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The Evolution of Debt and Equity
Markets in Economic Development

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Introduction

With some exceptions, it generally seems to be the case that economies with high levels of per capita output also have active equity markets. In addition, during historical periods where there was little government interference with the operation of financial markets, many economies experienced a large increase in equity market activity as a concomitant of strong real growth. According to Michie (1987), for example, about 25 percent of capital investment in the United Kingdom was financed by issuing equity in 1853. By 1913, that figure had risen to 33 percent. For the United States, Gurley and Shaw (1967, p. 259) report that the "stock of primary securities more than doubled relative to national wealth and income" in the nineteenth century, although it has remained relatively constant thereafter.

Cross-country comparisons indicate a similar relationship between equity market activity and levels of real development. Antje and Jovanovic (1992, Table 2), Demirguc-Kunt and Levine (1993, Table 4), and Levine and Zervos (1994) all report a positive correlation between the ratio of stock market capitalization to real output, and real output per capita.[1] Gurley and Shaw (1967, p. 259) argue that "primary securities issues approximate 1 or 2 percent of gross national product in the poorer countries. The ratio generally lies within the range of 10 to 15 percent in wealthier countries." These assessments seem consistent with the data in Table 1; while there are some very significant exceptions there, strong equity market activity is typically associated with high levels of economic development.

These kinds of observations have led many, including Gurley and Shaw (1955, 1960, 1967), to argue that the evolution of more sophisticated financial markets is an essential aspect of the process of economic growth. For example, Gurley and Shaw (1955, p. 515) assert that "development . . . is accompanied, too, by the 'institutionalization of saving and investment' that diversifies

channels for the flow of loanable funds and multiplies varieties of financial claims." But why should this be the case? We believe that the answer to this question is not currently well understood.

There has been significant progress in developing a better theoretical understanding of how financial markets contribute to the growth process. However, typical contributions to the literature have the feature that all external finance takes the form of either debt (typically bank loans; see for instance Greenwood and Jovanovic 1990, Bencivenga and Smith 1991, or Cooley and Smith 1992) or equity (Levine 1991 and Bencivenga, Smith, and Starr 1994a,b,c) but not both. Our objective in this paper is to present a framework in which capital formation is financed by issuing both debt and equity. We then use it to understand how the amount of activity in these markets influences—and is influenced by—the process of economic development. In doing so, we also consider a number of related issues. One of these is whether debt and equity markets are substitutes or complements for financing investment. In addition, we examine how choices among alternative kinds of investment opportunities change as equity markets evolve. And, we will also take up the topic of whether—and in what sense—the endogenous evolution of financial market activity as an economy grows supplies that economy with a more efficient set of capital markets.

In order to consider these issues, we need a model which allows for growth and capital accumulation. In addition, we need a model in which capital investment is financed with external debt and equity, and in which the development of debt and equity markets is related to the composition of capital investments. We, therefore, examine an economy in which this investment is undertaken by a set of agents who require external finance, and in which investment is subject to a standard costly state verification (CSV) problem. The CSV problem in our economy is very similar to that in the original CSV literature (for example, Townsend 1979, Diamond 1984, Gale and Hellwig 1985, and Williamson 1986, 1987), with one exception. In the traditional CSV environment, investors have access to only a single kind of investment opportunity. We, on the

other hand, give investors access to two kinds of investment technologies, both of which display stochastic constant returns to scale. One yields a return which is freely observable only by the initiating investor. The other yields a return which is publicly observable. Under the assumption that the expected return on the former technology exceeds that on the latter (gross of verification costs), agents undertaking capital investments face a trade-off. The technology with the unobservable return is (on average) more productive, but is also subject to a CSV problem. The less productive project can therefore be used to reduce verification costs. We allow investors to choose any convex combination of the two technologies.

A static version of this kind of economy is considered in detail by Boyd and Smith (1994d). They demonstrate that the optimal composition of firm investment depends on the relative expected returns on the two investment technologies, and on the cost of state verification. Higher verification costs, ceteris paribus, favor the increased use of the observable return technology. Moreover, as in the conventional CSV literature, Boyd and Smith (1994d) show that the use of the unobservable return technology is associated with debt finance. The use of the observable return technology, on the other hand, enables equity finance to be employed. Hence, under certain conditions, a firm will have a determinate optimal debt-equity ratio, which depends on the composition of its investments, on the cost of state verification, and on the distribution of returns on both kinds of investments.

In this paper we embed the model of Boyd and Smith (1994d) in the neoclassical growth model of Diamond (1965). Since the latter is a two period, overlapping generations model, all of the results regarding firm investment choices and the optimal debt-equity ratio can be carried over intact from the static environment into the growth context.

Under the assumption that state verification costs are born in the form of final goods and services,[2] we obtain the following results. As an economy moves along a growth path and accumulates capital, the relative price of capital will fall. Since investment projects produce capital

while state verification consumes final goods and services, the implication is that investors will see *relative* monitoring costs that rise as an economy develops. As a consequence, under weak conditions investors will tend to employ the observable return capital production technology more and more intensively as an economy grows. Since the use of the observable return technology is associated with equity issues, it will therefore often be the case that economic growth is accompanied by an increased volume of equity market activity, and a falling (aggregate) debt-equity ratio.[3] Indeed, we produce examples of economies where, at low levels of development, only the unobservable return technology is employed and (as in the standard CSV literature) only debt finance is used. However, as the economy grows, relative monitoring costs increase, and eventually the observable return technology comes into use to economize on verification. When this occurs, the economy will start to observe a positive level of activity in equity markets. In an economy of this type, equity market activity will not occur at all—for fully endogenous reasons—until the economy attains a critical level of development. Indeed, it is possible to produce examples of economies that experience no equity market activity at all until a critical level of development is reached, but that have the property that once equity market activity begins, its volume is very large. Section 6 illustrates various possibilities with respect to the evolution of equity market activity over time.

Our analysis suggests that a typical pattern of development will be that increases in the level of per capita output are associated with a greater volume of equity market activity, a lower (aggregate) ratio of debt-to-equity, and a composition of investment that more heavily favors the observable return technology. Thus, in less developed economies, investment projects with high expected returns (not inclusive of verification costs) will be relatively heavily utilized. More mature economies will tend to make greater use of lower expected return projects, but ones that demand less frequent verification.

The last observation has the following implication: as economies develop, fewer resources may be consumed by the necessity of state verification.[4] Thus, the severity of the CSV problem—as measured by the cost in final goods and services of its presence—will often be smaller for more than for less developed economies. It is often argued (for example, by McKinnon 1973 and Shaw 1973) that less developed economies are less developed because their financial market frictions are more severe than those of their more developed counterparts. In most of our examples, economies that are "farther along their growth paths" lose fewer resources due to the presence of the CSV problem than do less mature economies. In our model this is an endogenous outcome; financial market frictions can be expected to become less severe as a natural consequence of the development process. In this sense the natural evolution of financial markets as an economy develops does tend to create more efficient capital markets.[5]

Finally, our analysis bears on the issue of whether debt and equity markets are substitutes or complements. By issuing some equity, firms can make it cheaper to issue debt. This fact makes it possible for individual firms, and society, to economically employ high return capital production technologies whose use might otherwise not be feasible. Moreover, our analysis provides a sense in which debt and equity markets are likely to become more complementary as economies become more highly developed. This point is illustrated by example in Section 6.

The remainder of the paper proceeds as follows. Section 1 describes the model, and the nature of trade in factor markets. Section 2 analyzes firm investment and financing decisions. Section 3 demonstrates how an optimal set of capital investments can be financed by issuing a determinate combination of debt and equity. Section 4 considers the evolution of the capital stock and of financial markets over time in a general equilibrium, while some special topics are taken up in Section 5. Section 6 presents several numerical examples illustrating our results, and Section 7 concludes.

1. The Model

A. Environment

We consider an economy populated by an infinite sequence of two-period lived, overlapping generations, plus an initial old generation. Each generation consists of a continuum of agents with unit mass, and each young generation is identical in its composition.

In particular, agents in each young generation are divided into two types, which we term borrowers and lenders. All borrowers (lenders) are identical, ex ante, and borrowers (lenders) constitute a fraction $\alpha \in (0,1)$ $(1-\alpha)$ of the population. Lenders are endowed with one unit of labor when young, which they supply inelastically, and are retired when old. Borrowers are endowed with no labor, but they are endowed with access to individual-specific, high return investment projects, which are described below. All young agents, both borrowers and lenders, are assumed to be risk neutral and to care only about old period consumption. Thus all young period income is saved (invested).[6]

There is a single consumption good at each date, which is produced according to a standard, commonly available constant returns to scale production function with capital and labor as inputs. In particular, if K_t is the time t capital stock and L_t is time t labor input, then production of the final good is given by $F(K_t, L_t)$. In addition, if $k_t \equiv K_t/L_t$ is the capital-labor ratio at t, then $f(k_t) \equiv F(k_t, 1)$ is the intensive production function. We maintain standard assumptions on f; that is $f(0) \geq 0$, $f'(k) > 0 > f''(k)$ for all $k \geq 0$, and that f satisfies the usual Inada conditions.

Capital is produced at each date from the consumption good using one or more of the following three technologies. First, there is a commonly available, nonstochastic linear technology whereby one unit of current output invested at t yields $r > 0$ units of capital at $t + 1$. In addition, there are two stochastic linear technologies that convert current output into future capital. Technology o (for observable) produces y units of capital at $t + 1$ per unit invested at t, where y

is an iid (across agents and across time periods) random variable which is realized at $t + 1$. We assume that $y \in \{y_1, y_2, \ldots, y_N\}$, and we let $p_n \equiv \mathrm{prob}(y = y_n)$. Obviously $0 \le p_n \le 1$ for all n, and $\Sigma p_n = 1$. Finally, we assume that for any investor the amount of capital yielded by investments in technology o is publicly observable (at zero cost).

Technology u (for unobservable) is assumed to produce w units of capital at $t + 1$ per unit invested at t. w is a continuous, iid (across investors and periods) random variable with cdf G and pdf g. g is assumed to be continuously differentiable, and to have support $[0, \bar{w}]$. In addition, the return on investments in technology u can be observed (by any agent other than the initiating investor) only by bearing a fixed cost of $\gamma > 0$ units of the current consumption good. Thus a standard costly state verification (CSV) problem arises for investments in the technology u.

We assume that only borrowers are endowed with access to the investment technologies o and u. Ownership of these investment opportunities cannot be traded. Moreover, as is typical in CSV models (with fixed verification costs) we will want to impose an upper bound on the scale at which any borrower can operate the investment technologies. Thus we let i_t^o (i_t^u) be investment in technology o(u) by a representative borrower at t, and we let $i_t \equiv i_t^o + i_t^u$. Then each borrower faces the maximum scale of operation constraint $i_t \le q$, where q is an exogenously given parameter. Finally, we also assume that $i_t^o \ge 0$ and $i_t^u \ge 0$ must hold. For future reference, we define $\theta_t \equiv i_t^o/i_t$ to be the fraction of total investment done in technology o at t by a represent borrower. Clearly $\theta_t \in [0,1]$ must hold.

Define

$$\hat{y} \equiv \sum_n p_n y_n$$

to be the expected gross return (in units of capital) on investments in technology o, and

$$\hat{w} \equiv \int_0^{\bar{w}} wg(w)dw$$

to be the expected gross return (in units of capital), not inclusive of verification costs, on investments in technology u. We assume that

(a.1) $\hat{w} > \hat{y} > r.$

Thus the commonly available technology is relatively unproductive.

Once capital is received, we assume that it is used in production and that it then depreciates completely. This assumption prevents us from having to worry about capital resale markets. We also assume that the initial old agents are endowed with $K_0 > 0$ units of capital, per capita. No agents thereafter are endowed with either capital, or the consumption good.

Since borrowers have no young period income, they must obviously rely on external finance in order to fund their investments. Lenders can either provide this external finance to borrowers, or invest in the inferior, commonly available capital production technology, or both. We now describe how this trade occurs.

B. Trade

Two kinds of trade take place in this economy: capital and labor are rented in competitive factor markets, and funds are transferred from lenders to borrowers. Here we describe activity in factor markets; funds transfers are the subject of the next section.

The inherited capital stock (the proceeds of the previous period's investment) and labor are both supplied inelastically. Both factors are demanded by competitive producers, and hence are paid their marginal products. Thus if k_t is the aggregate capital-labor ratio at t, ω_t is the time t real wage rate, and ρ_t is the time t rental rate for capital, we have

(1) $\rho_t = f'(k_t)$

(2) $\omega_t = f(k_t) - k_t f'(k_t) \equiv \omega(k_t)$.

Clearly, $\omega'(k_t) > 0$ holds, for all k_t.

2. Financing of Investment

At this point we assume that borrowers obtain external funds by announcing general contracts specifying how repayments will occur in various contingencies. These contracts should be regarded as abstract objects; in Section 3 we describe how equilibrium contracts can be supported through the use of debt and equity markets. We also assume throughout that borrowers confront no credit rationing,[7] or in other words, that the supply of funds by lenders is always at least as great as the demand for funds by borrowers. The supply of funds by lenders (since lenders save their entire young period income) is $(1-\alpha)\omega_t$ at t, while the maximum demand for funds by borrowers is αq. Hence we assume that

(a.2) $(1-\alpha)\omega(k_t) \geq \alpha q$

for all "relevant" (see below) values of k_t. When (a.2) holds, any marginal savings must be invested in the commonly available technology, yielding r units of capital per unit invested. Thus the opportunity cost of funds at t is $r\rho_{t+1}$, since the rental value of the capital obtained at t + 1 is ρ_{t+1}. All agents are assumed to treat ρ_{t+1} parametrically.

A. Funding Contracts

A funding contract specifies a quantity of resources that will be transferred to a particular borrower (i_t), as well as how these resources will be allocated among technologies o and u (θ_t).[8] In addition, the contract specifies a set of state contingent repayments. Since the return y is observable, repayments can always be made contingent on it. However, since the return w can be observed externally only if monitoring (or verification) occurs, repayments can only meaningfully be made

contingent on w if state verification occurs. Hence a contact must specify a set of states $A_t(y)([0,\bar{w}] - A_t(y) \equiv B_t(y))$ in which monitoring will (will not) occur at $t + 1$; this set can obviously be conditioned on y. In monitoring states, repayment can be made contingent on w. We let $R_t(w,y)$ be the promised repayment at $t + 1$, in units of time $t + 1$ consumption and per unit borrowed, if $w \in A_t(y)$. In addition, we let $x_t(y)$ be the promised repayment at $t + 1$, in units of time $t + 1$ consumption and per unit borrowed, if $w \in B_t(y)$. Notice that we have restricted attention to nonstochastic monitoring.[9] Furthermore, we assume that the funding contract specifies a set of agents who are responsible for monitoring (if necessary), so that there is never a necessity for the duplication of monitoring effort.[10]

Funding contracts are assumed to be announced by borrowers, and such contracts are then either accepted or rejected by lenders. In order to avoid rejection, such contracts must satisfy a set of constraints, which we now describe.

First, contracts must be feasible or, in other words, specify nonnegative consumption levels for borrowers. Thus a borrower's repayment can never exceed his total available resources, or

(3.a) $R_t(w,y) \leq \rho_{t+1}[\theta_t y + (1-\theta_t)w]$

(3.b) $x_t(y) \leq \inf_{w \in B_t(y)} \rho_{t+1}[\theta_t y + (1-\theta_t)w]$

must hold. Note that ρ_{t+1} appears as it does in (3) since investment returns are in units of capital while repayments are in units of consumption. ρ_{t+1} gives the relative price of capital in terms of consumption. Second, contracts must be incentive compatible, so that borrowers must have an incentive to announce truthfully when a monitoring state has occurred. They will do so iff repayments are lower in monitoring than in nonmonitoring states. Therefore the incentive constraint takes the form

(4) $R_t(w,y) \leq x_t(y)$,   for all $w \in A_t(y)$.

Finally, since lenders can always invest in the commonly available technology, the expected repayment—per unit borrowed—must at least equal the opportunity cost of funds ($r\rho_{t+1}$) plus expected monitoring costs (which without loss of generality we assume are born by lenders). Thus contractual repayments must satisfy the expected return constraint

$$(5) \quad \left\{ \sum_n p_n \int_{w \in A_t(y_n)} R_t(w, y_n) g(w) dw + \sum_n p_n \int_{w \in B_t(y_n)} x_t(y_n) g(w) dw \right\} i_t$$

$$- \gamma \sum_n p_n \int_{w \in A_t(y_n)} g(w) dw \geq r\rho_{t+1}.$$

r is multiplied by ρ_{t+1} in (5) since r is in units of capital, which has a goods value of ρ_{t+1} at $t + 1$.

Borrowers announce funding contract terms to maximize their own expected utility, subject to the constraints (3)-(5). The expected utility of a borrower, as a function of contract terms, is simply the expected return on the borrower's investments, less the expected repayment implied by the contract. Thus borrowers choose contract terms to maximize

$$i_t \left\{ \rho_{t+1}[\theta_t \hat{y} + (1 - \theta_t) \hat{w}] - \sum_n p_n \int_{w \in A_t(y_n)} R_t(w, y_n) g(w) dw - \sum_n p_n \int_{w \in B_t(y_n)} x_t(y_n) g(w) dw \right\}$$

subject to (3)-(5).

Noting that (5) must hold with equality at an optimum, this problem can be transformed as follows. Substituting (5)—at equality—into the objective function yields the alternative maximand

$$i_t[\theta_t \hat{y} + (1 - \theta_t)\hat{w} - r]\rho_{t+1} - \gamma \sum_n p_n \int_{A_t(y_n)} g(w) dw$$

which the borrower seeks to maximize subject to (3)-(5). We now make several observations about the solution to the borrower's problem, which are summarized in the following lemma.

