allocation solves individual optimization problems given the prices, the quantities and prices are consistent with the two governments' budget constraints, and markets clear. There are six markets in the model: two world markets in food and bonds, and four domestic markets in country-specific services and money. By Walras's Law we eliminate separate consideration of the world bond market. In the remaining markets five price sequences are determined: the world real interest rate (ρ), the relative price of services to food in each country (Q and Q*), and the rate of change of food prices in each country (π^F and π^{F*}).

We next describe individual optimization problems, government budget constraints, and market clearing conditions. We conclude this subsection with a description of an equilibrium.

1. Individual Optimization Problems

We describe the optimization problems of the old at date 1 (generation 0) and of the young at all dates (generation $t > 0$). We only consider the home country, since the individuals solve parallel problems in the foreign country. Individuals take prices as given and have perfect foresight with respect to prices in their second period of life.

The old at $t = 1$ maximize:

$$\theta \log c_0^S(1) + (1-\theta) \log c_0^F(1)$$

subject to

$$Q(1)c_0^S(1) + c_0^F(1) = A_0;$$

where $Q(1) = [p^S(1)/p^F(1)]$ is the relative price of services to food at date 1. For old service providers, $A_0 = A^S/p^F(1)$, where $A^S > 0$ is a given nominal value of assets and $p^F(1)$ is the first period price of food, an endogenous variable. The assets consist of some combination of home country money and private and public bonds from either country. For old farmers, $A_0 = w_0^F(1) - D_0/p^F(1)$, where $D_0 > 0$ is a given nominal value of private debt consisting of bonds issued earlier. Note that for both types of old agents at $t = 1$, the real value of A_0 depends on $p^F(1)$, an endogenous variable subject to the effects of policy changes in period 1.

The optimization problem generates the usual Cobb-Douglas utility demand functions:

$$c_0^S(1) = \theta A_0/Q(1)$$

and

$$c_0^F(1) = (1-\theta)A_0,$$

where A_0 takes on the values specified above for old service providers and farmers, respectively.

Agents born at dates $t \geqq 1$ maximize

$$\theta \log c_t^S(t) + (1-\theta) \log c_t^F(t) + \delta\left[\theta \log c_t^S(t+1) + (1-\theta) \log c_t^F(t+1)\right]$$

subject to

$$W(t) = Q(t)c_t^S(t) + c_t^F(t) + \left[Q(t+1)c_t^S(t+1) + c_t^F(t+1)\right]/(1 + i(t)),$$

where

$$W(t) = \begin{cases} Q(t)w_t^S(t), & \text{for service providers} \\[2em] w_t^F(t) + \dfrac{w_t^F(t+1)}{1 + \rho(t)}, & \text{for farmers} \end{cases}$$

$$1 + i(t) = \begin{cases} 1 + \rho_s(t), & \text{for service providers} \\[1em] 1 + \rho(t), & \text{for farmers} \end{cases}$$

$r(t) \equiv$ the nominal interest rate on one-period bonds issued at t;

$\pi^F(t) \equiv p^F(t+1)/p^F(t)$, the rate of food price inflation from t to t + 1;

$\rho(t) \equiv [1 + r(t)]/[1 + \pi^F(t)] - 1$, the real rate of return on one-period bonds issued at t.

$\rho_s(t) \equiv [1 + (1-\lambda(t))r(t)]/[1 + \pi^F(t)] - 1$, the real rate of return on savings at t.

Note that the wealth of lenders (service providers) doesn't depend just on the real rate of return on bonds. Because lenders are legally required to hold a fraction of their assets as domestic currency, their net nominal rate of return is not r(t), the net nominal rate on bonds, but rather $[1 - \lambda(t)]r(t)$, where $\lambda(t)$ is the fraction of their assets that savers must hold in the form of money. The gap between the real return earned by lenders and that paid by borrowers depends on $\lambda(t)$ and r(t), and, through r(t), on inflation.

The optimization problem again generates typical Cobb-Douglas utility demand functions:

$$c_t^S(t) = \left[\frac{\theta}{1+\delta}\right]W(t)/Q(t),$$

$$c_t^F(t) = \left[\frac{1-\theta}{1+\delta}\right]W(t),$$

$$c_t^S(t+1) = \left[\frac{\theta\delta}{1+\delta}\right]\left[\frac{W(t)}{Q(t+1)}\right](1 + i(t)),$$

and

$$\tilde{c}^F_t(t+1) = \left[\frac{(1-\theta)\delta}{1+\delta}\right][W(t)(1 + i(t))].$$

2. Government Budget Constraint

The government at date t pays for current expenditures and the retirement of last period's bonds and money by issuing new bonds and money:

$$p^S(t)g^S(t) + p^F(t)g^F(t) + B^G(t-1) + H(t-1) = P^B(t)B^G(t) + H(t),$$

where

$P^B(\cdot)$ is the nominal price of government bonds, $P^B(t) = 1/(1 + r(t))$,

$B^G(\cdot)$ is the total nominal face value of government bonds, and

$H(\cdot)$ is the total quantity of money.

Let the real value of bonds and money in terms of food at date t be defined respectively by:

$$b^G(t) \equiv \frac{P^B(t)B^G(t)}{p^F(t)}$$

and

$$h(t) \equiv \frac{H(t)}{p^F(t)}.$$

In addition, let $\beta(t) \equiv B^G(t)/H(t)$.

Then the government budget constraint can be written

$$Q(t)g^S(t) + g^F(t) = \left[\frac{1 + r(t) + \beta(t)}{1 + r(t)}\right]h(t) - \frac{[1 + \beta(t-1)]}{[1 + \pi^F(t-1)]}h(t-1).$$

3. Price Levels, Exchange Rates, and Interest Rates

We first define selected price variables and then express relationships among them which must hold in equilibrium. The aggregate price level in a particular country is defined as:

$$P(t) \equiv p^S(t)^\theta \, p^F(t)^{(1-\theta)}$$

a measure of the minimal cost of one unit of utility.

The nominal exchange rate e defines how many units of the foreign country's currency are exchanged for one unit of the home country's currency. By the Law of One Price applied to the traded good, we get that $e(t)p^F(t) = p^{F*}(t)$, or $e(t) = p^{F*}(t)/p^F(t)$. The real exchange rate then is defined as the normal exchange rate divided by the ratio of the price levels, or:

$$\bar{e}(t) \equiv e(t)/[P^*(t)/P(t)] = [Q(t)/Q^*(t)]^\theta.$$

Since private and government bonds within a country are perfect substitutes, their nominal (as well as real) rates of return must be the same in equilibrium—a fact we already used in stating the optimization problems. Moreover, because domestic and foreign bonds are perfect substitutes, it follows that the real interest rates in equilibrium must be the same in each country:

$$\frac{1 + r(t)}{1 + \pi^F(t)} \equiv 1 + \rho(t) = 1 + \rho^*(t) \equiv \frac{1 + r^*(t)}{1 + \pi^{F*}(t)}.$$

We'll refer to this single real rate as $\rho(t)$.

4. Equilibrium

All equilibrium quantities and prices can be derived from the equilibrium values of the five sequences ρ, Q, Q^*, π^F, and π^{F*}. These sequences can be derived by equating demand and supply in the goods and money markets.

(1) Home Services Market

$$N^S\left(\hat{c}^S_{t-1:S}(t) + \hat{c}^S_{t:S}(t)\right) + N^F\left(\hat{c}^S_{t-1:F}(t) + \hat{c}^S_{t:F}(t)\right) + g^S(t) = N^S w^S_t(t),$$

where N^S and N^F are the numbers of service-providers and farmers, respectively, in a given generation in the home country and subscripts S and F refer to consumption by, respectively, service and food providers.

(2) Foreign Services Market (Parallel to (1))

(3) World Food Market

$$N^S\left(\hat{c}^F_{t-1:S}(t) + \hat{c}^F_{t:S}(t)\right) + N^F\left(\hat{c}^F_{t-1:F}(t) + \hat{c}^F_{t:F}(t)\right) + g^F(t) + N^{S*}\left(\hat{c}^{F*}_{t-1:S}(t) + \hat{c}^{F*}_{t:S}(t)\right)$$

$$+ N^{F*}\left(\hat{c}^{F*}_{t-1:F}(t) + \hat{c}^{F*}_{t:F}(t)\right) + g^{F*}(t)$$

$$= N^F\left(w^F_{t-1}(t) + w^F_t(t)\right) + N^{F*}\left(w^{F*}_{t-1}(t) + w^{F*}_t(t)\right),$$

where N^{S*} and N^{F*} are the numbers of service-providers and farmers, respectively, in a given generation in the foreign country.

(4) Home Money Market

$$H(t) = \lambda(t)N^S(t) \cdot \left[p^S(t)w^S_t(t) - p^S(t)\hat{c}^S_{t:S}(t) - p^F(t)\hat{c}^F_{t:S}(t)\right].$$

(5) Foreign Money Market (Parallel to (4))

The money market equilibrium conditions assume that the reserve requirements are binding; that is, money is held only to satisfy the reserve requirements. Since we consider only equilibria for which $\rho > 0$, bonds will dominate money in rate of return and this assumption will be warranted.

