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Uninsured Idiosyncratic Risk
and Aggregate Saving

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ABSTRACT

We find that precautionary saving accounts for only a modest (less than 3 percentage point) increase in the aggregate saving rate, at least for moderate and empirically plausible parameter values. This finding is based on a quantitative analysis of a reasonably parameterized version of the standard growth model modified to include a large number of agents who receive uninsured idiosyncratic labor endowment shocks. In contrast to representative agent models, asset trading is quite important to individuals. The model can also account qualitatively for the positive skewness of wealth and income distributions, and significantly greater wealth inequality compared to income inequality.

I. Introduction

This paper has two main goals. The first is to provide an exposition of models whose aggregate behavior is the result of market interaction among a large number of agents subject to idiosyncratic shocks. This class of models involves a considerable amount of individual dynamics, uncertainty, and asset trading which is the main mechanism (in the models) by which individuals attempt to smooth consumption. However, aggregate variables are unchanging. This contrasts with representative agent models in which individual dynamics and uncertainty coincide with aggregate dynamics and uncertainty. The exposition is motivated by two facts: (i) the behavior of individual consumptions, wealths and portfolios is strongly at variance with the complete markets model implicit in the representative agent framework, and (ii) recently several authors have found versions of such models useful for analyzing a variety of issues including asset pricing, monetary policy, business cycles, and taxation[1].

The exposition is built around the standard growth model of Brock and Mirman [1972] modified to include a role for uninsured idiosyncratic risk and liquidity/borrowing constraints. This is done by having a large number of agents who receive idiosyncratic labor endowment shocks which are uninsured, as in the models of Bewley [1986, undated][2]. As a result of this market incompleteness in combination with the possibility of being borrowing constrained in future periods, agents accumulate excess capital in order to

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[1]See, for example, Aiyagari [1993b], Diaz-Jimenez and Prescott [1992], Imrohoroglu [1989, 1991], Aiyagari and Gertler [1991], Huggett [1990].

[2]The absence of insurance markets is taken as given. There can be no doubt that private information and the resulting problems due to moral hazard and adverse selection have a lot to do with incomplete insurance. While it would be desirable to take explicit account of these features (see, for eg., Green 1987, Phelan and Townsend 1991, Taub 1990, Levine 1991, Atkeson and Lucas 1992), this is beyond the scope of this paper.

smooth consumption in the face of uncertain individual labor incomes.

The second main goal of this paper is to use such a model to study the quantitative importance of individual risk for aggregate saving. This study is motivated by the debate concerning the sources of aggregate capital accumulation, in particular, the suggestion that precautionary saving may be a quantitatively important component of aggregate saving[3]. For example, Modigliani [1988] argues that the pure bequest motive is likely important only for people in the highest income and wealth brackets and that (p.39), "Some portion of bequests, especially in lower income brackets, is not due to a pure bequest motive but rather to a precautionary motive reflecting uncertainty about the length of life, although it is not possible at present to pinpoint the size of this component." Several other authors have suggested that the precautionary motive may contribute importantly to wealth accumulation. For example, Zeldes [1989, p.289] has conjectured that, "...a significant fraction of the capital accumulation that occurs in the United States may be due to precautionary savings." Skinner [1988] and Caballero [1990] contain similar suggestions.

The results of this paper suggest that the contribution of uninsured idiosyncratic risk to aggregate savings is quite modest, at least for moderate and empirically plausible values of risk aversion, variability and persistence in earnings. The aggregate saving rate is higher by no more than 3 percentage points. However, for sufficiently high variability and persistence in earnings the aggregate saving rate could be higher by as much as 7-14 percentage points.

We should emphasize that the focus of this paper is on idiosyncratic

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[3]See Kotlikoff and Summers [1981], Kotlikoff [1988] and Modigliani [1988].

shocks. It is important to distinguish between risk that is due to aggregate shocks (and hence undiversifiable) and risk that is due to idiosyncratic shocks (which is diversifiable if complete insurance markets existed) because the assumption that consumers face a constant (in a steady state) interest rate is justified only when the shocks are purely idiosyncratic (and uninsured) since in that case there is no aggregate uncertainty. Therefore, the specification of the earnings process consumers face should only capture the idiosyncratic component of the uncertainty and not include the common (aggregate) component since this involves aggregate uncertainty and will be reflected in stochastic interest rates. Further, the empirically reasonable requirement that cross-section distributions of earnings, wealth, income, consumption, etc. (normalized by the respective per capita values) be stationary requires that the specification of the individual earnings process be trend stationary[4].

There are several additional distinguishing features of our exercise relative to the many analyses of precautionary saving in the literature. These are: (i) endogenous heterogeneity, (ii) aggregation, (iii) infinite horizons, (iv) borrowing constraint, and (v) general equilibrium, i.e., endogenously determined interest rate. For a given interest rate optimal individual saving behavior leads to a distribution of agents with different levels of assets reflecting different histories of labor endowment shocks. Aggregation implies some level of per capita assets. In a steady state

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[4]Since the shocks are purely idiosyncratic (by assumption), per capita earnings will be growing deterministically. Therefore, the existence of a stationary cross-section distribution for earnings normalized by per capita earnings implies that the individual earnings process must be trend stationary. Some authors (eg., Caballero 1990, Deaton 1991, Section 2.1) have used difference stationary earnings processes which have the empirically unattractive implication that cross-section distributions (normalized by per capita values) get more and more dispersed over time and stationary distributions do not exist.

equilibrium the per capita amount of capital must equal the per capita asset holdings of consumers and the interest rate must equal the net marginal product of capital (as determined by a standard neoclassical production function). These features in combination explain why the interest rate is necessarily less than the time preference rate and, hence, the aggregate capital stock and the saving rate are necessarily greater than under certainty (equivalently, complete markets). In particular, this is true regardless of the convexity of the marginal utility of consumption which has been the traditional criterion for generating precautionary saving.

Most analyses of precautionary saving are done in a single agent model with an exogenously specified interest rate. Consequently, the endogenous heterogeneity and aggregation issues are not addressed. The focus on the convexity of marginal utility in the traditional literature is entirely due to the focus on single agent problems who typically have a two period horizon and do not face a binding borrowing constraint. The convexity of marginal utility becomes unnecessary once features (i) - (v), especially (iii) and (iv), are taken into account. This is explained in Section III.

We wish to emphasize the point that general equilibrium effects may not be just second order considerations. As we will show, they can be very important in understanding why idiosyncratic shocks and liquidity constraints can fail to generate significant increases in aggregate saving.

Some additional implications of our analysis are as follows. In contrast to representative agent models (see Cochrane 1989), it turns out that access to asset markets is quite important in enabling consumers to smooth out earnings fluctuations. In one example, by optimally accumulating and decumulating assets, an individual can cut consumption variability by about half and enjoy a welfare gain of about 14 percent of per capita consumption, or about 8 percent of per capita GNP, compared to a situation

in which he had no access to asset markets[5].

The model is also consistent, at least qualitatively, with certain features of income and wealth distributions. The distributions are positively skewed (median < mean), the wealth distribution is much more dispersed than the income distribution, and inequality as measured by the Gini coefficient is significantly higher for wealth than for income.

The rest of this paper is organised as follows. In Section II we review the relevant empirical and theoretical-quantitative literature which suggests that the precautionary motive and liquidity constraints may be important for a variety of phenomena. In Section III we offer an exposition of models with uninsured idiosyncratic risk and liquidity constraints. In Section IV we describe the specification and parameterization and the computational procedure. Section V contains the results and Section VI concludes with some suggestions for further work. The Appendix contains several propositions and proofs.

II. Precautionary Motive and Liquidity Constraints

There is a considerable literature which emphasizes precautionary savings and liquidity/borrowing constraints for understanding household

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[5]The above calculation requires a complete model since good data on consumption at the individual level are not available. Otherwise, one could use the consumption data to get an idea of consumption variability at the individual level and combine it with some specification of the utility function to obtain an estimate for the welfare gain as in Lucas [1985]. Since data on earnings is available, this can be used together with a complete model to estimate how much individual consumption varies in a stochastic steady state equilibrium. The present model seems much more appropriate for addressing this type of a question than a representative agent model because in a representative agent model agents face only aggregate uncertainty, which seems quite unrealistic. The representative agent model may be a useful abstraction for other questions but not for this one.

consumption/saving behavior as well as a variety of aggregate phenomena[6]. The behavior of individual consumptions, wealths and portfolios are at considerable variance with the predictions of complete markets models[7].

Casual empiricism as well as formal evidence indicates that individual consumptions are much more variable than aggregate consumption (Barsky, Mankiw and Zeldes 1986, Deaton 1991). Further, individual consumptions are not very highly correlated either with each other or with aggregate consumption as would be the case with complete frictionless Arrow-Debreu markets. This suggests that heterogeneity due to incomplete markets may be important. Heterogeneity is clearly necessary for studying the importance of borrowing constraints.

Further and more detailed evidence for the importance of precautionary saving is described by Carroll [1991] and summarized below. Individual wealth holdings appear to be highly volatile with large fractions of households moving from one wealth decile to another over a few years[8]. It would be hard to explain such mobility across the wealth distribution over a fairly short period of time (suggesting that age and life cycle related factors are not the reasons) in the absence of temporary idiosyncratic shocks. Avery, Elliehausen and Canner [1984] present evidence to the effect

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[6]A partial list of such work includes: Aiyagari [1993b], Diaz-Jimenez and Prescott [1991], Aiyagari and Gertler [1991], Carroll [1991], Deaton [1991], Caballero [1990], Huggett [1990], Zeldes [1989a,b], Imrohoroglu [1989, 1991], Kimball and Mankiw [1989], and Scheinkman and Weiss [1986].

[7]See Aiyagari [1993a, pp. 22-24] for a lengthier discussion of the predictions of models with complete frictionless markets and their empirical shortcomings.

[8]According to Avery and Kennickell [1989], 60 per cent of households were in a different wealth decile in 1985 than in 1982. Approximately 30 per cent moved up and 30 per cent moved down. Only people in the topmost and the bottommost deciles were more likely to stay put than move to another decile.

that the ratio of median wealth to median income is higher for individuals in occupations with greater income uncertainty, eg., farming and self-employed businessman.

The evidence on portfolios indicates considerable diversity in portfolio compositions for households with different wealth levels. Mankiw and Zeldes [1991] present evidence that only about 25 percent of U. S. households own any stocks in spite of the fact that the expected return on stocks has been so much higher than the risk-free rate. According to evidence presented by Avery, Elliehausen and Kennickell [1988], the ownership of stocks is highly concentrated at the top end of the wealth distribution whereas the ownership of liquid assets is concentrated in the lower portion of the wealth distribution[9]. The portfolios of households with low wealth contain a disproportionately large share of low return risk-free assets and a disproportionately small share of high return risky assets. The portfolios of high wealth households exhibit the opposite characteristic[10]. Such wide disparities in portfolio compositions would be hard to explain under complete frictionless markets assuming individuals have roughly constant and equal relative risk aversion coefficients. Lastly, it would be

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[9]For example, the top one per cent of wealth holders own about sixty per cent of all equity but only about ten per cent of all liquid assets. In contrast, the bottom ninety per cent of households own about 53 per cent of all liquid assets and only about nine per cent of all equity. Greenwood [1983] presents similar evidence to the effect that the top five per cent of wealth holders own about 85 per cent of all corporate stock and about 60 per cent of all debt instruments (Table 4, p. 35 and Figure 2, p.34).