Lemma 1. The solution to the borrower's problem has

(a) $i_t = q$

(b) $A_t(y_n) = \left[0, \dfrac{[x_t(y_n)/\rho_{t+1}] - \theta_t y_n}{1 - \theta_t} \right)$

and

(c) $R_t(w, y_n) = \rho_{t+1}[\theta_t y_n + (1-\theta_t)w]$; $w \in A_t(y_n)$.

The Proof of Lemma 1 appears in Boyd .and Smith (1994d).[11] Lemma 1 asserts that borrowers

always invest up to their maximum scale of operation. In addition, they repay $x_t(y_n)$ if it is feasible

to do so, in which case no monitoring occurs. If it is infeasible to repay $x_t(y_n)$ (which occurs iff

$w \in A_t(y_n)$, as defined in (b)), then monitoring takes place. Part (c) of the lemma says that if

monitoring occurs, providers of external funding receive the entire net of monitoring cost proceeds

of the borrower's investments. All of these results mirror standard results in more traditional CSV

environments.

In order to further characterize optimal funding contracts, it will be useful to make the

following transformation. Define

(6). $z_{nt} \equiv \{[x_t(y_n)/\rho_{t+1}] - \theta_t y_n\}/(1-\theta_t)$; $n = 1, \ldots, N$.

Then Lemma 1 implies that $A_t(y_n) = [0, z_{nt})$ for all n, and in addition it implies that the borrowers'

problem can be reformulated as maximizing

$$\rho_{t+1}[\theta_t \hat{y} + (1-\theta_t)\hat{w} - r]q - \gamma \sum_n p_n G(z_{nt})$$

subject to

(7) $\displaystyle\sum_n p_n \rho_{t+1} \int_0^{z_{nt}} [\theta_t y_n + (1-\theta_t)w]g(w)dw + \sum_n p_n x_t(y_n)[1 - G(z_{nt})]$

$$- (\gamma/q)\sum_n p_n G(z_{nt}) = r\rho_{t+1}.$$

Rewriting equation (7) as

$$(7') \quad \rho_{t+1} \sum_n p_n \theta_t y_n G(z_{nt}) \; + \; \rho_{t+1}(1-\theta_t) \sum_n p_n \int_0^{z_{nt}} wg(w)dw$$

$$+ \; \sum_n p_n x_t(y_n)[1 \; - \; G(z_{nt})] \; - \; (\gamma/q) \sum_n p_n G(z_{nt}) \; = \; r\rho_{t+1}$$

and adding and subtracting $\Sigma p_n \theta_t y_n [1 - G(z_{nt})]$ from the left-hand side of (7'), we can reformulate

the borrower's problem as choosing $\theta_t \in [0,1]$ and $z_{nt} \in [0,\bar{w}]$, $n = 1, \dots, N$, to solve the problem

$$(P) \quad \max \; \theta_t \hat{y} \; + \; (1-\theta_t)\hat{w} \; - \; r \; - \; \psi_{t+1} \sum_n p_n G(z_{nt})$$

subject to

$$(8) \quad \theta_t \hat{y} \; + \; (1-\theta_t) \sum_n p_n \int_0^{z_{nt}} wg(w)dw \; + \; (1-\theta_t) \sum_n p_n z_{nt}[1 \; - \; G(z_{nt})] \; - \; \psi_{t+1} \sum_n p_n G(z_{nt}) \; = \; r$$

where $\psi_{t+1} \equiv \gamma/q\rho_{t+1}$ is the cost of state verification—measured in units of capital—per unit

invested.

In order to characterize the solution to the problem (P), it will be useful to have the following

result. The proof appears in Boyd and Smith (1994d).

Lemma 2. Define the function $\bar{z}(\theta;\psi)$ by

$$\bar{z}(\theta;\psi) \equiv \underset{z}{\mathrm{argmax}} \left\{ (1-\theta) \int_0^z wg(w)dw \; + \; (1-\theta)z[1 \; - \; G(z)] \; - \; \psi G(z) \right\}.$$

(a) The vector $(z_{1t}, z_{2t}, \dots, z_{Nt})$ solving (P) satisfies $z_{nt} \leq \bar{z}(\theta_t; \psi_{t+1})$, for all n. (b) $\bar{z}(\theta_t; \psi_{t+1}) \leq$

$\bar{z}(0; \psi_{t+1})$ holds, for all θ_t. (c) Therefore, $z_{nt} \leq \bar{z}(0; \psi_{t+1})$ holds, for all n.

Lemma 2 asserts that z_{nt} should never exceed the value of z that maximizes a lender's

expected return (conditional on a choice of θ_t). Since $\bar{z}_1(\theta;\psi) \leq 0$ holds, it follows that z_{nt} should

never be chosen to exceed the value of z that maximizes a lender's expected return, conditional on $\theta = 0$ holding.

Before characterizing the solution to (P), it will prove convenient to make assumptions implying that the borrower's objective function is a concave function of $(z_{1t}, z_{2t}, \ldots, z_{Nt})$, and similarly, that the constraint set is convex—at least for values of this vector that can conceivably be chosen by the borrower. In view of Lemma 2, the following assumption implies the satisfaction of both conditions for all "relevant" values of $(z_{1t}, z_{2t}, \ldots, z_{Nt})$:

(a.3) $g'(w) \geq 0$;   for all $w \in [0, \bar{z}(0; \psi_{t+1})]$.

Assumption (a.3) requires that g be nondecreasing for "small" values of w, where "small" means values less than the largest possible value of the expected return maximizing choice of z_{nt}.

In order to describe the solution to (P), it will be useful to proceed in two steps. First, we arbitrarily fix a value $\theta_t \in [0,1)$, and for that value of θ_t find the optimal choices $(z_{1t}, z_{2t}, \ldots, z_{Nt})$. Second, using these choices, we examine the problem of selecting $\theta_t \in [0,1]$ to maximize the expected utility of a borrower. Thus, in consonance with the first step, we fix θ_t and consider the problem

(P′) $\max \; \theta_t \hat{y} + (1 - \theta_t) \hat{w} - r - \psi_{t+1} \sum_n p_n G(z_{nt})$

subject to (8) and $z_{nt} \in [0, \bar{z}(0; \psi_{t+1})]$.[12] Letting μ_t denote the Lagrange multiplier associated with (8), and imposing (a.3), the following conditions are necessary and sufficient for an *interior* optimum:

(9) $g(z_{nt}) / \{ (1 - \theta_t)[1 - G(z_{nt})] - \psi_{t+1} g(z_{nt}) \} = \mu_t$; $n = 1, \ldots, N$.

Since (a.3) implies that the left-hand side of (9) is strictly increasing in z_{nt}, the N conditions in (9) imply that z_{nt} is independent of n; that is $z_{1t} = z_{2t} = \ldots = z_{Nt}$ holds.[13] Suppose we impose constancy of z_{nt} in (8), and define the function $z(\theta; \psi)$ by

(10) $\quad \theta \hat{y} + (1-\theta) \int_0^{z(\theta;\psi)} wg(w)dw + (1-\theta)z(\theta;\psi)\{1 - G[z(\theta;\psi)]\} - \psi G[z(\theta;\psi)] \equiv r$

and $z(\theta;\psi) \leq \bar{z}(\theta;\psi)$. Thus, $z(\theta_t;\psi_{t+1})$ is the common value of z_{nt}, given θ_t and ψ_{t+1}, that satisfies the expected return constraint at equality. Then (8) and (9) imply the following result:

(11) $\quad z_{nt} = z(\theta_t;\psi_{t+1}); \quad n = 1,\ldots,N.$

In light of the constancy of z_{nt}, suppose we define the function $\pi(z,\theta;\psi)$ by

(12) $\quad \pi(z,\theta;\psi) \equiv (1-\theta) \int_0^z wg(w)dw + (1-\theta)z[1-G(z)] - \psi G(z).$

Then the definition of $z(\theta;\psi)$ implies that that function satisfies the condition

(13) $\quad \pi[z(\theta;\psi),\theta;\psi] \equiv r - \theta \hat{y}.$

The function π, and the determination of $z(\theta;\psi)$, are depicted in Figure 1. Assumption (a.3) implies that π is a concave function of z for all $z \leq \bar{z}(0;\psi_{t+1})$, and

(a.4) $\quad \pi_1(0,\theta_t;\psi_{t+1}) > 0$

implies that π is increasing at $z = 0$. In addition, if equation (13) has more than one solution, the criterion of minimizing expected monitoring costs dictates that the smallest solution to (13) is the relevant one. Thus, since (by assumption) credit is not rationed,

(14) $\quad \pi_1[z(\theta_t;\psi_{t+1}),\theta_t;\psi_{t+1}] > 0$

holds, as does $z(\theta_t;\psi_{t+1}) < \bar{z}(\theta_t;\psi_{t+1})$.

We now turn our attention to a representative borrower's optimal choice of θ_t. Upon substituting (11) into the objective function in (P), we obtain the alternative maximand

$\theta_t \hat{y} + (1-\theta_t)\hat{w} - r - \psi_{t+1}G[z(\theta_t;\psi_{t+1})] \equiv H(\theta_t;\psi_{t+1}).$

Then the optimal choice for the composition of investment, which we denote by θ_t^*, satisfies

$\theta_t^* \equiv \underset{\theta \in [0,1]}{\text{argmax}} \ H(\theta;\psi_{t+1}) \equiv T(\psi_{t+1}).$

The next section describes the properties of the function T.

B. The Optimal Composition of Investment

We begin by stating conditions under which θ_t will and will not take on "extreme" values, and by describing what these extreme values are. Our first result is

PROPOSITION 1. (a) $T(\psi_{t+1}) \leq r/\hat{y} < 1$ holds, for all ψ_{t+1}. (b) $T(\psi_{t+1}) \in (0,r/\hat{y})$ holds if

(15) $\quad \hat{y} - \hat{w} - \psi_{t+1}g(0)z_1(r/\hat{y};\psi_{t+1}) < 0$

and

(16) $\quad \hat{y} - \hat{w} - \psi_{t+1}g[z(0;\psi_{t+1})]z_1(0;\psi_{t+1}) > 0.$

The Proof of Proposition 1 appears in Boyd and Smith (1994d). Part (a) of the proposition asserts that $\theta_t^* \leq r/\hat{y}$ always holds. It is easy to verify that $z(r/\hat{y};\psi_{t+1}) = 0$ for all ψ_{t+1}, and hence that there are no monitoring costs for $\theta_t \geq r/\hat{y}$. Since $\hat{w} > \hat{y}$, it is never optimal to increase θ_t beyond the point at which expected monitoring costs are driven to zero. Equations (15) and (16), of course, are just $H_1(r/\hat{y};\psi_{t+1}) < 0 < H_1(0;\psi_{t+1})$. Under the conditions stated, $H(\theta;\psi_{t+1})$ has the configuration depicted in Figure 2. It follows that $T(\psi_{t+1}) \in (0,r/\hat{y})$.[14] Straightforward differentiation of (13) establishes that equations (15) and (16) are equivalent to the conditions

(15′) $\quad (\hat{w}-\hat{y})[(\hat{y}-r)/\hat{y} - \psi_{t+1}g(0)] > \psi_{t+1}\hat{y}g(0)$

and

(16′) $\quad \{\hat{y} - r - \psi_{t+1}G[z(0;\psi_{t+1})]\}\psi_{t+1}g[z(0;\psi_{t+1})]$

$\quad\quad\quad > (\hat{w}-\hat{y})\{1 - G[z(0;\psi_{t+1})] - \psi_{t+1}g[z(0;\psi_{t+1})]\}.$

When (15) and (16) [or (15′) and (16′)] are satisfied, θ_t^* satisfies the first-order condition

(17) $\quad -\psi_{t+1}g[z(\theta_t^*;\psi_{t+1})]z_1(\theta_t^*;\psi_{t+1}) = \hat{w} - \hat{y}$

and the second-order condition

(18) $\quad -\psi_{t+1}g[z(\theta_t^*;\psi_{t+1})]z_{11}(\theta_t^*;\psi_{t+1}) - \psi_{t+1}g'[z(\theta_t^*;\psi_{t+1})][z_1(\theta_t^*;\psi_{t+1})]^2 < 0.$

Equation (17) implicitly defines the function $T(\psi_{t+1})$, while satisfaction of (18) implies that that function is differentiable. Implicitly differentiating (17), we obtain

(19) $\quad \{\psi g[z(\theta^*;\psi)]z_{11}(\theta^*;\psi) + \psi g'[z(\theta^*;\psi)][z_1(\theta^*;\psi)]^2\}T'(\psi)$

$$= -g[z(\theta^*;\psi)]z_1(\theta^*;\psi) - \psi g[z(\theta^*;\psi)]z_{12}(\theta^*;\psi) - \psi g'[z(\theta^*;\psi)]z_1(\theta^*;\psi)z_2(\theta^*;\psi).$$

Equation (17) implies that $z_1(\theta^*;\psi) < 0$ holds, and it is easy to verify that $z_2(\theta^*;\psi) > 0$ (see Boyd and Smith 1994d). Thus we have the following result.

PROPOSITION 2. $T'(\psi_{t+1}) > 0$ holds iff

(20) $\quad z_1(\theta_t^*;\psi_{t+1}) + \psi_{t+1}z_{12}(\theta_t^*;\psi_{t+1})$

$$+ \psi_{t+1}\{g'[z(\theta_t^*;\psi_{t+1})]/g[z(\theta_t^*;\psi_{t+1})]\}z_1(\theta_t^*;\psi_{t+1})z_2(\theta_t^*;\psi_{t+1}) > 0.$$

A sufficient condition for (20) is that $z_{12}(\theta_t^*;\psi_{t+1}) \leq 0$.

We now state a sufficient condition for $z_{12}(\theta_t^*;\psi_{t+1}) \leq 0$ to hold.

Lemma 3. $z_{12}(\theta_t^*;\psi_{t+1}) \leq 0$ holds if

$$(\hat{w}-\hat{y})\{(\hat{y}-r)/\hat{y} - \psi_{t+1}g[z(\theta_t^*;\psi_{t+1})]\} \geq \psi_{t+1}G[z(\theta_t^*;\psi_{t+1})]\{1 - G[z(\theta_t^*;\psi_{t+1})]\}.$$

The Proof of Lemma 3 is given by Boyd and Smith (1994d). Lemma 3, of course, gives a sufficient—but far from a necessary—condition for $T'(\psi_{t+1}) > 0$ to hold.

When we do have $T'(\psi_{t+1}) > 0$, increases in the cost of state verification result in an increase in θ_t^* or, in other words, in heavier use of the observable return investment technology. Thus it is certainly intuitively plausible that $T'(\psi_{t+1}) > 0$ is the "natural" situation whenever $\theta_t^* \in (0, r/\hat{y})$.

3. The Use of Debt and Equity

In this section we consider how the firm's optimal financing contract can be implemented through the issue of a determinate mix of debt and equity. This discussion summarizes a more elaborate one in Boyd and Smith (1994d), who also motivate in far more detail the interpretation of debt and equity given here.

Recall that an optimal funding contract specifies a (per unit) repayment of $x_t(y)$ at $t + 1$, as a function of the return on the investment in the observable return technology (y), if monitoring does *not* occur. Then, from equation (6),

$$x_t(y) = \rho_{t+1}[(1 - \theta_t^*)z(\theta_t^*; \psi_{t+1}) + \theta_t^* y]$$

in nonmonitoring states. Notice that this payment consists of a component $[\rho_{t+1}(1 - \theta_t^*)z(\theta_t^*; \psi_{t+1})]$ which is not contingent in any way on firm performance, as well as a component $[\rho_{t+1}\theta_t^* y]$ which does depend on that aspect of firm performance which is observable in nonverification states. Standard terminology in the existing CSV literature suggests that we should think of the first component as payments to debt-holders (per unit of funding) in such states, while we can then think of the second, performance related component as payments to equity-holders.

Now consider what happens in a monitoring state. The firm "owes" debt-holders $q(1 - \theta_t^*)z(\theta_t^*; \psi_{t+1})\rho_{t+1}$. If debt-holders receive this amount they do nothing, while if they receive less than this amount they claim the entire (net of monitoring cost) proceeds of the firm's investments. This event can be interpreted as a bankruptcy.

If a bankruptcy occurs, equity-holders receive nothing. If, on the other hand, debt-holders are fully repaid, equity-holders receive the residual returns on the firm's investments—$q[(1 - \theta_t^*)w + \theta_t^* y]\rho_{t+1} - q\rho_{t+1}(1 - \theta_t^*)z(\theta_t^*; \psi_{t+1})$. Of course, this requires that equity-holders monitor the firm. Equity-holders cannot, by definition, force a bankruptcy, but there are a number of ways in which

equity-holders can undertake costly actions to acquire information about the true value of a firm's assets.[15] Notice that, under this interpretation, debt-holders are "paid first," while equity-holders are true residual claimants. Insider owner-managers are subordinate both to owners of debt and external equity.

With this interpretation of how borrowers implement optimal funding contracts, it is infeasible to fully repay debt-holders iff $[\theta_t^* y + (1 - \theta_t^*) w] < (1 - \theta_t^*) z(\theta_t^*; \psi_{t+1})$. This condition is equivalent to

$$w < z(\theta_t^*; \psi_{t+1}) - [\theta_t^*/(1 - \theta_t^*)] y.$$

Defining the function $J(\theta, y; \psi)$ by

$$J(\theta, y; \psi) \equiv \max\{0, z(\theta; \psi) - [\theta/(1 - \theta)] y\},$$

a bankruptcy occurs iff $w \in [0, J(\theta_t^*, y; \psi_{t+1}))$. If $w \in [J(\theta_t^*, y; \psi_{t+1}), z(\theta_t^*; \psi_{t+1}))$, then debt-holders are fully repaid and equity-holders are responsible for monitoring, while if $w \geq z(\theta_t^*; \psi_{t+1})$, no state verification is required.