We wish to express the five equilibrium conditions in terms of the five sequences ρ, Q, Q*, π^F, and π^{F*} and basic parameters: preferences, endowments, and policies. We do this by substituting into conditions (1)–(5) the price relationships and demand functions derived in the

previous sections. But before proceeding, we make a simplification. Note that in the equilibrium conditions the N's just multiply a combination of corresponding w's and ĉ's and that in the consumption demand equations the ĉ's are just fractions of the w's. This implies that without loss of generality we can take all N's to be 1 (and take the new endowments to be the corresponding old N's times the old endowments).

Before examining the resulting equilibrium conditions, we note that the initial (t=1) equilibrium will depend on what we assume about the distribution of wealth among the initial old. In particular, the aggregate demand of the initial old is proportional to $\left[(A^S - D_0 + w_0^F(1)/p^F(1)\right]$ in the home country and to $\left[(A^{S^*} - D_0^* + w_0^{F^*}(1)/p^{F^*}(1)\right]$ in the foreign country. We limit our attention to the initial wealth distributions in which there is no international indebtedness, so that $A^S - D_0 = H(0) + B^G(0)$ and $A^{S^*} - D_0^* = H^*(0) + B^{G^*}(0)$. This assumption is convenient, in the sense that it avoids the issue of specifying whether initial cross-country debt is specified in home currency units, foreign currency units, or real units. It also emerges somewhat naturally below in our analysis of steady-states, were our pre-shock parameter values impose enough symmetry between the countries so that steady-states under the pre-shock parameters imply no cross-country debt.

With the substitutions and simplifications above, equilibrium conditions (1)-(5) can be expressed:

For t = 1,

(1a)
$$\frac{\theta}{1 + \delta} w_1^S(1) + \frac{\theta}{Q(1)} w_0^F(1) + \frac{\theta(H(0) + B^G(0))}{Q(1)p^F(1)}$$
$$+ \frac{\theta}{1 + \delta}\left[\frac{1}{Q(1)}\right]\left[w_1^F(1) + \frac{w_1^F(2)}{1 + \rho(1)}\right]g^S(1) = w_1^S(1)$$

where we have substituted $H(0) + B^G(0)$ for $A^S - D_0$. That is, the initial old farmers begin period 1 with their second-period endowment $w_0^F(1)$ and debt incurred at date 0, D_0, and the initial old service providers begin period 1 with debt incurred by their country's farmers, D_0, and by their country's government, $H(0) + B^G(0)$.

For $t \geq 2$,

(1b)

$$
\begin{aligned}
\theta \left[\frac{\delta}{1+\delta}\right] \left[\frac{Q(t-1)}{Q(t)}\right] & \left[(1 - \lambda(t-1))(1 + \rho(t-1)) + \frac{\lambda(t-1)}{1 + \pi^F(t-1)}\right] w_{t-1}^S(t-1) \\
& + \frac{\theta}{1+\delta} w_t^S(t) + \theta \left[\frac{\delta}{1+\delta}\right]\left[\frac{1}{Q(t)}\right](1 + \rho(t-1))\left[w_{t-1}^F(t-1) + \frac{w_{t-1}^F(t)}{1 + \rho(t-1)}\right] \\
& + \frac{\theta}{1+\delta}\left[\frac{1}{Q(t)}\right]\left[w_t^F(t) + \frac{w_t^F(t+1)}{1 + \rho(t)}\right] + g^S(t) = w_t^S(t).
\end{aligned}
$$

(2a) and (2b) are parallel to (1a) and (1b).

For $t = 1$,

(3a)

$$
\begin{aligned}
\left[\frac{1-\theta}{1+\delta}\right] Q(1)w_1^S(1) & + (1-\theta)w_0^F(1) + (1-\theta)(H(0) + B^G(0))/p^F(1) \\
& + \left[\frac{1-\theta}{1+\delta}\right]\left[w_1^F(1) + \frac{w_1^F(2)}{1 + \rho(1)}\right] + g^F(1) + [*] = w_0^F(1) + w_1^F(1) + [*],
\end{aligned}
$$

where [*] repeats all expressions on its side of the equation but substitutes corresponding terms for the foreign country.

For $t \geq 2$,

$$\left[\frac{(1-\theta)\delta}{1+\delta}\right]\left[\frac{Q(t-1)}{Q(t)}\right]\left[(1-\lambda(t-1))(1+\rho(t-1))+\frac{\lambda(t-1)}{1+\pi^F(t-1)}\right]Q(t)w^S_{t-1}(t-1)$$

(3b)
$$+\left[\frac{1-\theta}{1+\delta}\right]Q(t)w^S_t(t)+\left[\frac{(1-\theta)\delta}{1+\delta}\right](1+\rho(t-1))\left[w^F_{t-1}(t-1)+\frac{w^F_{t-1}(t)}{1+\rho(t-1)}\right]$$

$$+\left[\frac{1-\theta}{1+\delta}\right]\left[w^F_t(t)+\frac{w^F_t(t+1)}{1+\rho(t)}\right]+g^F(t)+[\ast]=w^F_{t-1}(t)+w^F_t(t)+[\ast].$$

For t = 1

(4a)
$$Q(1)g^S(1)+g^F(1)=\lambda(1)Q(1)\left[\frac{\delta}{1+\delta}\right]w^S_1(1)\left[1+\frac{\beta(1)}{(1+\rho(1))(1+\pi^F(1))}\right]$$

$$-\left[\frac{H(0)+B^G(0)}{p^F(1)}\right],$$

where use has been made of the government's budget constraint to write equation (4) in terms of the parameters and basic price sequences.

For t ≥ 2,

(4b)
$$Q(t)g^S(t)+g^F(t)=\lambda(t)Q(t)\left[\frac{\delta}{1+\delta}\right]w^S_t(t)\left[1+\frac{\beta(t)}{(1+\rho(t))(1+\pi^F(t))}\right]$$

$$-\lambda(t-1)Q(t-1)\left[\frac{\delta}{1+\delta}\right]w^S_{t-1}(t-1)\left[\frac{1+\beta(t-1)}{1+\pi^F(t-1)}\right].$$

(5a) and (5b) are parallel to (4a) and (4b).

Given initial values $[H(0)+B^G(0)]$ and $[H^*(0)+B^{G^*}(0)]$, given preference parameters δ and θ, endowments $w_t(t)$, $w^*_t(t)$, $w_t(t+1)$, and $w^*_t(t+1)$, $t \geq 1$, and policy parameters $g(t)$, $g^*(t)$, $\lambda(t)$, $\lambda^*(t)$, $\beta(t)$, $\beta^*(t)$, $t \geq 1$, an equilibrium is a set of initial prices $p(1)$, $p^*(1)$ and price sequences: $\rho(t)$, $Q(t)$, $Q^*(t)$, $\pi^F(t)$, $\pi^{F^*}(t)$, $t \geq 1$, such that equations (1a)-(5a) are satisfied at $t = 1$

and (1b)–(5b) are satisfied for $t \geq 2$ (the vectors $g(t)$, $g^*(t)$, $p(1)$, and $p^*(1)$ are defined as $x(m) \equiv \langle x^S(m), x^F(m) \rangle$). By construction, all markets clear at these prices, the governments' budget constraints are satisfied, and individuals maximize utility subject to their budget constraints.

Initial values are needed to determine nominal price levels. The initial value for each country is the total amount of outside government debt (money and bonds).

Policy Analysis

We use the equilibrium relationships introduced above to study the response of the world economy to policy shocks emanating from the home country, under the assumption that the foreign country doesn't respond with a policy change of its own. In particular, we study economies that can be viewed as entering time 1 on a steady-state trajectory and analyze the effects of permanent, unanticipated, one-time changes in monetary policy (β, λ) beginning at time 1.

Our methodology makes heavy use of properties of steady-states. We define steady-states and establish sufficient conditions for their existence and uniqueness. We then use these conditions to restrict the parameter space for our numerical simulations. For our policy analysis we begin by comparing steady-states under changes in one country's monetary policy for a number of sets of initial conditions. We examine over 500 sets for which the countries initially are identical and repeat the experiment to examine scale effects by letting the endowments differ in the two countries. Finally, for all policy simulations we also compute the true dynamic equilibrium paths to examine the transition of the economy from one steady-state equilibrium to the next after a policy perturbation.

A. Steady-States

Steady-states are defined for particular stationary endowment patterns and policies. Steady-state endowment patterns are of the form $w_t(t) = w_1$ and $w_t(t+1) = w_2$ for $t \geq 1$. Thus, service

providers of any generation are endowed with w_1^S of services when young and 0 when old. Similarly, farmers of any generation are endowed with w_1^F of food when young and $w_2^F > w_1^F$ when old. Parallel endowment patterns hold in the foreign country.

Steady-state policies are of the form $g(t) \equiv (g^S(t), g^F(t)) = (g^S, g^F)$, $\lambda(t) = \lambda$, and $\beta(t) = \beta$ for $t = 1, 2, \ldots$, and similarly for the foreign country. Thus, steady-state policies have, in each period, the same government consumption of each good, the same reserve requirement, and the same ratio of government bonds to money.

We define a steady-state as a solution of (1b)-(5b) for steady-state endowments and policies such that $\rho(t) = \rho$, $Q(t) = Q$, $Q^*(t) = Q^*$, $\pi^F(t) = \pi^F$ and $\pi^{F*}(t) = \pi^{F*}$ for $t \geq 2$. Given this solution for $t \geq 2$, we set $p^F(1) = p^{F*}(1) = 1$ and choose initial values for $H(0) + B^G(0)$ and $H^*(0) + B^{G*}(0)$ such that it is also a solution at $t = 1$. In the initial period we are choosing the real wealth distribution that obtains in the steady state, so that the equilibrium price distribution in that period must match the steady-state solution.