[10]Kessler and Wolff [1991] calculate that the lowest wealth quintile's portfolio contains over 80 per cent of liquid assets (currency, demand deposits and time deposits), only about 9 per cent of financial securities and corporate stock, and only about 3 per cent of other real estate (i.e. not including housing) and unincorporated business. In contrast, the highest wealth quintile's portfolio contains only about 15 per cent of liquid assets, about 22 per cent of financial securities and corporate stock, and over 42 per cent of other real estate and unincorporated business (Table 6, p.263). Similar evidence is presented in Mankiw and Zeldes [1991].

hard to reconcile the vast amount of trading in asset markets and the pattern of transaction velocities across assets with a complete frictionless markets story.

The above facts constitute quite strong *a priori* evidence in favor of the importance of uninsured idiosyncratic risk.

In the next section we provide an intuitive overview of the workings of general equilibrium dynamic economies with heterogeneous agents, uninsured idiosyncratic shocks and borrowing constraints.

III. Economies With Heterogeneous Agents, Uninsured Idiosyncratic Shocks and Borrowing Constraints: An Exposition

The chief ingredient in this class of models is the "income fluctuation problem"[11]. In this problem a single individual facing uncertain earnings and a constant return on assets makes consumption and asset accumulation/decumulation decisions optimally in order to maximize the expected value of the discounted sum of one-period utilities of consumption. The individual may be permitted to borrow (hold negative assets) up to some limit. Under some conditions this is a well-defined problem and gives rise to unique decision rules and a unique long-run distribution of asset holdings, and, hence, a unique long-run average asset holdings.

The solution of this problem can be turned into a stochastic steady state of a general equilibrium dynamic capital accumulation model in the following way. Imagine that there is a continuum of individuals (of size unity) subject to idiosyncratic earnings uncertainty and among whom asset holdings are distributed according to the long-run distribution mentioned

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[11]See, Shechtman and Escudero [1977]. The ensuing exposition is based on results from this paper and from Clarida [1987, 1990] and Bewley [undated, 1986]. See also Laitner [1992].

above. By construction, then, the cross-section distribution of assets will be constant over time even though individual asset holdings vary stochastically over time. Further, the long-run average asset holdings for an individual will equal the constant per-capita assets of the population.

We now introduce a neoclassical aggregate production function into this economy in which per-capita output depends on per-capita capital (the only outside asset) and per-capita labor supply. Idiosyncratic earnings uncertainty is generated by assuming that individual labor supplies are randomly inelastic and independent across agents. Due to the idiosyncratic nature of the labor supply shocks, the expected value of labor supply for an individual equals the per-capita labor supply. Therefore, per-capita labor supply is constant and may be normalized to unity[12]. Individual earnings are then given by the wage (which equals the marginal product of labor) times individual labor supply. Lastly, the return on assets faced by individuals must equal the net marginal product of capital and the per-capita amount of capital must equal per-capita asset holdings.

In the absence of earnings uncertainty (equivalently, with full insurance markets) all agents are alike and face no uncertainty. The model collapses to the representative agent Brock-Mirman [1972] model of capital accumulation whose steady state is characterized by an interest rate equal to the rate of time preference and per-capita capital given by the modified golden rule. However, with idiosyncratic earnings uncertainty and no insurance markets the combination of the precautionary motive and limited borrowing leads to an interest rate lower than the rate of time preference

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[12]An equivalent description is to imagine that individual labor supplies are inelastic at unity but that individual productivities are idiosyncratically random and that average productivity is normalized to unity.

and, therefore, to a per-capita capital higher than the modified golden rule capital[13]. Aggregate saving and the saving rate are higher.

To see these points more clearly, we start by describing the income fluctuation problem and some properties of its solution.

The Individual's Problem

For simplicity we assume that labor endowment shocks (equivalently, earnings) are i.i.d. over time[14]. We also permit some borrowing[15]. Let c_t, a_t, and l_t denote period t consumption, assets and the labor endowment. Let $U(c)$ be the period utility function, β be the utility discount factor with $\lambda = (1-\beta)/\beta > 0$ being the time preference rate, r be the return on assets, and w be the wage. The individual's problem is to maximize

(1a) $\qquad E_0 \left\{ \sum_{t=0}^{\infty} \beta^t U(c_t) \right\}$ subject to:

(1b) $\qquad c_t + a_{t+1} = wl_t + (1+r)a_t;\ c_t \geq 0,\ a_t \geq -b,$ almost surely (a.s.),

where, b (if positive) is the limit on borrowing and l_t is assumed to be i.i.d with bounded support given by $[l_{min}, l_{max}]$, with $l_{min} > 0$.

Some discussion of the borrowing constraint seems appropriate here. Clearly, if $r < 0$, some limit on borrowing is required; otherwise the

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[13]Throughout this paper we abstract from technical progress and aggregate growth. It is straightforward to incorporate this and is indicated later.

[14]In our quantitative analysis we permit the labor endowment shock to be serially correlated so that anticipation effects will be present.

[15]The purpose of permitting borrowing is to emphasize the point that it only serves to reduce aggregate saving and the saving rate. That is, the impact on the saving rate would be even less if borrowing were to be allowed.

problem is not well-posed and a maximum does not exist. The present value of earnings is infinite (a.s.) and nothing prevents the individual from running a Ponzi scheme. If $r > 0$, then a less restrictive alternative to imposing a borrowing limit is to impose present value budget balance (a.s.). This is equivalent to requiring $\lim_t a_t/(1+r)^t \geq 0$ (a.s.). In turn, this limit condition together with the nonnegativity of consumption is equivalent to the period by period borrowing constraint $a_t \geq -wl_{min}/r$ (a.s.)[16]. Consequently, if b exceeds wl_{min}/r, then the borrowing limit b will never be binding and b may be replaced by the smaller amount wl_{min}/r. Therefore, without loss of generality we may specify the limit on borrowing as

(2a) $\qquad a_t \geq -\phi$

(2b) $\qquad \phi = \min \{b, wl_{min}/r\}$, for $r > 0$, $\phi = b$, for $r \leq 0$.

If the limit on borrowing b is more stringent than wl_{min}/r (for instance, if b is zero so that no borrowing is permitted), then the borrowing limit b may be regarded as ad hoc in the sense that it is not a consequence of present value budget balance and nonnegativity of consumption. Note that ϕ is to be regarded as a function of b, w and r. The above form of the borrowing constraint will be referred to as a "fixed" borrowing limit. Figure IIa shows the typical shape of the borrowing limit as a function of r (curve marked ϕ).

We now define $\hat{a}_t$ and z_t.

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[16]See proposition 1 in the Appendix. Since $r > 0$, it is easy to see that the borrowing constraint implies the limit condition. To see the reverse, note that wl_{min}/r is the maximum amount that the consumer can repay for sure without violating nonnegativity of consumption.

(3a) $\hat{a}_t = a_t + \phi,$

(3b) $z_t = wl_t + (1+r)\hat{a}_t - r\phi,$

z_t may be thought of as the total resources of the agent at date t. Using (2) and (3) we can rewrite (1b) as follows.

(4a) $c_t + \hat{a}_{t+1} = z_t,$ $c_t \geq 0,$ $\hat{a}_t \geq 0,$

(4b) $z_{t+1} = wl_{t+1} + (1+r)\hat{a}_{t+1} - r\phi.$

Let $V(z_t, b, w, r)$ be the optimal value function for the agent with total resources z_t. This function is the unique solution to the following Bellman's equation[17].

(5) $V(z_t, b, w, r) \equiv \max \left\{ U(z_t - \hat{a}_{t+1}) + \beta \int V(z_{t+1}, b, w, r) dF(l_{t+1}) \right\}$

where the maximization on the right side is over $\hat{a}_{t+1}$ subject to (4).

The optimal asset demand rule for an agent is obtained by solving the maximization on the right side of (5). This yields the following single-valued and continuous asset demand function.

(6) $\hat{a}_{t+1} = A(z_t, b, w, r);$ (asset demand function)

Substituting (6) into (4b) we obtain the transition law for total resources z_t.

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[17]We assume that the utility function U(.) is bounded, continuously differentiable, strictly increasing and strictly concave. Then, the value function V(.) is well-defined and is also continuously differentiable, strictly increasing and strictly concave. The differentiability of the value function is established in proposition 2 in the Appendix which also characterizes the solution to the maximization problem in (5).

(7) $z_{t+1} = wl_{t+1} + (1+r)A(z_t,b,w,r) - r\phi.$

In Figures Ia and Ib we show some typical shapes for the functions on the right sides of (6) and (7), under the assumption that the interest rate r is less than the rate of time preference λ. Clearly, the agent would like to borrow but is limited by the borrowing limit. As total resources get smaller and smaller the individual borrows more and more in order to maintain current consumption and his debt approaches the borrowing limit. At some point when total resources are too low it would be optimal to borrow upto the limit and consume all of total resources. Thus, there exists a positive value $\hat{z} > z_{min} \equiv wl_{min} - r\phi \geq 0$, such that whenever $z_t \leq \hat{z}$, it is optimal to consume all of total resources (i.e., set $c_t = z_t$) and set $\hat{a}_{t+1}$ to its lowest permissible value which is zero (see proposition 3 in Aiyagari 1993a). That is, it is optimal to exhaust the borrowing limit. For $z_t \geq \hat{z}$, both c_t and $\hat{a}_{t+1}$ are strictly increasing in z_t, i.e., A(.) is strictly increasing with a slope less than unity. In this situation the borrowing limit is not currently binding[18].

Under some additional assumptions, the support of the Markov process defined by (7) is bounded; specifically there is a z^* such that for all $z_t \geq z^*$, $z_{t+1} \leq z_t$ with probability one (see Figure Ib)[19]. These conditions also

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[18]The excess sensitivity of consumption to a transitory earnings innovation is apparent here. In the liquidity constrained region consumption responds one to one even to transitory earnings shocks. Note that in this i.i.d. case given current consumption other currently known variables will not improve the forecast of future consumption. (This will not be true when the earnings shocks are serially correlated.) Thus, tests such as Hall's [1978] do not necessarily throw light on whether liquidity constraints are or are not important.

[19]See proposition 4 in the Appendix. The key condition here is that the relative risk aversion coefficient should be bounded . This condition is violated by, for example, negative exponential utility in which case there

guarantee that there exists a unique, stable stationary distribution for
$\{z_t\}$ which behaves continuously with respect to the parameters b, w, and r
(see proposition 5 in the Appendix). Let Ea_w (the subscript reflecting the
fact that for now w is being treated as fixed) denote long run average
assets. Using (3a) and (6) this is given by

(8) $Ea_w = E\{A(z,b,w,r)\} - \phi,$

where $E\{.\}$ denotes expectation with respect to the stationary distribution
of z.

Endogenous Heterogeneity and Aggregation

   The distribution of $\{z_t\}$ and the value of Ea_w reflect the endogenous
heterogeneity and the aggregation features mentioned in the introduction.
Ea_w represents the aggregate assets of the population consistent with the
distribution of assets across the population implied by individual optimal
saving behavior. In Figure IIa we show a typical shape of the graph of Ea_w
versus r.

   The most important feature of this graph is that Ea_w tends to infinity
as r approaches the rate of time preference λ from below[20]. This reflects the
infinite horizon of consumers. If r equals or exceeds the rate of time
preference then the individual will accumulate an infinitely large amount of

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exist values of r below λ and a probability distribution for $\{l_t\}$ with
bounded support such that the consumer's assets will wander off to infinity
a.s. (see Schechtman and Escudero 1977, pp. 159-161).

