Federal Reserve Bank of Minneapolis
Research Department

Asset Pricing Lessons
for Modeling Business Cycles

Michele Boldrin, Lawrence J. Christiano,
and Jonas D. M. Fisher*

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Abstract

We develop a model which accounts for the observed equity premium and average risk-free rate, without implying counterfactually high risk aversion. The model also does well in accounting for business-cycle phenomena. With respect to the conventional measures of business-cycle volatility and comovement with output, the model does roughly as well as the standard business-cycle model. On two other dimensions, the model's business-cycle implications are actually improved. Its enhanced internal propagation allows it to account for the fact that there is positive persistence in output growth, and the model also provides a resolution to the "excess sensitivity puzzle" for consumption and income. Two key features of the model are habit persistence preferences and a multisector technology with limited intersectoral mobility of factors of production.

1 Introduction

General equilibrium models with complete markets and optimizing agents have enjoyed a measure of success in accounting for business cycle fluctuations in quantities. However, these models have been notoriously unsuccessful in accounting for the behavior of asset prices.[1] Two failures in particular have attracted the most attention: the *equity premium puzzle*, the fact that returns on the stock market exceed the return on Treasury bills by an average of six percentage points; and the *risk-free rate puzzle*, the fact that Treasury bills on the average earn a very low return. For the most part, the response of business-cycle researchers has been to ignore the asset pricing implications of their models.

This is unfortunate. As emphasized most recently by Cochrane and Hansen (1992), business-cycle models assume that households equate intertemporal marginal rates of substitution in utility with intertemporal marginal rates of transformation. Under the complete markets hypothesis, asset returns offer a direct measure on these margins, and so should provide an excellent guide to constructing and evaluating business-cycle models.

This is the perspective adopted here.[2] We take the standard business-cycle model as our starting point, and modify it by replacing the power specification of utility with the habit persistence specification proposed by Constantinides (1990) and Sundaresan (1989).[3] There are two reasons why we do this. First, as demonstrated by Constantinides, habit persistence has the potential to account for both of the asset return puzzles, while implying only a modest degree of risk aversion on the part of households. Alternatives (for example, Abel's 1990 "catching up with the Jones" specification, power utility, and nonexpected utility) in practice require high-risk aversion to account for the asset pricing puzzles.[4] Throughout our analysis,

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[1]Influential early discussions of this include Hall (1978), Hansen and Singleton (1982, 1983), and Mehra and Prescott (1985).

[2]Since starting on this work we have become aware of independent research along similar lines. This includes the work of Danthine and Donaldson (1994), Jermann (1994), Lettau and Uhlig (1995), and Tallarini (1995). Among these papers, only Jermann considers habit persistence preferences. Below we discuss the similarities and differences between our papers.

[3]Other researchers have investigated a different set of perturbations to the complete markets model. See, for example, Nason (1988), Reitz (1988), David, Oh, Ostroy, and Shin (1992), and Tsionas (1994). Some have followed the suggestion of Mehra and Prescott (1985) by investigating the potential of market incompleteness to account for the equity premium and risk-free rate. See, e.g., Aiyagari and Gertler (1991), Danthine, Donaldson, and Mehra (1992), Heaton and Lucas (1992), Mankiw (1986), and Weil (1992).

[4]The analyses we have in mind here are based on pure exchange economies in which the equilibrium

we restrict the parameterization of habit persistence so that the coefficient of relative risk aversion roughly averages unity. Our second reason for studying habit persistence preferences is that, according to several econometric analyses, this form of preferences can reconcile US data on consumption and asset returns (see Ferson and Constantinides 1991, Burnside 1994, and Heaton 1995).

We show that introducing habit persistence preferences into the standard business-cycle model has *no* impact on the equity premium. After diagnosing the reasons for this, we address the following questions:

- How must the technology in this model be modified to account for the mean risk-free rate and equity premium?

- What are the business-cycle implications of the resulting model?

We develop a model that accounts for the equity premium and average risk-free rate. Our model's ability to account for the equity premium lies in producing the right business-cycle pattern in the price of capital. To generate this pattern, we adopt—in addition to habit persistence—a multisectoral technology with limited intersectoral mobility of factors of production.

Our model's successes in accounting for asset pricing phenomena do not come at the expense of its business-cycle implications. With respect to the conventional measures of business-cycle volatility and comovement with output, the model does roughly as well as the standard business-cycle model. On two other dimensions, the model actually outperforms the standard model. First, the dynamics in our model enhance its internal propagation of shocks, improving its ability to account for the observed persistence in output growth. Second, our model accounts for the so-called *excess sensitivity puzzle*: instrumental variable regressions

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consumption process is specified exogenously. The "catching-up-with-the-Jones" and nonexpected utility specifications studied by Campbell and Cochrane (1995) and Weil (1989, 1992) use risk aversion in excess of 40. Risk aversion in excess of 30 is required in the Mehra and Prescott (1985) model to simultaneously drive the risk-free rate below its empirical value and the equity premium above its empirical value (see Section 3 below). For recent evidence which suggests that levels of risk aversion this high are empirically implausible, see Barsky, Juster, Kimball, and Shapiro (1995). By resorting to nonstandard distributions for the equilibrium consumption process, it is possible to account for the asset pricing puzzles with power utility and lower risk aversion. See, for example, Kandel and Stambaugh (1991), Reitz (1988), and Tsionas (1994).

indicate that consumption growth is strongly related to income, while being relatively weakly related to interest rates (Hall 1988, and Campbell and Mankiw 1989, 1991). While this puzzle is an embarrassment for the standard business-cycle model, it is not a problem for ours.