Let d_t denote the value of the debt issued by a representative borrower at t, and let e_t be the value of the same borrower's external equity. d_t and e_t are determined by two requirements. First, $d_t + e_t = q$ must hold. Second, the expected return to both debt and equity-holders must equal $r\rho_{t+1}$. As demonstrated by Boyd and Smith (1994d) [see their Proposition 1], equity-holders earn an expected return of $r\rho_{t+1}$ iff e_t satisfies

$$
\begin{aligned}
e_t = (q/r) \Bigg\{ r &- \sum_n p_n(1 - \theta_t)[z(\theta_t; \psi_{t+1}) - J(\theta_t, y_n; \psi_{t+1})] \\
&- \sum_n p_n \pi[J(\theta_t, y_n; \psi_{t+1}), \theta_t; \psi_{t+1}] \Bigg\} \equiv E(\theta_t; \psi_{t+1}).
\end{aligned}
$$

(21)

$d_t = q - e_t$ then gives the value of debt issued by a representative firm. Boyd and Smith further establish that $0 = E(0; \psi_{t+1}) \leq E(\theta_t; \psi_{t+1}) \leq E(r/\hat{y}; \psi_{t+1}) = q$.

E gives the amount of equity issued by a typical firm as a function of θ_t, which is chosen by the firm, and of ψ_{t+1}, which is external to the firm. Thus the volume of equity issued depends on the composition of a firm's assets θ_t, and on its monitoring cost ψ_{t+1}. It also depends on parameters; in particular, on the entire distribution of y, and on the productivity of the commonly available investment technology r.

Boyd and Smith (1994d) show that, whenever $E_2(\theta;\psi)$ exists, it satisfies $E_2(\theta;\psi) \leq 0$. Thus, with a fixed composition of investment, higher monitoring costs favor the use of debt. $E_1(\theta;\psi)$ is harder to sign, when E_1 exists, but Boyd and Smith (1994d) state a variety of conditions sufficient to ensure that $E_1(\theta;\psi) > 0$, when E_1 exists. This result indicates that—holding monitoring costs constant—higher equity levels are associated with a more intensive use of technology o. As suggested by the standard CSV literature, the use of technology u is associated with debt-finance (and when $\theta_t = 0$, $d_t = q$), while the use of technology o permits some equity finance to be viable in equilibrium.

At an optimum, of course, $\theta_t = T(\psi_{t+1})$, and the volume of equity issued is given by

$$(22) \quad e_t^* = E[T(\psi_{t+1});\psi_{t+1}] \equiv E^*(\psi_{t+1}).$$

The sign of $E^{*\prime}$, when it exists, is generally ambiguous. However, a number of numerical examples calculated by Boyd and Smith (1994d) indicate that E^* can typically be expected to be an increasing function (see also Section 6 below). Thus as monitoring costs rise so—generally—does θ_t^*, along with the ratio of external equity to debt issued by a representative firm. In Section 6 we explore the implications of this observation.

4. Capital Accumulation and the Composition of Investment:

General Equilibrium

A. The Capital Stock

In an equilibrium with no credit rationing, all loan demand by borrowers is met. By Lemma 1, this demand—in per capita terms—is αq in each period. Any savings in excess of this amount must be invested in the publicly available capital investment technology. Since per capita savings is $(1-\alpha)\omega(k_t)$ at t, investment in the low return capital production technology is $(1-\alpha)\omega(k_t) - \alpha q$,[16] and the quantity of capital produced from this technology at t + 1 is $r[(1-\alpha)\omega(k_t) - \alpha q]$. Of the αq units of funds obtained by borrowers, a fraction $\theta_t^* = T(\psi_{t+1})$ is invested in technology o at t, yielding (on average) $\hat{y}$ units of capital per unit invested, while $1 - \theta_t^*$ is invested in technology u at t, yielding (on average) $\hat{w}$ units of capital per unit invested. Therefore the time t + 1 per capita capital stock, K_{t+1}, is given by

$$(23) \quad K_{t+1} = \alpha q\{T(\psi_{t+1})\hat{y} + [1 - T(\psi_{t+1})]\hat{w} - r\} + r(1-\alpha)\omega(k_t).$$

Since the aggregate capital-labor ratio at t + 1 is simply $k_{t+1} = K_{t+1}/(1-\alpha)$, the equilibrium law of motion for $\{k_t\}$ is given by

$$(24) \quad k_{t+1} = r\omega(k_t) + [\alpha/(1-\alpha)]q\{T(\psi_{t+1})\hat{y} + [1 - T(\psi_{t+1})]\hat{w} - r\}.$$

To further characterize the equilibrium law of motion for k_t, recall that $\psi_{t+1} \equiv \gamma/q\rho_{t+1} = \gamma/qf'(k_{t+1}) \equiv \psi(k_{t+1})$. Clearly ψ is an increasing function of k_{t+1} satisfying

$$\lim_{k \to 0} \psi(k) = 0$$

$$\lim_{k \to \infty} \psi(k) = \infty.$$

Moreover, using this definition of the function ψ, we can rewrite equation (24) as

$$(25) \quad k_{t+1} = [\alpha/(1-\alpha)]q\{\hat{w} - r - (\hat{w}-\hat{y})T[\psi(k_{t+1})]\} + r\omega(k_t).$$

Given the initial capital-labor ratio k_0, equation (25) describes the evolution of the equilibrium sequence $\{k_t\}_{t=0}^{\infty}$.

Equation (25) is depicted in Figure 3. Apparently,

$$dk_{t+1}/dk_t = r\omega'(k_t)/\{1 + [\alpha/(1-\alpha)]q(\hat{w}-\hat{y})T'(\cdot)\psi'(\cdot)\}.$$

Whenever it is feasible to set θ_t anywhere in the interval $[0,r/\hat{y}]$ (see Section 5), and whenever $T'[\psi(\cdot)] \geq 0$, as will typically be the case (see Section 6), equation (24) gives k_{t+1} as a continuous, monotonically increasing function of k_t. We denote this function by $k_{t+1} = \Phi(k_t)$.

1. Existence of a Steady State Equilibrium

We now state conditions sufficient to imply that an economy has a nontrivial steady state equilibrium capital-labor ratio (denoted k^s) satisfying $(1-\alpha)\omega(k^s) \geq \alpha q$. To do so, we define $\underline{k}$ by

$$\underline{k} \equiv \omega^{-1}[\alpha q/(1-\alpha)].$$

In addition, define $\bar{k}$ to be the largest solution to the equation

$$k - r\omega(k) = [\alpha/(1-\alpha)]q(\hat{w}-r).$$

We can then state the following existence result.

PROPOSITION 3. Suppose that it is feasible to set θ_t anywhere in the interval $[0,r/\hat{y}]$ for all $\psi_{t+1} \leq \psi(\bar{k})$ (see Section 5), and that

(a.5) $\hat{y}\alpha q \geq \underline{k}(1-\alpha)$.

Then there exists at least one nontrivial steady state equilibrium capital-labor ratio, k^s, satisfying $(1-\alpha)\omega(k^s) \geq \alpha q$. Moreover, there is at least one asymptotically stable steady state equilibrium if $T'[\psi(k)] \geq 0$ for all $k \geq \underline{k}$.

The Proof of Proposition 3 appears in the Appendix. The proposition states a condition under which there exists an asymptotically stable steady state equilibrium that is free from credit rationing. In Section 6 we exhibit several examples which satisfy the conditions of Proposition 3, as well as one which does not.

2. *Dynamical Equilibria*

Figure 3 depicts the evolution of the equilibrium sequence $\{k_t\}$, under the conditions of Proposition 3 and under the assumption that $T' \geq 0$. Assumption (a.5) implies that, if $k_t = \underline{k}$, $k_{t+1} \geq \underline{k}$ holds. Therefore, if $k_o \geq \underline{k}$ is satisfied, $\{k_t\}$ is a monotone increasing sequence, and $k_t \geq \underline{k}$ holds at each date. Thus credit is never rationed, and a steady state is asymptotically approached.

B. The Composition of Investment

Suppose that $k_o \in [\underline{k}, k^s)$ holds, and that $T' \geq 0$. Then $\{k_t\}$ is an increasing sequence. It follows that $\{\psi_{t+1}\} = \{\psi(k_{t+1})\}$ is an increasing sequence as well, so that investors perceive monitoring costs that are rising over time. The reason this occurs is that capital investment projects yield physical capital, and the goods value of capital is simply its rental rate, $f'(k_{t+1})$. As k_{t+1} rises, the relative price of capital falls. However, monitoring consumes final goods and services.[17] Thus as k_{t+1} rises, the costs of monitoring rise relative to the value of investment returns.[18]

This fact has a strong implication. If $T'(\psi_{t+1}) \geq 0$ holds (which it does uniformly in our examples; see Section 6), then $\{\theta_t^*\}$ is also an increasing sequence. Thus, as an economy moves along its growth path, an increasing fraction of investment by borrowers (as well as of social investment) will take place in technologies that have low (gross of monitoring costs) expected returns. In other words, the development process will be accompanied by a declining (gross of verification cost) return on investment, even measuring these returns in units of capital.

This shift in the composition of investment does allow for the possibility that total resources consumed by state verification decline as an economy grows. In particular, resources used in monitoring in real terms are given by $\alpha\gamma G[z(\theta_t^*;\psi_{t+1})]$, per capita. As k_{t+1} rises, $z(\theta_t^*;\psi_{t+1})$ can either rise or fall; in our examples (Section 6) $z(\theta_t^*;\psi_{t+1})$ does not increase in the development process as long as $\theta_t^* > 0$. Thus more advanced economies (ones that are "farther along their growth paths") will typically use less resources (and a smaller fraction of total resources) in dealing with costs that arise due to informational frictions. In this sense, less developed economies will appear to face financial market frictions that are more severe than those confronting their better developed counterparts. However, in the model, this is not because their financial markets are intrinsically more severely flawed; rather it is a purely endogenous outcome.

C. The Equilibrium Equity Ratio

The equilibrium ratio of equity issued to total external finance is simply $e_t/q = E^*(\psi_{t+1})/q$. $E^{*\prime}$ (when it exists) generally has a theoretically indeterminate sign. However, in a series of numerical examples, Boyd and Smith (1994d) found that E^* seems typically to increase with ψ_{t+1}. This finding is confirmed by the examples reported below (Section 6). Thus as our example economies develop, they appear to employ equity markets in an increasingly heavy way.

5. Some Other Possibilities

The analysis of Sections 2–4 was predicated on two assumptions: (a) that it is feasible to set θ_t anywhere in the unit interval, and (b) that $\theta_t^* \in (0,r/\hat{y})$. We now briefly describe how the analysis must be modified when either assumption is violated.

A. Feasibility

For a given value of ψ_{t+1}, it is feasible for firms to select a particular value θ_t iff that choice of θ_t permits them to offer investors an expected return of at least $r\rho_{t+1}$. It is straightforward to verify that the largest expected return a borrower can offer, as a function of θ, is given by

$$\rho_{t+1}\{\pi[\bar{z}(\theta;\psi_{t+1}),\theta;\psi_{t+1}] + \theta\hat{y}\} \equiv \rho_{t+1}Q(\theta;\psi_{t+1}).$$

Then, for any value of $\theta \in [0,r/\hat{y}]$, it is feasible to offer investors an expected return of $r\rho_{t+1}$ iff $Q(\theta;\psi_{t+1}) \geq r$ holds.

It is easy to check that the function Q has one of the two configurations depicted in Figure 4.

Case 1. $Q_1(\theta;\psi_{t+1}) \geq 0$ for all $\theta \in [0,r/\hat{y}]$. Then Q has the configuration represented in Figure 4a. In particular, Q is an increasing function, and increases in ψ shift Q downward. Thus, for low values of ψ_{t+1} (such as ψ_1 in the figure) it is feasible to set θ anywhere in the unit interval. As ψ_{t+1} rises (say to ψ_2), feasibility requires that $\theta \geq \underline{\theta}_2$ hold. Further increases in ψ reduce the set of feasible choices of θ even further. Thus, if $\psi_{t+1} = \psi_3 > \psi_2$, feasibility requires that $\theta \geq \underline{\theta}_3$ hold. Finally we note that the set of feasible choices of θ is always nonempty, since $Q(r/\hat{y};\psi_{t+1}) > r$ holds, for all ψ_{t+1}.

When $Q_1(\theta;\psi_{t+1}) \geq 0$ holds everywhere, the set of feasible values of θ is a closed interval. It is easy to verify that $Q_1(\theta;\psi_{t+1}) \geq 0$ holds for all θ iff

(26) $\hat{y} \geq \pi[\bar{z}(0;\psi_{t+1}),0;\psi_{t+1}] + \psi_{t+1}G[\bar{z}(0;\psi_{t+1})].$

Case 2. $Q_1(0;\psi_{t+1}) < 0$ holds. This case obtains when (26) is violated. Here it is possible to show that Q necessarily has the "u-shape" depicted in Figure 4b. Again, increases in ψ_{t+1} reduce the maximum expected return that a borrower can offer. Thus when $\psi_{t+1} = \psi_1$, any value of θ is

feasible. As ψ_{t+1} increases to ψ_2, the feasible set consists of two disjoint intervals, while as ψ_{t+1} increases further (say to ψ_3), feasibility requires that $\theta \geq \underline{\theta}_3$.

When the feasible set consists of two disjoint intervals, as when $\psi_{t+1} = \psi_2$ in Figure 4b, borrowers face a nonconvex constraint set. This nonconvexity can lead to some interesting dynamical behavior of the debt-equity ratio, as we illustrate in Section 6.

Clearly, the correct formulation of the borrowers' problem in either Case 1 or Case 2 is that θ_t must be chosen to maximize $H(\theta;\psi_{t+1})$, subject to $Q(\theta;\psi_{t+1}) \geq r$. It is straightforward to show that, under the conditions stated in Section 2b, there is some function T such that the optimal choice of θ satisfies $\theta_t^* = T(\psi_{t+1})$, where T has the properties described there. Thus, in particular, if $T' \geq 0$ holds whenever assumptions (a) and (b) are satisfied, $T' \geq 0$ holds when those conditions are violated as well.

B. "Corners"

When $Q(0;\psi_{t+1}) \geq r$ holds, it is feasible to set $\theta_t = 0$. It is optimal to set $\theta_t^* = 0$ at date t only if

$$(27) \quad (\hat{w}-\hat{y})\{1 - G[z(0;\psi_{t+1})]\} \geq \{\hat{w} - r - \psi_{t+1}G[z(0;\psi_{t+1})]\}\psi_{t+1}g[z(0;\psi_{t+1})].$$

Since $z_2(0;\psi) \geq 0$, evidently (27) can obtain only when ψ_{t+1} is relatively small or, equivalently, only when k_{t+1} is relatively low. It follows from this observation that we would expect to see $\theta_t^* = 0$ holding, if at all, only for low values of k_{t+1} and k_t. This fact raises the following possibility: $\theta_t^* = 0$ may hold at low levels of the current capital stock. However, once the capital stock exceeds a critical level, $\theta_t^* > 0$ will be satisfied.

Suppose that $\theta_t^* = 0$ for all $t < T$, and that $\theta_t^* > 0$ thereafter. Since the equilibrium quantity of equity issued at t is given by $E(\theta_t^*;\psi_{t+1})$, since $E(0;\psi_{t+1}) \equiv 0$, and since $E(\theta;\psi_{t+1}) > 0$ for all $\theta > 0$, it follows that an economy with this sequence $\{\theta_t^*\}$ will have no equity market activity for

$t < T$. At T equity markets will become active, and they will remain so at all subsequent dates. Thus it can easily occur that an economy has no equity market activity until it reaches a critical level of development and that, thereafter, its equity markets form. Indeed, it is possible that when equity markets form the level of activity in them becomes very large immediately, so that there is a dramatic spurt rather than slow growth in measures of equity market activity. We produce examples illustrating various possibilities of this kind in Section 6.

When $Q(0;\psi_{t+1}) < r$ holds, it is not feasible to set $\theta_t = 0$. In this case there is a minimum feasible value of θ, denoted by $\underline{\theta}_t$, which satisfies $Q(\underline{\theta}_t;\psi_{t+1}) \equiv r$. Then $\theta_t^* \geq \underline{\theta}_t$ must hold, and it is easy to show that $d\underline{\theta}_t/d\psi_{t+1} > 0$. In this event, if $\{k_t\}$ is an increasing sequence, the set of feasible choices of θ (that is, the interval $[\underline{\theta}_t, r/\hat{y}]$) will shrink over time. We have found no examples in which $\theta_t^* = \underline{\theta}_t$.

Finally, it can occur that $\theta_t^* = r/\hat{y}$. This condition obtains only if

(28) $(\hat{w}-\hat{y})(\hat{y}-r)/\hat{w}g(0) \leq \psi_{t+1}$.

Obviously (28) requires that ψ_{t+1} (and hence k_{t+1}) be large, and clearly (28) rules out $\theta_t^* = r/\hat{y}$ if $g(0) = 0$. Thus only relative developed economies can conceivably experience $\theta_t^* = r/\hat{y}$; if this ever occurs, $\theta^* = r/\hat{y}$ will also hold at all subsequent dates. Moreover, clearly $T[\psi(k^s)] < r/\hat{y}$ will hold if $g(0)$ is sufficiently small.

