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ON THE RELATION BETWEEN THE EXPECTED VALUE  
AND THE VOLATILITY OF THE NOMINAL  
EXCESS RETURN ON STOCKS

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ABSTRACT

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Earlier researchers have found either no relation or a positive relation between the conditional expected return and the conditional variance of the monthly excess return on stocks when they used the standard GARCH-M model. This is in contrast to the negative relation found when other approaches were used to model conditional variance. We show that the difference in the estimated relation arises because the standard GARCH-M model is misspecified. When the standard model is modified allow for (i) the presence for seasonal patterns in volatility, (ii) positive and negative innovations to returns to having different impacts on conditional volatility, and (iii) nominal interest rates to affect conditional variance, we once again find support for a negative relation. Using the modified GARCH-M model, we also show that there is little evidence to support the traditional view that conditional volatility is highly persistent. Also, positive unanticipated returns result in a downward revision of the conditional volatility whereas negative unanticipated returns result in an upward revision of conditional volatility of a similar magnitude. Hence the time series properties of the monthly excess return on stocks appear to be substantially different from that of the daily excess return on stocks.

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**ON THE RELATION BETWEEN THE  
EXPECTED VALUE AND THE VOLATILITY OF  
THE NOMINAL EXCESS RETURN ON STOCKS**

The tradeoff between risk and return has long been an important topic in asset valuation research. Most of this research has examined the tradeoff between risk and return among different securities within a given time period. The intertemporal relation between risk and return has recently been examined by several authors. This paper extends that research.

There is general agreement that investors, within a given time period, will require a larger expected return from a security that is more risky. However, there is no such agreement as to whether investors will require a larger risk premium on average for investing in a security during times when the security is more risky. At first blush, it may appear that rational risk-averse investors would require a relatively larger risk premium during times when the payoff from the security is more risky. A larger risk premium may not be required, however, because time periods which are relatively more risky could coincide with time periods when investors are better able to bear particular types of risk. Further, a larger risk premium may not be required because investors may want to save relatively more during periods when the future is more risky. If all the productive assets available for transferring income to the future carry risk and no risk-free investment opportunities are available, then the price of the risky asset may be bid up considerably, thereby reducing the risk premium.<sup>1</sup> Hence a positive as well as a negative sign for the covariance between the conditional mean and the conditional variance of the excess return on stocks would be consistent with theory.

Since there are conflicting predictions about this aspect of the tradeoff between risk and return, it is important to empirically characterize the nature of the relation between the conditional mean and the conditional variance of the excess return on stocks as a group.<sup>2</sup>

The empirical literature on this topic has attempted to characterize the nature of the linear relation between the conditional mean and the conditional

variance of the excess return on stocks. However, the reported findings are also somewhat conflicting.<sup>3</sup>

Most of the support for a zero or positive relation has come from studies that use the standard GARCH-M model of stochastic volatility. (See Bollerslev, Chou, and Kroner 1992 for an extensive survey of GARCH and GARCH-M models in finance.) Other studies have documented a negative relation between expected return and conditional variance. In order to resolve this conflict we examine the possibility that the standard GARCH-M model may not be rich enough to capture the time series properties of the monthly excess return on stocks. We consider a more general specification of the GARCH-M model. In particular, (i) we incorporate dummy variables in the GARCH-M model to capture seasonal effects using the procedure first suggested by Glosten, Jagannathan and Runkle (1988), (ii) we allow for asymmetries in the conditional variance equation, following the suggestions of Glosten, Jagannathan and Runkle (1988), (iii) we include nominal interest rate in the conditional variance equation, and (iv) we consider the EGARCH-M specification suggested by Nelson (1991) with the modifications mentioned in (i) and (iii) above. We find a weak but statistically significant negative relation between conditional variance and expected return.

Our other findings are somewhat at odds with the existing literature. First, our data provides little evidence to support the belief that the conditional volatility of the monthly excess return on stocks is highly persistent.<sup>4</sup> Second, both unexpected positive and negative excess return on stocks change next period's conditional volatility of the excess return on stocks by the same magnitude. However, unexpected positive returns result in a downward revision while unexpected negative returns result in an upward revision.

In contrast, Nelson (1991) and Engle and Ng (1991), using daily data on stock-index returns, find that large positive as well as negative unanticipated returns lead to an upward revision in the conditional volatility, although negative shocks of similar magnitude lead to larger revisions. Hence the time series properties of monthly excess returns are somewhat different from those of daily returns reported in Nelson (1991) and Engle and Ng (1991).

There are no theoretical reasons for the properties of the monthly and daily returns to be the same. For example, Nelson (1990) argues that as the

frequency at which data are sampled becomes very high, persistence should become larger. Thus, our results for monthly data along with the results for daily data reported by others provide a more complete characterization of the time series properties of stock index returns.

The remainder of the paper proceeds as follows. Section I describes the model that forms the basis for our empirical analysis. Section II discusses the econometric issues involved and our estimation methods. Section III contains the empirical results. Section IV concludes.

## **I. The relation between the conditional mean and the conditional variance of the excess return on stocks**

Consider the relation between conditional variance and conditional mean given by:

$$E_t[x_{t+1}] = \beta\sigma_t^2 \quad (1)$$

When  $x_{t+1}$  is the excess return on the aggregate wealth portfolio, and  $\sigma_t^2$  captures most of the economic uncertainty that agents care about, the model in (1) is the approximation to the true risk-return relation derived by Merton (1980).

In our empirical work, we will assume that (1) holds even for nominal returns. We will consider the following general model for estimation.

$$E(x_{t+1}|F_t) = \alpha + \beta\text{Var}(x_{t+1}|F_t) \quad (2)$$

where  $F_t$  denotes the information set of agents. Campbell (1992) provided sufficient conditions for the relation given in (2) to hold approximately in equilibrium, where  $x_t$  is the excess return on the market-index portfolio. However,  $\beta$  will not in general be a measure of the risk-aversion coefficient of the representative agent and  $\alpha$  will not in general equal zero. The relation in (2) forms the basis for our empirical work.

## **II. Estimating the model**

### **A. Econometric issues:**

The parameter  $\beta$  in the model given by (2) cannot be estimated without specifying how variances change over time, since  $\text{Var}(x_{t+1}|F_t)$  can not be directly

observed by the econometrician. To appreciate the difficulties involved, project both sides of (2) on  $G_t$ , the econometrician's information set, which is a strict subset of the agents' information set  $F_t$ . Doing so, we get

$$E(x_t | G_{t-1}) = \alpha + \beta E(v_{t-1} | G_{t-1}). \quad (3)$$

Hence, we can write

$$x_t = \alpha + \beta E(v_{t-1} | G_{t-1}) + \eta_t, \text{ where}$$

$$\eta_t = u_{t-1} + \epsilon_t, \quad u_{t-1} = \beta [v_{t-1} - E(v_{t-1} | G_{t-1})], \text{ and}$$

$$\epsilon_t = x_t - E(x_t | F_{t-1}).$$

Since, by definition

$$E(\epsilon_t^2 | F_{t-1}) = v_{t-1},$$

$$u_{t-1} = \beta [E(\epsilon_t^2 | F_{t-1}) - E(\epsilon_t^2 | G_{t-1})].$$

Note that  $E(\eta_t | G_{t-1}) = E(u_{t-1} | G_{t-1}) = E(u_{t-1} \epsilon_t | G_{t-1}) = 0$ .