The following section provides a brief, nontechnical overview of our paper and the main results. After that comes the formal analysis, followed by concluding remarks.

2   Overview of the Analysis

Our analysis begins with a version of the pure exchange economy studied in Lucas (1978) and Mehra and Prescott (1985). We use this to establish a benchmark and to identify the key channels by which changes in preferences affects the equity premium and the risk-free rate. The insights obtained here are then applied to business-cycle modeling. The following two subsections summarize our basic results for the exchange economy and for business-cycle models, respectively.

2.1   Overview of Findings for the Exchange Economy

Consistent with the results in Constantinides (1990), we show that habit persistence with low risk aversion can account for both the equity premium and risk-free rate puzzles in an exchange economy. Here we review our results with respect to the equity premium.

It is useful to recall a classic covariance formula: the equity premium is negatively related to the conditional covariance between the one-period-ahead marginal utility of consumption and the rate of return on equity. A change in the specification of the model changes both arguments in the covariance term. Thus when we switch from power utility to habit persistence there are two effects that raise the equity premium. On the one hand, it increases the spread, across states of nature, of the one-period-ahead marginal utility of consumption. When other things are held constant, this raises the equity premium. We refer to this mechanism as the *curvature channel,* because it is determined by the degree of curvature in the utility function. On the other hand, the type of consumption-smoothing motive inherent in habit persistence gives rise to a particular pattern of demand for assets across states of nature: when consumption opportunities are high, households seek to buy assets, and when

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consumption opportunities are low, they seek to sell. Because the stock of physical capital is fixed in the exchange economy, variation in the demand for equity translates into large fluctuations in the price of capital across states of nature, with large capital gains in states when consumption is high, and small or negative capital gains when consumption is low. When other things are held constant, this also raises the equity premium. We refer to this as the *capital gains channel.*

The curvature channel is the exclusive focus of much of the empirical literature on the equity premium (we have in mind here the work stimulated by Hansen and Singleton 1982, 1983). The literature takes the empirical process for consumption and the rates of return as given and evaluates alternative specifications of utility for their ability to reconcile the two. In analyses of general equilibrium economies the rate of return on equity is endogenous, and so the capital gains channel also plays a role.

There are two reasons why this channel warrants considerable attention. First, in all of our computational experiments, the capital gains channel plays by far the most important role quantitatively. Second, because the price of capital reflects the outlook for events extending into the distant future, it is influenced by many other features of the environment in addition to the curvature properties of the utility function. (We measure curvature by the elasticity of the marginal utility of consumption with respect to consumption.) These features include such things as households' preferences over the intertemporal pattern of consumption, and the persistence properties of households' consumption opportunities. Thus although our computational experiments suggest that high curvature is a necessary ingredient for getting the equity premium, it is by no means sufficient. We dramatize this point by discussing examples in which there is high curvature, yet the equity premium is negative.

These considerations suggest that the following two ingredients are crucial for a general equilibrium model if it is to generate an equity premium: (i) a strong incentive on the part of households to buy assets when the marginal utility of consumption is low, and to sell assets when the marginal utility of consumption is high; and (ii) a technology that frustrates these desires. The ability of the exchange economy to account for asset returns in part reflects the extreme position it takes on (ii): capital supply is completely inelastic and labor supply cannot be varied to offset the consumption impact of unfavorable shocks.

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2.2 Overview of Findings for Modeling Business Cycles

Armed with this intuition, we proceed to analyze the introduction of habit persistence into the standard real business-cycle model. As noted previously, we find that this modification has essentially *no* impact on the equity premium. But this is not surprising, in view of the intuition developed above. The real business-cycle model in effect assumes that the supply of capital is infinitely elastic, so that its equilibrium price is a constant. As is well-known, the payoff on capital (its rental rate) in the standard business-cycle model fluctuates very little. As a result, fixing the price of capital essentially shuts down the capital gains channel. The curvature channel is more complicated in the production economy because of the endogeneity of consumption.[5] Still, it plays a negligible quantitative role.

Our interpretation of the absence of an equity premium in this model is that households have an unrealistically large number of opportunities to smooth consumption. These reflect their ability to flexibly exploit three margins: variations in labor effort, variations in the rate of capital accumulation, and variations in the allocation of factors of production to the consumption and investment goods sectors. We proceed to study a multisector production model in which households have less flexibility.