C. Remarks

Suppose that $Q(0;\psi_{t+1}) < r$ holds at t. Under this condition, if borrowers were constrained to set $\theta = 0$ (or if they had access only to technology u), it would be infeasible for them to operate their investment projects. Thus access to technology o, along with the use of the equity finance that is associated with investments in that technology, can permit borrowers (and society) to employ high return investment projects in a way that would not be possible if only technology u were available.

Moreover, if $\theta_t^* < r/\hat{y}$, it will be optimal for borrowers to issue some debt to finance their investments in technology u. The reason it is optimal to issue some debt when $\theta^* > 0$ is that the use of equity makes it cheaper to issue debt than would be the case if firms were entirely debt financed. In this sense, debt and equity markets are complementary in the kinds of environments under consideration here.

6. A Class of Examples

We now present a class of examples designed to illustrate how debt and equity market activity evolve as an economy moves along its growth path. All of our examples are based on the following triangular distribution for w:

$$(29) \quad g(w) = \begin{cases} \varsigma w; & 0 \leq w \leq \bar{w}/2 \\ \varsigma(\bar{w}-w); & \bar{w}/2 \leq w \leq \bar{w}. \end{cases}$$

With $\varsigma \equiv 4/\bar{w}^2$, $G(\bar{w}) = 1$ holds, and it is easy to verify that $\hat{w} = \bar{w}/2$.

Figure 5 depicts the density function described by equation (29). This density function is very tractable to use in calculating numerical examples, and it obviously represents a simple, piecewise linear approximation to any symmetric unimodal distribution for w. It therefore seems clear that this is an empirically more plausible distribution to use in generating examples than, say, the even simpler uniform distribution.

We have assumed previously that g is everywhere differentiable. We have also imposed (a.3) in many of our derivations. For the density function given in (29), (a.3) is equivalent to

$$(30) \quad \bar{z}(0;\psi_{t+1}) < \bar{w}/2$$

for all "relevant" values of ψ_{t+1}. If (30) holds, then g′ exists for all values of w that are relevant to a borrower's decision making. Thus (30) guarantees satisfaction of our assumptions "where they matter."

The following result is proved in Boyd and Smith (1994d).

PROPOSITION 4. Equation (30) holds iff $\psi_{t+1} > \hat{w}/2$.

Since $\psi_{t+1} = \psi(k_{t+1})$ is an increasing function of k_{t+1}, Proposition 4 asserts that (30) is satisfied for sufficiently large values of k_{t+1} (and hence, whenever $dk_{t+1}/dk_t > 0$, for sufficiently large values of k_t). Indeed, if $T' \geq 0$ and

$$(31) \qquad \psi[\hat{y}\alpha q/(1-\alpha)] > \hat{w}/2$$

then (30) holds for all $k_t \geq \underline{k}.$[19] We henceforth maintain the assumption that (31) holds.

For this class of examples, $G(w) = (\zeta/2)w^2$ holds for $w \leq \bar{w}/2$. In addition, whenever (30) is satisfied, the function $\pi(z,\theta;\psi)$ is given by

$$(32) \qquad \pi(z,\theta;\psi) = (1-\theta)z - (1-\theta)(\zeta/6)z^3 - \psi(\zeta/2)z^2.$$

Equations (13) and (32) implicitly define the function $z(\theta;\psi)$. Once this function is obtained, equation (17) implicitly defines $T(\psi_{t+1})$. Equation (25) and the definition of $\psi(k_{t+1})$ then describe the equilibrium law of motion for k_t, which we trace out and represent diagrammatically for each of our numerical examples. Finally, once k_{t+1} is obtained for each value of k_t, we can compute the values $z_t = z[T(\psi_{t+1});\psi_{t+1}]$ and $e_t/q = E^*(\psi_{t+1})$ corresponding to each value of the current capital-labor ratio. This allows us to depict how the equity ratio (e_t/q) and, by implication, the ratio of equity to debt, evolves as an economy moves along its growth path. In addition, total resources consumed by monitoring (measured in units of current consumption) at t are simply $\alpha\gamma G(z_{t-1})$. Thus we can show how the resource loss implied by the existence of the credit market friction changes as an economy develops. For four of our five examples, $\{z_t\}$ is a monotone decreasing sequence, so that the resource losses attributable to the CSV problem typically diminish with increased levels of real activity. This observation gives a sense in which more highly developed economies (ones that are "farther along their growth paths") have less severe financial market frictions than their less

developed counterparts. This can be the case even though the state verification technologies (and investment opportunities) are identical across different economies.

In order to derive an equilibrium law of motion for k_t, it is necessary to specify a set of parameters $(\hat{w}, \hat{y}, r, \alpha, q, \gamma)$, along with a production function $f(k)$. In each of our examples, we assume that $f(k) = Bk^\beta$, with $\beta \in (0,1)$. Then a specification of the vector $(\hat{w}, \hat{y}, r, \alpha, q, \gamma, \beta, B)$ is adequate to allow us to derive the values k_{t+1}, $\psi(k_{t+1})$, $\theta_t^* \equiv T[\psi(k_{t+1})]$, and $z_t \equiv z\{T[\psi(k_{t+1})];$ $\psi(k_{t+1})\}$ corresponding to each value of $k_t \geq \underline{k}$. Appendix B gives the system of equations that determine these values. We then depict diagrammatically the pairs (k_t, k_{t+1}) and (z_t, k_{t+1}) for each example. Equation (21) then gives e_t/q for each value of k_t; clearly in order to obtain the equity-ratio it is necessary to specify the entire distribution of y. Thus, in particular, while allocations depend only on $\hat{y}$, e_t/q depends on the entire vector $(y_1, y_2, \ldots, y_N, p_1, p_2, \ldots, p_N)$. This vector must be specified for each of our numerical examples.[20]

A. Example 1

For this example we set $\hat{w} = 1.6$, $\hat{y} = 1.2$,[21] $r = 0.6$, $q = 1$, $\gamma = 5/3$, $\alpha = 0.1268$, $B = 1.1$, and $\beta = 0.5$. For these parameter values, $\underline{k} = 0.07$. The upper right-hand quadrant of Figure 6 depicts the equilibrium law of motion for k_t (that is, equation (25)) for this set of parameter values, and for $k_t \geq \underline{k}$. As is apparent from the figure, the equilibrium law of motion gives k_{t+1} as a monotone increasing function of k_t. There is a unique asymptotically stable steady state equilibrium (with no credit rationing); here $k^s \in (0.30, 0.31)$.

The lower right-hand quadrant of Figure 6 depicts what the equilibrium equity-ratio (e_t/q) would have to be for each current value of k_t. For $k_t = \underline{k}$ in this example, $e_t/q = 0.56$ would have to hold. At the nontrivial steady state $(k_t = k^s)$, the equilibrium equity ratio is 0.72. Moreover, as is clear from the figure, the equity-ratio increases monotonically as k_t rises. Thus increased

economic development is accompanied by an increase in the level of equity market activity, and also by an increase in the (aggregate) ratio of equity-to-debt. Clearly, for this example, the equity-debt ratio can rise dramatically as an economy moves along its growth path.

The upper left-hand quadrant of Figure 6 depicts $z_t = z\{T[\psi(k_{t+1})];\psi(k_{t+1})\}$ as a function of k_{t+1}. Since $z_1(\theta_t^*;\psi_{t+1}) < 0 < z_2(\theta_t^*;\psi_{t+1})$ holds, it is not obvious a priori how z_t will vary with k_{t+1}. However, for this example, z_t declines monotonically as the economy accumulates capital. Thus the resources lost due to the CSV problem decline as the economy develops.

It is interesting to contrast the behavior of z_t with that of $\tilde{z}_t \equiv z(0;\psi_{t+1})$. $\tilde{z}_t$ is, of course, the realization of w that triggers monitoring when $\theta_t = 0$, and $G(\tilde{z}_t)$ is the fraction of projects monitored at $t + 1$ when $\theta_t = 0$. Of course in standard single investment project CSV environments, such as those studied by Gale and Hellwig (1985) and Williamson (1986, 1987), $\theta_t \equiv 0$ holds. Since (as is easily shown), $z_2(\theta_t;\psi_{t+1}) > 0$, if $\theta_t \equiv 0$ is exogenously imposed, increases in k_{t+1} necessarily raise $\tilde{z}_t$, and hence necessarily increase the amount of monitoring that occurs. Thus those models—when embedded in a growth context—would imply that the real resources consumed by state verification *increase* as an economy moves along a growth path. This result—that with fixed θ economic development leads to a more severe resource loss due to the CSV problem—may be overturned (as it is in this example) when θ can be varied in response to increases in ψ. We take this observation to be an illustration of the importance of allowing the composition of investment to be endogenous in models designed to study the role of financial market frictions in influencing capital accumulation.

B. Example 2

This example is identical to Example 1, except that $\hat{y} = 1.3.$[22] A comparison of Examples 1 and 2 permits us to infer some of the consequences of raising the expected return to investments in the observable return capital production technology, ceteris paribus.

Figure 7 depicts various aspects of an equilibrium for this example. Here an increase in $\hat{y}$ shifts the equilibrium law of motion for k_t upwards, albeit by a very small amount. This is to be expected (although see the discussion in Example 3), since we have increased the productivity of one of the capital investment technologies. However, the effect of the increase in $\hat{y}$ (which increases by 8.3 percent between Examples 1 and 2) for the capital stock is very small; the steady state per capita capital stock (and capital-labor ratio) rises by about 1.7 percent.

The effect of an increase in $\hat{y}$ for the equilibrium value of the equity-ratio (e_t/q) is far more dramatic. At $k_t = \underline{k} = 0.07$, $e_t/q = 0.73$ for this example, a 30 percent increase relative to Example 1. At the steady state, $e_t/q = 0.8$, an 11 percent increase relative to Example 1. The reason for this change in the equity-ratio is that higher values of $\hat{y}$ make technology o more attractive relative to technology u, ceteris paribus, so that θ_t^* increases at each value of k_t (see below). Since investments in technology o are, typically, associated with the use of equity, the equity-ratio rises along with $\hat{y}$.

The lower left-hand quadrant of Figure 7 depicts this change in the equilibrium value of θ_t^*. Evidently the effect of an increase in $\hat{y}$ for θ_t^* is larger at larger values of k_t (and hence k_{t+1}); this occurs because as k_{t+1} increases so does $\psi(k_{t+1})$. Hence monitoring costs loom larger at higher values of k_{t+1}, and thus the increased attractiveness of technology o is magnified as k_{t+1} rises.

The upper left-hand quadrant of Figure 7 shows the effect on z_t of the increase in $\hat{y}$. As a comparison of Figures 6 and 7 will indicate, the increase in $\hat{y}$ has a quite dramatic effect on z_t at all values of k_{t+1}, and the steady state value of z_t (that is, $z\{T[\psi(k^s)];\psi(k^s)\}$) declines by 29 percent

between Examples 1 and 2 (from about 0.286 to about 0.201). The result is a nearly 50 percent reduction in the value of resources consumed by monitoring in the steady state. Thus while an increase in $\hat{y}$ of slightly over 8 percent has a quite small effect on the steady state capital stock, it has a huge proportional effect on the quantity of resources consumed due the CSV problem. Interestingly, this is true even though the change occurred in the technology that is not subject to this problem.

1. Interest Rate Differentials

The differential between the rate of interest on loans and the rate of interest on deposits is often employed as a measure of the severity of frictions in financial markets. Interestingly, in our model economy such a rate differential is not an absolutely reliable measure. Specifically, this interest rate differential can behave nonmonotonically even when $\{z_t\}$ is monotone decreasing—meaning that the amount of resources consumed by monitoring is monotone decreasing as well. What appears to be true is that the trend behavior of the loan-deposit rate differential does reflect the trend behavior of resources used in monitoring. To make these relationships precise, we next analyze the loan-deposit interest rate differential for the present example.

Payments to debt-holders (which could be loan payments to financial intermediaries) by *nondefaulting* borrowers are given by $q(1-\theta_t^*)z(\theta_t^*;\psi_{t+1})\rho_{t+1}$, while payments per unit of debt issued are simply $q(1-\theta_t^*)z(\theta_t^*;\psi_{t+1})\rho_{t+1}/d_t = q(1-\theta_t^*)z(\theta_t^*;\psi_{t+1})\rho_{t+1}/[1 - (e_t/q)]$. This is a measure of the average loan rate. The rate of interest received by depositors is $r\rho_{t+1}$. Thus the ratio of the two rates is given by $(1-\theta_t^*)z(\theta_t^*;\psi_{t+1})/r[1 - (e_t/q)]$.

The behavior of this ratio for the present example is depicted in Figure 8. Overall, increased development tends to be associated with a decline in the interest rate differential. However, this decline is not monotonic so that the loan-deposit rate differential can rise even as the resource loss

due to the presence of the CSV problem declines. Although we do not present them here, the results are qualitatively similar for Examples 1, 3, and 4. However, the magnitude of the nonmonotonicity in the interest rate differential varies considerably across the different examples. An important topic for future research is a more systematic analysis of when this interest rate differential does (does not) adequately represent the extent of financial market frictions.

C. Example 3

The economy of this example is identical to that of Example 2, except that now we set $r = 0.7$. Here, a comparison of Examples 2 and 3 permits us to examine some of the consequences of raising the return on the commonly available investment technology, and hence of raising (other things equal), the opportunity cost of funds to borrowers.

Figure 9 depicts the usual aspects of an equilibrium for this example. One natural conjecture would be that the increase in r uniformly shifts the equilibrium law of motion for k_t upward, since it improves the productivity of one capital production technology while leaving all others unaltered. And, indeed, if θ_t were fixed this conjecture would be correct. However, a comparison of the lower left-hand quadrants of Figures 7 and 9 will indicate that the increase in r uniformly raises θ_t^* at each value of k_t; this is a consequence of the fact that borrowers face (ceteris paribus) a higher opportunity cost of funds. Thus some investment is transferred from technology u to technology o, and this effect operates to shift the equilibrium law of motion for k_t downwards.[23] Which effect dominates is theoretically ambiguous; for this example the equilibrium law of motion shifts downward with an increase in r for $k_t \in [0.07, 0.13)$ and shifts upward with an increase in r for $k_t \geq 0.13$. One result of the increase in r is a higher steady state value of the capital-labor ratio, which rises from less than 0.31 to in excess of 0.33.

As a comparison of the lower right-hand quadrants of Figures 7 and 9 will demonstrate, the increase in r also acts to substantially increase the equity-ratio. Indeed between Examples 2 and 3, the steady state value of the equity ratio rises by about 11 percent (from 0.8 to 0.89). This, of course, is a consequence of the increased opportunity cost of funds to borrowers. Finally, an examination of the upper left-hand quadrants of Figures 7 and 9 indicates that the increase in r (and the associated increase in θ_t^*) again substantially reduces z_t—and hence the quantity of resources consumed by monitoring—at each value of k_{t+1}. Here, then, an increase in the productivity of the commonly available capital production technology both results in a higher steady state capital stock, and in fewer resources being lost due to the presence of the CSV problem.

This example also has an interesting feature which is *not* shared by Examples 1 and 2. In particular, it is possible to show that $Q[0;\psi(k_{t+1})] < r$ holds for all relevant values of k_{t+1}. This fact, in turn, means that it is not feasible for borrowers to offer an expected return of at least $r\rho_{t+1}$ at any date if $\theta_t = 0$. Such a result indicates the importance of allowing borrowers to make an endogenous choice of the composition of their investments. If there was only a single investment opportunity ($\theta_t \equiv 0$), as in the standard CSV literature, or if θ_t was exogenously fixed at some positive—but low—level, borrowers would be driven out of capital markets altogether. Here that does not happen, however, because borrowers can adjust θ_t in a way that always permits them to offer the expected return necessary to obtain funds. As a result, society does not lose the benefit of the high return investment technologies possessed by borrowers.

D. Example 4

For this example, we set $\hat{w} = 2.5$, $\hat{y} = 2.2805$,[24] $r = 0.5$, $q = 1.0$, $\gamma = 0.8$, $\alpha = 0.5$, $B = 2.0$, and $\beta = 0.5$. For these parameter values, $\underline{k} = 1.0$ holds.

Figure 10 depicts the usual set of equilibrium relationships for $k_t \geq \underline{k}$. As in all of the previous examples, the equilibrium law of motion for k_t is a monotone increasing function, and there is a unique, asymptotically stable steady state equilibrium with no credit rationing. In addition, θ_t^* declines as k_t and k_{t+1} increase and z_t is a monotonically decreasing function of k_{t+1}. Thus, as in all of our examples where $\theta_t^* > 0$, capital accumulation is associated with a reduction in the volume of resources consumed by the presence of the CSV problem. In this sense as our example economies develop, the evolution of their debt and equity markets provides them with a more efficiently functioning financial system.

A particularly interesting feature of the present example is that, at low values of k_t, $e_t/q = 0$. More specifically, for $k_t \in [1.0, 1.2)$, $e_t/q = 0$ holds. This, of course, is a reflection of the fact that, for these values, $\theta_t^* = 0$ holds as well, so that only technology u is in use. And—as in the usual CSV environment—firms are 100 percent debt financed. However, once $k_t \geq 1.2$ holds, k_{t+1} and $\psi(k_{t+1})$ are large enough so that technology o comes into use to economize on monitoring costs, $\theta_t^* > 0$ holds, and so does $e_t/q > 0$.