A steady-state is then a solution to the following steady-state versions of (1b)-(5b):

$$
\text{(1ss)} \quad \left\{ \delta \left[(1-\lambda)(1+\rho) + \frac{\lambda}{1 + \pi^F} \right] + \left[1 - \left[\frac{1+\delta}{\theta} \right] \right] \right\} Qw_1^S
$$

$$
+ \left[1 + \delta(1+\rho) \right] w_1^F + \left[\frac{1 + \delta(1+\rho)}{1+\rho} \right] w_2^F + \left[\frac{1+\delta}{\theta} \right] Qg^S = 0.
$$

(2ss) is parallel to (1ss).

$$
\text{(3ss)} \quad \left\{ 1 + \delta \left[(1-\lambda)(1+\rho) + \frac{\lambda}{1 + \pi^F} \right] \right\} Qw_1^S + \left[\delta(1+\rho) - \left[\frac{1+\rho}{1-\theta} \right] \right] w_1^F
$$

$$
+ \left[\frac{1}{1+\rho} - \left[\frac{1+\delta\theta}{1-\theta} \right] \right] w_2^F + \left[\frac{1+\delta}{1-\theta} \right] g^F + [*] = 0,
$$

where [*] means repeat all terms but substitute foreign country variables.

$$(4ss) \quad \left[\frac{\lambda\delta}{1+\delta} \right] \left[\frac{\pi^F}{1 + \pi^F} - \frac{\beta\rho}{(1+\rho)(1+\pi^F)} \right] Qw_1^S - Qg^S - g^F = 0.$$

(5ss) is parallel to (4ss).

If there is a solution to (1ss)-(5ss), then initial values can be chosen for $H(0) + B^G(0)$ and $H^*(0) + B^{G^*}(0)$ so that the solution also satisfies (1a)-(5a); that is, the steady-state solution ρ_0, Q_0, Q_0^*, π_0^F, $\pi_0^{F^*}$ is also a solution at $t = 1$. To show this, we note that if ρ_0, Q_0, Q_0^*, π_0^F, $\pi_0^{F^*}$ is a solution to (1ss)-(5ss), it is also a solution to (1a)-(5a) when the initial values in the home country are

$$\frac{H(0) + B^G(0)}{p^F(1)} = \left[\frac{\delta}{1+\delta} \right] \left[(1-\lambda)(1+\rho_0) + \frac{\lambda}{1 + \pi_0^F} \right] Q_0 w_1^S$$

$$+ \left[\frac{\delta}{1+\delta} \right] (1+\rho_0) w_1^F - \left[\frac{1}{1+\delta} \right] w_2^F$$

$$\frac{H^*(0) + B^{G^*}(0)}{p^{F^*}(1)} = \text{same as above with } * \text{ variables.}$$

We now state sufficient conditions for the existence and uniqueness of solutions to (1ss)-(5ss). We consider solutions for which $\rho > 0$ to insure that the reserve requirement is binding. Let γ be a vector of parameter values: preferences, endowments, and policies. We are able to rewrite (1ss)-(5ss) in the form:

$$E(\rho;\gamma) = 0,$$

$$Q = \psi(\rho;\gamma),$$

$$Q^* = \psi^*(\rho;\gamma),$$

$$\pi^F = \phi(\rho,Q;\gamma), \text{ and}$$

$$\pi^{F^*} = \phi^*(\rho,Q^*;\gamma); \text{ where the functions } \psi, \psi^*, \phi, \text{ and } \phi^* \text{ map into } \mathbf{R_+}, \mathbf{R_+}, \mathbf{R}, \text{ and } \mathbf{R},$$

respectively.

Thus, to show existence and uniqueness for the system, we need only show it for the first equation, $E(\rho;\gamma) = 0$. We state our theorems and provide sketches of the proofs. Detailed proofs are available from the authors.

We first define feasible budget policies as elements of the set of nonnegative government consumption bundles that can be financed at a zero real interest rate and infinite inflation rate. (We can use zero interest and infinite inflation to define this set because savers in our model save a constant fraction of their wealth, independent of rate of return. Thus there is no Laffer curve, and government seignorage is everywhere increasing in inflation and decreasing in ρ. See Equation 4ss.) That is, for the home country feasible budget policies g^S and g^F must be individually nonnegative and satisfy the following budget constraint for $\rho = 0$ and π^F as $\pi^F \to \infty$:

$$0 < \psi(0;\gamma)g^S + g^F \leq \lambda\left[\frac{\delta}{1+\delta}\right]\psi(0;\gamma)\,w_1^S,$$

where

$$\psi(0;\gamma) \equiv Q(\text{given } \rho=0) = \frac{\theta}{1-\theta}\left[\frac{w_1^F + w_2^F - g^F}{w_1^S - g^S}\right].$$

Theorem (Existence). For all feasible budget policies, a positive real interest rate equilibrium exists if

1. $5/8 < \delta < 1$

2. $\delta \leq (1-\theta)/\theta < 1 + \delta$

3. $\lambda < 1/(1+\beta)$

4. $\lambda^* < 1/(1+\beta^*)$

5. $w_2^F > kw_1^F$

6. $w_2^{F*} > k^*w_1^{F*}$

where

$$-k \equiv \left[\frac{\delta(1-\lambda)/(1 + \delta(1-\lambda))b + \delta/(1+\delta)}{\delta(1-\lambda)/(1 + \delta(1-\lambda))b) - 1/(1+\delta)} \right] < 0,$$

and k* is defined similarly.

Proof. The proof has three parts. We show that given feasible budget policies and conditions 1-6

    A. $E(0;\gamma) < 0$,

    B. $E(\rho;\gamma)$ is continuous with respect to ρ for $0 \le \rho < \rho_c < 1$, and

    C. $E(\rho;\gamma) \to \infty$ as $\rho \to \rho_c$.

In order to gain some economic intuition for our conditions, it helps to think why an equilibrium with a positive interest rate may not exist in this model. There seem to be three possibilities. First, the government could consume more than it can raise in seignorage. We rule out this possibility by restricting budget policies to a set that is feasible at zero real interest rates and then by bounding δ and θ away from zero (left side of condition 1 and right side of condition 2) so that demand for credit does not drive interest rates too far from zero. Second, the demand for loans could be less than the supply of loans at a zero real interest rate. To prevent this possibility, farmers need a second-period endowment sufficiently greater than their first-period endowment (conditions 5 and 6) to make them want to borrow enough. And, farmers must be impatient enough (right side of condition 1) to want to borrow, while service providers must be relatively poor enough (left side of condition 2) to make them not want to lend too much. Third, the amount of reserves required could be more than the amount of money supplied. To prevent this, the reserve requirement must be sufficiently small relative to the money portion of total government debt (conditions 3 and 4).

In order to state our uniqueness theorem we first make some definitions. Let

$\omega \equiv \langle w_1^S, w_1^F, w_2^F, g^S, g^F \rangle$, similarly for ω^*, and

$\langle \kappa, \omega, \kappa^*, \omega^* \rangle \equiv \gamma$, a partition of parameters according to country.

<u>Theorem</u> (Uniqueness). For all feasible budget policies there exists exactly one equilibrium with a positive real interest rate if

    A. Conditions 1, 2, 5, and 6 of the previous theorem are satisfied,

    B. Conditions 3 and 4 of the previous theorem are modified to

        3'. $\lambda < \min\{1/3, 1/(1+\beta)\}$

        4'. $\lambda^* < \min\{1/3, 1/(1+\beta^*)\}$,

    C. $\kappa = \kappa^*$ and $\omega = z\omega^*$ for $z \gtreqless 0$.

The theorem basically states that when a positive real interest rate equilibrium exists, it will be unique if the only difference in the two economies is one of scale.

<u>Proof</u>. We show that $E(\rho;\gamma)$ can be written:

$$E(\rho;\gamma) = H(\rho;\kappa,\omega)\left[K_3(\kappa,\omega)\rho^3 + K_2(\kappa,\omega)\rho^2 + K_1(\kappa,\omega)\rho + K_0(\kappa,\omega)\right]$$
$$+ H(\rho;\kappa^*,\omega^*)\left[K_3(\kappa^*,\omega^*)\rho^3 + K_2(\kappa^*,\omega^*)\rho^2 + K_1(\kappa^*,\omega^*)\rho + K_0(\kappa^*,\omega^*)\right].$$

If the conditions of the theorem are satisfied, the functions H and K_i have the properties

    1. $H > 0$ for $0 \le \rho < \rho_c$,

    2. $K_3 > 0$ and $K_0 < 0$,

    3. $H(\rho;\kappa,z\omega) = zH(\rho;\kappa;\omega)$, and

    4. $K_i(\kappa,z\omega) = z^2 K_i(\kappa,\omega)$.

We first show that for given parameters the cubic in ρ given by $K_3\rho^3 + K_2\rho^2 + K_1\rho + K_0$ has a single positive root. By 2 above $K_3 > 0$ and $K_0 < 0$. It then follows that if $K_2 \gtreqless 0$ or

$K_1 \lesssim 0$ the cubic expression has a single positive root. Suppose instead that $K_2 < 0$ and $K_1 > 0$. The cubic then has positive slope at $\rho = 0$, where the slope is simply $3K_3\rho^2 + 2K_2\rho + K_1$. The slope does not change sign with respect to ρ if $K_2^2 - 3K_1K_3 < 0$. With $K_3 > 0$, $K_2 < 0$, and $K_1 > 0$ the slope is positive for all ρ if there exists an $x > 0$ such that $xK_1 + K_2 > 0$ and $(3/x)K_3 + K_2 > 0$. We show that the last two inequalities hold with $x = 1$.