For our first modification, we assume that capital and consumption are nonhomogeneous goods and that capital inputs must be assigned to the production of the two goods prior to the realization of the current period technology shock. We assume, for example, that an oven used to bake bread cannot be instantaneously transformed into a bulldozer. Introducing this form of *ex post* inflexibility converts the model into a two-sector model. Because there are diminishing returns in varying the labor input, this places curvature in the production transformation frontier between consumption and investment goods, making the supply of capital less than infinitely elastic. This change has a positive impact on the model's equity premium, though it does not get it even close to its empirically observed value.

Next, we assume that the sectoral labor inputs also have to be chosen one period in advance, before the technology shock is realized. This assumption captures the various real-

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[5]For example, in the context of the exchange economy, only events during the life of the one-period equity claim play a role in the curvature channel, because equilibrium consumption is exogenous. In a business-cycle model, consumption is influenced in part by views about the future, and so the future also plays a role in the curvature channel.

world factors that make finding work or changing jobs a time-consuming process. As a result of these assumptions, capital supply is completely inelastic in the period of a shock.[6] At least in the short run, the model resembles the exchange economy in that there are no opportunities to insulate consumption from contemporaneous shocks. The cumulative effect of these modifications is to raise the equity premium to 2 percent.

Finally, we introduce an amount of leverage suggested by the empirical analysis in Benninga and Protopapadakis (1990). When we do so the equity premium jumps to around 5 percent. At the same time, the risk-free rate is 2.7 percent. Based on the statistical information provided in Cecchetti, Lam, and Mark (1993), we argue that these values are not significantly different from their empirical counterparts.

The last step in our analysis is to study the business-cycle implications of this model. We find that they are surprisingly good. Where the model does poorly, it does not do substantially worse than the standard business-cycle model. On the plus side, the short-term rigidities in our model and its multisectoral structure enhance its internal propagation by delaying the full response by factors of production to shocks. Habit persistence also plays a role here. For example, when a shock generates a positive innovation in consumption, habit persistence creates an incentive to apply factors of production in such a way as to keep consumption high for several periods.[7]

Our model's ability to resolve the excess sensitivity of consumption growth to income reflects that, under habit persistence, the intertemporal Euler equation relates consumption growth to lagged consumption growth, as well as to expectations of future consumption growth. In this case, the apparent excess sensitivity to income reflects income's statistical

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[6]See Goolsbee (1994) for empirical evidence that short-run capital supply is very inelastic.

[7]Among the papers mentioned in the introduction that are related to ours, the closest is that of Jermann (1994), who also studies habit persistence preferences. Still, his paper differs in many respects from ours. For example, the version of Jermann's model that incorporates habit persistence holds aggregate labor effort constant, while the intersectoral allocation of capital and labor are free to respond to shocks. Consistent with the analysis here, Jermann finds that capital supply must be less than perfectly elastic if he is to get an equity premium. Unlike the analysis here, his strategy for reducing this elasticity is to impose adjustment costs on the installation of new capital. One important difference between the two papers lies in the interpretation of the results: in analyzing the impact of changing preferences on the equity premium, Jermann does not emphasize the capital gains channel, whereas we stress its central role. Finally, there is a difference in focus between the two papers. Ours focuses relatively more on the business-cycle implications of the model, while Jermann focuses on a broader set of asset pricing implications.

role as a proxy for these variables. The model's ability to account for the lack of sensitivity of consumption growth to interest rates is perhaps not surprising, in view of the fact that our assumption of high curvature is equivalent to the assumption of low intertemporal substitution in consumption. Because agents in our model have low risk aversion, our framework provides a formal basis for Hall's (1988) suggestion that the weak empirical relation between consumption growth and the interest rate should be interpreted as reflecting low intertemporal substitution in consumption instead of high risk aversion.

3 The Exchange Economy

In this section we analyze versions of the exchange economy studied in Lucas (1978) and Mehra and Prescott (1985). We accomplish the following three objectives. First, we describe the model economy, establish the notation and market decentralization used throughout the paper, and document that the model is consistent with the observed mean risk-free rate and equity premium. Second, we review some key properties of habit persistence preferences and compare them with power utility and the "catching up with the Jones" preferences recently studied by Abel (1990), Campbell and Cochrane (1995), and Lettau and Uhlig (1995). Third, we present experiments designed to shed light on the channels by which changes in preferences affect the risk-free rate and equity premium in equilibrium. Our primary result is that the main channel by which a change in preferences affects the equity premium is the capital gains channel discussed in the previous section.

3.1 The Model Economy

Households

The economy is composed of a continuum of infinitely lived, identical households who maximize expected discounted utility. Let $\mathcal{E}_t$ denote the expectation operator, conditional on the information available at time t. At every date t, the representative household values

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consumption from that point forward according to:[8]

$$(1) \quad \mathcal{E}_t \sum_{j=t}^{\infty} \beta^{j-t} \frac{(C_j - X_j)^{1-\phi} - 1}{1 - \phi},$$

where X_j represents the *habit stock,* which evolves as follows:

$$(2) \quad X_j = h X_{j-1} + b C_{j-1}.$$

In (1), $0 < \beta < 1$ is the household's discount factor. For the purposes of our analysis, we define *power utility preferences* as the case $\phi > 0$, $h = b = 0$, and *habit persistence preferences* as the case $\phi \equiv 1$, and either h or $b \neq 0$.