This observation has an interesting implication. Suppose that $k_0 \in [\underline{k}, 1.2)$. Then early in its development process this economy will have *no* equity market activity. However, $\{k_t\}$ will be an increasing sequence, and at some date $T > 0$, k_T will be large enough so that equity markets begin to be active. These markets will remain active at all subsequent dates, and indeed the steady state level of the equity ratio for this example (that is, the value of $E^*[\psi(k^s)]$) is about 0.103. Thus this economy can undergo a transition from having no equity market activity to a situation where equity finances a fairly substantial fraction of investment activity. And, for purely endogenous reasons, equity market activity will not be observed until a critical level of real activity has been attained.

This last observation is reminiscent of Gurley and Shaw's (1955, 1960, 1967) account of the coevolution of the real and financial sectors of an economy during the growth process. At relatively low levels of development, only debt-financed and self-financed investments (here investments by lenders in the commonly available technology) are observed. However, as an economy grows, eventually equity (and other) markets become active. This is certainly a feature of the present example; a feature that we reemphasize is a property of the economy's growth path dynamics. It requires no changes in the cost or availability of any intermediation technologies, or any other exogenous events.

Example 4 illustrates one possible evolution of equity market activity in the development process: at some critical level of real activity there is a fairly continuous transition from inactive to active but small equity markets. Much more discontinuous transitions are also possible, as the next example demonstrates.

E. Example 5

Often economies display long periods with little or no equity market activity. However, once equity market activity begins, it sometimes increases dramatically within a short period rather than growing slowly over time.[25] Our model can easily replicate this pattern of equity market development when the set of feasible choices of θ for borrowers consists of two disjoint intervals for some range of relative monitoring costs (as discussed in Section 5a). We now illustrate this possibility with an example.

The present example has $\hat{w} = 2$, $\hat{y} = 1.1$, $r = 0.96$, $q = 1$, $\gamma = 1.4175$, $\alpha = 0.27$, $B = 2$, and $\beta = 0.5$.[26] For $\psi_{t+1} = \psi(k_{t+1}) \leq 1.4819$, and hence for $k_{t+1} \leq 1.0929$, it is feasible to set $\theta_t = 0$ [that is, $Q(0;\psi_{t+1}) \geq r$ holds for all $\psi_{t+1} \leq 1.4819$.] It is easy to verify that, for all $\psi_{t+1} \leq$

1.4819, (27) holds. Numerical calculations then verify that $\theta_t^* = 0$ gives the optimal choice of θ for borrowers for all such ψ_{t+1}.

When $\theta_t^* = 0$, and when $k_t \geq \underline{k} = 0.1368$, the capital-labor ratio evolves according to

$$(33) \quad k_{t+1} = r(1-\beta)Bk_t^\beta + [\alpha/(1-\alpha)]q(\hat{w}-r) \equiv \lambda(k_t).$$

Since $\lambda^{-1}[\psi^{-1}(1.4819)] = 0.5443$ for the parameters of the example, $k_{t+1} = \lambda(k_t)$ must hold for all $k_t \in [0.1368, 0.5443] \equiv N$.

Obviously, $\theta_t^* = 0$ holds for all $k_t \in N$. Hence $e_t/q \equiv \tilde{E}[\psi(k_{t+1})] \equiv \tilde{E}\{\psi[\lambda(k_t)]\} = 0$ holds for all $k_t \in N$ as well. Thus whenever $k_t \leq 0.5443$, there is no equity market activity in this economy.

It is readily verified that, for the kinds of values of ψ_{t+1} under consideration, Q has the configuration depicted in Figure 4b. Thus, for $\psi_{t+1} \in (1.4819-\epsilon, 1.4819]$, the set of feasible choices of θ consists of two disjoint intervals, as is the case with the intermediate locus in Figure 4b. When $\psi_{t+1} > 1.4819$ holds, it is easy to check that $Q(0;\psi_{t+1}) < r$ holds, so that $\theta_t \geq \underline{\theta}_t > 0$ must obtain. (This situation is represented by the lower locus in Figure 4b.) Obviously, then, for $\psi_{t+1} > 1.4819$ ($k_{t+1} > 1.0929$), $\theta_t^* > 0$, and $E^*[\psi(k_{t+1})] > 0$ must hold as well. In short, if $k_{t+1} \leq 1.0929$ there will be no equity market activity, while when $k_{t+1} > 1.0929$ equity market activity must be observed. Thus, as in Example 4, there is a critical level of development at which equity markets become active.

Indeed, for this example, not only do equity markets become active for $k_{t+1} > 1.0929$; they become very active. Numerical calculations indicate that, for all such k_{t+1}, $\theta_t^* = T[\psi(k_{t+1})] > \underline{\theta}_t$ holds, so that there is an interior optimum for θ_t^*. The equilibrium choice of θ_t^* corresponding to each value of k_{t+1} is shown in the lower left-hand quadrant of Figure 11: evidently, when $k_{t+1} > 1.0929$, $\theta_t^* \geq 0.8531$ holds. This observation suggests what Figure 11 confirms, that the equilibrium

equity ratio is also extremely high whenever equity markets are active. Thus, when this economy makes the transition to positive levels of equity market activity, it will see e_t/q go from zero to at least 0.97. While this example is obviously constructed to be very dramatic with respect to the evolution of e_t/q, it does illustrate that equity market activity may begin with a bang rather than with a slow evolution.

The upper left-hand quadrant of Figure 2 depicts the evolution of $\{z_t\}$ and, hence, of monitoring costs for this example. Evidently as equity markets become active, there will be a dramatic decline in z_t and, by implication, in the quantity of resources consumed due to the presence of the CSV problem.

1. The Law of Motion for k_t

When $k_{t+1} > 1.0929$ holds, $\theta_t^* = T[\psi(k_{t+1})]$ as before. Hence the law of motion for k_t is given by equation (25). It follows that

$$(34) \quad k_{t+1} = \begin{cases} \lambda(k_t); & k_{t+1} \leq 1.0929 \\ \Phi(k_t); & k_{t+1} > 1.0929 \end{cases}.$$

For $k_{t+1} > 1.0929$, $\Phi' > 0$ holds. Thus the law of motion for k_t is given by the monotone locus in the upper right-hand quadrant of Figure 11.

This example depicts a possibility that did not exist in the previous examples: in particular, if $k_t \in (\lambda^{-1}(1.0929), \Phi^{-1}(1.0929)]$ for any t, then there is no competitive equilibrium. The potential for nonexistence of equilibrium derives from the nonconvexity of the constraint set for borrowers implicit in the constraint $Q(\theta; \psi_{t+1}) \geq r$. This nonconvexity was not present in the previous examples. Thus, in particular, if $k_0 \in N$, and if $\lambda^t(k_0) \in (\lambda^{-1}(1.0929), \Phi^{-1}(1.0929)]$ for any t—where λ^t denotes λ composed with itself t times—no competitive equilibrium exists. When $\lambda^t(k_0) \notin (\lambda^{-1}(1.0929), \Phi^{-1}(1.0929)]$ for all t, on the other hand, a competitive equilibrium does exist, and it is easy to show that the set of values $k_0 \in N$ such that a competitive equilibrium exists

is nonempty for this example. When a competitive equilibrium does exist, $\{k_t\}$ will be a monotone increasing sequence if $k_0 < k^s$. However, in general, the existence of an equilibrium can depend on the value of the initial capital stock k_0. When existence depends on k_0 in this way, it will typically transpire that all equilibria starting from certain initial conditions will display an abrupt and dramatic expansion of equity market activity at some point in their evolution.

7. Conclusions

We have developed a model in which capital is produced by investors who make use of two technologies. One yields a high expected return, gross of monitoring costs, but is subject to a CSV problem. The other yields a lower expected return, but has the advantage of full public observability. Investors must make a decision regarding how heavily to utilize each technology. This decision depends, among other things, on the relative price between capital and the resources used in state verification.

As an economy moves along its growth path, investors will perceive a relative cost of monitoring that rises over time. As a result, under conditions that we typically expect to prevail, less use will be made of the unobservable return, and more use will be made of the observable return technology. Since investment in the unobservable return technology is generally associated with the use of debt finance—while the use of the observable return technology is associated with equity—we also typically expect the ratio of equity finance to rise an economy develops. This intuition is confirmed by each of our numerical examples.

Moreover, it is possible to produce parameter values such that—at low levels of development—there will be *no* use of equity markets. Equity market activity can be observed for such parameters only once the economy attains a critical level of real development. Such examples

support the conclusion of Gurley and Shaw (1960, p. 92) that "the selection of financial assets evolves in the growth process," and that the variety of financial claims increases as well.

It is also the case that, in almost all of our numerical examples (and whenever $\theta_t^* > 0$), the quantity of resources consumed by monitoring declines (in real terms) as an economy develops. In this context, the evolution of debt and equity markets that occurs during the development process provides an economy with a more efficiently functioning set of financial markets.

Our analysis provides a sense in which debt and equity markets function as complements rather than substitutes. A case against the importance of equity markets in financing real development is often made on the basis that credit markets are close substitutes for equity markets. Our analysis calls the validity of such arguments into serious question. Indeed, Boyd and Smith (1994d) produces examples in which, if equity markets are exogenously precluded, it may be infeasible for borrowers to operate at all. This possibility arises because, in the absence of equity markets, it may simply be prohibitively expensive to issue debt. When this circumstance arises, the absence of equity markets would prevent society from employing its most productive capital production technologies. If this were to occur, it is easy to imagine that there would be large associated welfare losses.

There are a number of directions along which the current analysis could profitably be extended. One possible extension would be to give investors some resources of their own, so that capital investments are not entirely externally financed. Existing results in CSV models which allow for some internal finance (Bernanke and Gertler 1989 and Boyd and Smith 1994b,c) suggest that the introduction of these considerations is likely to allow much more scope for volatility as an economy develops, as well as for multiple steady state equilibria, and hence for development traps. If such traps can occur, then they might have the feature that some economies "get stuck" in steady state equilibria with low levels of real activity, and low levels of equity market activity as well.

Another possible extension would be to allow for more ex ante heterogeneity among investors. Some empirical evidence suggests that while less developed economies in the aggregate have less active equity markets than do more developed economies, large firms in LDCs often make more use of equity markets than do their counterparts in the developed world (Hamid and Singh 1992). It would be interesting to be able to investigate why this is the case.

Finally, it would be natural to introduce money and to analyze the consequences of various monetary policies for the evolution of debt and equity markets. It is often argued (Tun Wai and Patrick 1973) that high rates of inflation are detrimental to the development of equity markets. It would be interesting to analyze this possibility in a modified version of the present model.

Appendix

Proof of Proposition 3. When $k_t = \underline{k}$, all capital investment is done by borrowers. Hence $k_{t+1} \geq$ $\hat{y}\alpha q/(1-\alpha)$ holds. Thus if (a.5) is satisfied, the equilibrium (k_t, k_{t+1}) combination does not lie below the 45° line when $k_t = \underline{k}$. In addition, the assumption that it is feasible to select any $\theta_t \in [0, r/\hat{y}]$ for all $\psi_{t+1} \leq \psi(\bar{k})$ implies that (25) is continuous for all $k_t \in [\underline{k}, \bar{k}]$. Moreover, as is readily verified, $\Phi(\bar{k}) \leq \bar{k}$ must hold, so that equation (25) necessarily lies below the 45° line for large enough values of k_t. Thus, by continuity, (25) must cross 45° line at least once (from above), establishing the existence of the desired value k^s. If $T'[\psi(k)] \geq 0$ for all $k \geq \underline{k}$ holds, the existence of an asymptotically stable steady state follows from $dk_{t+1}/dk_t > 0$. $\square$

System of Equations for the Numerical Examples

Equations (13) and (32) imply that $z(\theta;\psi)$ is the smallest nonnegative solution to

(A1) $(1-\theta)z - (1-\theta)(\zeta/6)z^3 - \psi(\zeta/2)z^2 = r - \theta\hat{y}$

for each combination (θ,ψ), if such a solution exists. (If there is no such solution for a given pair (θ,ψ), then it is infeasible to set θ too low. We will exhibit examples with this property.) Straightforward differentiation of (A1) then establishes that

(A2) $z_1(\theta;\psi) = \{z(\theta;\psi) - (\zeta/6)[z(\theta;\psi)]^3 - \hat{y}\}/\{1 - \theta - \psi\zeta z(\theta;\psi) - (1-\theta)(\zeta/2)[z(\theta;\psi)]^2\}$.

In addition, the borrower's first order condition (17), along with the distributional assumption (29), implies that an interior optimum satisfies

(A3) $-z_1(\theta_t^*;\psi_{t+1}) = (\hat{w}-\hat{y})/\zeta z(\theta_t^*;\psi_{t+1})\psi_{t+1}$.

If $k_t \geq \underline{k}$ holds, credit cannot be rationed. Then for each value of k_t, the time $t + 1$ capital-labor ratio is given by

(A4) $k_{t+1} = [\alpha/(1-\alpha)]q[\hat{y}\theta_t^* + \hat{w}(1-\theta_t^*) - r] + r\omega(k_t)$.

In addition, for the class of examples under consideration,

(A5) $\quad \psi_{t+1} \equiv \psi(k_{t+1}) \equiv \gamma k_{t+1}^{1-\beta}/q\beta B.$

For each value of $k_t \geq \underline{k}$, equations (A1)–(A5) allow us to solve for θ_t^*, k_{t+1}, ψ_{t+1}, and $z_t = z(\theta_t^*;\psi_{t+1})$.

In the calculation of the numerical examples, we always impose $k_t \geq \underline{k}$ (no credit rationing), and (31), which implies that (a.3) is satisfied for each value $k_t \geq \underline{k}$. In addition, we check that, given ψ_{t+1}, θ_t^* corresponds to a global (as opposed to merely a local) maximum of the borrower's objective function.

Footnotes

[1]Demirguc-Kunt and Levine report a highly significant correlation of 0.55. Their data include only developing countries.

[2]Our results would be strengthened if state verification consumed labor effort. However, letting state verification costs take the form of time leads to a more complicated model. Therefore, we retain the simpler formulation that verification utilizes final output.

[3]This statement requires some qualification. Theoretically, the debt-equity ratio can either increase or decrease as an economy develops. All of our numerical examples (Section 6) indicate that it increases.

[4]Again, this is not a theoretically unambiguous result. However, it is an accurate statement about *all* of our numerical examples whenever the equity/debt ratio is nonzero.

[5]Such an outcome does appear to be observed in practice. According to Watson et. al. (1986, p. 10), "intermediation costs have been sharply reduced (in OECD countries) by the substitution for bank credits of direct transactions in securities." In our model, all debt is held by intermediaries, so that our results are consistent with their observation.

[6]There is no difficulty created by giving lenders more general utility functions and letting them make a nontrivial consumption-savings decision. However, this adds to notational requirements without yielding any additional insights. Thus we retain the simpler specification of the text.

[7]Credit rationing in this context is described by Gale and Hellwig (1985) and Williamson (1986, 1987).

[8]Since $\hat{y} > r$, a borrower will never invest in the commonly available technology. Also, note that we are assuming that i_t and θ_t are observable objects for external investors.

[9]While this is a real restriction, Boyd and Smith (1994a) have shown that—for realistic parameter values—the utility gains resulting from stochastic monitoring are negligible.

[10]For a further discussion of the issue of duplication of monitoring, see Diamond (1984), Boyd and Prescott (1986), Williamson (1986), or Krasa and Villamil (1992).

[11]It is straightforward to demonstrate that $x_t(y_n) \geq \rho_{t+1}\theta_t y_n$ always holds, so that $A_t(y_n)$ is well-defined. In addition, we might note that Boyd and Smith (1994d) consider a two period economy where investment projects yield final goods rather than capital. Hence, for them, $\rho_{t+1} = 1$. However, it is easy to check that allowing for general values of ρ_{t+1} does not alter their results.

[12]Lemma 2 implies that appending the latter constraint to the problem (P') does not affect its solution. Also, for the present, we assume that it is feasible for a borrower to set θ_t anywhere in the unit interval. Conditions under which this is the case are discussed in Section 5. We also discuss there what happens when it is not possible to choose θ_t arbitrarily.

[13]This conclusion does not depend on the existence of an interior optimum. The issue of noninterior optima is discussed in Section 5.

[14]An alternative strategy for guaranteeing the existence of an interior optimum is to assume that $H_1(r/\hat{y};\psi_{t+1}) < 0$, as in equation (15), and that $H(0;\psi_{t+1}) \leq H(r/\hat{y};\psi_{t+1})$ holds, while relaxing (16). The latter condition is equivalent to $\psi_{t+1}G[z(0;\psi_{t+1})] \geq r(\hat{w}-\hat{y})/\hat{y}$. Under this assumption, the function $H(\theta;\psi_{t+1})$ has the configuration depicted in Figure 2a. Again, this configuration guarantees the existence of an interior optimum. Noninterior optima are considered in Section 5.

[15]For example, equity owners may—as a group—hire outside auditors to ascertain a firm's actual value. They may also seek to replace incumbent management when returns are low, even if the firm can fully repay its debt-holders. Actions of both types consume resources and involve the acquisition of information about firm performance. In this sense these activities constitute costly state verification.

In the formal analysis, we assume that equity-holders, as well as debt-holders, have coordinated their efforts so as to avoid wasteful, redundant monitoring. This assumption is discussed in more detail in Boyd and Smith (1994d). It is also necessary to make sure that equity-holders possess adequate resources to carry out their monitoring responsibilities, when this is called for. Equity owners always have the requisite resources if they are perfectly diversified, or if equity ownership is intermediated—say through a mutual fund. Either situation is consistent with our analysis.

[16]Of course (a.2) implies that this quantity is positive.