Therefore,

$$E(\eta_t^2 | G_{t-1}) = E(u_{t-1}^2 | G_{t-1}) + E(\epsilon_t^2 | G_{t-1}). \quad (4)$$

The term on the left is the variance of the error in forecasting  $x_t$  based on the econometrician's information set. The first term on the right is the variance of the measurement error,  $(v_{t-1} - E[v_{t-1} | G_{t-1}])$ , and the second term is the expected value of the conditional variance  $v_{t-1}$  based on the econometrician's information set  $G_{t-1}$ . Unless the variance of the measurement error is a constant, we cannot obtain a consistent estimate of  $\beta$ . This problem was first pointed out by Pagan and Ullah (1988). Also notice that the intercept term  $\alpha$  in equation (2) is not identifiable based on the smaller information set available to the econometrician, since  $E(v_{t-1} | G_{t-1})$  may involve a constant term.

To see this problem more clearly, consider now the special case where

$$E(v_{t-1} | G_{t-1}) = b_0 + b_1 z_{t-1}$$

for some  $z \in G$  where  $b_1$  is a row vector and  $z_{t-1}$  is a column vector. Then

$$E(\eta_t^2 | z_{t-1}) = \beta^2 \text{Var}(v_{t-1} | G_{t-1}) + (b_0 + b_1 z_{t-1}).$$

The left side is the variance of the excess return, conditional on observing the instrument  $z$  alone. The first term on the right side is the variance of the measurement error  $\beta[v_{t-1} - E(v_{t-1} | G_{t-1})]$ , and the second term is the variance of  $\epsilon_t$  given  $z_{t-1}$ .

There have been several approaches to the estimation of this general econometric model. One approach was suggested by Campbell (1987), and assumes that  $\text{Var}(v_{t-1} | G_{t-1})$  is an arbitrary constant, while  $z_{t-1}$  is a vector of observable variables. If  $\text{Var}(v_{t-1} | G_{t-1})$  is a constant, then we can test whether  $\beta$  is positive. We can estimate the regression equations

$$x_t = c_0 + c_1 z_{t-1} + \eta_t \quad \text{and} \quad (5)$$

$$\eta_t^2 = d_0 + d_1 z_{t-1} + \zeta_t. \quad (6)$$

Since the estimated slope coefficient  $c_1$  is a consistent estimate of  $\beta b_1$ , and  $d_1$  provides a consistent estimate of  $b_1$ , the ratio of any two corresponding elements of  $c_1$  and  $d_1$  will provide a consistent estimate of  $\beta$ . If  $z_{t-1}$  is not a scalar, then we may impose the constraint that the slope coefficients in (5) and the slope coefficients in (6) differ only by the scale factor  $\beta$  in estimating  $\beta$ . Such a restriction also provides a natural test for the validity of the model specification. We will denote this approach as **Campbell's instrumental variable model**.

Another approach, the GARCH-M model, assumes that  $\text{Var}(v_{t-1} | G_{t-1})$  is identically zero, and that  $z_{t-1}$  consists of innovations and variances that, while unobservable, can be estimated by the econometrician. A generalization of the GARCH-M approach, maintains the assumption that  $\text{Var}(v_{t-1} | G_{t-1})$  is zero but allows  $z_{t-1}$  to consist both of observable instruments and lagged values of estimated variances and innovations. We will denote this approach as the **Modified GARCH-M model**. Since the specification of the information set is crucial for the Modified GARCH-M model, we will first describe the information set used in this study and then proceed to describe the GARCH-M models we examine.

**B. Specification of the econometrician's information set:**

Implementation requires taking a stand on the variables that make up the instrument vector,  $z_t$ . In our investigations we focus attention on the volatility information in the following variables: (a) nominal interest rate, (b) October and January seasonal dummies, and (c) unanticipated part of the excess return to stocks. In what follows we provide some justification as to why we focus our attention on these variables.

The use of nominal interest rates in conditional variance models has some intuitive appeal. It has been well known since Fischer (1981) that the variance of inflation increases with its level. To the extent that short-term nominal interest rates embody expectations about inflation, they could be a good predictor of future volatility in excess returns. Using the information contained in nominal interest rates, Fama and Schwert (1977), Campbell (1987), and Breen, Glosten, and Jagannathan (1989) have demonstrated that it is possible to forecast time periods when the excess return on stocks is relatively large and significantly less volatile. Singleton (1989) also examined the ability of nominal interest rates to predict changes in the volatility of stock returns.

Including deterministic seasonal dummies is motivated by the seasonal patterns in volatility of stock returns reported in Lakonishok and Smidt (1988) and Keim (1985). Table I presents the summary statistics for the monthly excess returns on the CRSP value-weighted stock-index portfolio during the post-Treasury Accord period for the months of October, January, and other calendar months.<sup>5</sup> An apparent increase in October and January volatility is suggested by results presented in panels A and C.

During the period 1951:4 to 1989:12, monthly excess continuously compounded returns on the CRSP value-weighted index of stocks, during months other than October and January, had a mean of 0.48 percent and a standard deviation of 3.83 percent. The standard deviation of January excess returns is 5.19 percent (i.e., 1.35 times that in other months) and the standard deviation of October excess returns is 6.17 percent (1.61 times that in other months). While October and January are both months of relatively larger volatility, October, unlike January, has relatively lower excess returns on average than other months.<sup>6</sup>

There are several potential contributing explanations for the excess volatility of October and January excess returns on stocks. Information about the fall harvest starts coming in during October. Even though agriculture is no longer a substantial part of the U.S. economy, the multiplier effect could be there since agricultural output forms an important input into several industries. Further, important political elections occur in the first week of November. Information about the likely outcome of these elections come in during October.

Relatively more news arrives in January since most firms (almost two thirds) use the calendar year as their fiscal year. Such firms will close their books on December 31. The annual reports are typically more informative since they are done more carefully and are audited. Information from such reports starts leaking in during the month of January. Further consumer sales exhibit pronounced quarterly seasonal patterns. This pattern arises because the fourth quarter is the important holiday season. Comprehensive and reliable information about consumer spending during this period typically becomes available during January.

There may be stories for other months as well. Further investigations should examine other months. Laurie Cohen<sup>7</sup>, writing in a Wall Street Journal article attributes to Mark Twain the observation, "October is one of the peculiarly dangerous months to speculate in stocks. The others are: July, January, September, April, November, May, March, June, December, August and February."