At every date t, the household must satisfy the following budget constraint:

$$(3) \quad B_t + S_t + C_t \leq (1 + r_t^e) S_{t-1} + (1 + r_{t-1}^f) B_{t-1},$$

where B_t and S_t denote period t acquisition of two types of one-period assets, denominated in consumption units. Their rates of return are $1 + r_t^f$ and $1 + r_{t+1}^e$, respectively. The rate of return on S_t is conditional on the realization of the date $t+1$ state of nature, and the rate of return on B_t is not. The problem of the household is as follows: at every date t, it takes S_{t-1}, B_{t-1}, X_t and $\{r_j^e, r_{j-1}^f; j \geq t\}$ as given and maximizes (1) subject to (2), and (3) by choice of $\{B_j, S_j, C_j; j \geq t\}$.

Firms

The technology for converting capital, K_t, into output, Y_t, is as follows:

$$(4) \quad Y_t = Z_t K_t,$$

where

$$(5) \quad Z_t = Z_{t-1} \exp(\theta_t).$$

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[8]This (standard) specification of the habit persistence utility function has the distinctive feature that the present discounted value of the utility of a consumption sequence is nonmonotone in the consumption of any particular period. This reflects the fact that, although the period utility function is increasing in current consumption, period utility at later dates is decreasing in current consumption. This latter effect dominates at high values of consumption. In the simulations computed for this paper, consumption is always in the region of increasing marginal utility.

The random variable θ_t follows the autoregressive process

$$(6) \quad \theta_t = (1 - \rho)\overline{\theta} + \rho\theta_{t-1} + \epsilon_t,$$

and $\epsilon_t \sim N(0, \sigma^2)$, for all $t \geq 0$. Capital does not depreciate, and there exists no technology for increasing or decreasing its magnitude. The aggregate, per capita stock of capital is a constant, equal to $K > 0$.

We assume that firms have a one-period planning horizon. In order to operate capital in period $t + 1$, a firm must purchase it in period t. To do so, it issues equity S_t, subject to the following financing constraint:

$$(7) \quad P_{k,t}K_{t+1} \leq S_t,$$

where $P_{k,t}$ is the date t price of capital, denominated in consumption units, and K_{t+1} represents the quantity of capital the firm plans to use. Let π_{t+1} denote the firm's period $t + 1$ revenues, net of expenses, denominated in period $t + 1$ consumption units. Revenues include the sale of output, Y_{t+1}, plus the sale of the capital stock, $P_{k,t+1}K_{t+1}$. The firm's expenses are limited to its obligations on equity, $(1 + r_{t+1}^e)S_t$. Its choice variables are S_t and K_{t+1}, and it takes $P_{k,t}$ and the state contingent objects r_{t+1}^e and $P_{k,t+1}$ as given. The firm's outlays in each state of the world must not exceed its revenues:

$$(8) \quad \pi_{t+1} = Y_{t+1} + P_{kt+1}K_{t+1} - (1 + r_{t+1}^e)S_t \geq 0.$$

The firm's problem at date t is to maximize, by choice of S_t and K_{t+1}, the value of π_{t+1} across states of the world, subject to (4)-(8). This implies that the financing constraint, (7), is satisfied as a strict equality in equilibrium. Linear homogeneity of the firm's objective, together with the weak inequality in (8), imply the equilibrium condition, $\pi_{t+1} = 0$ for all $t + 1$, and for all states of nature, so that:

$$(9) \quad 1 + r_{t+1}^e = \frac{Z_{t+1} + P_{k,t+1}}{P_{k,t}}.$$

Equilibrium

We adopt the normalization that the number of households and firms is one. Then the resource constraints for this economy can be expressed as follows:

$$(10) \quad C_t \leq Y_t, \quad K_t \leq K.$$

A sequence-of-markets competitive equilibrium is defined in the usual way.

The objects in equilibrium are obtained as follows. First, $C_t = Z_t K$. We find prices by combining the household's first-order condition for S_t with (5) and (9) to get:

$$(11) \quad p_{k,t} = \mathcal{E}_t p_{c,t+1} \exp(\theta_{t+1})[1 + p_{k,t+1}],$$

where $p_{k,t} \equiv P_{k,t}/Z_t$. In addition,

$$(12) \quad p_{c,t+1} = \beta \frac{\Lambda_{c,t+1}}{\Lambda_{c,t}},$$

where $\Lambda_{c,t}$ denotes the derivative of (1) with respect to C_t. This is computable given the solution for C_t described above. We then find $p_{k,t}$ by specifying it to be a function of θ_t and solving for the fixed point of the functional equation, (11). To approximate the solution to this and other functional equations, we use the nonlinear methods described in Judd (1992) and Christiano and Fisher (1994). Given $P_{k,t} = p_{k,t} Z_t$, we solve for r_{t+1}^e using (9). Finally,

$$(13) \quad 1 + r_t^f = \frac{1}{\mathcal{E}_t p_{c,t+1}}.$$

3.2 Preferences and Asset Returns

Here we review key properties of habit persistence preferences that are relevant for asset prices. We also discuss the implications of habit persistence for the risk-free rate, and present our formal decomposition of the equity premium into curvature and capital gains channels.