   [20]See Bewley [undated, Figure 1, p.4], Clarida [1990, proposition 2.4,
p.548]. Ea_w is a continuous function of r (and also of b and w) but need not
be monotone in r.

assets and Ea_w may be thought to be infinity. Intuitively, if r exceeds λ then the individual wants to postpone consumption to the future and be a lender. The consumption profile will be upward sloping and the agent will accumulate an infinitely large amount of assets to finance an infinitely large amount of consumption in the distant future[21]. This conclusion carries over to the borderline case of r equal to λ. In this case the consumer attempts to maintain a smooth marginal utility of consumption profile. At the margin it is costless for the consumer to acquire an additional unit of the asset. However, since there is a positive probability of getting a sufficiently long string of bad draws of labor shocks, maintaining a smooth marginal utility of consumption profile is only possible if the consumer has an arbitrarily large amount of assets to buffer the shocks[22].

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[21]The first order necessary condition (the Euler equation) for individual optimization is: $U'(c_t) \geq \beta(1+r)E_t[U'(c_{t+1})]$ with equality if $\hat{a}_{t+1} > 0$. Therefore, on average marginal utility is decreasing at least geometrically over time and will converge to zero (a.s.). More formally, $\beta^t(1+r)^t U'(c_t)$ is a nonnegative super-martingale and converges a.s. to a finite random variable. Since $\beta(1+r) > 1$, it follows that U' must converge to zero a.s. and, hence, c_t must converge to ∞ a.s. It follows that z_t and $\hat{a}_t$ also converge to infinity, a.s. The conclusion holds also for the borderline case of $\beta(1+r) = 1$. See, Chamberlain and Wilson [1984] for the details of the arguments.

[22]It is easy to see that there cannot be a fixed point in Figure Ib corresponding to $l_{t+1} = l_{max}$ when $\beta(1+r)$ is unity. If there is such a finite fixed point (denoted z_{max}) then we have (combining the envelope condition with the first order condition for problem 5 and using the strict concavity of the value function): $V'(z_{max}) \geq E_t\{V'(y_{t+1}+(1+r)A(z_{max}))\} > V'(y_{max}+(1+r)A(z_{max})) = V'(z_{max})$ which is a contradiction. Consequently, the support of the distribution of $\{z_t\}$ is unbounded. In fact, z_t, a_t, and c_t all go to infinity a.s. Thus, there is a crucial difference between the solutions to the individual problem with and without uncertainty when $\beta(1+r)$ equals unity.

Next, note that for values of $r < \lambda$, Ea_w is always higher under uncertainty than if earnings were certain, atleast as long as r is not too much below λ, i.e., assets are not too costly to hold. This result is independent of whether U' is convex or concave and arises due to the borrowing constraint and the infinite horizon. In a single consumer problem with a two period horizon there is no heterogeneity and the borrowing constraint may be ignored by making suitable assumptions about the time profile of earnings. However, with an infinite horizon, repeated shocks, and $r < \lambda$ neither the hetrogeneity nor the borrowing constraint can be ignored.

If earnings were certain (or, equivalently, markets complete) Ea_w would equal $(-\phi)$ for all $r < \lambda$. That is, per capita assets under certainty are at their lowest permissible level since all agents are alike and everyone is constrained. However, in a steady state under incomplete markets there is a distribution of agents with different total resources reflecting different histories of labor endowment shocks. Those with low total resources will continue to be liquidity constrained whereas those with high total resources will accumulate assets beyond the constrained level regardless of the convexity of marginal utility simply because their current total resources are quite high relative to average future total resources[23]. Aggregation then implies that per capita assets must necessarily exceed its level under certainty[24].

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[23]It should be noted that for $r \in (0,\lambda)$ such that $wl_{min}/r < b$, the borrowing constraint under incomplete markets is more stringent than under complete markets since under complete markets the borrowing limit would be min $\{b,w/r\}$ since $E\{l\} = 1$. This, in itself, makes per capita assets potentially higher under incomplete markets.

[24]As can be seen in Figure IIa, if r is too low then even under uncertainty everyone continues to be liquidity constrained and per capita assets are the same as in the certainty case. This will happen for values of r which satisfy: $U'(wl_{max}-r\phi) \geq \beta(1+r)E_t\{U'(wl_{t+1}-r\phi)\}$. A related question

General Equilibrium

The crucial features which explain how uninsured idiosyncratic shocks and borrowing constraints lead to higher aggregate saving are that Ea_w is finite only if r is less than λ and that it tends to infinity as r approaches λ from below. To see this, let $f(k,1)$ denote per capita output as a function of per capita capital (k) and per capita labor (which equals unity) and let δ be the depreciation rate of capital. Now consider the curve labeled $K(r)$ in Figure IIb. This is a graph of k versus r defined by the marginal condition arising from producer profit maximization, $r = f_1(k,1)-\delta$, which must hold in a steady state of this economy. Under standard assumptions, this curve is downward sloping, tends to ∞ as r tends to $(-\delta)$ and tends to zero as r tends to ∞. In addition, we can express the wage w as a function of r since w equals $f_2(k,1)$. Denote this by $\omega(r)$ which is a decreasing function under standard assumptions, tends to zero as r tends to ∞, and tends to ∞ as r tends to $(-\delta)$. For a given r, let Ea denote the value of Ea_w when w equals $\omega(r)$ and let NI (for no insurance) be the graph of $Ea(r)$ versus r[25]. A steady state of this economy is then characterized by the

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is what happens to Ea_w if uncertainty is increased. Sibley [1975] and Miller [1976] show that if U' is convex then a mean preserving spread in the distribution of $\{l_t\}$ will lower the consumption function, equivalently, raise the asset demand function (6). (In their papers the borrowing constraint is of the form: $a_t \geq -wl_{min}/r$.). That is, the consumer will consume less and save more for each level of total resources z_t. (This may be thought of as the infinite horizon analogue of the standard two period analyses of precautionary saving.). However, whether this will increase per capita assets or decrease it is hard to say since a shift in the asset demand function changes the stationary distribution of total resources and, thereby per capita assets in a complicated way. The quantitative results reported in Section V always indicated that an increase in the variability of $\{l_t\}$ shifted the Ea_w curve to the right.

[25]This curve also has the same general shape as the Ea_w curve in Figure

condition $K(r) = Ea(r)$. This is shown in Figure IIb by the intersection point (labeled e^n) of these two curves[26]. Intuitively, one may think of $K(r)$ as the capital desired by firms at the interest rate r, and $Ea(r)$ as the capital supplied by households at the interest rate r.

Now consider what the steady state of the economy would be if there were no uncertainty, or, equivalently, there were full insurance markets. Then the economy consists of a representative agent who receives the constant earnings w in each period. If r is less than λ the agent would always be up against his borrowing limit and his asset holdings would be $(-\phi)$. If r equals λ his asset holdings equal his initial holdings whatever they may be. If r exceeds λ then he would accumulate an infinitely large amount of assets. Therefore, the right angled line labeled FI (for full insurance) consisting of the horizontal segment at the height λ and the vertical segment corresponding to the binding borrowing constraint represents the individual's desired asset holdings as a function of r. The steady state of this full insurance economy is at the point e^f.

It is clear from the preceding argument that the aggregate capital stock is higher and the interest rate is lower in the economy with uninsured idiosyncratic shocks and borrowing constraints as compared to the standard

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IIa in which w was treated as a separate parameter, instead of as a function of r. It is also continuous and tends to ∞ as r tends to λ from below. It need not be monotonic. To understand this, suppose the borrowing limit is zero, r is fixed and the utility function is iso-elastic. Then it is easy to see that an increase in w leads to a proportional increase in $\hat{a}$, z, c, and a. That is, the Ea_w curve shifts to the right with an increase in w. However, since $\omega(r)$ is decreasing in r, it is quite possible that the NI curve is non-monotonic in r.

[26]Note that since the NI curve need not be monotonic there is no guarantee that the steady state is unique.

economy[27]. The saving rate which is given by $\delta k/f(k,1)$ is also higher[28].

The above analysis also explains why general equilibrium considerations in combination with the shape of the $Ea(r)$ curve can play an important role in limiting the impact of idiosyncratic risk and liquidity constraints on aggregate saving. Since Ea approaches infinity as r approaches λ from below average household assets are extremely sensitive to slight variations in the interest rate when it is close to (but below) the rate of time preference. In a partial equilibrium analysis of a single household r is chosen arbitrarily and if one chooses an interest rate close enough to the rate of time preference one can generate arbitrarily large precautionary saving in excess of the certainty case. For instance, Zeldes [1989] considers a finite horizon partial equilibrium model and assumes that r equals λ. If the horizon is long enough then the fact that r equals λ will imply a substantial accumulation of assets. However, a general equilibrium analysis does not permit r to be chosen arbitrarily. Instead r is determined endogenously as described earlier and is always less than λ. How much r is

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[27]Deterministic growth in the aggregates can easily be accommodated in the same way as it can be in the standard growth model assuming that the utility function is isoelastic (see section 5.4, pp. 105-107 in Stokey and Lucas with Prescott 1989) . Assume that due to labor augmenting technical progress, effective per capita labor supply, and, hence, the wage, grow as $(1+g)^t$. Assume that the borrowing limit also grows as $(1+g)^t$ and let μ be the elasticity of marginal utility. Then the steady state interest rate with complete markets will equal $[(1+\lambda)((1+g)^{\mu} -1]$ whereas with incomplete markets the interest rate would be less than $[(1+\lambda)(1+g)^{\mu} -1]$. The distributions of earnings, total resources, assets, consumption, and income after normalizing by their respective per capita values would be stationary.

[28]This follows since $k/f(k,1)$ is an increasing function of k. It is strictly increasing if the capital share is less than unity. Note that since there is no growth in this economy net saving is zero in the steady state.

reduced relative to the certainty case and, thereby, how much aggregate saving is increased is then a quantitative issue. Our quantitative analysis in the next two sections indicates a modest impact on aggregate saving when r is endogenously determined[29].

We will briefly describe the effects of varying the borrowing limit b. This is related to two other features of the Ea_w curve in Figure IIa. When r equals negative unity (so that the gross return on assets is zero), assets will always equal $(-b)$. When r equals zero, b is not an argument of the asset demand function $A(.)$. Hence, Ea_w decreases one to one with increases in b when r is zero. These two features suggest that the Ea_w curve shifts to the left when b increases. Therefore, permitting a higher borrowing limit serves to lower aggregate capital and raise the interest rate towards the time preference rate (see Figure IIb). The intuition behind this conclusion is that when borrowing is permitted individuals need not rely solely on holdings of capital to buffer earnings variation. Borrowing can also be used to buffer these shocks and, hence, leads to smaller holdings of capital[30].

It follows from the previous remarks that if uninsured idiosyncratic risk and no borrowing $(b = 0)$ lead to a small increase in aggregate capital relative to the certainty case, then permitting some borrowing $(b > 0)$ will lead to an even smaller increase in aggregate capital.

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[29]Clearly, the elasticity of the $K(r)$ curve plays an important role in determining whether the impact of uninsured idiosyncratic shocks and borrowing constraints is mainly on the interest rate or on aggregate saving. However, it should be kept in mind that the $K(r)$ and $w(r)$ functions are related through the production function. Therefore, the $K(r)$ and $Ea(r)$ functions are not independent of each other.