We also allow positive and negative innovations to returns to have different impact on conditional variance. To see why this is desirable, suppose discount rates are constant and have no relationship to anticipated future volatility. Any unanticipated decrease in expected future cashflows will decrease the stock price. If the variance of the future cashflows remains the same or does not fall proportionately to the fall in stock prices, the variance of future cashflows per dollar of stock price will rise and future returns will be more volatile. Hence, if most of the fluctuations in stock prices are caused by fluctuations in expected future cashflows and the riskiness of future cashflows does not change proportionally when investors revise their



expectations, then unanticipated changes in stock prices and returns will in general be negatively related to unanticipated changes in future volatility.

This argument can be made as rigorous as follows. Consider a representative-agent endowment economy. Let  $y_t$  denote the date  $t$  endowment of the single consumption good where

$$y_t = a_0 + a_1 y_{t-1} + e_t; \quad E_{t-1}[e_t] = 0; \quad E_{t-1}[e_t^2] = \sigma^2.$$

Suppose that the representative agent is risk neutral and that the price of a unit discount bond is a constant,  $\delta$ ; that is, the risk-free interest rate is a constant,  $(1-\delta)/\delta$ , per period. Without loss of generality we may assume that the dividends are paid by a single firm entirely owned by the representative agent. In this economy, the expected return on any financial asset is the same as the riskless return, and there is no intertemporal relationship between risk and return.

It can be verified that the price of the single firm is given by

$$P_t = k_0 + k_1 y_t; \quad k_0 = \delta a_0 / [(1-\delta)(1-a_1\delta)]; \quad k_1 = a_1 \delta / (1-a_1\delta).$$

The variance of the return,  $R_t \equiv (P_t + y_t) / P_{t-1}$  is given by

$$\text{Var}_t[R_{t+1}] \equiv \text{Var}_t[(P_{t+1} + y_{t+1}) / P_t] = a_1^2 (1+k_1)^2 \sigma^2 / (k_0 + k_1 y_t)^2.$$

Here  $R_t$  and  $\text{Var}_t[R_{t+1}]$  are negatively related.<sup>8</sup>

Black (1976) and Christie (1982) have suggested a different reason for the negative effect of current returns on future variance: a decrease in today's stock price changes a firm's capital structure by increasing leverage. This increased leverage causes higher expected variance in the future. Both Black and Christie find support for their predictions in the relation between expected return and variance for individual stocks.

### C. Modified GARCH-M model for the variance

#### C.1. Model specifications

The GARCH model assumes that the information set of investors and the econometrician coincide. The general Modified GARCH-M model can be written as:

Equation for the conditional mean

$$E[x_{t+1}|G_t] = \mu(G_t)$$

where  $\mu(\cdot)$  is a function that describes the nature of the dependence of the conditional mean on the elements of the information set  $G_t$ . Hence, we can write

$$x_{t+1} = \mu(G_t) + \epsilon_{t+1}, \text{ with } E[\epsilon_{t+1}|G_t] = 0.$$

Equation for the conditional variance

$$\text{Var}[x_{t+1}|G_t] \equiv \text{Var}[\epsilon_{t+1}|G_t] = V(G_t)$$

where  $V(\cdot)$  is a function that describes the nature of the dependence of the conditional variance on the elements of the information set  $G_t$ . It is convenient to assume that the conditional variance function can be decomposed in the following way:

$$V(G_t) = f_m(G_{t-1}) + f(G_t|G_{t-1})$$

where  $f_m(G_{t-1})$  is that part of the conditional variance,  $V(G_t)$ , that depends only on information known as of date  $t - 1$ , and  $f(G_t|G_{t-1})$  is that part of the conditional variance that depends on the new information,  $G_t|G_{t-1}$ , that becomes available at date  $t$ .

Now consider the standard GARCH-M process suggested by Bollerslev (1986) for stock excess return,  $x_t$ , given by

MODEL 1

$$x_t = a_0 + a_1 v_{t-1} + \epsilon_t \quad (7)$$

$$v_{t-1} = b_0 + b_1 v_{t-2} + g_1 \epsilon_{t-1}^2 \quad (8)$$

where  $E_{t-1}[\epsilon_t] = 0$  and  $E_{t-1}(\epsilon_t^2) = v_{t-1}$ . The GARCH-M model specifies that the conditional mean function  $\mu(G_{t-1}) = a_0 + a_1 v_{t-1}$  and that the conditional variance function  $V(G_{t-1}) = b_0 + b_1 v_{t-2} + g_1 \epsilon_{t-1}^2$ . That is,  $f_m(G_{t-2}) = b_0 + b_1 v_{t-2}$  and  $f(G_{t-1}) = g_1 \epsilon_{t-1}^2$ . The univariate GARCH-M model assumes that the econometrician's information set consists only of the past innovations to the excess return  $x_t$ . Hence, the only new information that becomes available at date  $t - 1$  is  $\epsilon_{t-1}$ . The

model further assumes that the function  $f(\epsilon_{t-1}) = g_1 \epsilon_{t-1}^2$ . As we have argued earlier, there are a priori reasons to suspect that the assumption that the function  $f(\epsilon_{t-1}) = g_1 \epsilon_{t-1}^2$  may not be reasonable.

If future variance is not a function only of the squared innovation to current return, then a simple GARCH-M model will be misspecified and any empirical results based on it alone are not reliable. In Model 2 we assume that the impact of  $\epsilon_{t-1}^2$  on conditional variance  $v_{t-1}$  will be different when  $\epsilon_{t-1}$  is positive (i.e., when  $I_{t-1}$  in (9) is 1) than when  $\epsilon_{t-1}$  is negative (i.e., when  $I_{t-1}$  in (9) is 0). This leads to

#### MODEL 2

$$v_{t-1} = b_0 + b_1 v_{t-2} + g_1 \epsilon_{t-1}^2 + g_2 \epsilon_{t-1}^2 I_{t-1}. \quad (9)$$

In models 3 through 5 we will relax the assumption that the information set  $G_t$  consists only of past realizations of the excess return on the stock-index portfolio. Including the risk-free interest rate<sup>9</sup> leads to

#### MODEL 3

$$v_{t-1} = b_0 + b_1 v_{t-2} + b_2 r_t + g_1 \epsilon_{t-1}^2 + g_2 \epsilon_{t-1}^2 I_{t-1}. \quad (10)$$

For reasons we mentioned earlier, we expect October and January to be relatively more volatile than other months. We therefore introduce January and October seasonal dummies in the variance of stock-index excess returns. For this purpose we assume that the seasonal effects amplify the underlying *fundamental volatility* (which does not by definition exhibit any seasonal patterns) in the months of October and January by a constant month specific scale factor. We also assume that the fundamental volatility next period depends only on the fundamental part of the excess return innovation.