Risk Aversion and Curvature

To understand the impact of habit persistence on the equity premium, it is important to distinguish the concept of relative risk aversion (RRA) from measures of the curvature of the utility function. The concept of curvature that we use is $-C\Lambda_{cc}/\Lambda_c$, where Λ_{cc} is the derivative of Λ_c with respect to C, and where absence of a time subscript indicates the value in nonstochastic steady state. With preferences like power utility, or "catching up with the Jones" curvature and risk aversion are identical. This is why researchers who seek to account for the equity premium by increasing curvature, simultaneously encounter counterfactually high levels of risk aversion. Constantinides (1990) pointed out that, in

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contrast, habit persistence preferences disentangle these two concepts. For example, for β close to 1, RRA is close to unity, *independent* of b and h. At the same time, by increasing the values of these parameters, curvature —and the equity premium—are both raised. (See the Appendix for further discussion.)

To gain intuition into why curvature can be high while RRA is low under habit persistence, recall the definition of RRA: it measures how much an individual household is willing to pay to avoid a fair bet on its wealth. This magnitude is directly related to the utility loss that the household suffers in the adverse state of the world. If the household were forced to accept an immediate drop in consumption, the loss of such a bet would be very painful, given the short-term exogeneity of the habit stock and the assumed high curvature. However, the habit persistence household can avoid this. Though the present value of its total lifetime consumption must fall, recourse to credit markets enables the household to slow the fall in actual consumption so that the habit stock can fall. This is why the disutility occasioned by the loss of a bet on wealth may be relatively small for a household with habit persistence preferences.

It is revealing to compare the implications for risk aversion of habit persistence with those of "catching up with the Jones" preferences. For the latter type of household, the habit stock is exogenous for all time, and so recourse to credit markets represents a much less effective cushion against the loss of a bet. As a result, the level of risk aversion implied by this utility function is very high. For example, in the formulation studied by Campbell and Cochrane (1995), risk aversion is 48 (see also Weil 1992). This contrasts with risk aversion of roughly unity for the habit persistence preferences studied in this paper.

The Risk-Free Rate

Consider (13) along a nonstochastic steady-state growth path in which $C_t = C_{t-1} \exp(\bar{\theta})$:

$$(14) \quad 1 + r_t^f = \begin{cases} \frac{\exp(\phi\bar{\theta})}{\beta}, & \text{for power utility} \\ \frac{\exp(\bar{\theta})}{\beta}, & \text{for habit persistence.} \end{cases}$$

As is well-known, the elasticity of intertemporal substitution in consumption is the inverse of our measure of curvature. In the appendix, we show that with habit persistence utility, intertemporal substitution is reduced by increasing b or h. Also it is well-known that raising

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ϕ reduces intertemporal substitution in consumption in the case of a power utility function. Thus, we infer from (14) that reducing intertemporal substitution has a very different impact on the risk-free rate, depending on whether one adopts habit persistence or power preferences.

The intuition for this difference between the two utility functions is simple. With power utility and positive consumption growth, the future marginal utility of consumption is low compared with the marginal utility of present consumption. Increasing ϕ intensifies this, so that a higher interest rate is required to discourage households from attempting to reallocate consumption from the future to the present. The impact of increasing b or h is quite different. This has the effect of increasing the future habit stock and, other things the same, this raises the marginal utility of future consumption, reducing the incentive to reallocate consumption toward the present.

In sum, accounting for the equity premium by increasing curvature is more likely to avoid counterfactual implications for the risk-free rate if it is done by increasing b or h, than if it is done by increasing ϕ. For a further discussion of related issues, see Weil (1989, 1992) and Campbell and Cochrane (1995).

The Equity Premium

The curvature and capital gains channels correspond to the two arguments in the conditional covariance expression for the equity premium:

$$(15) \quad r_t^{ep} = \frac{\mathcal{E}_t(1 + r_{t+1}^e)}{1 + r_t^f} = 1 - Cov_t\left(\beta \frac{\Lambda_{c,t+1}}{\Lambda_{c,t}}, \frac{Z_{t+1} + P_{k,t+1}}{P_{k,t}}\right),$$

where $Cov_t(x, y)$ denotes the date t conditional covariance between x and y. Let $\Delta E r_t^{ep}$ denote the change in mean of the equity premium due to a change in preferences. Our decomposition is:

$$(16) \quad \Delta E r_t^{ep} = \delta_\Lambda E r_t^{ep} + \delta_{P_k} E r_t^{ep},$$

where $\delta_\Lambda E r_t^{ep}$ and $\delta_{P_k} E r_t^{ep}$ measure the curvature channel and the price of capital channel, respectively. We define $\delta_\Lambda E r_t^{ep}$ as the change in the mean equity premium due to a change in the utility function, holding fixed the distribution of $(Z_{t+1} + P_{k,t+1})/P_{k,t}$ and C_{t+1} across dates and states of nature. The capital gains channel, $\delta_{P_k} E r_t^{ep}$, is simply defined as the residual: $\delta_{P_k} E r_t^{ep} = \Delta E r_t^{ep} - \delta_\Lambda E r_t^{ep}$.