[30]Note that the effects of an increase in b occur only in the range of interest rates for which $wl_{min}/r > b$. If $wl_{min}/\lambda < b$, then there is a range of values of r below and near λ for which marginal changes in b do not affect Ea_w or the $Ea(r)$ curves. Therefore, if the steady state is in this range then marginal changes in b have no effect on the steady state.

We briefly describe the effects of setting $a_t \geq -\phi^* = -wl_{min}/r$ and restricting r to be positive. As noted earlier this is the appropriate form of the borrowing constraint implied by the present value budget balance and nonnegativity of consumption when $r > 0$, and will be referred to as the "present value" borrowing constraint. If l_{min} is zero, this is equivalent to the case of no borrowing (b = 0). If, however, l_{min} is positive, then Ea_w tends to $(-\infty)$ as r tends to zero (see Figure IIa, curve marked $Ea_w(\phi^*)$). This can be seen by substituting wl_{min}/r for ϕ in equations (3) - (8). The first term on the right side of (8) remains finite whereas the second term tends to $(-\infty)$ as r tends to zero. Intuitively, as r becomes smaller the borrowing limit becomes larger permitting the individual to carry large amounts of debt. That is, the present value of minimum earnings is tending to infinity enabling the individual to service large amounts of debt. The main difference between this case and the case of a fixed borrowing limit is that under the present value borrowing constraint there always exists a steady state with a positive interest rate. With a fixed borrowing limit there may be no steady state with a positive interest rate though there does exist a steady state with a negative interest rate[31].

Some Alternative Interpretations

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[31]This can be seen by referring to Figure IIb. With a fixed borrowing limit b > 0, the steady state interest rate will be negative if Ea is positive when r equals zero and the curve K(r) is such that it intersects the Ea curve at a negative r. (This occurs in one of our numerical examples.). When the borrowing limit is wl_{min}/r there is always a positive interest rate at which Ea is zero, ensuring that there is always a steady state with a positive interest rate (see Figure IIa). Again, note that the Ea_w curves corresponding to a fixed b and corresponding to the variable limit wl_{min}/r may coincide for a range of values of r near and below λ if it happens that $wl_{min}/\lambda < b$. This is because, for values of r in this range the borrowing limit b is not binding (see equation 2).

The model of individual optimization in (1) and (2) can be turned into a pure exchange model with government debt in order to analyze the effects of changing the level of government debt. Let the government have outstanding a constant per capita amount of debt denoted d interest on which is financed by an equal (across agents) lump sum tax τ (= rd). Then the consumer's budget constraint (2) is altered to: $c_t + a_{t+1} = wl_t - rd + (1+r)a_t$. The steady state equilibrium condition is $Ea_w = d$, where Ea_w denotes per capita asset holdings in the steady state.

In this model, whether debt neutrality holds or not depends crucially on how the borrowing constraint is specified. With a fixed borrowing limit as in (2) debt neutrality will not hold. However, with a present value borrowing constraint, i.e., the borrowing limit equals the present value of minimum earnings *adjusted for tax obligations* so that we have $a_t \geq -(wl_{min}-rd)/r$, then debt neutrality does hold. The equilibrium interest rate, the distribution of asset holdings net of government debt, and consumption are invariant to the level of d. This can be seen by using the transformation $a_t^* = a_t - d$ in the above equations. Thus, the validity of debt neutrality in this framework with incomplete markets is entirely dependent on whether the borrowing limit takes account of changing tax obligations.

The model of individual optimization in (1) and (2) can also be turned into an "optimum quantity of money" model as in Bewley [1983] by interpreting a_t as $(m_t-1)/p$, b as $1/p$, and r as the interest paid on money, where m_t is an agent's nominal money holding at the beginning of period t, p is the price level, and the per capita nominal money supply is constant at unity. Note that r/p is the real value of per capita lump sum taxes (equal across agents) levied to finance interest payments on money. The borrowing constraint is equivalent to nonnegativity of money holdings. The steady state equilibrium condition is that per capita asset holdings must be zero;

equivalently, per capita nominal money holdings equal unity. The problem can be posed as finding an equilibrium p for a given r (as Bewley posed it), or as finding an equilibrium r for a given p (as is done here). The latter way of posing the problem makes it easier to see why it is not possible to have monetary equilibria with r being arbitrarily close to the rate of time preference (Bewley 1983).

In Figure IIa, let r# be the interest rate at which average assets are zero corresponding to the Ea_w curve with variable borrowing limit given by wl_{min}/r. This will be a monetary equilibrium with a price level p# = $r\#/(wl_{min})$; equivalently a borrowing limit b# = 1/p#. One cannot support an interest rate higher than r# by lowering the price level (raising real balances, or, equivalently, raising the borrowing limit) since the portion of the curve Ea_w at and above r# is unaffected. This is because when p < p# and r > r#, we have $wl_{min}/r < wl_{min}/r\# = 1/p\# < 1/p$. Therefore, the constraint $a_t \geq -1/p$ will never bind. Raising the price level above p# (equivalently lowering the borrowing limit) only serves to increase Ea_w in a neighborhood of r# and, hence, lowers the interest rate.

In the next section we describe model specification, parameterization and the computation procedure for a version of the capital accumulation model with serially correlated labor endowment shocks. In the section after that we describe the results on the contribution of precautionary saving to aggregate saving and some other results.

IV. Model Specification, Parameterization and Computation

The model period is taken to be one year and the utility discount factor β is chosen to be 0.96. The production function f(.) is assumed to be Cobb-Douglas with the capital share parameter (denoted α) taken to be 0.36. The depreciation rate of capital (δ) is set at 0.08. The period utility

function is of the constant relative risk aversion (CRRA) type, i.e., U(c) = $[c^{1-\mu} - 1]/(1-\mu)$, where μ is the relative risk aversion coefficient. Results are reported for three different values of $\mu \in \{1, 3, 5\}$. The above technology and preference specifications and parameter values are chosen to be consistent with aggregate features of the post war U. S. economy and are commonly employed in aggregative models of growth and business cycles[32].

For the labor endowment shocks we use a Markov chain specification with seven states to match the following first order autoregressive representation for the logarithm of the labor endowment shock (equivalently earnings).

$$(9) \qquad \log(l_t) = \rho\log(l_{t-1}) + \sigma(1-\rho^2)^{1/2}\varepsilon_t, \quad \varepsilon_t \sim Normal(0,1)$$

$$(10) \qquad \sigma \in \{0.2, 0.4\}, \quad \rho \in \{0, 0.3, 0.6, 0.9\}$$

The coefficient of variation equals σ and the serial correlation coefficient equals ρ. We then follow the procedure described in Deaton [1991, p.1232] and Tauchen [1986] to approximate the above autoregression by a seven state Markov chain[33]. Table I at the end reports the σ and ρ values

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[32]See, for example, Prescott [1986].

[33]We divide the real line into seven intervals as follows. $I_1 = (-\infty, -5\sigma/2)$, $I_2 = (-5\sigma/2, -3\sigma/2)$, $I_3 = (-3\sigma/2, -\sigma/2)$, $I_4 = (-\sigma/2, \sigma/2)$, $I_5 = (\sigma/2, 3\sigma/2)$, $I_6 = (3\sigma/2, 5\sigma/2)$, and $I_7 = (5\sigma/2, \infty)$. The state space of $\ln(l_t)$ is taken to be the finite set $\{-3\sigma, -2\sigma, -\sigma, 0, \sigma, 2\sigma, 3\sigma\}$ so that $l_i = \exp[(i-4)\sigma]$, i = 1,2,...,7. We then compute the transition probabilities $\pi_{ij} = prob\{\ln l_{t+1} \in I_j | \ln l_t = \log l_i\}$ by numerical integration using the Normal (0,1) density for ε_t assumed in (4.1). We then compute the stationary probability vector θ associated with the probability transition matrix π and the expected value of the labor endowment $El = \Sigma_i \theta_i l_i$. Per capita labor endowment is normalized

implied by the Markov chain and shows that the approximation is quite good.

The values of σ and ρ were based on the following studies. Kydland [1984] reports that the standard deviation of annual hours worked from PSID data is about 15 per cent. Abowd and Card [1987, 1989] use data from the PSID and NLS and calculate that the standard deviations of per cent changes in real earnings and annual hours are about 40 per cent and 35 per cent, respectively. The implied value for the coefficient of variation (c.v.) in earnings depends on the serial correlation in earnings. If earnings are i.i.d. this yields a figure of 28 per cent for the c.v. of earnings. Positive correlation would lead to a larger figure[34]. The covariances reported in Abowd and Card [1987, Table 3, p.727 and 1989, Tables IV, V, VI, pp.418-422] suggest a first order serial correlation coefficient of about 0.3. This would give a figure of 34 per cent for the c.v. of earnings. Heaton and D. Lucas [1992] also use PSID data to estimate several versions

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to unity by scaling the support of the labor shock distribution by El. That is, we define $1'_i = 1_i/El$. The Markov chain for the labor endowment shock is defined by the state space $\{1'_i\}$ together with the probability transition matrix π. Note that $1'$ will have the same coefficient of variation and serial correlation coefficient as 1. Table 1 at the end shows that the approximation is quite good for moderate values of σ, though for high values of σ the Markov chain had a somewhat higher coefficient of variation. We also tried the following alternative for calculating the transition probabilities: $\pi_{ij} = \text{prob}\{\ln 1_{t+1} \in I_j | \ln 1_t \in I_i\}$. This procedure yielded a very good approximation to σ even for high values. However, its approximation to ρ (especially for the high values) was not so good. The values of ρ based on the Markov chain were somewhat lower.

[34] Let y be the log of earnings, σ_y be the standard deviation (s.d.) of y, and σ_g be the s.d. of $(y_t - y_{t-1})$. Suppose that y follows the first order process: $y_t = \text{trend}_t + \rho y_{t-1} + \varepsilon_t$, where ε is i.i.d. It is straightforward to calculate that $\sigma_y/\sigma_g = [2(1-\rho)]^{-1/2}$.

of equation (9). Their estimates (see their Tables A.2 – A.5) indicate a range of 0.23 to 0.53 for ρ and a range of 0.27 to 0.4 for σ. These studies suggest that a value of σ of 20-40 per cent at an annual rate may be reasonable.

Note that we have made no allowance for the possibility that the reported earnings variabilities contain significant measurement error. As the discussion in the papers by Abowd and Card suggests, this is a serious possibility, and the relevant degree of idiosyncratic earnings variability may be somewhat lower. However, this is balanced by the possibilities that the data do not include uninsured losses and taste shocks. In addition since the agents in the model are infinitely rather than finitely lived a larger value of σ may be needed to capture the relevant degree of variability in permanent income[35].

Lastly, the borrowing limit b is set to zero, i.e., borrowing is prohibited. As explained in the previous section permitting some borrowing would lead to even smaller effects on the aggregate saving rate.