In particular, we assume that we can write the excess return innovation in any calendar month as a scale multiple of some underlying fundamental innovation that does not exhibit any seasonal patterns, as follows:

$$\epsilon_t = (1 + \lambda_1 \text{OCT}_t + \lambda_2 \text{JAN}_t) \eta_t$$

where  $\eta_t$  does not exhibit any deterministic seasonal behavior. Let  $h_{t-1} = E_{t-1}[\eta_t^2]$  denote the conditional variance of  $\eta_t$ . We postulate that  $h_t$  evolves over time according to

MODEL 4

$$h_{t-1} = b_0 + b_1 h_{t-2} + g_1 \eta_{t-1}^2 + g_2 \eta_{t-1}^2 I_{t-1} \text{ and} \quad (11)$$

MODEL 5

$$h_{t-1} = b_0 + b_1 h_{t-2} + b_2 r_{ft} + g_1 \eta_{t-1}^2 + g_2 \eta_{t-1}^2 I_{t-1}. \quad (12)$$

Notice that Model 1 is obtained from Model 5 by imposing the restriction that  $\lambda_1 = \lambda_2 = b_2 = g_2 = 0$ . Similarly, Models 2, 3, and 4 can be considered as restricted versions of Model 5.

Our approach to modelling seasonals is different from the one used by Baillie and Bollerslev (1989). In our specification, we assume that we can deseasonalize the excess return innovation,  $\epsilon_t$ , to get  $\eta_t$ . The realized value of the deseasonalized innovation,  $\eta_t$ , influences the conditional variance of the distribution from which the deseasonalized innovation for the next period is drawn from. In contrast, in Baillie and Bollerslev (1989), the seasonal part of the innovation to this period return will affect the variance of the deseasonalized innovation next period.

Because inference in GARCH-M models depends on correct specification of the information set and the validity of the functions used to represent the conditional mean and the conditional variance, we estimate three additional models to check our specification. First, we check for nonlinearity in the mean equation by adding  $v_{t-1}^{1/2}$  to Model 2 and Model 4. These models are then called Model 6 and Model 7. If the coefficient on  $v_{t-1}^{1/2}$  is significantly different from zero, that difference is evidence of misspecification.

In the above models, there are a priori reasons to suspect that the coefficient  $g_2$  as well as  $g_1 + g_2$  will be negative, since empirical evidence suggests that a positive innovation to stock return is associated with a decrease in return volatility. However, if  $g_1 + g_2$  is negative, conditional variance can

potentially become negative for some realization of  $\epsilon$ . Hence we will also follow the suggestion of Engle (1982) and Nelson (1991) and consider the exponential form for the law of motion for conditional variance, as given below.

MODEL 2-L

$$\log(h_{t-1}) = b_0 + b_1 \log(h_{t-2}) + g_1 \eta_{t-1}/\sqrt{h_{t-2}} + g_2 \eta_{t-1}I_{t-1}/\sqrt{h_{t-2}}. \quad (13)$$

Following Nelson (1991), we use  $\eta_{t-1}/\sqrt{h_{t-2}}$  instead of functions of  $\eta_{t-1}^2$  in equation (13) to minimize the impact of extremely large realizations in absolute value so that the stochastic process for  $h_t$  will be well behaved.<sup>10</sup> Model 1-L is the same as model 2-L but with  $g_2$  restricted to be zero.

Since we also want to test whether the risk-free rate helps predict conditional variance using the log specification, we also estimate

MODEL 3-L

$$\log(h_{t-1}) = b_0 + b_1 \log(h_{t-2}) + b_2 r_t + g_1 \eta_{t-1}/\sqrt{h_{t-2}} + g_2 \eta_{t-1}I_{t-1}/\sqrt{h_{t-2}}. \quad (14)$$

Model 4-L and 5-L add deterministic seasonals to the variance equation of Model 2-L and 3-L in the manner adopted for the level specification. Two additional models were estimated to test the specification of the EGARCH-M model. Model 6-L adds  $v_{t-1}^{1/2}$  to the mean equation for Model 4-L. Model 7-L is identical to Model 2-L, except for the starting values of the parameters used in estimation, i.e., Model 7-L corresponds to a local maximum whereas Model 2-L corresponds to a global maximum.

**C.2. Estimation and inference and diagnostic tests**

We estimate all models discussed in this section by maximizing the log-likelihood function for the model, assuming that  $\epsilon_t$  is conditionally normally distributed. Even if this assumption is incorrect, as long as the conditional means and variances are correctly specified, the quasi-maximum likelihood estimates will be consistent and asymptotically normal, as pointed out by Glostén, Jagannathan and Runkle (1988) and Bollerslev and Wooldridge (1989). All our inference is based on robust standard errors from quasi-maximum likelihood estimation.<sup>11</sup>

We also use a variety of diagnostic tests to determine whether various aspects of our different models are correctly specified. First, we examine whether the residuals of the estimated models display excess skewness and kurtosis. Properly specified GARCH-M and EGARCH-M models should be able to significantly reduce the excess skewness and kurtosis evident in nominal excess returns. We test for excess skewness and kurtosis, under the null hypothesis that the errors are drawn from a conditional normal distribution. These tests were previously applied to GARCH-M models by Campbell and Hentschel (1991).

Second, we examine whether the squared standardized residuals from the estimated models,  $(\epsilon_t/h_{t-1})^2$  are independent and identically distributed. We use the three tests proposed by Engle and Ng (1991): the Sign-Bias Test, the Negative-Size-Bias Test, and the Positive-Size-Bias Test.

In the Sign-Bias Test, the squared standardized residuals are regressed on a constant and a dummy variable, denoted as  $S_t^+$  that takes a value of one if  $\epsilon_{t-1}$  is negative and zero otherwise. The Sign-Bias Test Statistic is t-statistic for the coefficient on  $S_t^+$ . This test shows whether positive and negative innovations affect future volatility differently from the prediction of the model.

In the Negative Size-Bias test, the squared standardized residuals are regressed on a constant and  $S_t^-\epsilon_{t-1}$ . The Negative-Size Bias Test Statistic is the t-statistic for the coefficient on  $S_t^-\epsilon_{t-1}$ . This test shows whether larger negative innovations are correlated with larger biases in predicted volatility.

In the Positive Size-Bias test, the squared standardized residuals are regressed on a constant and  $S_t^+\epsilon_{t-1}$ , where  $S_t^+=1-S_t^-$ . The Negative-Size Bias Test Statistic is the t-statistic for the coefficient on  $S_t^+\epsilon_{t-1}$ . This test shows whether larger negative innovations are correlated with larger biases in predicted volatility.

There is one additional comparison that we make among the models, although it is not formally a diagnostic test. Because the parameterization of the models differ so much, it is hard to compare the amount of persistence in variance that these models predict. One way to compare persistence in variance across models is to regress  $h_t$  on a constant and  $h_{t-1}$ . We report the coefficients and t-statistics on  $h_{t-1}$  in the regressions for each model.

### III Empirical results

As mentioned earlier, our objective is to examine the role of model specification in determining the estimated relation between risk and return. The discussion above suggests that we can estimate this relation using either Campbell's Instrumental Variable model or a variety of Modified GARCH-M and EGARCH-M models. While our focus is on the latter, we first present results using the first approach and find that Campbell's general conclusions are replicated in our data. We then present the results for the various GARCH-M models.