3.3 Quantitative Results

In this section we present our quantitative results for the exchange economy. First we discuss our method for assigning values to the model parameters. Second, we document the importance of the capital gains channel. We do this by exhibiting the sensitivity of the equity premium to the persistence of consumption growth in the power utility model. Also, we use the decomposition in (16) to quantify the magnitude of the curvature and capital gains channels under habit persistence. Third, we document the ability of habit persistence preferences to account for key features of asset prices in our exchange economy.

Parameter Values

We adopt the normalization, $K = 1$. The equilibrium consumption process (that is, the technology shock Z_t) was chosen to be consistent with the observed mean, standard deviation, and with the autocorrelation of quarterly U.S. per capita consumption growth.[9] This requires setting

$$(17) \quad \overline{\theta} = 0.0045, \sigma = 0.0053, \rho = 0.34.$$

We set $\beta = 0.99999$ to maximize the model's ability to account for the observed risk-free rate. We adopt this value of β throughout the analysis.

Conditional on these parameter values, the two habit persistence parameters were set to optimize the model's implications for the mean equity premium and risk-free rate. Our metric for this is $\mathcal{L}(\psi)$, where:

$$(18) \quad \mathcal{L}(\psi) = [\hat{\nu}_T - f(\psi)] \, \hat{V}_T^{-1} \, [\hat{\nu}_T - f(\psi)]',$$

and $\psi = (b, h)$. Also, $\hat{\nu}_T$ is the 2×1 vector of point estimates for the risk-free rate and the equity premium reported in Cecchetti, Lam, and Mark (1993) (CLM), and the 2×2 matrix $\hat{V}_T$ is their estimate of the underlying sampling variance. Finally, f is the model's implied

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[9]Our measure of consumption is private consumption of nondurables and services, plus a measure of the service flow from the stock of durables. The data cover the period 1959.1 to 1989.4, and are discussed in Christiano (1988) and Fisher (1994). Consumption growth at different levels of time aggregation have different autocorrelation patterns (see Heaton 1993, 1995). Accounting for this phenomenon is beyond the scope of this paper.

risk-free rate and equity premium, given ψ. We executed this mapping by computing the average of these variables across 500 artificial data sets, each of length 120. We considered $\psi \in \Psi$, a grid of points, b, h, in the unit box, having the property that $C_t \leq X_t$ and $\Lambda_{c,t} \leq 0$ are never observed in the Monte Carlo simulations used to evaluate f. Let

$$(19) \quad J = \mathcal{L}(\hat{\psi}_T),$$

where $\hat{\psi}_T$ minimizes $\mathcal{L}(\psi)$ over $\psi \in \Psi$. In practice, we could not find values of ψ that set $J = 0$. We find that $\hat{\psi}_T = (0.58, 0.3)$, with $J = 0.37$.

Power Utility

The results of analyzing the economy with power utility are summarized in Figure 1. That figure indicates (see "U.S. data") the sample averages for the risk-free rate and the equity premium taken from CLM. In addition, we report 1 percent and 5 percent confidence ellipses, based on CLM's reported $\hat{V}_T$ matrix. Results for several versions of the exchange economy with power utility are presented.

The curve marked "power utility, consumption growth autocorrelation = 0.34" adopts the parameter values in (17). Moving from left to right, each letter "o" reports $f(\psi)$ for a different value of ϕ, with $\phi = 1, 2, 3, 4, 5, 10, 15, 20, 25, 30, 35$. There are two basic findings here. First, consistent with the nonstochastic analysis reported above, increasing curvature with power utility preferences produces a rise in the average risk-free rate. Second, increasing curvature results in a *fall* in the equity premium. For ϕ exceeding 5, the equity premium is negative, with equity actually being a good hedge against risk.

To understand this result, we studied three other versions of our model. First, we repeated the calculations with $\rho = -0.34$ (see "power utility, consumption growth autocorrelation = -0.34"). This change in the autocorrelation of consumption growth has essentially no effect on the monotone relationship between the risk-free rate and curvature, but the effect on the equity premium is substantial. Now the equity premium rises monotonically with curvature. Second, we simulated a version of the model in which the parameters of the equilibrium consumption growth process are taken from Mehra and Prescott (1985). They based their parameter values on annual U.S. data covering the period 1889–1978, in which the first-order

autocorrelation of consumption growth is -0.14 (see "Mehra-Prescott, consumption growth autocorrelation $= -0.14$"). Note that now the risk-free rate initially rises, then falls, as curvature rises. In addition, consistent with the version of our model with $\rho = -0.34$, the equity premium rises monotonically with curvature. Third, we altered the Mehra-Prescott parameterization by switching the sign on ρ (see "Mehra-Prescott, consumption growth autocorrelation $= 0.14$"). This has a very large impact on the equity premium. It now falls sharply with increased curvature. Significantly, none of the perturbations considered places the power utility model anywhere close to the U.S. data.