We next observe that by 1 the expression $H(K_3\rho^3 + K_2\rho^2 + K_1\rho + K_0)$ has a single positive root. This implies that if the countries are identical there is a unique positive real interest rate equilibrium.

We now show that when condition C of the theorem holds there is a unique positive real interest rate equilibrium. So, suppose $\kappa = \kappa^*$ and $\omega = z\omega^*$. Then by 3 and 4 $E(\rho;\gamma) = (1+z^3)H(K_3\rho^3 + K_2\rho^2 + K_1\rho + K_0)$ which has a single positive root. Note the equilibrium for the two economies differing only by the scale factor z is the same as that for two identical economies with size $\omega' = ((1+z^3)/2)\omega$.

B. Comparison of Steady-State Equilibria

We compute steady-state derivatives for the monetary policy parameters β' ($\equiv \beta/(1+r)$) and λ.[2] Points that satisfy the sufficient conditions for uniqueness of equilibria[3] are randomly sampled from a uniform distribution over the parameter space. For each point we compute the equilibrium. We then increase the policy parameter by 1 percent and compute the differences in equilibrium values of selected variables. We repeat the exercise for about 500 points with identical economies and also for about 500 points with the scale of the home country being 10 percent of the foreign economy.[4]

1. Qualitative Steady-State Effects of an Increase in β' (Open-Market Sale)

a. Identical Economies (Text Refers to Figure 5a)

In all cases an increase in β' raises the real interest rate ρ. A sale of government bonds increases the demand for credit. Since the supply of credit, or savings, is perfectly inelastic with respect to changes in rates of return in our economy, the increased government demand must drive out private credit demands. This requires a rise in the real interest rate.

In all cases an increase in β' causes inflation π to rise more in the home country than in the foreign country. (Since relative prices are constant in the steady-state, all prices increase at a common rate $\pi = \pi^F$). That result follows from unpleasant monetarist arithmetic, since the base for the inflation tax has been lowered in the home country relative to the foreign country.

Higher inflation in the home country than abroad implies that home country savers get a lower real rate of return on their savings $\rho_S \equiv [(1 + (1-\lambda)r)/(1+\pi)] - 1$ than do foreign savers $\rho_S^* \equiv [(1 + (1-\lambda^*)r^*)/(1+\pi^*)] - 1$. Higher inflation in the home country can cause the real rate of return on home savings to fall even though the real rate of return on bonds rises. That is, with ρ_S rewritten as $\lambda/(1+\pi) + (1-\lambda)(1+\rho) - 1$, it is possible that π will rise enough to lower ρ_S even though ρ rises. An increase in β' produces this result for roughly 10 percent of our randomly drawn parameter vectors.

With borrowers in both countries facing the same real interest rate and lenders in the home country receiving a lower real rate than do lenders abroad, there has in a sense been a transfer of wealth from the home country to the foreign country. It seems conceivable that the additional wealth in the foreign country could reverse the effect of a higher real interest rate on its inflation rate. We find that outcome obtains for roughly 5 percent of our randomly drawn parameter vectors.

Since residents of the home country are relatively poorer, they consume relatively less of the traded good. For them to consume the same amount of the nontraded good and less of the traded

good, the relative price of nontraded to traded goods Q must fall. By similar reasoning Q* must rise. Since the real exchange rate $\bar{e}$ is just $(Q/Q^*)^\theta$, it must fall. And, with the home country residents consuming less of the traded good, its trade balance $w_1^F + w_2^F - c_1^F - c_2^F - g^F$ must move toward surplus.

The welfare effects in the steady-state of an increase in β' follow from its effects on real rates of return and relative prices. Lenders/service providers are made better off with a higher real rate of return on savings and a higher relative price of nontraded to traded goods. That is, we can express the utility of a service provider as $U^S = \psi^S(\rho_S, Q)$ with $\psi_1^S > 0$ and $\psi_2^S > 0$. Borrowers/farmers, meanwhile, are made worse off with a higher real rate of interest on loans and a higher relative price of nontraded to traded goods, that is, $U^F = \psi^F(\rho, Q)$ with $\psi_1^F < 0$ and $\psi_2^S < 0$. In our simulations a higher β' always leads to a higher ρ and Q* and to a lower Q. It leads to a higher ρ_S in roughly 90 percent of the draws and to a higher ρ_S^* in over 99 percent of the draws. In all the draws we examine foreign service-providers are made better off with a higher ρ_S^* and Q* and foreign farmers are made worse off with a higher ρ and Q*.[5] In the home country the welfare effects are more complicated. Service-providers are made better off by an increase in ρ_S, but ρ_S rises in only 90 percent of the draws. They are always made worse off by a fall in Q. On net, the effect of a rising real interest rate dominates, as they are made better off by an increase in β' in roughly 75 percent of the draws. Home country farmers are hurt by the increase in ρ but helped by the fall in Q. However, in the identical economy case, the real interest rate effect again dominates, as farmers are made worse off by an increase in β' in all draws.

b. Home Economy is 10 percent of Foreign Economy (Text refers to Figure 5b)

The reasoning about how an increase in β' affects price variables seems unaffected by scale. And that is largely verified by our results. However, the comparative magnitudes of the changes in

real rates of return and relative prices do seem likely to be affected by scale. When the home economy is small relative to the rest of the world, the change in its supply of bonds is small relative to the world supply and, hence, is likely to cause only a small change in the real interest rate. However, a given percentage increase in β' still shrinks the home country's base for the inflation tax as it did before. Thus, the difference in inflation rates between the two countries following a change in β' is likely to be larger when the home country is small. It follows that the difference in the returns on savings will be larger and that Q will have to fall more.

We find the relative price transmission channel of monetary policy to be much more important when the economy is small compared to the rest of the world. In the home country the fall in the relative price of nontraded to traded goods tends to be more important in terms of welfare than the rise in the real interest rate. For example, an increase in β' now leads home service-providers to be better off in only 6 percent of the draws, compared to 75 percent in the identical economy case. It causes home farmers to be worse off in only 42 percent of the draws, compared to 100 percent in the identical economy case. The welfare effects of a change in β' in the rest of the world are qualitatively unaffected by scale, since the real interest rate and relative prices abroad move in the same direction.

2. Qualitative Steady-State Effects of an Increase in λ (Higher Reserve Requirement)

a. Identical Economies (Text Refers to Figure 6a)

The positive economic effects of an increase in the reserve requirement λ are much like those of an open-market sale, with the exception of the home country's inflation rate. The similarity between the two is not surprising because both types of monetary policy actions have similar effects on the real interest rate. With a higher λ less savings are supplied to the bond market, so that the real interest rate must rise to correspondingly reduce the demand for credit.

In the home country an increase in λ tends to lower the inflation rate, even though it has two opposing effects. One effect is that the resulting higher real interest rate requires the government to raise more seignorage, which tends to cause the inflation rate to rise. However, with a higher λ the base for the inflation tax rises, which allows a given amount of seignorage revenue to be raised at a lower inflation rate. In over 98 percent of the draws the tax base effect dominates the real interest rate effect, so that the increase in λ results in a lower inflation rate.

An increase in λ results in a lower real rate of return on savings in the home country compared to the foreign country. In the foreign country the rise in the real interest rate is quantitatively more important than the rise in the inflation rate in the sense that the real rate of return on savings rises all the time. In the home country a fall in the inflation rate and rise in real interest rate by themselves cause the real rate of return on savings to rise. However, there is an opposing effect of a higher λ, since savings are shifted from bonds having a relatively high real rate of return to money having a relative low rate of return. This shifting effect of a higher λ causes the real rate of return on savings to decline in 9 percent of the draws. It also causes the real rate of return on savings in the home country to fall below that in the foreign country all of the time, which, once again, can be considered a transfer of wealth from the home country to the foreign country.

The economic effects of the transfer in wealth from an increase in reserve requirements are much like what they were for an open-market sale. Because the home country is poorer, it consumes less of the traded good, and that requires a fall in the relative price of nontraded to traded goods Q. Because the foreign country is richer, the reverse occurs there, so that Q* rises. It then follows that the real exchange rate ē falls. With home country residents consuming less of the traded good, its trade balance moves to surplus. The transfer of wealth to the foreign country conceivably can overturn the effect on foreign inflation of a higher real interest rate, but that occurs in only 5 percent of the draws.