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[35]Suppose that earnings (y_t) follow the process: $y_{t+1} = (1-\rho)y^a + \rho y_t + \sigma(1-\rho^2)^{1/2}\varepsilon_{t+1}$, where ε_t is i.i.d. with zero mean. Let $\gamma = 1/(1+r)$ be the market discount factor and T be the horozon. Then permanent income (y_t^p) is given by: $y_t^p = y^a + (y_t - y^a)[(1-\gamma)/(1-\gamma\rho)][1-(\gamma\rho)^T]/(1-\gamma^T)$. Therefore, the variability of permanent income as measured by the standard deviation (s.d.) is higher when the horizon is finite as compared to when the horizon is infinite. For illustrative purposes if we take r = 0.04, ρ = 0, and T = 50, then the s.d. of permanent income is higher by a factor of 1.16 as compared to the infinite horizon case. Note that higher values of ρ reduce this adjustment factor. This suggests that to capture the effects of the observed variability in earnings in a model with infinitely lived agents the standard deviation of earnings in the model needs to scaled up by a factor of about 1.2. The Markov chain approximation that we use tends to deliver this automatically for the high value of σ (0.4); see Table I at the end.

Computation

We approximate the asset demand as a function of total resources (for each of seven possible current labor endowment shocks) by a continuous, piece-wise linear function over an interval. The minimum value of total resources z_{min} equals $w_{min}l_{min}$, where w_{min} is the wage corresponding to an interest rate equal to the time preference rate[36]. The maximum value of total resources z_{max} is set equal to the maximum possible value of per-capita total resources which equals $f(k_{max},1) + (1-\delta)k_{max}$, where k_{max} is the maximum sustainable capital stock[37]. The interval $[z_{min}, z_{max}]$ is divided into 25 sub-intervals not of equal length. Finer sub-intervals were chosen at the lower end of the interval and coarser sub-intervals at the upper end of the interval[38]. Figure 3 shows the asset demand functions corresponding to l_{min} and l_{max} for a particular set of parameter values.

The algorithm for approximating the steady state uses simulated series and the bisection method. We start with some value of r (say, r_1) close to but less than the rate of time preference (see Figure IIb). We then compute the asset demand function as described above. We then simulate the Markov chain for the labor endowment shock using a random number generator and obtain a series of 10,000 draws[39]. These are used with the asset demand

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[36] Since the equilibrium interest rate is never higher than the rate of time preference, the wage cannot be lower than w_{min}.

[37] k_{max} is the unique positive solution to $f(k,1) = \delta k$.

[38] The reason is that for low levels of total resources assets will be zero since the borrowing constraint will bind. At some critical level of total resources assets will become positive. This introduces a high degree of nonlinearity in the asset demand function. Consequently, it is important to have a finer grid at the lower end of the interval to obtain a good approximation. It turned out that throughout the upper half of the interval the asset demand function was very nearly linear so that a small number of grid points was adequate to obtain a good approximation in this region.

[39] We repeated all the calculations using 20,000 draws and found that the

function to obtain a simulated series of assets. The sample mean of this is taken to be Ea. We then calculate r_2 such that $K(r_2)$ equals Ea. If r_2 exceeds the rate of time preference it is replaced by the rate of time preference. Now note that by construction r_1 and r_2 are on opposite sides of the steady state interest rate r*. Without loss of generality we may suppose that $r_1 < r* < r_2$ (by relabeling, if necessary). We then define $r_3 = (r_1 + r_2)/2$ (this is the bisection part) and calculate Ea corresponding to r_3. If Ea exceeds $K(r_3)$ then r_2 is replaced by r_3 and we use bisection again. If Ea is less than $K(r_3)$ then r_1 is replaced by r_3 and we use bisection again. Typically, this yields an excellent approximation to the steady state within ten iterations. Figure 4 shows the graphs of Ea(r) and K(r) for a particular case.

Once the steady state is approximated we use the solution to calculate the following objects of interest. We calculate the mean, median, standard deviation, coefficient of variation, skewness, and serial correlation coefficient for labor income, asset (capital) holdings, net income, gross income, gross saving, and consumption[40]. These descriptive statistics are based on the simulated series obtained in the manner described before. We also calculate measures of inequality for each of these variables. We use the simulated series for each variable to construct its distribution and then we compute the Lorenz curves and calculate the associated Gini coefficients.

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changes in the results were very minor.

[40]The skewness measure is (mean-median)/standard deviation. This is 1/3 of Pearson's second coefficient of skewness. For a log-normal distribution with standard deviation σ the skewness measure is approximately $\sigma/2$. Net income is defined as $wl_t + ra_t$. Gross income is net income plus depreciation which is δa_t. Gross saving is gross income minus consumption.

V. RESULTS

Aggregate Saving

In Tables IIA and IIB we present the net return to capital in percent (before the /) and the saving rates in percent (following the /) for σ (coefficient of variation of earnings) equal to 0.2 and 0.4, and various values of ρ (serial correlation in earnings) and μ (the relative risk aversion coefficient)[41]. For comparison it is easy to calculate that the full insurance net return to capital is 4.17 percent and the saving rate is 23.67 percent[42].

The main point to note is that the differences between the saving rates with and without insurance are quite small for moderate and empirically plausible values of σ, ρ and μ. However, for high values of σ, ρ and μ the presence of idiosyncratic risk can raise the saving rate quite significantly by up to 7 percentage points. The extreme case when σ equals 0.4, ρ equals 0.9 and μ equals 5 leads to a considerable increase in the saving rate of almost 14 percentage points.

These results may be related to the concepts of relative prudence (RP) and equivalent precautionary premium (EPP) developed by Kimball [1990]. For the CRRA preferences used here RP equals $(\mu+1)$ and EPP equals $RP(\sigma_c)^2/2$

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[41]The σ and ρ values reported in the Tables are the ones used in computing the Markov chain approximation to the labor endowment shock – not the values of σ and ρ implied by the Markov chain approximation. These are described in Table I. Note that for high values of σ, the Markov chain based value of σ is higher and hence indicates even greater earnings variability than is indicated in Tables II.

[42]Recall that the production function is Cobb-Douglas with α being the capital share parameter. The saving rate equals $\delta k/f(k,1)$ which may be written as $\delta[kf_1/f]/f_1$ which equals $\delta\alpha/(r+\delta)$. With full insurance, r equals λ.

where σ_c is the coefficient of variation of consumption. For a given value of r increases in earnings variability (σ) or persistence (ρ) shift the Ea curve in Figure IIB to the right and also increase σ_c and, hence, the EPP. As can be seen from Table II, the equilibrium interest rate falls and the saving rate goes up. An increase in μ also shifts the Ea curve to the right and directly increases RP and the EPP. Again, from Table II, the equilibrium interest rate falls and the saving rate goes up[43].

Some studies (eg., Caballero 1990, Deaton 1991, Section 2.1) use earnings processes that are difference stationary instead of being trend stationary. This may be approximated by making ρ approach unity and simultaneously letting σ approach infinity in such a way as to keep $\sigma[2(1-\rho]^{1/2}$ fixed and positive (see note 33). Table II suggests that this would depress the return to capital and increase the saving rate enormously. However, as noted in the introduction the limiting model is not well behaved.

Variabilities

The results for the variabilities (measured by the coefficient of variation) of consumption, income (net and gross), gross saving and assets are in Tables IIIA and IIIB at the end. The main points to note are the following. Consumption varies about 50-70 percent as much as income. Saving and assets are much more volatile than income. Saving varies about three times as much as income and assets vary about twice as much as income. Risk

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[43]As noted in the introduction and explained in Section III, infinite horizons and binding borrowing constraints play important roles in our results. Further the Ea curve represents the result of aggregating the saving behavior of a large number of consumers. Kimball's concepts and results are developed for a single consumer with a two period horizon and ignoring borrowing constraints.

aversion tends to reduce the variabilities of all these variables. Variability in earnings (σ) has a relatively smaller effect on the variability of consumption and relatively larger effects on the variabilities of other variables. Consumption variability rises with persistence in earnings and falls with risk aversion. Variability of consumption relative to income behaves similarly.

Importance of Asset Trading

An approximate expression for the welfare loss from consumption variability, measured as the percentage of average consumption the consumer is willing to give up, is given by $\mu\sigma_c^2/2$, where μ is the relative risk aversion coefficient and σ_c is the coefficient of variation of consumption. In contrast to representative agent models (see Cochrane 1989) the results here imply that consumers are able to accomplish a significant amount of consumption smoothing by accumulating and decumulating assets and, hence, enjoy significant welfare benefits from participating in asset markets. To see this consider how variable consumption would be if an individual could not trade in asset markets. Suppose that a consumer held a fixed quanitity of assets equal to the per capita amount and consumed his earnings plus the return on the assets. In this case σ_c would be given by $\sigma/[1+\alpha r/\{(1-\alpha)(r+\delta)\}]$[44]. If, as an example, we take $\mu = 3$, $\sigma = 0.4$, and $\rho = 0.6$ then σ_c with a fixed amount of assets equals 0.35. Actual consumption variability from Table IIIB at the end is 0.17. Thus by optimally accumulating and depleting assets consumption variability is cut in half

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[44]This may be derived as follows. With fixed assets equal to k individual consumption $c_t = rk+wl_t$. Hence, $\sigma_c = \sigma/[1+rk/w]$. Now note that, $k/w = (kf_1/f)/\{f_1(w/f)\} = \alpha/\{(r+\delta)(1-\alpha)\}$, since the production function is Cobb-Douglas.

yielding a welfare benefit of about 14 percent of per capita consumption, or about 8 percent of per capita GNP.

Cross Section Distributions and Inequality Measures

Since long run distributions for an individual coincide with cross-section distributions for the population, results for variabilites of individual consumption, income, and assets have immediate implicatons for cross-section distributions. These results are qualitatively consistent with casual empiricism and more careful empirical observation; there is much less dispersion across households in consumption compared to income and much greater dispersion in wealth compared to income (see Tables IIIA and IIIB, and Figures 5A and 5B for an example). In all cases, the fraction of liquidity constrained households is close to zero[45]. Skewness coefficients reveal another aspect of inequality. All of the cross-section distributions are positively skewed (median < mean)[46]. However the degree of skewness is somewhat less than in the data. For example, median net income, gross income and capital are all over 90 percent of their respective mean values. In contrast, U. S. median household income is about 80 percent of U. S. mean household income[47]. Lorenz curves and Gini coefficients (see Figure 6 for an

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[45]Of course, it is the possibility of being constrained (and its utility cost to the household) that affects behavior and leads to this result.

[46]Note that the labor shock distribution is positively skewed since it is an approximation of a log-Normal distribution. However, we found that even when the labor shock distribution is symmetric the mechanics of the model naturally generate positive skewness (median < mean) in the wealth distribution, even though the income and consumption distributions were roughly symmetric.

[47]This figure is for 1985 from the *Statistical Abstract of the United States 1988*, using numbers for median and mean household incomes from Tables 693 and 694, p.424. The corresponding ratio for male persons in 1985 is 0.79, and for female persons in 1985 is 0.71 (both from Table 710, p. 432, *ibid*). It should be possible to lower the ratio of median to mean income in

example) show that the model does generate empirically plausible relative degrees of inequality. Consumption exhibits the least inequality followed by net income, gross income and then capital, and saving exhibits the greatest inequality. However, the model cannot generate the observed degrees of inequality. For example, when μ is 5, ρ is 0.6 and σ is 0.2 the Gini coefficients for net income and wealth are 0.12 and 0.32, respectively (Figure 6). In U. S. data, however, the Gini coefficient for income is about 0.4 and that for net wealth is about 0.8[48].