We infer from these computational experiments that the autocorrelation of consumption growth is critical for determining whether higher curvature produces a positive or negative equity premium. This finding impressively illustrates the importance of the capital gains channel in determining the equity premium, since, by construction, the curvature channel plays no role with variations in ρ.

Insight into the role of ρ can be obtained by making use of a simple permanent income-type argument, and the covariance formula in (15). The sign of r_t^{ep} depends on the sign of the conditional covariance between $\Lambda_{c,t+1}$ and $Z_{t+1} + P_{k,t+1}$. When technology growth is positively autocorrelated, a date $t+1$ state of nature in which Z_{t+1} is high signals an even greater rise in technology at later dates, and thus a rise in households' long-run consumption opportunities. Under power utility, households have an incentive to adjust consumption immediately to its long-run potential, and this implies a large jump in desired C_{t+1}. However, to increase consumption by more than output, households must reduce their accumulation of equity; this in turn translates into a reduced demand for capital. The latter, in view of the fixed supply of capital, translates into a fall in its price, $P_{k,t+1}$. If this price effect is strong enough to overcome the jump in Z_{t+1} itself—a result that is more likely, the greater ϕ is—then the conditional covariance in (15) would be positive, implying the negative equity premiums that we see in Figure 1.

If technology growth is negatively autocorrelated instead, then a high Z_{t+1} signals a smaller increase—perhaps even a reduction—in long-run consumption prospects. Under these circumstances, adjusting consumption to its long-run potential dictates shifting C_{t+1} up by less than the rise in date $t + 1$ output, thus giving rise to an increased demand for

capital. This drives up $P_{k,t+1}$, guaranteeing that the covariance in (15) is negative and that the equity premium is positive.[10]

We think these results make it clear that, to understand the sensitivity of the equity premium to ρ, one must understand the impact of changes in ρ on the dynamics of the price of capital, that is, the capital gains channel.

Habit Persistence

Results for analyzing the economy with habit persistence utility are summarized in Figure 2. The figure reproduces the empirical observations and confidence ellipsoids from Figure 1. The mean equity premium and risk-free rate, corresponding to a subset of $(b, h) \in \Psi$, are also reported. To gain insight into the relation of b and h with asset returns, we find it useful to arrange the results in a particular way. That is, we consider instances of (b, h) that imply nonstochastic steady-state values of X_t/C_t, denoted by x, equal to 0.85, 0.83, 0.81, and 0.30. For the last two values of x, we consider $h = 0, 0.10, 0.20, ..., 0.90, 0.95$.[11] For $x = 0.85$ and $x = 0.83$, only the first 9 and 8 values of h are reported, respectively. Note that for each value of x, the equity premium/risk-free rate combinations form a half-ellipse. For small values of h, the equity premium and risk-free rate are both decreasing in h, for given x. For larger values of h, the equity premium continues to be decreasing in h, but the risk-free rate now begins to increase. Note also that the half-ellipses shift down and get smaller with decreasing x. As x gets even smaller, the half-ellipses converge to an equity premium of 0.03 percent and a risk-free rate of 1.8 percent, after rounding.

The point in Figure 1 (and among all $b, h \in \Psi$) closest to the U.S. data is $b = 0.58$, $h = 0.3$, with $x = 0.83$. At this point, the risk-free rate is 1.68 percent and the equity premium 6.86 percent. This is close to the U.S. numbers, once sampling uncertainty is taken

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[10]These observations can be illustrated with a simple example. Let $C_{t+1}/C_t = \exp(\theta_{t+1})$, with $\theta_t = \bar{\theta} + \varepsilon_t + \rho\varepsilon_{t-1}$, and $\varepsilon_t \sim IIN(0, \sigma_\varepsilon)$. Then

$$P_{k,t+1} = \mathcal{E}_{t+1} \sum_{j=1}^{\infty} \beta^j \frac{\Lambda_{c,t+1+j}}{\Lambda_{c,t+1}} C_{t+j+1}.$$

In the power utility case, this implies $d\log(P_{k,t+1})/d\varepsilon_{t+1} = 1 + (1 - \phi)\rho$, which may be negative if $\rho > 0$ and $\phi > 1$.