[17]This chain of reasoning would be strengthened if monitoring consumed labor, whose relative price is rising over time. However, allowing labor to be employed in state verification would complicate the determination of factor prices. Therefore, we retain the simpler specification of the text.

[18]This observation does *not* imply that total resources consumed by monitoring rise as an economy develops. Total resources (per capita) used in state verification are simply $\alpha \gamma G[z(\theta_t^*; \psi_{t+1})]$. This quantity may rise or fall over time; in most of our examples (see Section 6), it falls as an economy moves (upwards) along its growth path.

Clearly our analysis has the feature that—in the absence of credit rationing—the total quantity of funds transferred to borrowers is constant. Thus a decline in the cost of funds transfers per unit of funding extended must translate into a decline in the per capita quantity of resources consumed by the financial system. We conjecture that, in a model where the quantity of funds transferred is not constant, economic development can be associated with a decline in costs per unit of funding extended, but with an increase in the quantity of resources utilized by the financial sector. Obviously, however, this issue is outside the scope of the present model.

[19]If $k_t = \underline{k}$, then $k_{t+1} \geq \hat{y}\alpha q/(1-\alpha)$ must hold. It follows that $\psi_{t+1} = \psi(k_{t+1}) \geq$ $\psi[\hat{y}\alpha q/(1-\alpha)]$ also obtains. Therefore (31) guarantees that the condition of Proposition 4 is satisfied.

[20]Examples constructed in Boyd and Smith (1994d) suggest that increases in the variance of y, ceteris paribus, tend to increase the equity-ratio.

[21]In order to compute e_t/q, it is necessary to specify the entire distribution of y. Here we assume that $y \in \{y_1, y_2\}$, that $y_1 = 0.01$, $y_2 = 2.39$, and that $p_1 = p_2 = 0.5$. Examples computed in Boyd and Smith (1994d) suggest that e_t/q is not strongly sensitive to changes in the specification of the y distribution.

[22]As in Example 1, $y \in \{y_1, y_2\}$. Here $y_1 = 0.01$, $y_2 = 2.59$, and $p_1 = p_2 = 0.5$.

[23]The same forces are at work in Example 2, but there the change in θ_t^* is not large enough to shift the equilibrium law of motion downward for any value of k_t.

[24]Here $y \in \{y_1, y_2\}$, with $y_1 = 0.01$, $y_2 = 4.551$, and $p_1 = p_2 = 0.5$.

[25]Korea is an example of the kind of economy we have in mind. From 1982 through 1985, the ratio of the value of stock market transactions to GDP never exceeded 0.045. In 1986 this ratio was 0.103; by 1989 it was 0.568. Thus the value of transactions in these markets relative to GDP increased by a factor of 12.6 in a four year period. Parenthetically, nominal GDP *rose* by about two-thirds over the same period, while the average annual inflation rate never exceeded 7.1 percent.

[26]As in the previous examples $y \in \{y_1, y_2\}$, with $y_1 = 0.9$, $y_2 = 1.9$, $p_1 = 0.8$, and $p_2 = 0.2$.

References

Antje, Raymond, and Jovanovic, Boyan. 1992. Stock markets and development. Manuscript. New York University.

Bencivenga, Valerie R., and Smith, Bruce D. 1991. Financial intermediation and endogenous growth. *Review of Economic Studies* 58, 195–209.

Bencivenga, Valerie R., Smith, Bruce D., and Starr, Ross M. 1994a. Liquidity of secondary capital markets: Allocative efficiency and the maturity composition of the capital stock. *Economic Theory*, forthcoming.

_____. 1994b. Transactions costs, technological choice, and endogenous growth. *Journal of Economic Theory*, forthcoming.

_____. 1994c. Liquidity of secondary capital markets, capital accumulation, and the term structure of asset yields. Manuscript. Cornell University.

Bernanke, Ben S. and Gertler, Mark. 1989. Agency costs, net worth, and business fluctuations. *American Economic Review* 79, 14–31.

Boyd John H., and Prescott, Edward C. 1986. Financial intermediary coalitions. *Journal of Economic Theory*.

Boyd, John H., and Smith, Bruce D. 1994a. How good are standard debt contracts? Stochastic versus nonstochastic monitoring in a costly state verification environment. *Journal of Business* 67, 539–62.

_____. 1994b. Capital market imperfections, international credit markets, and nonconvergence. Manuscript. Federal Reserve Bank of Minneapolis.

_____. 1994c. Capital market imperfections in a monetary growth model. Manuscript. Federal Reserve Bank of Minneapolis.

_____. 1994d. The use of debt and equity in optimal financial contracts. Manuscript. Federal Reserve Bank of Minneapolis.

Cooley, Thomas F., and Smith, Bruce D. 1992. Financial markets, specialization, and learning by doing. Manuscript. Cornell University.

Demirguc-Kunt, Asli, and Levine, Ross. 1993. Stock market development and financial intermediary growth: A research agenda. Manuscript. The World Bank.

Diamond, Douglas W. 1984. Financial intermediation and delegated monitoring. *Review of Economic Studies* 51, 393–414.

Diamond, Peter A. 1965. National debt in a neoclassical growth model. *American Economic Review* 55.

Gale, Douglas, and Hellwig, Martin. 1985. Incentive-compatible debt contracts: The one-period problem. *Review of Economic Studies* 52, 647–63.

Gurley, John G., and Shaw, Edward S. 1955. Financial aspects of economic development. *American Economic Review* 45, 515–38.

_____. 1960. *Money in a theory of finance.* Brookings Institute, Washington, D.C.

_____. 1967. Financial structure and economic development. *Economic Development and Cultural Change* 15, 257–68.

Hamid, Javed, and Singh, Ajit. 1992. Corporate financial structures in developing countries. Technical Paper 1. International Finance Corporation.

Krasa, Stefan, and Villamil, Anne P. 1992. Monitoring the monitor: An incentive structure for a financial intermediary. *Journal of Economic Theory* 57, 197–221.

Levine, Ross. 1991. Stock markets, growth, and tax policy. *Journal of Finance* 46, 1445–65.

Levine, Ross, and Zervos, Sara. 994. Stock markets and banks: Revving the engines of growth. Manuscript. The World Bank.

McKinnon, Ronald I. 1973. *Money and capital in economic development*. Brookings Institute, Washington, D.C.

Michie, R. C. 1987. *The London and New York Stock Exchanges, 1850-1914*. Allen and Unwin. London.

Shaw, Edward C. 1973. *Financial deepening in economic development*. New York: Oxford University Press.

Townsend, Robert M. 1979. Optimal contracts and competitive markets with costly state verification. *Journal of Economic Theory* 21, 265-93.

Tun Wai, U., and Patrick, Hugh T. 1973. Stock and bond issues and capital markets in less developed countries. *International Monetary Fund Staff Papers 20*, 253-317.

Watson, C., Maxell, G., Kincaid, R., Atkinson, C., Kalter, E., and Folkerts-Landau, D. 1986. International capital markets: Developments and markets. International Monetary Fund, World Economic and Financial Surveys.

Williamson, Stephen D. 1986. Costly monitoring, financial intermediation, and equilibrium credit rationing. *Journal of Monetary Economics* 18, 159-79.

_____. Costly monitoring, loan contracts, and equilibrium credit rationing. *Quarterly Journal of Economics* 18, 159-79.

World Bank. 1989. World development report 1989. New York: Oxford University Press.

Table 1

Country	Average Market Capitalization[a] (percentage of GNP)
High-Income Countries	
Japan	92
United Kingdom	80
United States	58
Germany, Fed. Rep. of	21
France	18
Developing Countries	
Jordan	60
Malaysia	58
Chile	27
Korea, Republic of	19
Portugal	10
Zimbabwe	10
Thailand	9
Mexico	8
Brazil	7
Philippines	7
Venezuela	7
India	6
Greece	5
Pakistan	5
Nigeria	4
Columbia	3
Turkey	3
Argentina	2

[a]Average market capitalization is a five quarter average of the total value of listed stocks, based on year-end data, assuming constant exponential growth during the year.

Source: The World Development Report, 1989, Table 7.1

Figure 1

THE FUNCTION π
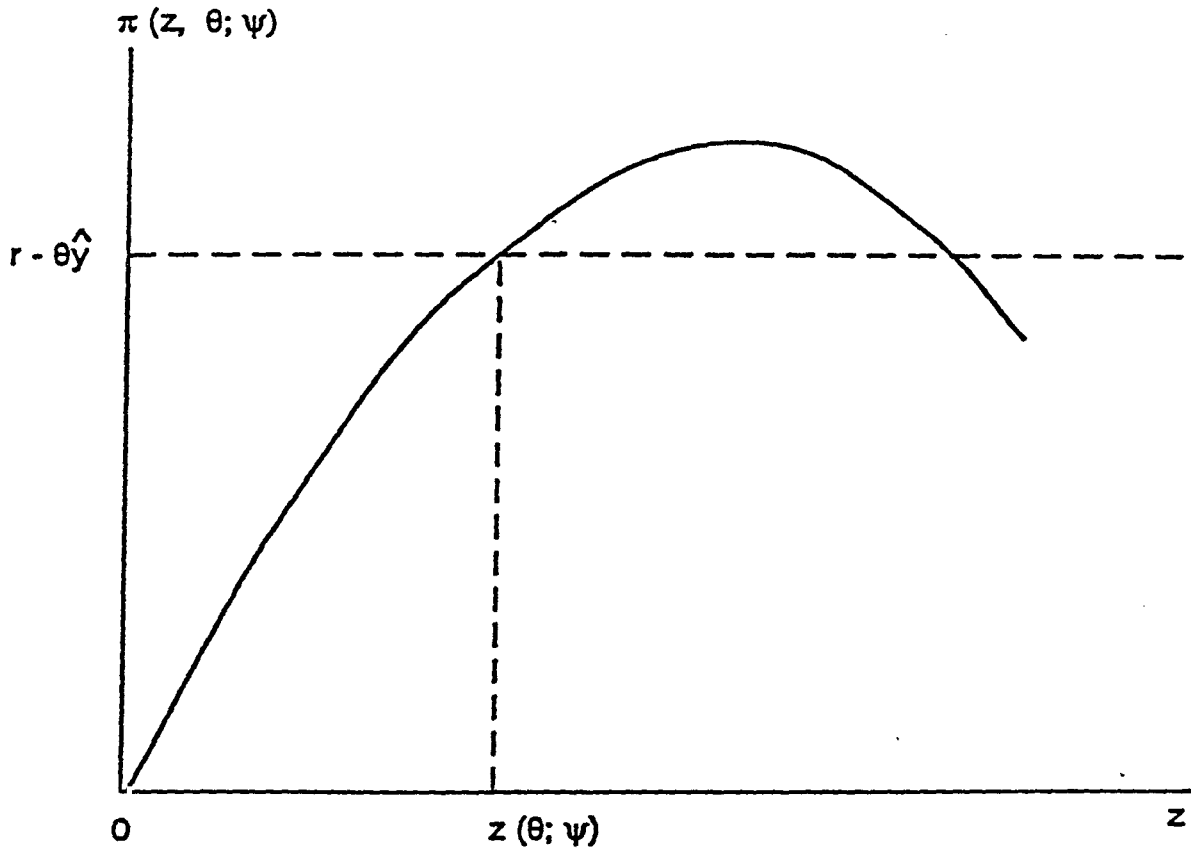
Figure 2

BORROWER'S OBJECTIVE FUNCTION

$H(\theta:\psi)$

0   θ^* $r/\hat{y}$ θ
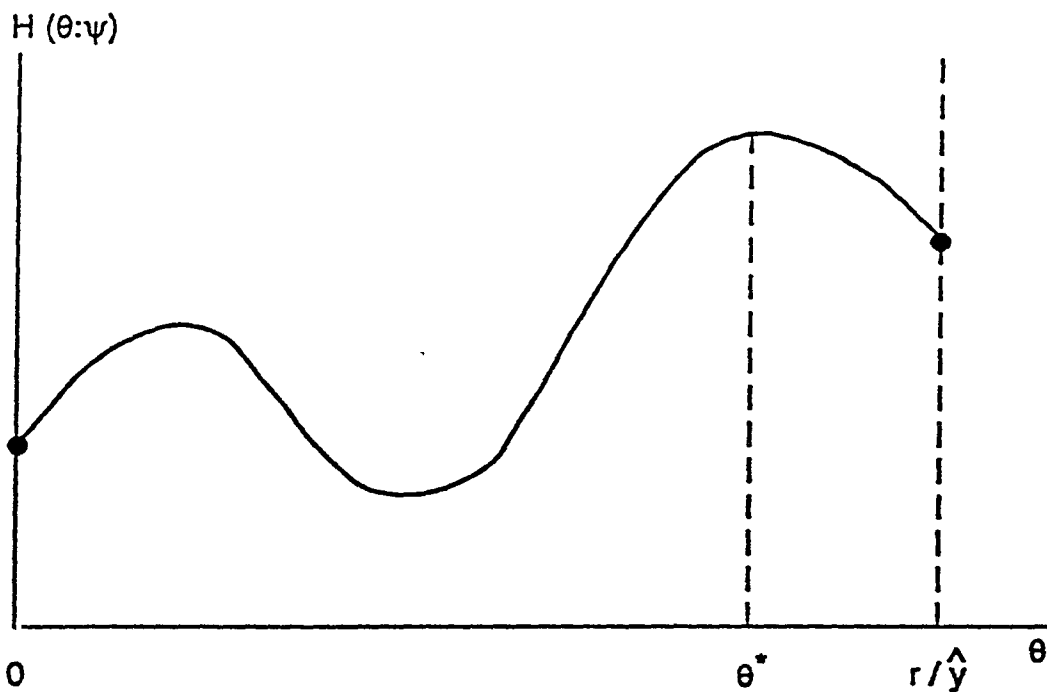
Figure 2.a

BORROWER'S OBJECTIVE FUNCTION

$H(\theta:\psi)$

0   θ^* $r/\hat{y}$ θ
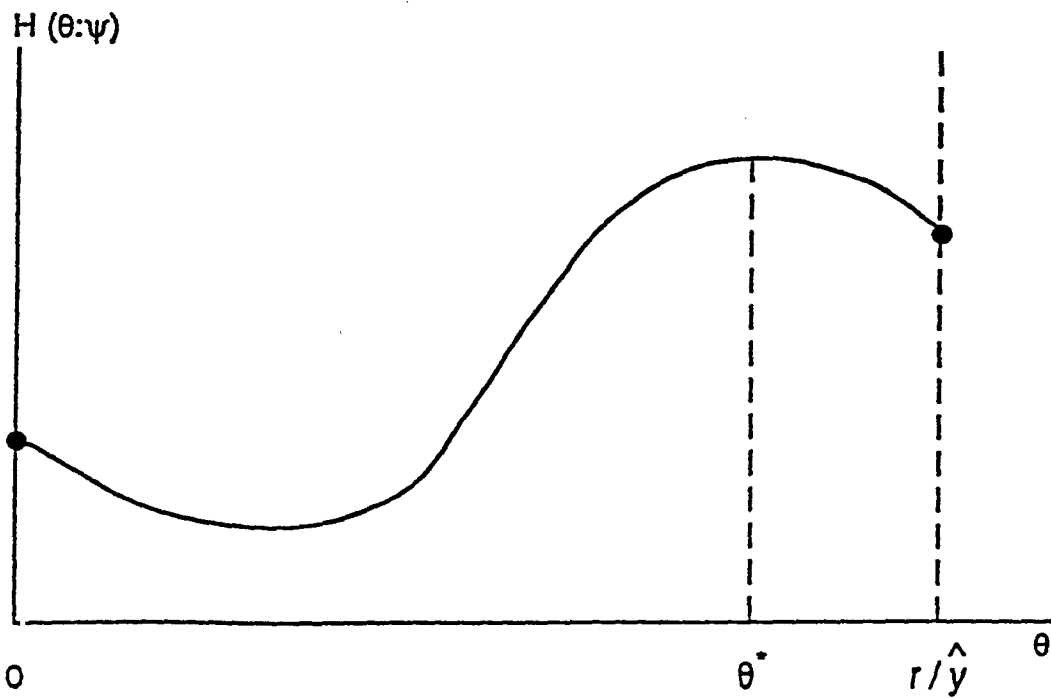
Figure 3.