#### A. Campbell's Instrumental Variable Model:

Table II provides the empirical results obtained when the CRSP value-weighted index of stocks on the New York Stock Exchange is used as the stock-index portfolio. We limit attention to 1951:4 to 1989:12, which is the post-Treasury Accord period. The estimated value of the slope coefficient for the risk-free rate in equation (5) for expected excess return is -2.31 ( $t = -3.42$ ). The estimated value of the slope coefficient for the risk-free rate in the variance equation given by (6) is 0.18 ( $t = 2.74$ ).

The  $t$ -statistics were computed using the procedures suggested by Newey and West (1987). Since there is substantial persistence in the residuals of equation (6), we report the  $t$ -statistics corresponding to a lag length of 20. In this sample, the  $t$ -statistics decrease as the number of lags increases, and hence these  $t$ -statistics are relatively conservative when compared to the  $t$ -statistics obtained when this serial correlation is ignored. Note that the residuals in equations (5) and (6) can be serially correlated, since the econometrician's information set can be strictly smaller than that of economic agents.

We also estimated the model by imposing the constraint that the slope coefficients in equation (5) are a scalar multiple of the slope coefficients in equation (6). The estimated value of the scalar,  $\beta$ , is -12.75 ( $t = 2.43$ ). With this restriction are two over-identifying restrictions. The null hypothesis that the over-identifying restrictions are not binding leads to a chi-square (D.F. = 2) value of 3.22 with an associated  $p$ -value of 0.20. Hence, based on these results, we can not reject the hypothesis that there is a negative relation

between the conditional mean and conditional variance of the excess return on stocks.

The natural question that arises at this stage is why the findings reported by French et. al. (1987) for the standard GARCH-M model are different than the conclusions in this section. We address this issue in the next section.

#### B. Modified GARCH-M models:

Table III, Panels A and B presents the estimates for Models 1 through 7. These results, and the accompanying specification tests, show that significant asymmetry exists in the conditional variance equation when predicted variance is also conditioned on the t-bill rate.

A comparison of Model 1 and Model 2 illustrates the restrictive nature of the standard GARCH-M specification for the conditional variance equation. Model 1 presents the results for a standard GARCH-M model. Both positive and negative innovation to excess returns results in an upward revision of the conditional variance ( $g_1$  is positive). Also, time periods with relatively large variances are associated with relatively larger returns on average ( $a_1$  is positive). However, the association is weak and not statistically significant at conventional levels.

These relations change as soon as positive and negative unanticipated returns are allowed to have different effects on conditional variance. Model 2 allows for such a difference by estimating the parameter  $g_2$ . A simple specification test comparing Model 1 and Model 2 shows that the standard GARCH-M model is too restrictive. If the parameters that the two models share are compared, using a generalized specification test, computed using robust standard errors, the value of the test statistic is 12.829.<sup>12</sup> Under the null hypothesis that Model 1 is correctly specified, this test statistic should be asymptotically distributed as a  $\chi^2_3$  random variable. Thus, we can reject the null hypothesis at the five-percent level.

Chart 1 shows the relation between the conditional variance,  $h_t$ , and the unanticipated excess return,  $\eta_t$ , when all other variables that appear in the conditional variance equation are set equal to their sample average. This is the news-impact curve developed by Engle and Ng (1991). Note that in Model 2 an



unexpected negative return sharply increases conditional variance of the next period excess return, while an unexpected positive return decreases conditional variance. Model 1 does not allow for such a possibility.

There is another important difference in the estimated variance equations between Model 1 and Model 2. Table III, Panel B also shows that for Model 2, the estimated persistence of variance from one period to the next, as measured by the first-order autoregressive coefficient for  $h_t$  is smaller than it is for Model 1.

Even though Model 2 seems less restrictive than Model 1, there are two reasons that we should not be satisfied with it. First, the robust standard error suggests that the coefficient  $g_2$  is imprecisely estimated. In fact, it is not significantly different from zero. Second, Model 2 does little better than Model 1 in any of the diagnostic tests.

Models 3, 4 and 5 attempt to solve the deficiencies in Model 2 by including the effect of the risk-free interest rate and deterministic seasonals on conditional variance. Each of these models results in statistically significant asymmetry in the conditional variance equation (i.e.,  $g_2$  is not equal to zero).

In Model 3, the risk-free rate is included as an explanatory variable in the conditional variance equation in Model 2. Note that  $a_1$  and  $g_2$  both become statistically significant, once the risk-free rate enters into the conditional variance equation. However, the coefficient on the risk-free rate itself,  $b_1$ , is not significant. And Table III, Panel B shows that excess skewness and kurtosis remain quite severe after the risk-free rate is included.

In Model 4, deterministic seasonals are added to Model 2. The test statistic for the hypothesis that  $\lambda_1 = \lambda_2 = 0$  is 6.38. Under the null, this statistic should be asymptotically distributed as a  $\chi^2_2$  random variable. Thus, we can reject the hypothesis that  $\lambda_1 = \lambda_2 = 0$  at the five-percent level. There are three other important characteristics of this model worth noting. First, with the inclusion of deterministic seasonals, both  $g_1$  and  $g_2$  are statistically significant. Second, this method of modeling seasonals in variance greatly reduces the excess kurtosis in the residuals. Finally, note that the amount of persistence in the conditional variance, as measured by the first-order autoregressive coefficient for  $h_t$  is much smaller than in any of the previous models.

Model 5 adds both the risk-free rate and deterministic seasonals to Model 2. As in Model 4, both  $g_1$  and  $g_2$  are significantly different from zero. Unlike in Model 3,  $b_2$ , the coefficient on the risk-free rate is significantly different from zero. Note that the serial correlation in the estimated conditional variances is much smaller than for the standard GARCH-M model, Model 1.

Table III, Panel B shows that the level of excess skewness and kurtosis have been significantly reduced—although the null hypothesis of no excess skewness or kurtosis can be rejected at the five-percent level. Model 5 is also the first model that does not fail the Sign-Bias Test at the five-percent level. In addition, it is the first model not to fail the joint test of sign bias, negative-size bias, and positive-size bias at the five-percent level. These diagnostics suggest that Model 5 is the most satisfactory model considered thus far.

Despite the success of Model 5, it is still quite fragile. We attempted to check the robustness of the specification by adding  $v_{t-1}^{1/2}$  to the mean equation. Even with great effort, we were not able to get the parameter estimates from that model to converge. Models 6 and 7 show the effects of adding  $v_{t-1}^{1/2}$  to Models 2 and 4, respectively. In neither case was the coefficient on  $v_{t-1}^{1/2}$  statistically significant. Note also that the estimated coefficients in the conditional variance equation are relatively close in all of these models.

We therefore come to the following three conclusions from our examination of the eight different GARCH-M specifications.

- (a) The relation between conditional mean and conditional variance is negative, and statistically significant.
- (b) The risk free rate contains information about future volatility, within the Modified GARCH-M framework.
- (c) The October and January seasonals in volatility are statistically significant.
- (d) Conditional volatility of the monthly excess return is not highly persistent as reported in other studies.