The welfare effects of an increase in λ are qualitatively the same as those of an increase in β'. Since in the home country the real interest rate and relative price of nontraded to traded goods move in opposite directions, the utilities of service-providers and farmers can change in either direction depending on the relative strengths of the two effects. In the identical economy case the real interest rate effect dominates, so that the utility of service providers increases in 75 percent of the draws while the utility of farmers declines in all of the draws. In contrast, since the real interest rate and relative price of nontraded to traded goods both rise abroad, the effects on utility of foreign residents is unambiguous. The utility of service providers there always rises, while the utility of foreign farmers always falls.

b. Home Economy is 10 percent of Foreign Economy (Text refers to Figure 6b)

Most of the price effects of an increase in λ are in the same direction for the small economy case as they were for the identical economy case. However, there are some differences. In the small economy case the effect of shifting savings from high return bonds to low return money tends to dominate the effect of higher rates of return on both assets, so that the real rate of return on savings tends to decline. It declines in roughly 86 percent of the draws for the small economy case, whereas it declined in only 9 percent of the draws for the identical economies case. The other major difference is, as with the increase in β', relative price effects are more important when the economy is small. Once again, we can see that with the effects of an increase in λ on home country residents. Service providers are made better off in only 6 percent of the draws in the small economy case, while they were made better off in 75 percent of the draws in the identical economies case. They are subjected more frequently to a fall in the real rate of return on savings, but it also seems likely that they are affected by a larger fall in the relative price of nontraded to traded goods. That last surmise is supported by the effects on home country farmers. In the small economy case they are

made worse off by an increase in λ in only 42 percent of the draws, while in the identical economy case they were made worse off all of the time.

3. Qualitative Comparison of Transition Paths

The addition of nontraded goods to the Miller-Wallace model complicates the dynamics following policy perturbations. In the original model the equilibrium immediately moves from one steady-state to another following a change in β or λ. When there are nontraded goods, however, the convergence occurs over a number of periods. Moreover, compared to their initial values, some of the variables change in opposite directions in early periods from how they change in steady state. As in the original model, a change in monetary policy has different effects on the initial price level and the inflation rate. Since the real wealth of the initial old depends on the price level, their demand for goods is likely to respond differently following a change in monetary policy than are the demands of the young and future generations. In the original model the difference in demands just determined how the pie was being divided but had no effect on real rates of return. When there are nontraded goods, however, the change in the wealth of the old affects relative prices. This effect dissipates over time as the economy approaches its new steady-state.

The difference between initial period and steady-state effects of a change in monetary policy can be illustrated with any of our policy experiments (Figures 5-6). We have assumed no international lending in the initial steady state. This means that the aggregate net wealth of the initial old (borrowers plus lenders) in a given country equals the outstanding government debt in that country. When policy is tightened (β' or λ is increased), the price level in the home country falls (since $H(1) = \lambda p^S(1)[\delta/(1+\delta)]w_1^S$ and $p^F(1)$ is set to 1). The price level in the foreign country rises, however, as the higher real interest rate raises its governments deficit inclusive of interest, its total outside debt $H^*(1) + B^{G*}(1)$, (and with a fixed β^*) its initial money stock $H^*(1)$. These price level

changes cause period one capital gains on home government debt and capital losses on foreign government debt. In the first period, then, there is a transfer of relative wealth to the initial old of the home country from the initial old of the foreign country. In all of our policy experiments, in period one this price level transfer effect outweighs the opposing inflation rate transfer effect that will eventually dominate in steady-state, as discussed above. In all cases the initial period transfer allows the home country to increase its consumption of the traded good. In order to consume more of the traded good and the same amount of the nontraded good, the relative price of nontraded to traded goods must rise. The reverse occurs in the foreign country. Since the old are not affected by changes in the real interest rate, their utility is determined by the change in relative prices. Thus, when monetary policy is tightened in the home country, the old service-providers are always made better off, and the old farmers are always made worse off, while in the foreign country, the reverse occurs.

The initial effects of policy changes quickly dissipate in our policy experiments, and the economy after a few periods approaches a new steady-state. The signs of derivatives of key variables following a policy change largely match the signs of steady-state derivatives after 3 or 4 periods. The dynamic path after 7 periods about coincides with the new steady-state path.

4. Quantitative Effects: A Case Study

Through its effects on relative wealth levels, monetary policy in our model has initial, transitional, and steady-state effects on real and nominal variables that can be interpreted as qualitatively in line with U.S. experience in the post-1972 floating exchange rate era. Quantitatively, however, the model appears to be able to match nominal data well, real interest rate data fairly well, and real exchange rate data hardly at all.

We draw from this conclusion from a set of 95 simulations designed to loosely parallel U.S. experience from mid 1979 to about 1985. During this period, the ratio of outside federal debt to monetary base experienced a sharp increase from about 3 to about 6. (See Figure 3.) At about the same time, nominal and real U.S. interest rates jumped several basis points (Figure 1) and both the nominal and the real foreign exchange value of the dollar rose by about 60 to 70 percent (Figure 2).

Our 95 simulations are set to match the increase from 3 to 6 in the outside debt/base ratio. That is, while the remaining parameters are picked randomly as in our previous symmetric country simulations, the value of β and β^*—the home country and foreign country (respectively) outside debt/base ratio—is fixed at 3 in computing the initial steady state. β is then changed to 6 at time 1, and we check to see if the time 1, or initial period, effects of this monetary tightening quantitatively reflect the U.S. experience of the early 1980s.

The simulations match experience reasonably well for nominal variables. Across the 95 simulations, the shocked time 1 nominal interest rate is, on average, about 35 percentage points higher than in the initial steady state. Considering that the time periods in our model correspond to generations, not years, this is in the ballpark. For example, when we "annualize" the shocked and steady-state gross nominal interest rates by exponentiating them by .04 (25 year generations) or .1 (10 year generations), we get that the increase in annualized nominal interest rates averages 1.3 or 4.5 percentage points, respectively. For nominal exchange rates, the average time 1 appreciation across the 95 simulations is about 59 percent. Thus, both the increase in annualized nominal interest rates and the increase in the nominal exchange rate are, on average, of the same magnitude as the increases experienced in the early 1980s.

The model appears to capture only some of the observed increases in real interest rates. In our 95 simulations, the increase in annualized real interest rates averages only 12 basis points even using 10 years per generation. For some parameter settings the effect on real interest rates was

higher, of course. The maximum increase we found, which was in a previous simulation separate from the randomly chosen 95, was about 1.5 percentage points, from a baseline annual percentage interest rate of 11.7 to 13.2 in the first post-shock period, using 10 years per generation to annualize. (Using 25 years per generation gives a baseline of 4.5 percent annual real interest rate and an initial post-shock annual real interest rate of 5.1 percent.) This example suggests that our model can explain, depending on its parameterization, a nonnegligible portion of the 1980s' increase in real interest rates, though far from all of it.

For real exchange rates, the model explains very little of the change in the 1980s. The average percentage increase in the real exchange rate in our 95 simulations was less than one percent, and there was little dispersion around this average. These effects are tiny compared to the increases shown in Figure 2.

What are the implications of these discrepancies? One interpretation is purely negative: Our research is not useful for explaining the behavior of real exchange rates since 1972, and only moderately helpful with respect to real interest rates. Not surprisingly, we take a more positive view, for several reasons.

One reason is that even from the narrowest perspective—focusing solely on our model in its present form—our quantitative results help to answer an obvious and interesting question: Is it easy to account for the size of recent real interest rate and real exchange rate movements as a response to a change in monetary policy merely by adding nontraded goods to a simple two-country general equilibrium model of money. It is useful to know that the answer is "No," at least when the Miller-Wallace model of money is used as a starting point.

We also view our quantitative results as not too discouraging because several of our simplifying assumptions probably limit the quantitative impacts of monetary policy. It is quite possible that obvious (but possibly hard to solve) generalizations of our model—such as the

introduction of 3 or more periods of life (and correspondingly a richer set of financial contracts and motives for holding money), production, capital accumulation, and alternative utility functions—could greatly magnify the sensitivity of real variables to monetary policy. Alternatively, we could magnify money's time 1 wealth effect either by assuming a larger stock of outstanding home government debt or by assuming an asymmetric distribution of initial asset holdings. (Asymmetric initial asset holdings might be relevant to the U.S. versus Latin American situation in the early 1980s. Suppose we modify our previous assumptions of no outstanding international debt at time 1 and now assume that the initial U.S. old have net nominal-dollar-denominated claims on the initial foreign old. Then the lower-than-expected U.S. price level also produces a capital gain on this component of the net aggregate wealth of the initial U.S. old, and a corresponding capital loss in the foreign country. Under our previous assumptions, by contrast, the capital gains of initial U.S. lenders on their private sector loans were exactly offset by the capital losses of U.S. borrowers, and thus private lending yielded no effect on the aggregate wealth of the initial old in either region.)

Even more broadly, we view our qualitative results as interesting despite the quantitative findings. Our model provides an explicit intertemporal framework for thinking about the channels through which monetary policy is transmitted; for instance, the relative price versus interest rate effects that monetary policy has on sectors like agriculture and manufacturing. Many of the effects we discuss within our model would also operate in other models that include borrowers and lenders as well as traded and nontraded goods sectors.

Footnotes

[1]These initial price level effects on the terms of trade are similar to those in Rotemberg (1985).

[2]For computational efficiencies we defined $\beta' \equiv P^B \cdot B^G/H \equiv B^G/(1+r)H \equiv \beta/(1+r)$ and did the experiments with β' and β'^* as the monetary policy parameters. The parameter β is the ratio of the face value of bonds to money, while the parameter β' is the ratio of the market value of bonds to money. When β is fixed and the government's financing requirement changes, it is being assumed that the government must forecast the interest rate to know how much bonds to sell to raise the required revenue. When β' is fixed, it is being assumed that the government does not have to forecast the interest rate; it just sells all the bonds required to raise the revenue at whatever interest rate it faces. If the conditions for existence and uniqueness of an equilibrium are satisfied for a set of parameters that include β and β^*, they will also be satisfied for β' and β'^*, since $\beta' < \beta$ and $\beta'^* < \beta^*$ imply that conditions 3 and 4 will still be satisfied. We found that the steady-state derivatives of the key price variables with respect to a change in monetary policy in the home country were qualitatively the same whether the monetary policy parameter was β or β'.