EQUILIBRIUM LAW OF MOTION FOR k_t
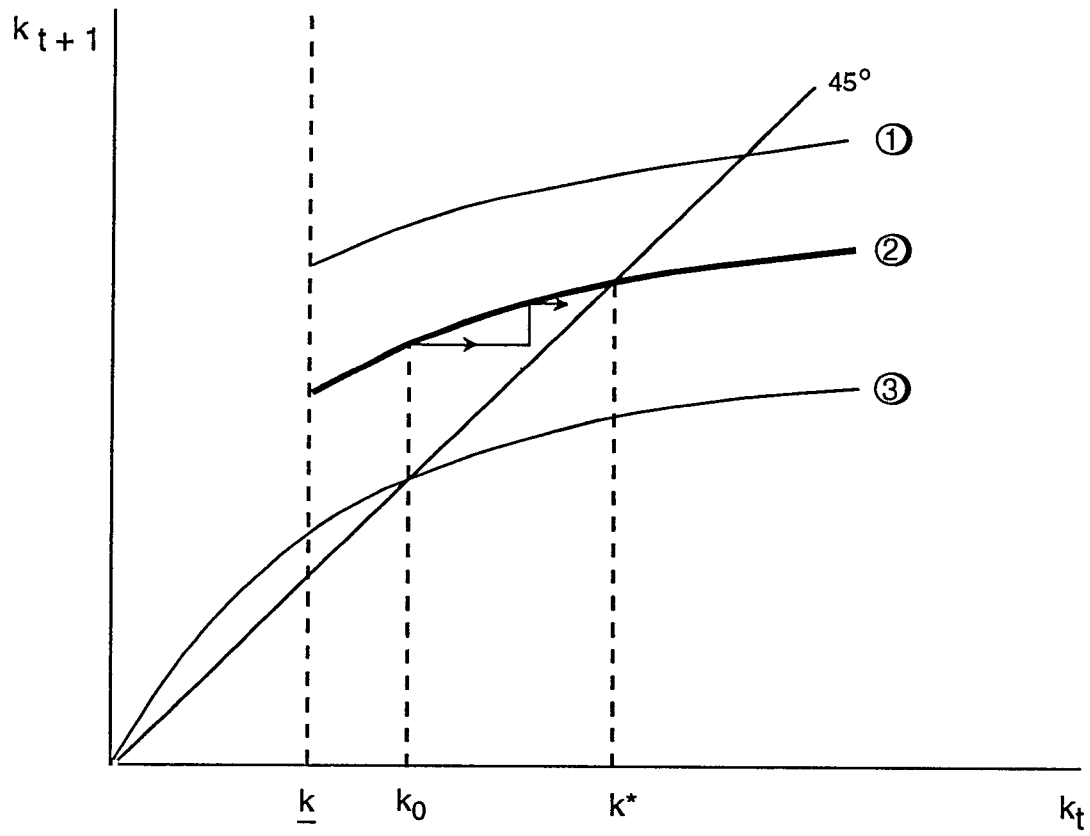
① $k_{t+1} = r\,\omega\,(k_t) + [\alpha\,/\,(1-\alpha)]\,q\,(\widehat{w} - r)$

② $k_{t+1} = r\,\omega\,(k_t) + [\alpha\,/\,(1-\alpha)\,]\,q\,[\theta_t^*\,\widehat{y} + (1-\theta_t^*)\,\widehat{w} - r]$

③ $k_{t+1} = r\,\omega\,(k_t)$

Figure 4.a

$Q(\theta; \psi)$

$Q(\theta; \psi_1)$

$Q(\theta; \psi_2)$

$Q(\theta; \psi_3)$

r

$\underline{\theta}_2$ $\underline{\theta}_3$ $r/\hat{y}$ θ

$\psi_1 < \psi_2 < \psi_3$

Figure 4.b

$Q(\theta; \psi)$

$Q(\theta; \psi_1)$

$Q(\theta; \psi_2)$

$Q(\theta; \psi_3)$

r

$\underline{\theta}_3$ $r/\hat{y}$ θ

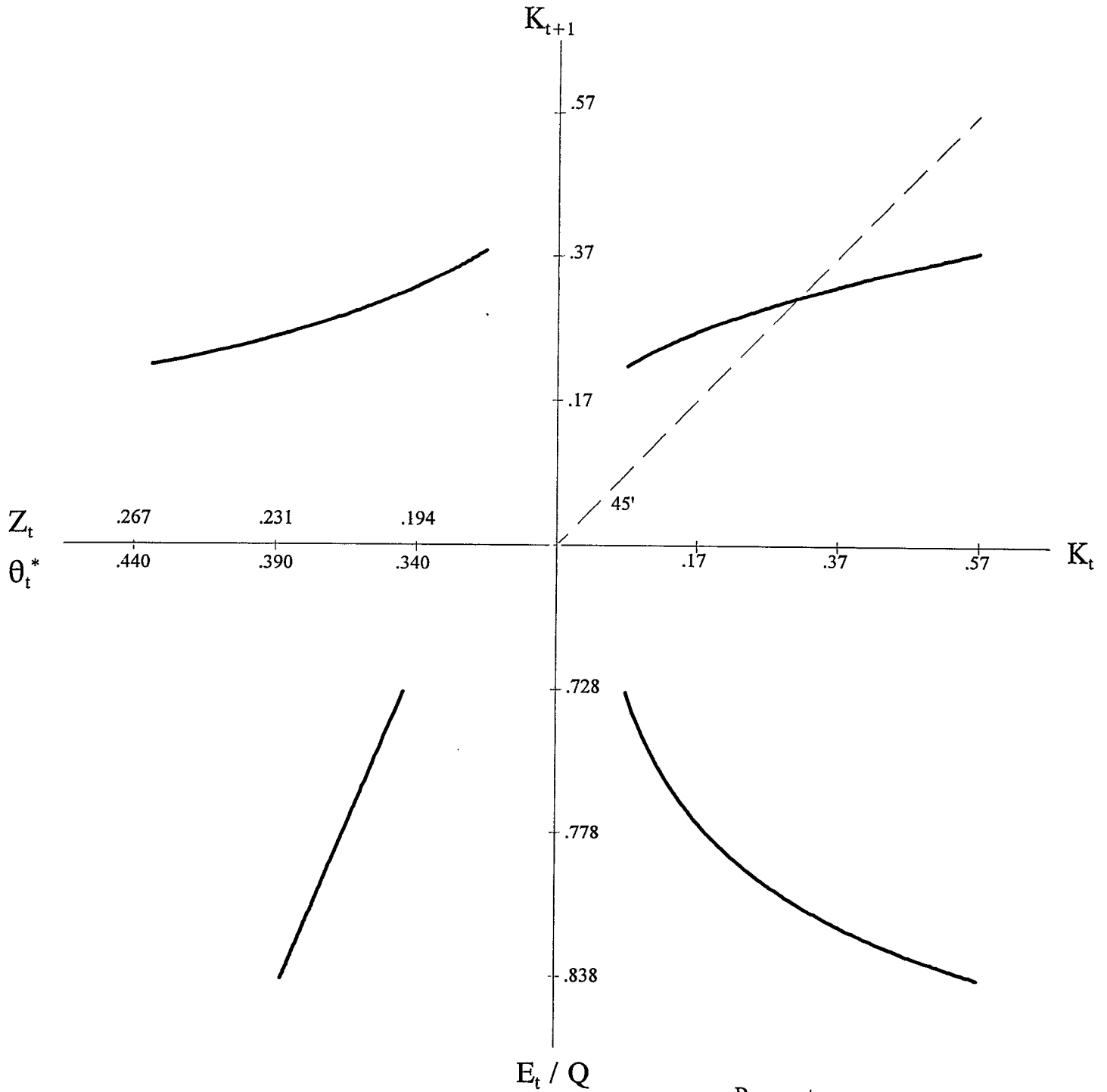$\psi_1 < \psi_2 < \psi_3$
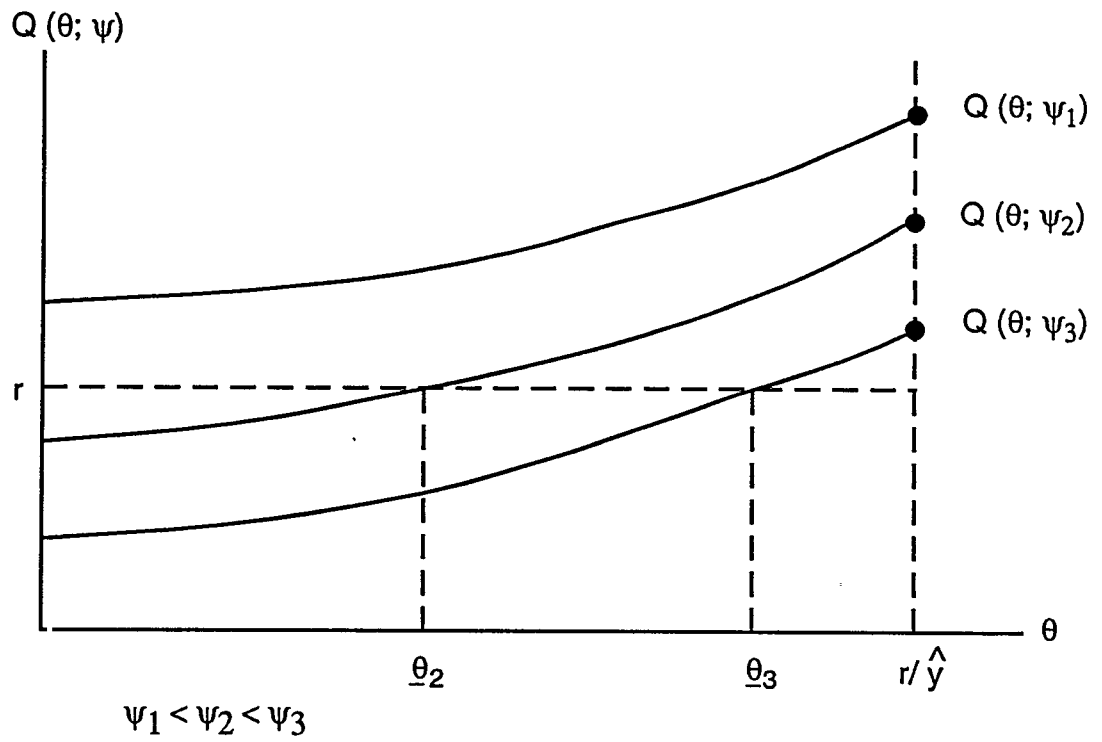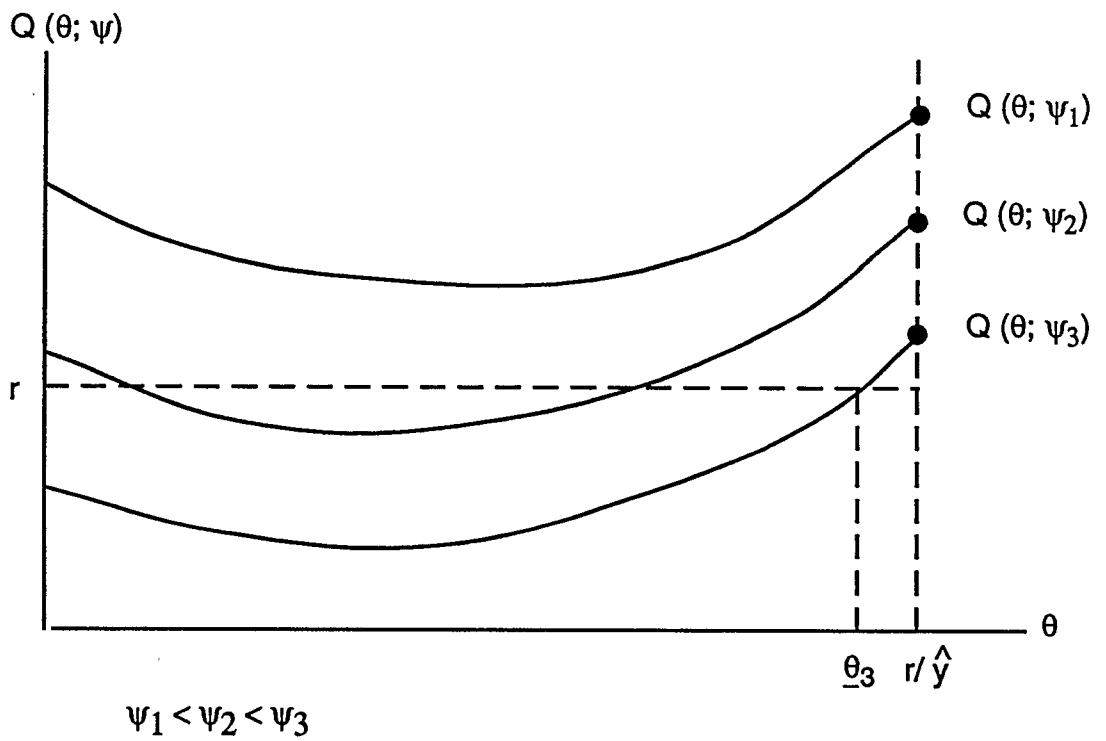
Figure 5

TRIANGULAR DISTRIBUTION
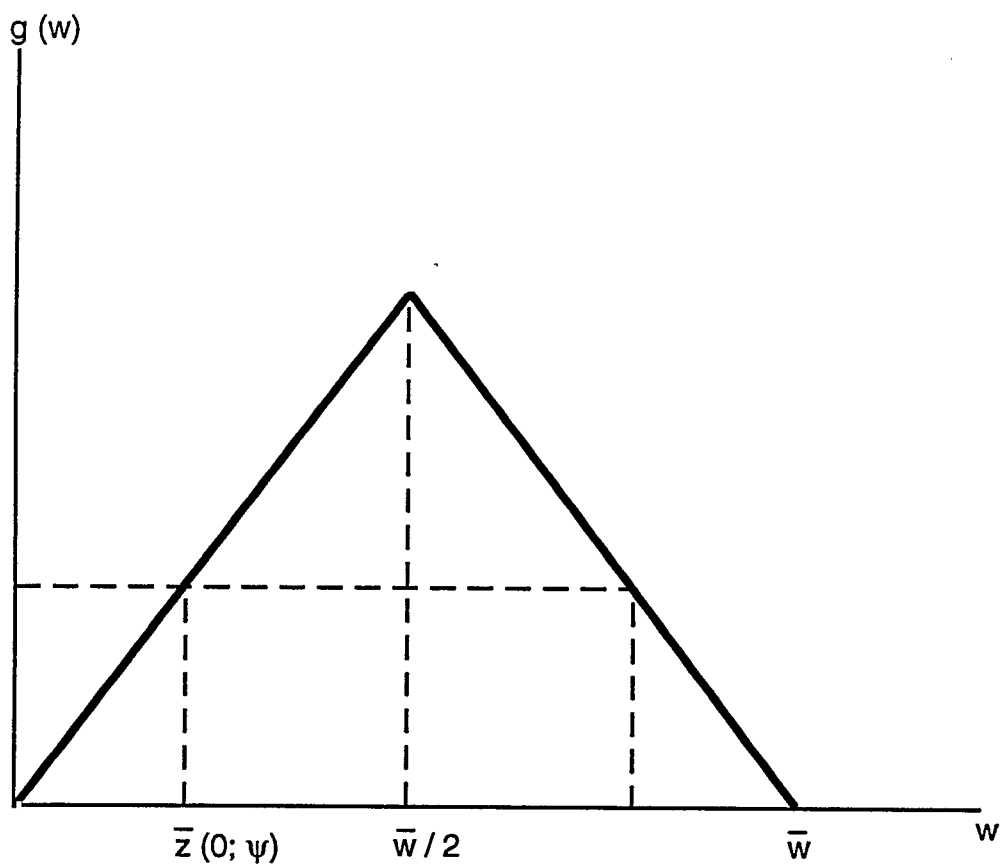
Example 1 / Figure 6
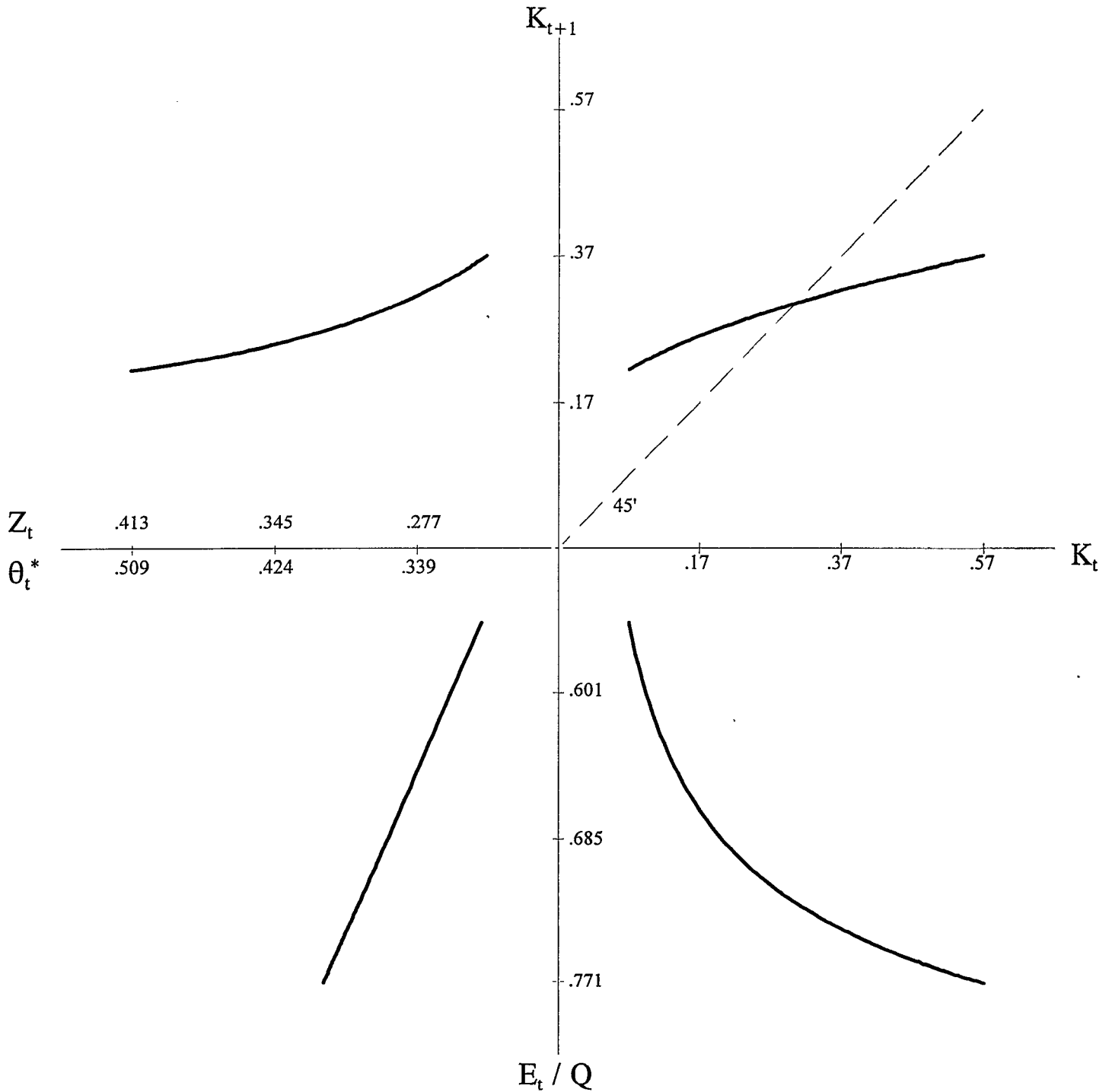
Parameters:

$\hat{w} = 1.6$	$q = 1.0$
$\hat{y} = 1.2$	$\gamma = 1.67$
$r = 0.6$	$\alpha = 0.1268$
$B = 1.1$	$\beta = 0.5$

Example 2 / Figure 7

K_{t+1}

.57

.37

.17

Z_t
.267　　　.231　　　.194

θ_t^*
.440　　　.390　　　.340

.17　　　.37　　　.57　　　K_t

45'

.728

.778

.838

E_t / Q

Parameters:

$\hat{w} = 1.6$　　　$q = 1.0$
$\hat{y} = 1.3$　　　$\gamma = 1.67$
$r = 0.6$　　　$\alpha = 0.1268$
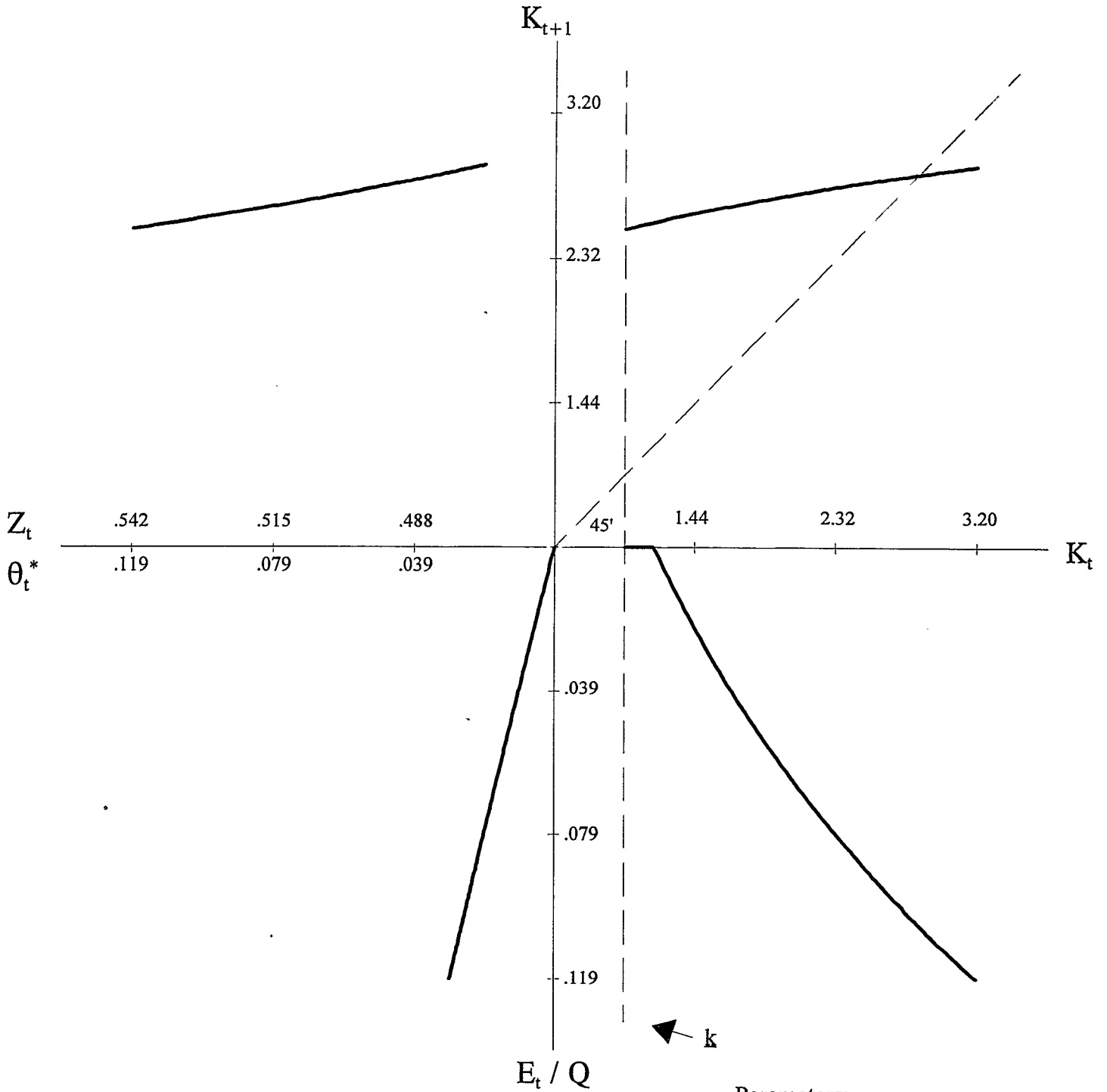$B = 1.1$　　　$\beta = 0.5$

Figure 8
Interest Rate Differentials
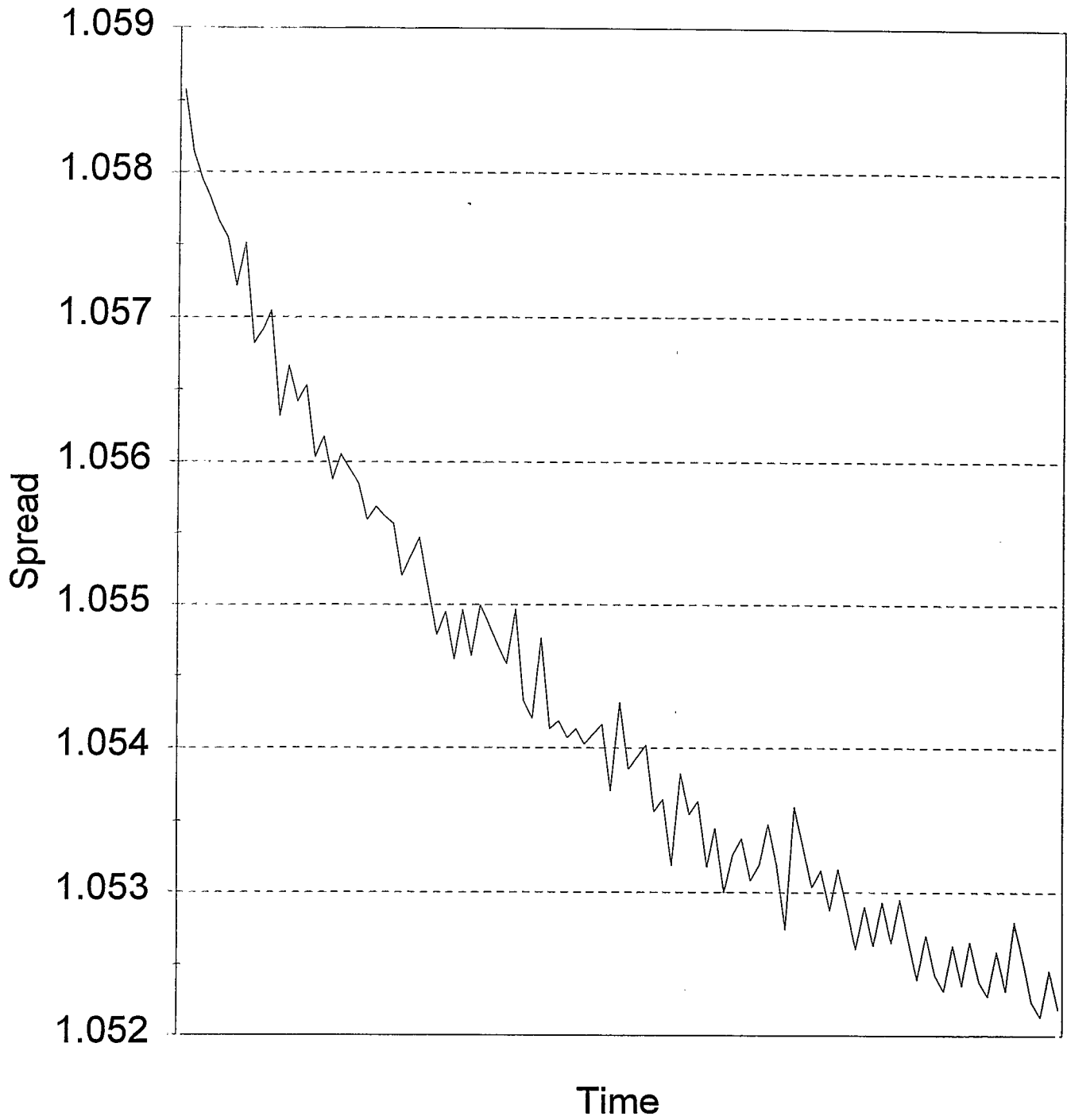
Example 3 / Figure9
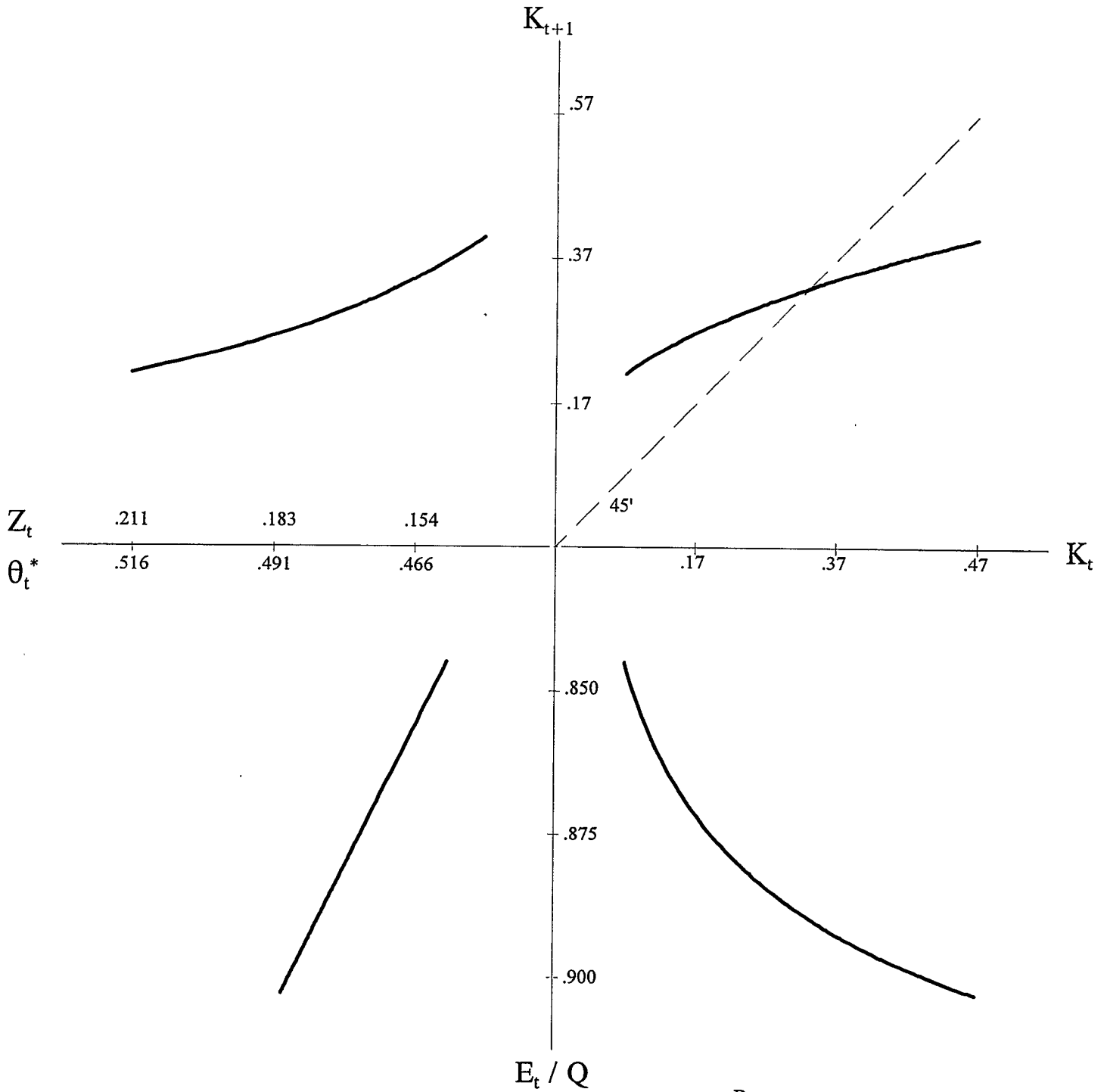
Parameters:

$\hat{w} = 1.6$ $q = 1.0$
$\hat{y} = 1.3$ $\gamma = 1.67$
$r = 0.7$ $\alpha = 0.1268$
$B = 1.1$ $\beta = 0.5$
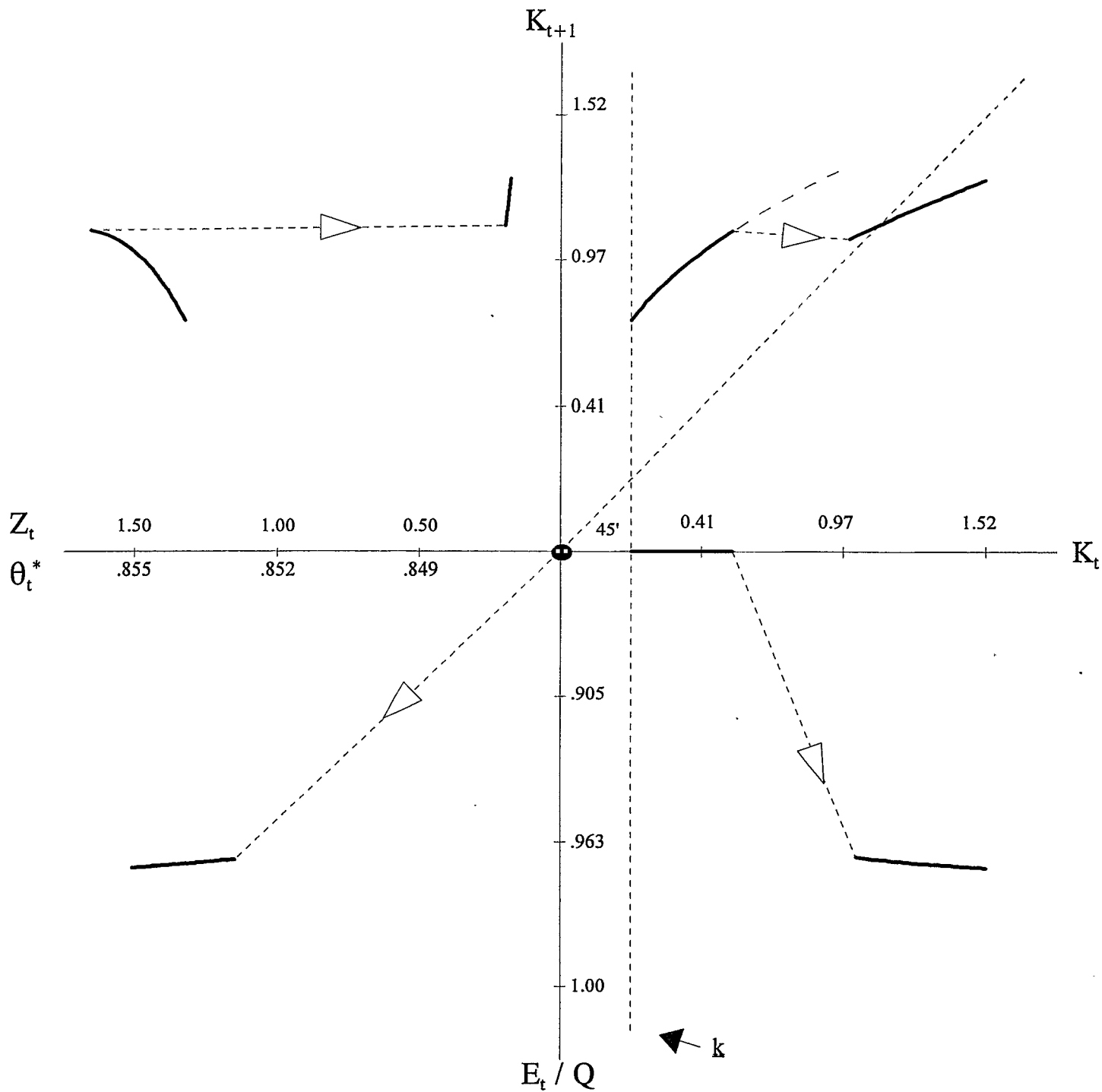
Example 4 / Figure 10

Parameters:

$\hat{w} = 2.5$ $q = 1.0$
$\hat{y} = 2.2805$ $\gamma = 0.8$
$r = 0.5$ $\alpha = 0.5$
$B = 2.0$ $\beta = 0.5$

Example 5 / Figure 11

Parameters:

$\hat{w} = 2.0$	$q = 1.0$
$\hat{y} = 1.1$	$\gamma = 1.417$
$r = 0.96$	$\alpha = 0.27$
$B = 2.0$	$\beta = 0.5$