Because even the best of these models displays excess skewness and kurtosis, we also estimated different EGARCH-M models. The estimates for Models 1-L to 7-L are shown in Table IV, Panels A and B. Models 1-L and 2-L are based on the EGARCH-M model proposed by Nelson (1991). However, the results in these models, using monthly data, are quite different from those found by Nelson. Note that  $g_2$ , the coefficient detecting asymmetry in the conditional variance equation, has a very small t-statistic. Table VI shows that both Model 1-L and Model 2-L display excess skewness and kurtosis, and that both fail the Sign-Bias Test. Note also that the first-order serial correlation of the  $h_t$ 's in both models is quite low.

Unlike in the level models, the coefficient on  $g_2$  remains insignificant, regardless of which additional variables were included in the conditional variance equation. As a result, we do not report these regressions. Instead, we try to address the deficiencies in Model 1-L in Models 3-L, 4-L and 5-L by including the effect of the risk-free interest rate and deterministic seasonals on conditional variance. Note that each of these models, Models 3-L to 5-L, impose the restriction that  $g_2=0$ .

Model 3-L adds the risk-free rate to the conditional variance equation in Model 1-L. Unlike in the Model 3, coefficient on the risk-free rate has a large t-statistic, even without deterministic seasonals. However, excess skewness and kurtosis are still a problem in this model. Note that there is no significant sign bias in this model. In fact, none of the Engle-Ng tests show any evidence of misspecification in this model.

Model 4-L adds deterministic seasonals to Model 1-L, using the same method adopted for the level models. The test statistic for the hypothesis that  $\lambda_1=\lambda_2=0$  is 7.66. Under the null, this statistic should be asymptotically distributed as a  $\chi_2^2$  random variable. Thus, we can reject the hypothesis that  $\lambda_1=\lambda_2=0$  at the five-percent level. Although the amount of excess kurtosis is much lower for Model 4-L than for any of the previous models, we can still reject the hypothesis of no excess kurtosis at the five-percent level. Note also, that Model 4-L appears to have sign bias.

Model 5-L adds both the risk-free interest rate and deterministic seasonals to Model 1-L. The coefficients on all of those terms are statistically significant. The test statistic for the hypothesis that  $\lambda_1=\lambda_2=0$  is 7.31, while the test statistic for the hypothesis that  $\lambda_1=\lambda_2=b_2=0$  is 15.53. Thus, we can reject both hypotheses at the five-percent level. Table VI shows that we cannot reject the hypothesis that there is no excess kurtosis in the estimated residuals from Model 5-L. Model 5-L also shows no signs of sign bias, negative-size bias, or positive-size bias.

Since Model 5-L passed more of the diagnostic tests than any other model, it is our preferred specification. However, we also estimated Model 6-L, by adding  $v_{t-1}^{1/2}$  to the conditional mean equation. The coefficient on  $v_{t-1}^{1/2}$  is not statistically significant, and the results for the conditional variance equation are qualitatively the same as for Model 5-L. However, the diagnostic tests show that model 6-L performs worse than Model 5-L in some important ways. Model 6-L has a statistically significant amount of excess kurtosis, and it fails the Sign-Bias Test. This suggests there is little evidence of misspecification in Model 5-L, and that that model should be the preferred specification. Note also that the first-order serial correlation of  $h_t$  in Model 5-L is still relatively low: 0.338.<sup>13</sup>

Since the finding of low persistence of conditional variance is so different from results reported in the literature (except for Campbell and Hentschel 1991), it needs some explanation. One explanation can be seen by comparing Model 7-L to Model 2-L. The two models use exactly the same data and have exactly the same variables, but the parameter estimates from the two models are very different. The difference in the estimates arises from the use of different starting values for the parameters in the estimation procedure. That is this model has two local maxima, one of which is also the global maxima. Note that the log-likelihood of Model 2-L, which corresponds to the global maximum, is higher than that of Model 7-L.

Although it would seem logical to completely dismiss Model 7-L because it is only a local maximum, the differences between Model 7-L and Model 2-L should be noted, because it is much easier to get the EGARCH-M model to converge to the

parameter estimates shown for Model 7-L than to the ones shown for Model 2-L. This suggests that without careful attention, researchers could report incorrect results for EGARCH-M models in monthly data.

#### IV. Conclusion

There is a positive but insignificant relation between the conditional mean and conditional volatility of the excess return on stocks when the standard GARCH-M framework is used to model the stochastic volatility of stock returns. In this paper we empirically show that the standard GARCH-M model is misspecified. This misspecification arises from the fact that both positive and negative unanticipated returns of the same magnitude are forced to have the same impact on conditional variance in the standard GARCH-M model. When the model is modified to allow positive and negative unanticipated returns to have different impacts on the conditional variance, we find a negative relation between the conditional mean and the conditional variance of the excess return on stocks. This relation becomes stronger and statistically significant when conditional variance is allowed have deterministic monthly seasonals and depend on the nominally risk-free interest rate. Hence our results are consistent with the negative relation between volatility and expected return reported in Fama and Schwert (1977), Campbell (1987), Breen, Glosten and Jagannathan (1989), and Harvey (1991). We show that our conclusions do not change when we use Nelson's EGARCH-M model, which also allows for such a possibility.

We also find that the time series properties of monthly excess returns are substantially different from the reported properties of daily excess returns. First, persistence of conditional variance in excess returns is quite low in monthly data. Second, positive and negative unexpected returns have vastly different effects on future conditional variance. These results differ substantially from previous results in the literature. We also show that misspecifying the conditional variance equation can cause errors in analyzing the relation between conditional variance and future expected return.

## Footnotes

<sup>1</sup>To our knowledge, Backus and Gregory (1992) were the first to show, using an equilibrium model, that the risk premium on the market portfolio of all assets could be relatively lower during relatively riskier times.

<sup>2</sup>See Schwert (1989) for a careful and extensive documentation of the nature of the comovement between conditional volatility of monthly stock returns and the level and volatility of several interesting economic variables.

<sup>3</sup>For example, Campbell and Hentschel (1991) and French, Schwert, and Stambaugh (1987) concluded that the data are consistent with a positive relation between conditional expected excess return and conditional variance, whereas Fama and Schwert (1977); Campbell (1987); Pagan and Hong (1988); Breen, Glosten, and Jagannathan (1989); Turner, Startz, and Nelson (1989); and Nelson (1991) found a negative relation. Chan, Karolyi and Stulz (1992) find no relation. Harvey (1989) provides empirical evidence suggesting that there may be some time variation in the relation between risk and return. Since the Modified GARCH models we use allow us to parameterize the conditional variance more flexibly than Harvey's model, our model may capture the relation between night return without time variation.

<sup>4</sup>For example, see Braun, Nelson and Sunier, using a bivariate E-GARCH model find that the conditional mean of the industry portfolio returns to be highly persistent. The differences between our results and those reported in the literature could also be due to fact that the sample period studied are different.

<sup>5</sup>A careful examination of the data in Lakonishok and Smidt (1988) suggests that the monthly seasonal patterns in volatility is unlikely to be captured adequately by treating months other than October and January as being similar. However, our objective in this paper is limited to showing how to model seasonals in a way different than what has been done in the literature. Characterizing the nature of the monthly patterns in volatility is left as an exercise for the future.