[3]With a 1 percent margin of safety, to allow numerical differentiation.

[4]In each case, 505 points were chosen, but usually a few points failed to yield results because of numerical problems in the solution algorithm.

[5]Foreign service providers are made better off even in the few cases where ρ_S^x declines.

References

Barro, R. J. 1976. Rational expectations and the role of monetary policy. *Journal of Monetary Economics* 2: 1-32.

Blanchard, O. J. 1984. The Lucas critique and the Volcker deflation. *American Economic Association Papers and Proceedings* 74: 211-215.

Dornbusch, R. 1988. Real exchange rates and macroeconomics: A selective survey. NBER working paper #2775.

Frankel, J. A. 1989. International financial integration, relations among interest rates and exchange rates, and monetary indicators. In *International financial integration and U.S. monetary policy*, C. Pigott, ed., New York: Federal Reserve Bank of New York.

Krugman, P. R. 1989. *Exchange-rate instability*. Cambridge, MA: MIT Press.

Miller, P. J. and Roberds, W. 1991. The quantitative significance of the Lucas critique. *Journal of Business and Economic Statistics*.

Miller, P. J., and Wallace, N. 1985. International coordination of macroeconomic policies: a welfare analysis. *Federal Reserve Bank of Minneapolis Quarterly Review* 9: 14-32.

Rotemberg, J. J. 1985. Money and the terms of trade. *Journal of International Economics* 19: 141-160.

Sargent T. J. and Wallace, N. 1975. Rational expectations, the optimal monetary instrument, and the optimal money supply rule. *Journal of Political Economy* 83: 241-254.

Wallace, N. 1984. Some choices for monetary policy. *Federal Reserve Bank of Minneapolis Quarterly Review* 8: 15-24.

## Glossary of Variable Names in Figures 5 and 6

RHO: Worldwide real rate of interest on bonds. Corresponds to ρ in the text.

RHOS: Real rate of return to domestic savers. Corresponds to $\rho_s \left[(1 + (1-\lambda)r)/(1+\pi^F)\right] - 1$ in the text.

RHOS-RHOS*:   Real rate of return to domestic savers minus real rate of return to foreign savers. Correspond to $\rho_s - \rho_s^*$ in the text.

TRBAL: Domestic balance of trade. Corresponds to $\{w_{t-1}^F(t) + w_t^F(t) - \left[\hat{c}_t^F(t) + \hat{c}_{t-1}^F(t)\right] - \left[\hat{c}_t^F(t) + \hat{c}_{t-1}^F(t)\right] - g^F(t)\}$ in the text.
(service providers)   (food providers)

Q: Domestic relative price of nontraded to traded good. Corresponds to Q in the text.

Q*:   Foreign relative price of nontraded to traded good. Corresponds to Q* in the text.

FINF: Rate of inflation of domestic food prices. Corresponds to π^F in the text.

FINF*:   Rate of inflation of foreign food prices. Corresponds to π^{F*} in the text.

FXRATE: Nominal foreign exchange rate of home country. Corresponds to $e = p^{F*}/p^F$ in the text.

RFXRATE: Real foreign exchange rate of home country. Corresponds to $\bar{e} = e/[p^*/p]$ in the text.

SU: Lifetime utility of newborn domestic savers/service providers. Corresponds to $U(c_t)$ (for domestic savers) in the text.

FU: Lifetime utility of newborn domestic borrowers/food providers. Corresponds to $U(c_t)$ (for domestic borrowers) in the text.

FIGURE 1: THREE-MONTH U.S. TREASURY BILL RATES
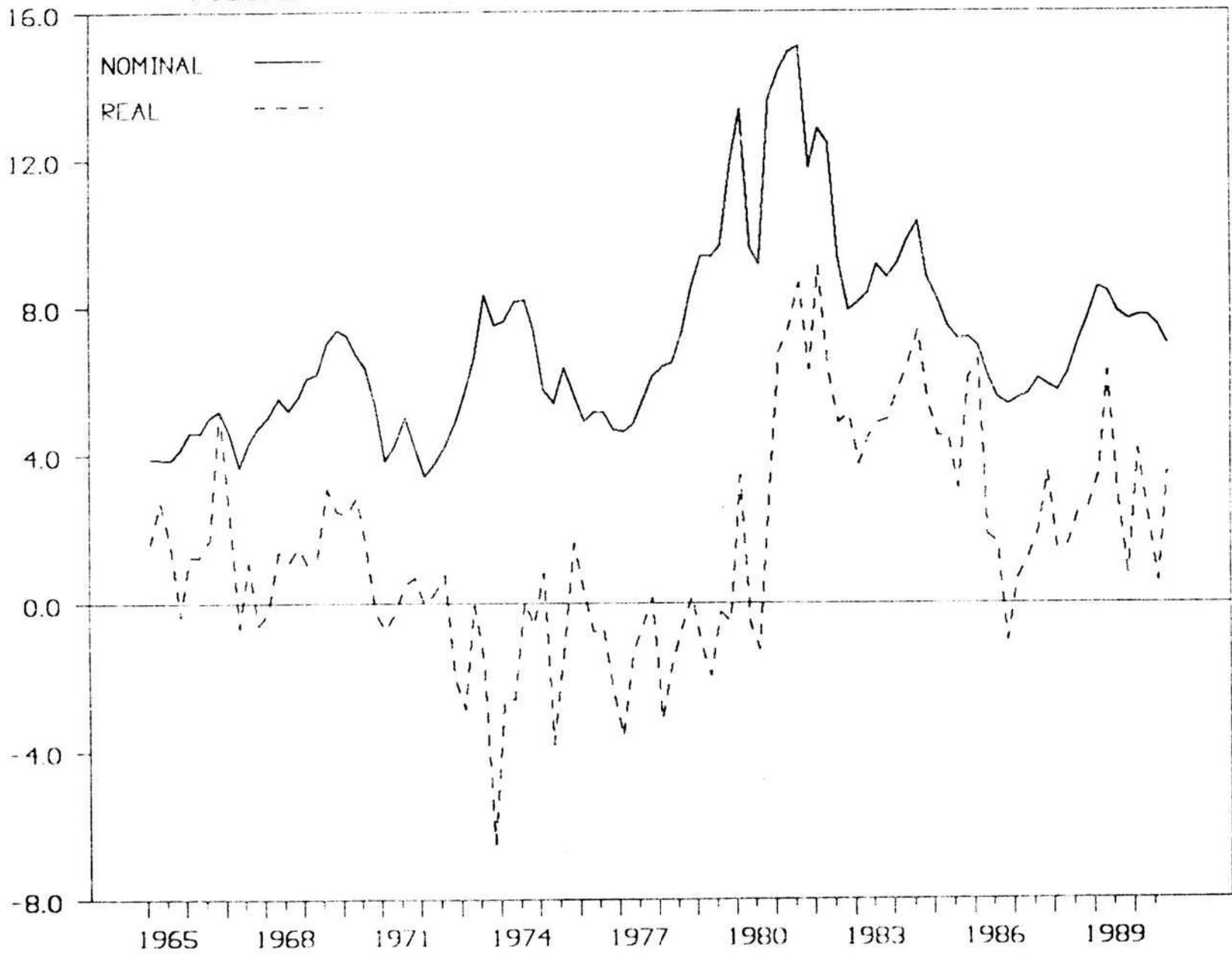
Note: Real rate calculated from the quarterly average nominal interest rate minus the rate of inflation in the succeeding quarter. Inflation was measured by the rate of increase in the consumption deflator.