VI. Concluding Remarks

In this paper a version of the Brock-Mirman growth model with a large number of agebts subject to uninsured idiosyncratic shocks was described and its qualitative and quantitative implications for the contribution of precautionary saving to aggregate saving, importance of asset trading, and income and wealth distributions were analyzed. This class of models may also be useful in understanding various asset return puzzles. Mehra and Prescott

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the model by adjusting σ, the coefficient of variation of earnings. Since earnings are approximately log-Normal, median/mean $\approx \exp(-\sigma^2/2)$. This ratio equals 0.98 when σ is 0.2, and it equals 0.92 when σ is 0.4.

[48]See, for example, Greenwood [1983, Table 9, p.41], Kessler and Wolff [1991, Table 4, p.260]. According to Greenwood [1983, Table 4, p.35, Table 7, p.40], in 1973, the top 5 percent of wealth holders held close to 60 percent of net wealth, and the top 5 percent of income earners earned about 23 percent of total income. Kessler and Wolff [1991, Table 3, p.259] present a very similar number for net wealth in 1983. There are several reasons for the discrepancy between the model and the data. First, the way income and wealth distribution data are put together does not match well with the model. Since the model is one of infinitely lived agents a household in the model should probably be thought of as consisting of members of all the different generations of a family. Implicitly, the model assumes that the different generations of a family are linked by bequests and, therefore, focuses only on total family income, wealth and consumption where the family is more broadly defined than in the data. Second, the model focuses on only one source of inequality (that due to different histories of labor endowment shocks) and abstracts from other sources of inequality like differences in the endowments of human capital (broadly interpreted).

[1985] suggested that these puzzles cannot be "accounted for by models that abstract from transactions costs, liquidity constraints and other frictions absent in the Arrow-Debreu set-up." However, this requires that the models be generalized to include aggregate dynamics and uncertainty[49]. This is a very hard problem computationally since the distribution of assets across households can no longer be taken to be constant. Instead, the cross-section distribution is part of the state vector which evolves stochastically over time in response to aggregate shocks. This is an issue that remains to be explored.

This class of models can also differ from the infinite lived agent complete markets model on some important policy issues. For instance, with complete markets dynamic optimal factor taxation leads to the result that the capital income tax should be zero in the long run (see Chamley 1986). However, Aiyagari [1993b] shows that with idiosyncratic shocks and incomplete markets the capital income tax is strictly positive even in the long run. Therefore, the large welfare gains of reducing the capital income tax to zero calculated by Lucas [1990] in a complete markets model may well turn out to be welfare losses in an incomplete markets model.

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[49]See Aiyagari and Gertler [1991] for an early attempt based on differential transactions costs but without aggregate uncertainty. Because of the absence of aggregate uncertainty that model could only analyze the risk-free rate and the transactions premium but not the risk premium.

Table I

Markov Chain Approximation to the Labor Endowment Shock

Markov Chain σ/Markov Chain ρ

σ\ρ	0	0.3	0.6	0.9
0.2	0.21/0	0.21/0.3	0.21/0.59	0.24/0.9
0.4	0.43/0	0.43/0.28	0.44/0.58	0.49/0.89

Table IIA

Net Return to Capital in %/Aggregate Saving Rate in % ($\sigma = 0.2$)

ρ\μ	1	3	5
0	4.1666/23.67	4.1456/23.71	4.0858/23.83
0.3	4.1365/23.73	4.0432/23.91	3.9054/24.19
0.6	4.0912/23.82	3.8767/24.25	3.5857/24.86
0.9	3.9305/24.14	3.2903/25.51	2.5260/27.36

Table IIB

Net Return to Capital in %/Aggregate Saving Rate in % ($\sigma = 0.4$)

ρ\μ	1	3	5
0	4.0649/23.87	3.7816/24.44	3.4177/25.22
0.3	3.9554/24.09	3.4188/25.22	2.8032/26.66
0.6	3.7567/24.50	2.7835/26.71	1.8070/29.37
0.9	3.3054/25.47	1.2894/31.00	-.3456/37.63

Table IIIA

Coefficients of Variation

consumption/net income/gross income
gross saving/assets

$\sigma = 0.2$

$\rho \backslash \mu$	1	3	5
0	.1/.2/.24 .75/.56	.09/.2/.24 .73/.53	.09/.19/.23 .71/.51
.3	.1/.2/.24 .73/.54	.09/.2/.23 .72/.52	.09/.19/.23 .7/.5
.6	.13/.22/.28 .82/.66	.12/.21/.27 .79/.62	.11/.21/.26 .75/.59
.9	.22/.28/.40 1.03/.97	.2/.26/.38 .94/.87	.18/.25/.36 .86/.79

Table IIIB

Coefficients of Variation

consumption/net income/gross income
gross saving/assets

$\sigma = 0.4$

$\rho \backslash \mu$	1	3	5
0	.12/.37/.35 1.2/.59	.11/.37/.34 1.17/.54	.1/.38/.33 1.12/.5
.3	.15/.38/.37 1.2/.64	.13/.38/.36 1.14/.58	.12/.39/.34 1.1/.53
.6	.2/.41/.42 1.24/.77	.17/.41/.40 1.13/.67	.15/.41/.38 1.03/.59
.9	.37/.49/.59 1.33/1.13	.31/.48/.54 1.13/.94	.25/.48/.51 .98/.8

APPENDIX

Proposition 1: Let $r > 0$. Then $a_t \geq -wl_{min}/r$ (a.s.) iff $\lim a_{t+1}/(1+r)^t \geq 0$ (a.s.).

Proof: By repeatedly using (2) we can write $\Sigma_{j=0}^t c_j/(1+r)^j = \Sigma_{j=0}^t wl_j/(1+r)^j + (1+r)a_0 - a_{t+1}/(1+r)^t$. The first summation on the right converges (as $t \to \infty$) to a well defined random variable which is finite since $r > 0$ and $\{l_t\}$ has bounded support. Further $a_t \geq -wl_{min}/r$ (a.s.) implies $\lim \inf a_{t+1}/(1+r)^t \geq 0$ (a.s.). Therefore, $\Sigma_{j=0}^t c_j/(1+r)^j$ converges to a well defined and finite random variable since $c_t \geq 0$. Hence, $\lim a_{t+1}/(1+r)^t$ is well defined and $\lim a_{t+1}/(1+r)^t \geq 0$ (a.s.). To see the reverse, suppose, if possible that $a_t = -wl_{min}/r - \varepsilon$, for some t, where $\varepsilon > 0$. With positive probability $l_t \leq l_{min} + r\varepsilon/2$. Therefore, using (2) and the nonnegativity of consumption we have that $a_{t+1} \leq -wl_{min}/r + r\varepsilon/2 - (1+r)\varepsilon$, with positive probability. Repeating the above argument for successive t's, we have that $a_{t+n} \leq -wl_{min}/r + (r\varepsilon/2)[1+(1+r)+(1+r)^2+\ldots+(1+r)^{n-1}] - (1+r)^n\varepsilon$, with positive probability. Therefore, $a_{t+n}/(1+r)^n \leq -[wl_{min}/r+\varepsilon/2]/(1+r)^n - \varepsilon/2 < -\varepsilon/2$, with positive probability. Now fix a n large such that $(wl_{max}/r)/(1+r)^{n-1} < \varepsilon/4$. By repeated application of (2) and the nonnegativity of consumption, we can conclude that $a_{t+n+m}/(1+r)^{n+m} \leq (wl_{max}/r)/(1+r)^{n-1}[1 - (1+r)^{-m-1}] + a_{t+n}/(1+r)^n$. Therefore, $\lim_{m\to\infty} a_{t+n+m}/(1+r)^{n+m} \leq (wl_{max}/r)/(1+r)^{n-1} + a_{t+n}/(1+r)^n < \varepsilon/4 - \varepsilon/2 = -\varepsilon/4$, with positive probability. This violates the limit condition.

Proposition 2: Let $z_{min} = wl_{min}-r\phi$. (a) $c_t > 0$ whenever $z_t > z_{min}$; (b) $V(.)$ is continuously differentiable in z for all $z > z_{min}$; (c) The solution to the maximization problem in (5) is characterized by: $V'(z_t) = U'(c_t) \geq \beta(1+r)E_t\{V'(z_{t+1})\}$ with equality of $\hat{a}_{t+1} > 0$.

Proof: (a) Since V is concave it has left and right derivatives. If for some $z\# > y_{min}$ we have $c\# = 0$ then we must have: $U'(0) \leq \beta RE\{V'_-(wl'-r\phi+Rz\#)\}$, where $R = 1+r$. Therefore, by concavity of V we must have $c(z) = 0$ for $z \in [z_{min}, z\#]$. Hence, $V(z) = U(0) + \beta E\{V(wl'-r\phi+Rz)\}$ for $z \in [z_{min}, z\#]$. There are two cases to consider. Case (i): $R \geq 1$. Then $wl'-r\phi+Rz \geq z$. Hence, $V'_+(z) = \beta RE\{V'_+(wl'-r\phi+Rz)\} \leq \beta RV'_+(z) < V'_+(z)$ which is a contradiction. Case (ii): $R < 1$. Then $z_{min} > 0$. Then $V'_+(z) = \beta RE\{V'_+(wl'-r\phi+Rz)\} \leq \beta RV'_+(z_{min}) < V'_+(z_{min})$. This results in a contradiction as we let $z \to z_{min}$.

(b) From (a) it follows that $\hat{a}_{t+1} < z_t$ whenever $z_t > 0$. Therefore, the

result follows from the Benveniste-Scheinkman [1979] theorem.

(c) The first equality follows from the Benveniste-Scheinkman [1979] theorem and the second inequality is just the first order necessary (and sufficient by strict concavity) condition for maximization on the right side of (5).

Proposition 3: Assume that either $U'(0) < \infty$ or $z_{min} = wl_{min} - r\phi > 0$. Then there is a $\hat{z} > z_{min}$ such that for all $z_t \le \hat{z}$, $c_t = z_t$ and $\hat{a}_{t+1} = 0$.

Proof: In either case $U'(z_{min})$ is finite and hence $V'(z_{min})$ is finite. To see the existence of such a $\hat{z}$ suppose to the contrary that the borrowing constraint is never binding. From proposition 1 (c) and (4b) we have: $V'(z_t) = \beta(1+r)E_t\{V'(z_{t+1})\} \le \beta(1+r)V'(z_{min}) < V'(z_{min})$. If we let $z_t \rightarrow z_{min}$ this results in a contradiction.

Note: If $U'(0) = \infty$ and $wl_{min} - r\phi = 0$, then the consumer would never exhaust the borrowing limit.

Proposition 4 (Schechtman and Escudero 1977, Theorems 3.8 and 3.9): Let $R = (1+r)$, $\beta R < 1$, and assume that: (i) $\{l_t\}$ has bounded support, (ii) $(-cU''/U')$ is bounded above for all c sufficiently large. Then there exists z^* such that whenever $z_t \le z^*$, $z_{t+1} \le z_t$.