<sup>6</sup>For example, the following papers document deterministic temporal patterns in conditional moments of stock returns. The relatively high average January return was documented in Keim (1985). The relatively low variance during nontrading hours was documented in French and Roll (1986). The relatively large volatility of stock returns in October was documented in Lakonishok and Smidt (1988). Seasonal patterns are present in returns to portfolios constructed using futures contracts as well. For example, McCurdy and Morgan (1989) documented day of the week effect in foreign currency futures.

<sup>7</sup>"That frightening month is here again," by Laurie Cohen, *The Wall Street Journal*, October 6, 1992, page C1, column 4.

<sup>8</sup>It is not true however that increased future volatility is the cause of lower returns today. Note also that this model assumes no contemporaneous relation between  $R(r_t)$  and  $\text{Var}_{t-1}(R_t)$ .

<sup>9</sup>Other researchers have also considered the use of nominal interest rates in the law of motion for variance. For example, see Giovannini and Jorion (1989).

<sup>10</sup>Potential negative values for the constructed conditional variances are not the only possible reason for using the log specification. It may also be true that the log model simply models the true conditional variance better than the level model. For more on this issue, see Engle and Ng (1991).

<sup>11</sup>Since we use dummy variables which take the value of one or zero, it may appear as though we may be violating the differentiability assumptions underlying the derivation of the robust standard errors. Note, however, that since the dummy variables are multiplied by the corresponding squared innovations, the differentiability conditions will be satisfied for the modified GARCH-M models we consider. Although, the differentiability conditions will be violated for the modified versions of Nelson's E-GARCH model we consider, this is unlikely to be an issue since points at which the differentiability assumptions are not satisfied will occur with zero probability, and the numerical derivatives we compute are always bounded.

<sup>12</sup>For more on generalized specification tests, see Newey (1985).

<sup>13</sup>We also tested the robustness of our conclusions in two other ways. First, we estimated Model 5-L with a sample ending in December 1986 to see whether the October 1987 stock-market crash had an undue influence on our estimates. Second, we estimated Model 5-L using equally-weighted returns. Both sets of results were qualitatively similar to those for Model 5-L.



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**TABLE I****SUMMARY STATISTICS FOR DATA RECORDED IN THE STANDARD PERIOD:****1951:4-1989:12**

	OCT.	JAN.	OTHER MONTHS
Number of Observations	39	38	388
<b>A: Continuously Compounded Monthly Return on the CRSP Value-Weighted Index of Status on the NYSE</b>			
mean (x100)	0.25	1.77	0.91
std. dev. (x100)	6.16	5.18	3.80
skewness	-1.35	0.21	-0.44
kurtosis	6.59	-0.41	0.64
<b>B: Continuously Compounded Monthly Return on Treasury Bills from Ibbotson &amp; Associates</b>			
mean (x100)	0.46	0.43	0.43
std. dev. (x100)	0.26	0.22	0.26
skewness	0.83	0.66	0.96
kurtosis	0.54	-0.03	0.90
<b>C: <math>x_t</math>; Excess of the Continuously Compounded Monthly Return on the CRSP Value-Weighted Index of Status on the NYSE over that on Treasury Bills</b>			
mean (x100)	-0.21	1.34	0.48
std. dev. (x100)	6.17	5.19	3.83
skewness	-1.40	0.18	-0.48
kurtosis	6.70	-0.43	0.65
mean/variance	0.56	0.50	3.28

**TABLE II**

Temporal Relation Between Risk And Return On The CRSP Value-Weighted Index of Status on The NYSE: Campbell's Instrumental Variable Approach. Period: 1951:4-1989:12

Model A

Mean Equation  $x_t = c_0 + c_1 OCT_t + c_2 JAN_t + c_3 r_{ft} + \epsilon_t$

Variance Equation  $\epsilon_t^2 = d_0 + d_1 OCT_t + d_2 JAN_t + d_3 r_{ft} + \zeta_t$

	$d_0$ $\times 10^4$	$d_1$ $\times 10^4$	$d_2$ $\times 10^4$	$d_3$	$c_0$ $\times 100$	$c_1$ $\times 100$	$c_2$ $\times 100$	$c_3$
Coefficient	7.25	21.68	13.11	0.18	1.48	-0.63	0.86	-2.31
t-statistic	2.70	1.35	2.25	2.74	5.21	-0.56	1.01	-3.42

Model B

Restricted Mean  $x_t = c_0 + \beta d_1 OCT_t + \beta d_2 JAN_t + \beta d_3 r_{ft} + \epsilon_t$  Equation

	$d_0$ $\times 10^4$	$d_1$ $\times 10^4$	$d_2$ $\times 10^4$	$d_3$	$c_0$ $\times 100$	$\beta$	ChisQ(2)	p-value
Coefficient	8.25	9.23	4.19	0.15	1.44	-12.75	3.22	0.20
t-statistic	3.49	0.87	1.42	2.90	5.02	-2.43		

- Notes:
1.  $x_t$  is the differential between the continuously compounded monthly return on the CRSP value weighted index of status on the NYSE and the continuously compounded monthly return on treasury bills from Ibbotson & Associates.
  2. t-statistics were computed, allowing for conditional heteroskedasticity and using the procedures in Hansen (1982). Although the t-statistics decline at first with the number of lags used in computing the covariance matrix, they stabilize after about 10 lags. The reported t-statistics are for 20 lags.

**TABLE III**

**PANEL A**

Temporal Relation Between Risk And Return On The CRSP Value-Weighted Index of Status On The NYSE,  $x_t$ : Modified GARCH-M models. Period: 1951:4-1989:12

$$x_t = a_0 + a_1 v_{t-1} + a_2 v_{t-1}^{1/2} + \epsilon_t; \epsilon_t = (1 + \lambda_1 \text{OCT} + \lambda_2 \text{JAN}) \eta_t$$

$$v_{t-1} = \text{Var}_{t-1}(\epsilon_t) \quad h_{t-1} = \text{Var}_{t-1}(\eta_t)$$

$$I_{t-1} = 1 \text{ if } \eta_{t-1} > 0 \text{ and } 0 \text{ otherwise}$$

$$h_{t-1} = b_0 + b_1 h_{t-2} + b_2 r_{t-1} + g_1 \eta_{t-1}^2 + g_2 \eta_{t-1}^2 I_{t-1} + g_3 \eta_{t-2}^2 + g_4 \eta_{t-2}^2 I_{t-2}$$