FIGURE 2: EXCHANGE RATE OF THE U.S. DOLLAR
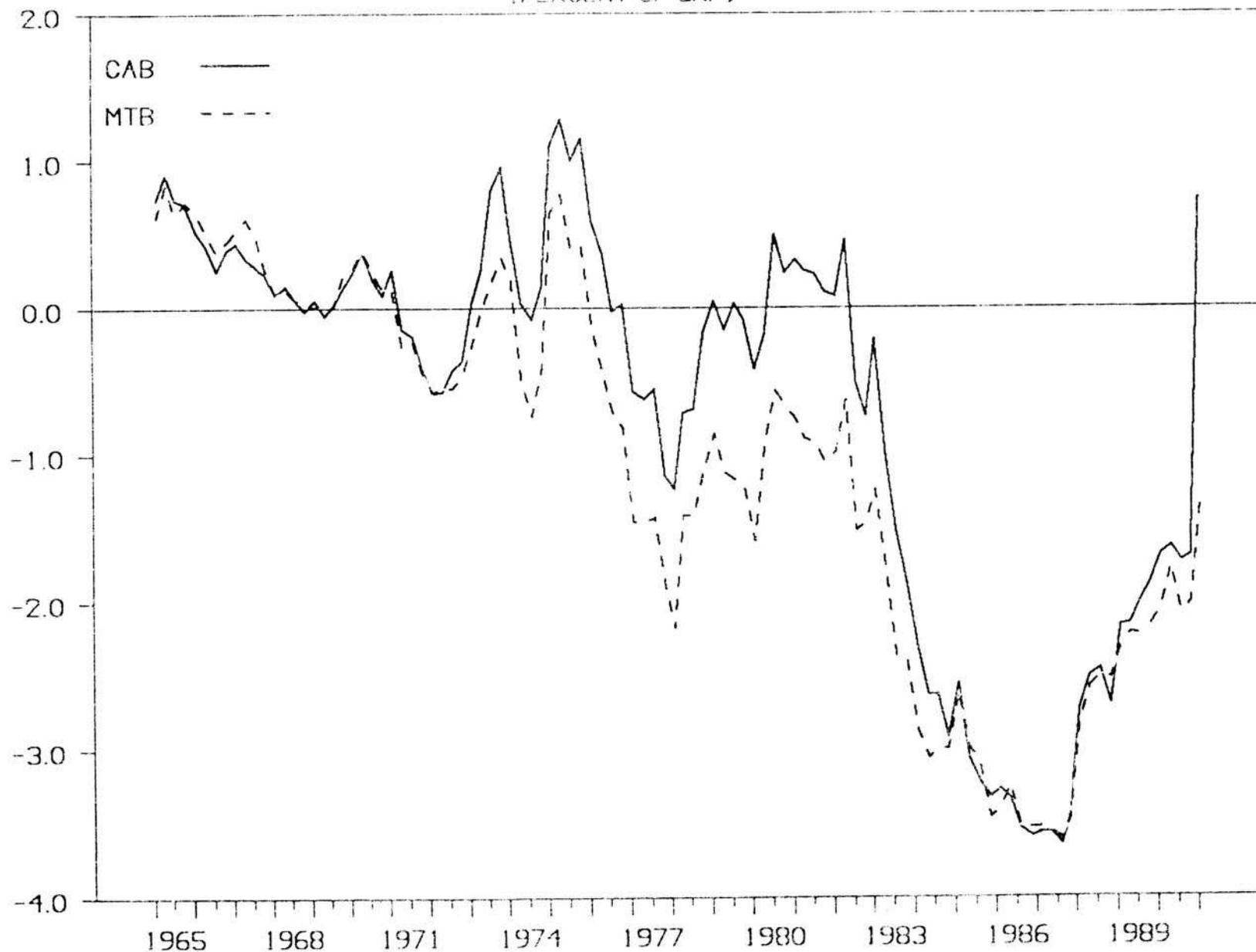(BASKET OF FOREIGN CURRENCIES PER DOLLAR, MARCH 1973 = 100)
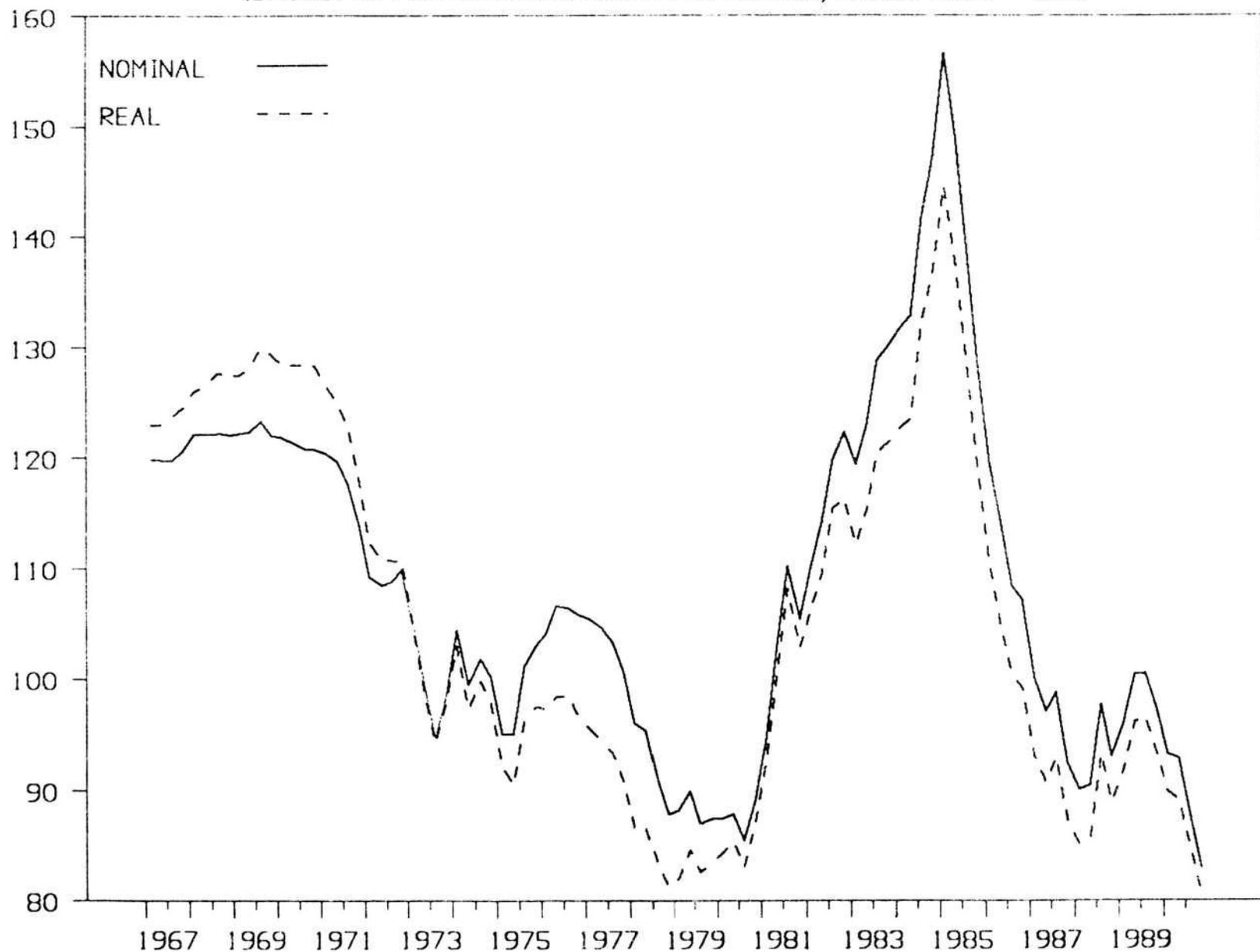
Source: Federal Reserve System's trade weighted average of dollar/foreign currency exchange
rates for 10 industrialized countries.

FIGURE 3: INDEX OF RELATIVE TIGHTNESS OF MONETARY POLICY

(RATIO OF OUTSIDE FEDERAL DEBT TO MONETARY BASE, QUARTERLY)
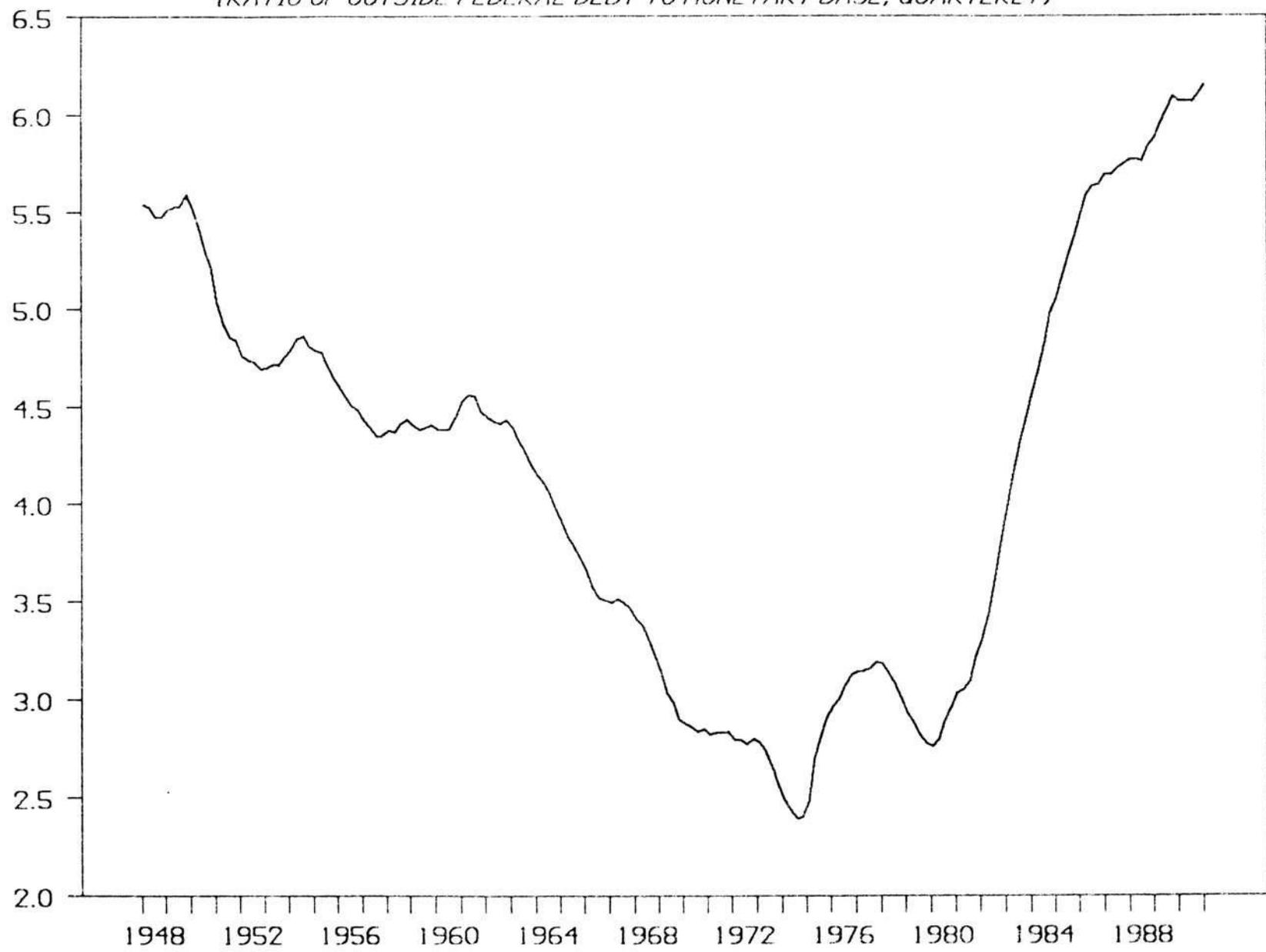
FIGURE 4: CURRENT ACCOUNT AND MERCHANDISE TRADE BALANCES
(PERCENT OF GNP)