Proof: For convenience let y denote $(wl-r\phi)$. We will show that there exists a $z^* < \infty$ such that whenever $z_t \ge z^*$, $y_{max} + RA(z_t) \le z_t$. If $A(z)$ is bounded so that $A(z) \le K$ for all $z \ge 0$, then we can take $z^* = RK + y_{max}$. So suppose that $A(z) \rightarrow \infty$ as $z \rightarrow \infty$. Note that $V'(z) = U'(c(z))$ where $c(z) = z-A(z)$. Therefore, we have: $E_t V'(z_{t+1})/V'(y_{max} + RA(z_t)) \le V'(y_{min} + RA(z_t))/V'(y_{max} + RA(z_t)) = U'(c(y_{min} + RA(z_t)))/U'(c(y_{max} + RA(z_t))) \le [c(y_{max} + RA(z_t))/c(y_{min} + RA(z_t))]^\mu$, where μ is an upper bound on relative risk aversion $(-cU''/U')$. Now, $c(y_{max} + RA(z_t)) = c(y_{min} + RA(z_t)) + w(l_{max} - l_{min})h(y)$, where $0 \le h(y) \le 1$, since both $c(z)$ as well as $A(z)$ are increasing in z. Therefore, $E_t V'(z_{t+1})/V'(y_{max} + RA(z_t)) \le [1 + w(l_{max} - l_{min})h(y)/c(y_{min} + RA(z_t))]^\mu$. As $z_t \rightarrow \infty$, $c_t \rightarrow \infty$ because $U'(c_t) = \beta R E_t\{V'(y_{t+1} + RA(z_t))\}$ (equality holds since $A(.)$ is positive), $A(z_t) \rightarrow \infty$, and $V(.)$ is bounded, increasing and concave. Therefore, we have: $\lim_{z_t \rightarrow \infty} E_t V'(z_{t+1})/V'(y_{max} + RA(z_t)) = 1$. Now choose $\varepsilon < (1-\beta R)/(\beta R)$ and note that there exists z^* sufficiently large such that $E_t V'(z_{t+1})/V'(y_{max} + RA(z_t)) \le 1+\varepsilon$ for all $z_t \ge z^*$. From the FONC (with equality since $A(.)$ is positive)

for the maximization on the right side of (5) and the envelope condition we have: $V'(z_t) = U'(c_t) = \beta RE_t V'(z_{t+1}) \leq \beta R(1+\epsilon)V'(y_{max}+RA(z_t)) \leq V'(y_{max}+RA(z_t))$ for all $z_t \geq z^*$. By strict concavity of $V(.)$ we then have that $y_{max}+RA(z_t) \leq z_t$ for all $z_t \geq z^*$.

Note: Condition (ii) in proposition 4 arises from a consideration of the exponential utility case which has constant absolute risk aversion and, hence, unbounded relative risk aversion. In this case there exist values of r below $(1-\beta)/\beta$ and a probability distribution for $\{l_t\}$ with bounded support such that the consumer's assets will wander off to infinity a.s. (see Schechtman and Escudero 1977, pp. 159-161).

Proposition 5: Under the assumptions of proposition 4, there exists a unique invariant distribution for $\{z_t\}$ which is stable[50]. Further the invariant distribution behaves continuously with respect to the parameters b, r, and w.

Proof: (Schechtman and Escudero 1977, Clarida 1987, Stokey and Lucas with Prescott, hereafter SLP 1989, section 12.4, **Monotone Markov Processes**) The Markov process defined by (7) is monotone because $A(z)$ is increasing in z which implies that a higher value of z_t makes higher values of z_{t+1} more likely. Therefore, if $g(.)$ is an increasing function then $E\{g(z_{t+1})|z_t\}$ is an increasing function of z_t. Further, it satisfies the Feller property since $A(z)$ is continuous. That is, if $g(.)$ is a continuous function then $E\{g(z_{t+1})|z_t\}$ is a continuous function of z_t. Lastly it satisfies a "mixing" condition which ensures that there is a unique ergodic set. This condition requires that there exist $z \in [z_{min}, z_{max}]$, $\epsilon > 0$, and $T \geq 1$ such that: $prob\{z_T \in [z, z_{max}]|z_0 = z_{min}\} \geq \epsilon$, and $prob\{z_T \in [z_{min}, z]|z_0 = z_{max}\} \geq \epsilon$ (see assumption 12.1, p.381, SLP 1989). Figure 1B suggests that this is the case. The key properties of this picture are that y_{min} is the unique fixed point when $l_{t+1} = l_{min}$ and it is strictly less than the smallest fixed point when $l_{t+1} = l_{max}$. To see the former note that at a fixed point for l_{min} the borrowing constraint must bind. Otherwise we have : $V'(z) = \beta RE\{V'(y'+RA(z))\} \leq \beta RE\{V'(y_{min}+RA(z))\} = \beta RV'(z)$. This contradicts the fact that $\beta R < 1$. Hence $z = y_{min}$ is the unique fixed point for l_{min}. It follows

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[50]That is, starting from any initial distribution the sequence of distributions generated by the Markov probability transition function converges to the unique invariant distribution.

that any fixed point for l_{max} exceeds y_{min}. In Brock and Mirman's [1972] terminology the fixed point configuration in figure 1B is stable and there is a unique ergodic set. Theorem 12.12, p.382 in SLP [1989] then delivers the existence of a unique, stable stationary distribution. Further, Theorem 12.13, p.384 in SLP [1989] shows that the distribution behaves continuously with respect to the parameters b, r and w.

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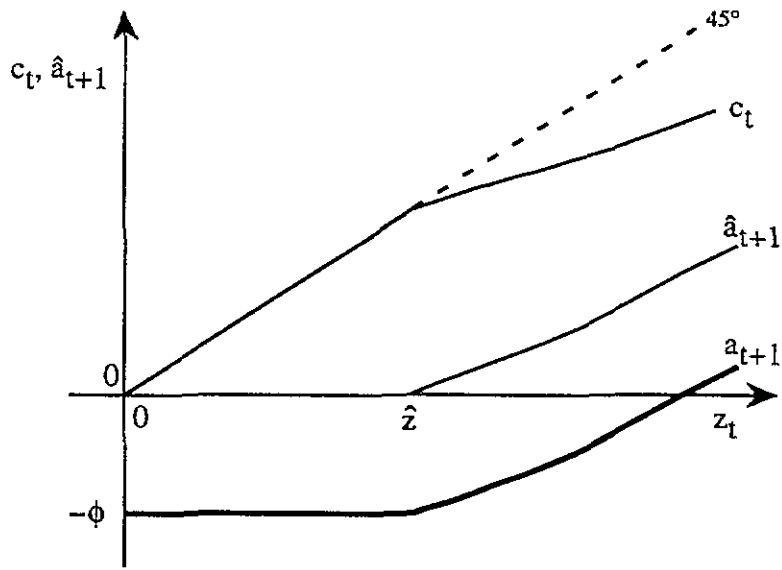
FIGURE Ia
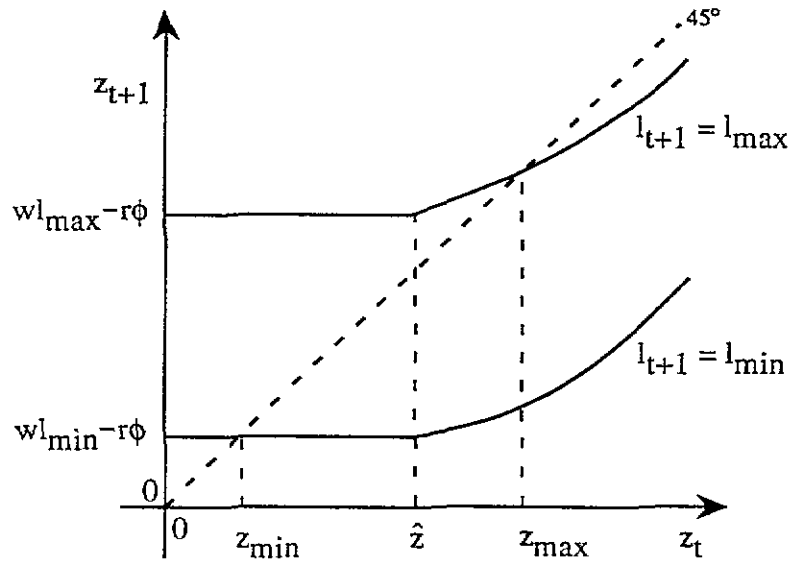Consumption and Assets as Functions of Total Resources
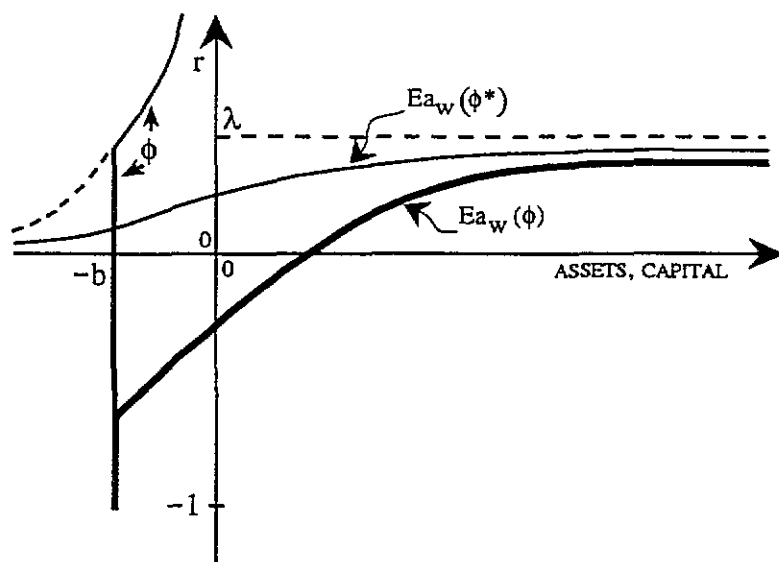