	MODEL 1	MODEL 2	MODEL 3	MODEL 4	MODEL 5	MODEL 6	MODEL 7
<b>a0</b> (x100)	-0.453 [-0.575]	1.064 [1.947]	1.850 [4.232]	1.071 [2.398]	1.854 [4.419]	5.730 [2.064]	7.782 [2.289]
<b>a1</b>	5.926 [1.307]	-2.843 [-0.878]	-7.625 [-2.621]	-3.165 [-1.131]	-8.019 [-2.828]	16.893 [1.163]	30.349 [1.652]
<b>b0</b> (x100)	0.016 [2.058]	0.074 [1.653]	0.035 [1.595]	0.026 [2.467]	0.030 [2.477]	0.059 [1.902]	0.018 [3.295]
<b>b1</b>	0.842 [16.758]	0.483 [1.572]	0.334 [0.824]	0.769 [8.801]	0.506 [2.927]	0.623 [2.830]	0.848 [17.299]
<b>b2</b>			0.159 [1.433]		0.078 [2.221]		
<b>g1</b>	0.070 [2.541]	0.257 [1.709]	0.188 [2.109]	0.153 [2.590]	0.177 [2.268]	0.161 [1.502]	0.121 [2.734]
<b>g2</b>		-0.340 [-2.270]	-0.248 [-2.602]	-0.227 [-3.570]	-0.252 [-3.272]	-0.267 [-2.683]	-0.207 [-3.749]
<b>λ1</b>				0.677 [3.638]	0.454 [3.120]		0.606 [3.600]
<b>λ2</b>				0.269 [1.816]	1.254 [1.795]		0.306 [1.936]
<b>a2</b>						-1.983 [-1.576]	-3.123 [-1.947]
Log Like	1248.202	1252.917	1266.288	1268.422	1276.518	1254.276	1271.824

Note: Robust t-statistics are in brackets.

TABLE III

PANEL B  
Diagnostic Tests

	Model 1	Model 2	Model 3	Model 4	Model 5	Model 6	Model 7
Skewness	-.781	-.701	-.463	-.455	-.289	-.709	-.478
t-stat	-6.855	-6.157	-4.065	-3.992	-2.532	-6.225	-4.190
Kurtosis	3.177	3.359	1.918	.927	.557	3.527	1.154
t-stat	13.880	14.675	8.380	4.052	2.431	15.408	5.040
Signif. Bias	.745	.988	.776	.710	.429	.841	.673
t-stat	2.280	2.992	2.756	2.932	1.895	2.515	2.613
Neg. Size Bias	.013	8.117	5.758	5.539	3.861	4.243	5.477
t-stat	.0028	1.612	1.329	1.483	1.052	.837	1.389
Pos. Size Bias	-3.866	2.138	1.342	.015	-1.220	1.988	-.826
t-stat	-.596	.345	.256	.0035	-.314	.316	-.166
Joint Test	16.797	11.854	10.851	13.050	6.459	9.415	11.997
sign. level	.0008	.0079	.013	.0045	.091	.024	.0074
AR(1) Coef. of de- seasonalized conditional variance	.897	.494	.609	.265	.374	.677	.355

Note: See Section II.C.2 for a description of the diagnostic tests.



**TABLE IV**

**PANEL A**

Temporal Relation Between Risk And Return On The CRSP Value-Weighted Index of Status on The NYSE,  $x_t$ : Modified EGARCH-M models. Period: 1951:4-1989:12

$$x_t = a_0 + a_1 v_{t-1} + a_2 v_{t-1}^{1/2} + \epsilon_t \quad \epsilon_t = (1 + \lambda_1 \text{OCT} + \lambda_2 \text{JAN}) \eta_t \quad v_{t-1} = \text{Var}_{t-1}(\epsilon_t)$$

$$h_{t-1} = \text{Var}_{t-1}(\eta_t) \quad H_{t-1} = \log(h_{t-1}) \quad I_{t-1} = 1 \text{ if } \eta_{t-1} > 0 \text{ and } 0 \text{ otherwise}$$

$$H_{t-1} = b_0 + b_1 H_{t-2} + b_2 r_t + g_1(\eta_{t-1}/\sqrt{h_{t-2}}) + g_2(\eta_{t-1}/\sqrt{h_{t-2}})I_{t-1} + g_3(\eta_{t-2}/\sqrt{h_{t-3}}) + g_4(\eta_{t-2}/\sqrt{h_{t-3}})I_{t-2}$$

	MODEL 1-L	MODEL 2-L	MODEL 3-L	MODEL 4-L	MODEL 5-L	MODEL 6-L	MODEL 7-L
a0	1.195	1.199	1.604	1.097	1.536	3.890	0.381
(x100)	[2.932]	[2.290]	[4.176]	[2.657]	[4.158]	[1.231]	[0.471]
a1	-3.947	-4.022	-6.387	-3.486	-6.119	12.157	0.791
	[-1.532]	[-1.537]	[-2.483]	[-1.305]	[-2.426]	[0.707]	{0.165}
a2						-1.366	
						[-0.908]	
b0	-5.583	-5.567	-5.728	-5.035	-5.102	-4.479	-1.004
(x100)	[-6.476]	[-6.472]	[-5.719]	[-4.513]	[-4.390]	[-2.654]	[1.671]
b1	0.133	0.133	0.183	0.235	0.281	0.321	0.861
	[0.999]	[0.992]	[1.271]	[1.386]	[1.701]	[1.248]	[9.010]
b2			99.600		81.002		
			[3.624]		[3.127]		
g1	-0.456	-0.427	-0.383	-0.378	-0.338	-0.355	-0.269
	[-6.448]	[-3.599]	[-5.195]	[-5.249]	[-4.811]	[-3.738]	[-3.077]
g2		-0.052					0.286
		[-0.274]					[2.352]
l1				0.426	0.349	0.451	
				[2.573]	[2.392]	[2.579]	
l2				0.318	0.299	0.346	
				[2.027]	[2.035]	[2.017]	
Log Like	1262.937	1262.274	1272.081	1270.970	1279.081	1271.538	1255.802

Note: Robust t-statistics are in brackets.

TABLE IV

PANEL B

Diagnostic Tests

	Model 1-L	Model 2-L	Model 3-L	Model 4-L	Model 5-L	Model 6-L	Model 7-L
Skewness	-.484	-.491	-.400	-.416	-.337	-.436	-.874
t-stat	-4.249	-4.306	-3.506	-3.654	-2.954	-3.825	-7.668
Kurtosis	1.372	1.409	1.095	.620	.441	.722	4.186
t-stat	5.995	6.157	4.786	2.710	1.925	3.153	18.287
Signif. Bias	.555	.577	.364	.507	.409	.549	.682
t-stat	2.109	2.180	1.446	2.183	1.833	2.311	1.913
Neg. Size Bias	4.111	3.794	3.244	3.676	3.575	4.342	4.804
t-stat	1.004	.919	.816	1.018	1.011	1.192	.905
Pos. Size Bias	7.469	8.257	3.054	5.982	3.449	4.910	-4.637
t-stat	1.535	1.693	.660	1.409	.861	1.103	-.678
Joint Test	4.632	5.074	2.139	4.813	2.407	5.408	9.515
sign. level	.201	.166	.544	.186	.333	.144	.023
AR(1) Coef. of de- seasonalized conditional variance	.078	.089	.430	.062	.335	.102	.828

Note: See Section II.C.2 for a description of the diagnostic tests.

# CHART 1

*News Impact Curve--Level Models*