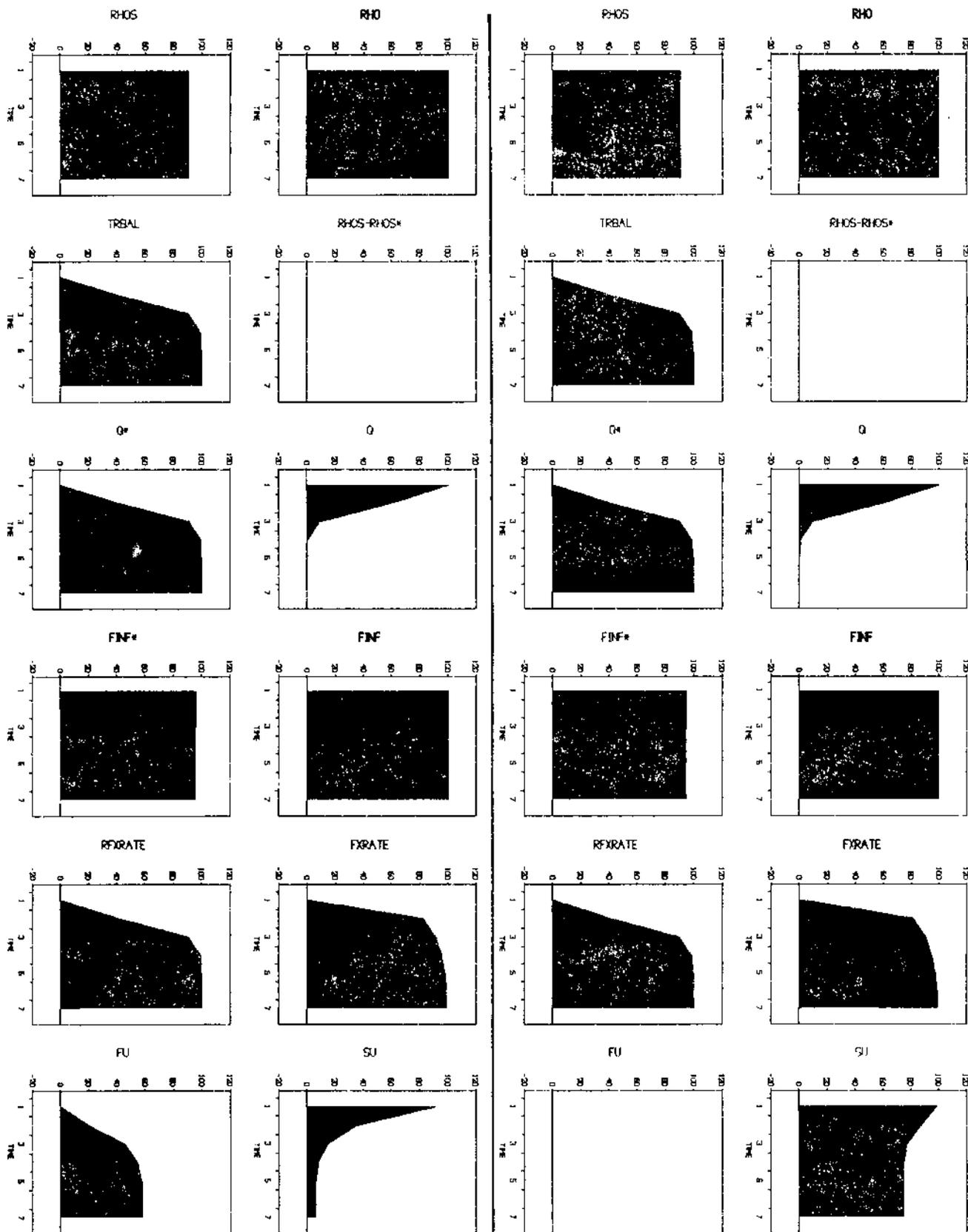Legend:
- CAB ———
- MTB – – –

Figure 5: Percentage of Positive Responses to an Open Market Tightening of Monetary Policy, By Time Period (the Home Country's Bonds-to-Monetary-Ratio is Shocked Up 1 Percent at Time 1)

B: Foreign Country 10 Times Bigger   A; Foreign Country Same Size

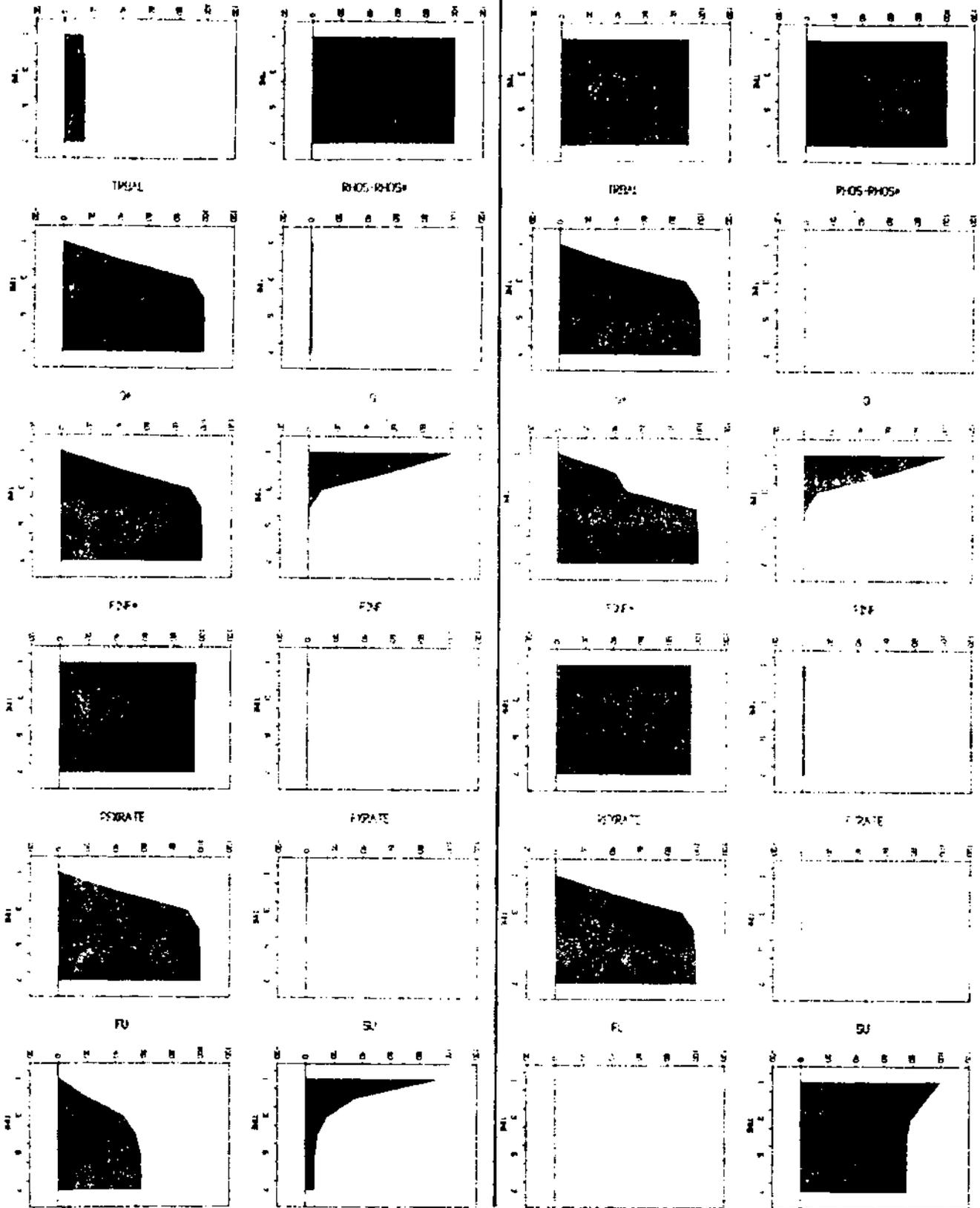Note: See Glossary of Variable Names after Figure 6

Figure 6: Percentage of Positive Responses to a Tightening of Reserve Requirements, by Time Period (the Home Country's Reserve Requirement is Shocked Up 1 Percent at Time 1)