FIGURE Ib
Evolution of Total Resources

FIGURE IIa
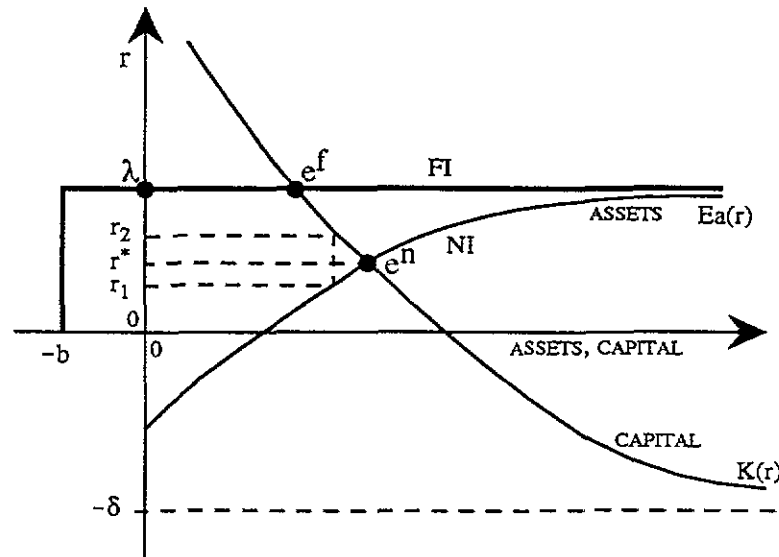Interest Rate versus Per-Capita Assets
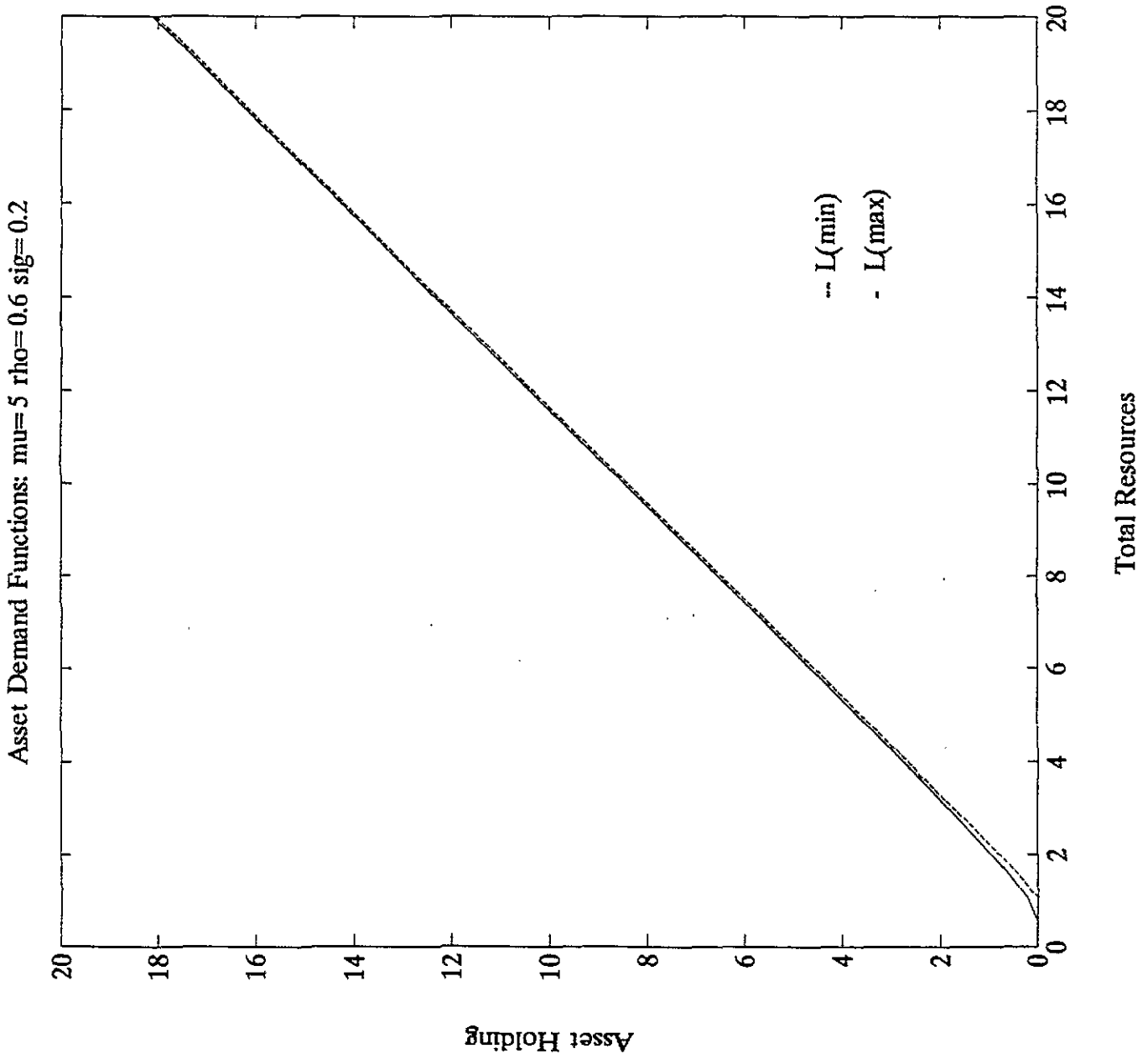

FIGURE IIb
Steady State Determination

FIGURE 3

FIGURE 4

Steady State: mu= 5 rho= 0.6 sig= 0.2

Net Return to Capital (%)

-- Assets -- Capital

1/B
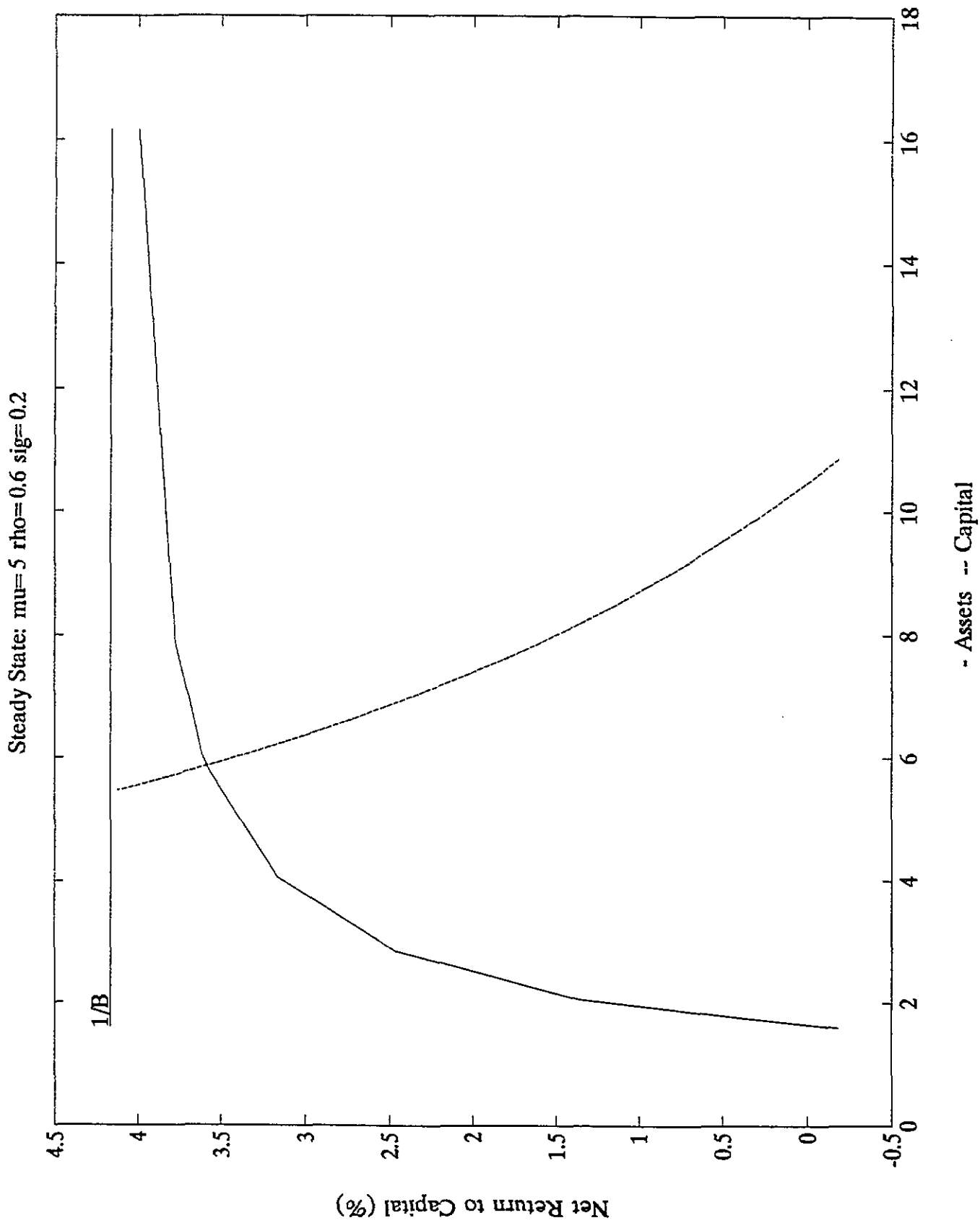
FIGURE 5A

Other Densities: mu= 5 rho= 0.6 sig= 0.2

-- s gross -- y net .. con + y gross

Density of Assets: mu=5 rho=0.6 sig=0.2

0% zero capital
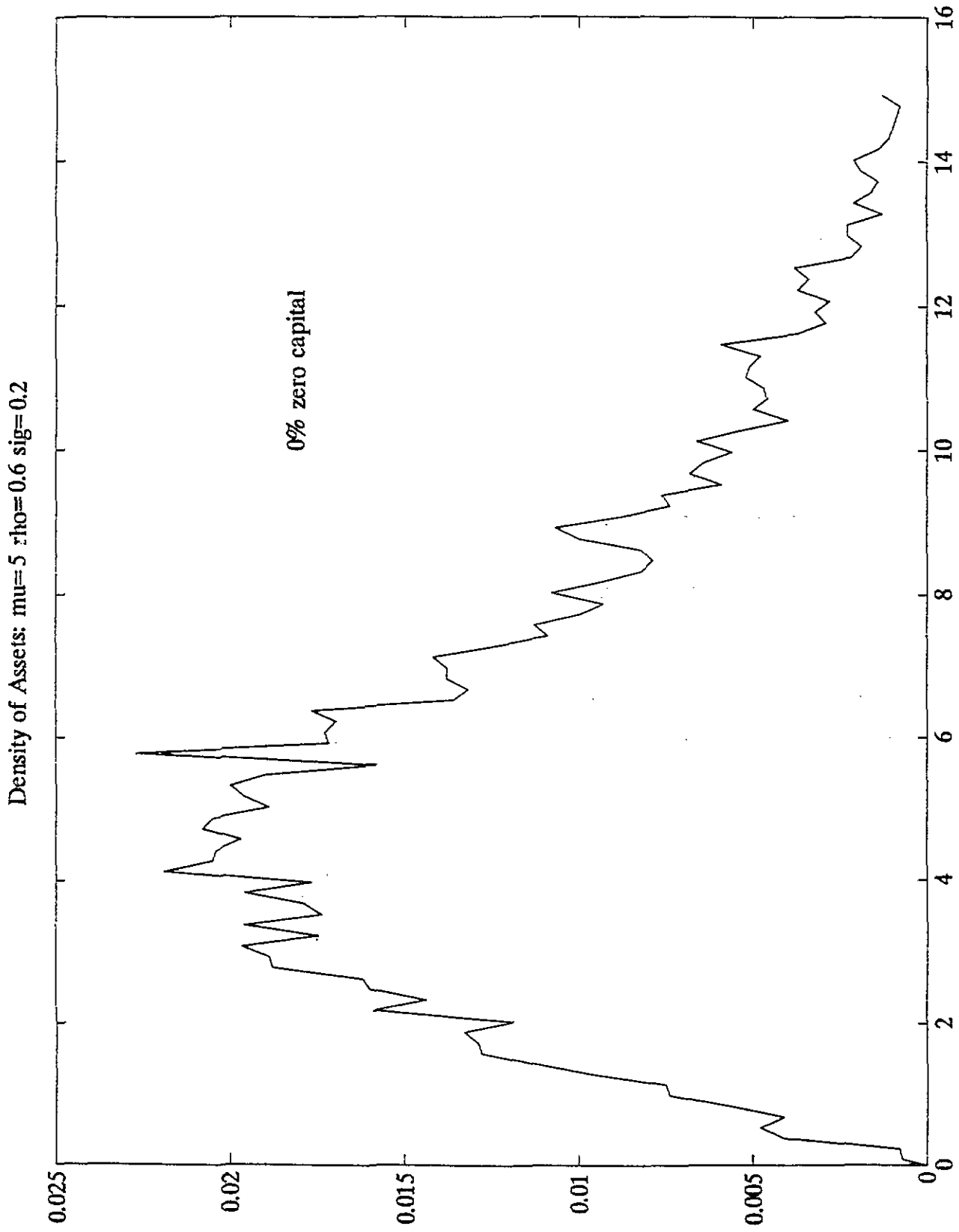
FIGURE 6

Lorenz Curves: mu= 5 rho= 0.6 sig= 0.2

Gini Coefficients:
Cap   0.3189
S gross 0.4175
Y net   0.1163
Consump 0.061
Y gross 0.1445

- cap -- s gross .. y net -. cons + y gross