[11]Given values of x and h, the value of b is implied by the condition $x = b/(\exp(\bar{\theta}) - h)$.

into account. Statistical results for this model economy are provided in Table 1a in the columns marked "Exchange Economy." The column labeled "No Habit" corresponds to the parameterization $\phi = 1$, $b = h = 0$, while "Habit" corresponds to the model with habit persistence, evaluated at the estimated parameters. The U.S. numbers are also reported in Table 1a. Note that, although the model does well in accounting for the mean risk-free rate and the equity premium, it does less well in accounting for the variance of these objects.

Table 1a also reports dynamic properties of the model's P_k, after logging and Hodrick-Prescott filtering. The volatility of this variable jumps to around 9.5 percent with the introduction of habit persistence, and its correlation with output (the latter also being logged and HP filtered) is about 0.5. To evaluate the empirical plausibility of these implications, we compare them with the properties of several measures of stock prices, which are reported in Table 1b (the results in Table 1b are multiplied by 2 to get σ_{P_k}). We find that the model's implied volatility of P_k conforms well with its empirical counterpart. However, the model slightly overstates the correlation of stock prices with GDP, which is in the neighborhood of about 0.30. We infer that, apart from the second moment properties of asset returns, this model does reasonably well.

Finally, to quantify the curvature channel, we considered the impact of the change, $(b = h = 0)$ to $(b = 0.8, h = 0)$. For this, $\delta_\Lambda E r_t^{ep} = 1.0003831 - 1.0000281$, and $\Delta E r_t^{ep} = 1.005244 - 1.0000281$, so that $\delta_\Lambda E r_t^{ep} / \Delta E r_t^{ep} = 0.07$. Thus, of the full increase in the equity premium (here expressed at a quarterly rate), 7 percent is accounted for by the curvature channel, and 93 percent is accounted for by the capital gains channel.

4 The One-Sector Production Economy

In this section, we modify the production side of our economy by allowing for capital accumulation and an elastic labor supply. We show that these modifications essentially eliminate the equity premium, even with habit-persistence preferences.

4.1 The Model

At date 0 the household's preferences are:

$$(20) \quad \mathcal{E}_0 \sum_{t=0}^{\infty} \beta^t \frac{[(C_t - X_t)(1 - h_t)^\nu]^{1-\phi} - 1}{1 - \phi}.$$

where h_t denotes time t labor, X_t is the habit stock, defined in (2), and:

$$(21) \quad 0 \leq h_t \leq 1, \quad \forall t \geq 0.$$

The resource constraint is:

$$(22) \quad K_t^\alpha (Z_t h_t)^{1-\alpha} \geq C_t + K_{t+1} - (1 - \delta)K_t,$$

where $0 < \delta, \alpha < 1$. The variable Z_t is a technology shock satisfying

$$(23) \quad Z_t = \exp(\theta_t) Z_{t-1}, \qquad \theta_t \sim N(\overline{\theta}, \sigma^2), \quad \forall t \geq 0.$$

We consider the same sequence-of-markets equilibrium concept used in the previous section, suitably modified to include the investment and labor-leisure choices. The quantities in this allocation are known to solve the planning problem: maximize (20) subject to (21)-(23) and the nonnegativity constraints $K_{t+1}, C_t \geq 0$. The rate of return on the risk-free asset B_t is computed using (12) and (13), and the rate of return on equity is:

$$(24) \quad r_{t+1}^e = \alpha \left(\frac{Z_{t+1} h_{t+1}}{K_{t+1}} \right)^{1-\alpha} - \delta.$$

4.2 Result: No Equity Premium

We used the following parameter values for the version of our model with habit persistence: $\alpha = 0.36$, $\delta = 0.021$, $\overline{\phi} = 1$, and $\nu = 2$. For empirical evidence on the first two of these, see Christiano and Eichenbaum (1992). The given value for ν was chosen to assure that the model implies households work 1/3 of available time. Finally, the parameters for the technology process, $\overline{\theta}$ and σ, were set equal to 0.40 percent and 1.8 percent, respectively. These correspond to empirical point estimates for the mean and standard deviation of the Solow residual (see, for example, Christiano and Eichenbaum 1992). Figure 3 reports the risk-free rate and equity premium combinations associated with a grid of b and h that

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are designed to cover the feasible set of these parameters. Note that no value of b and h comes even close to accounting for the U.S. data. Thus the production economy stands in striking contrast to the exchange economy, in that the equity premium and risk-free rates are essentially invariant to whether there is habit persistence or log utility.

The financial statistics pertaining to the cases of $b = h = 0$ and $b = 0.80, h = 0$ are reported in Table 1a in the columns labeled "No Habit" and "Habit," respectively. The business-cycle properties of quantity variables are reported in Table 3. Note from Table 1a that the equity premium drops from 6.86 percent in the exchange economy to essentially zero in the production economy.

4.3 Diagnosing the Result

To understand why there is no equity premium in this economy, we begin by analyzing the general equilibrium effects on consumption and other quantity variables of converting from $b = h = 0$ to $b = 0.8, h = 0$. These effects are substantial because a household with habit persistence preferences has a strong incentive to smooth the response of consumption to a shock. Moreover, the technology described in (22) offers at least three mechanisms through which this can be accomplished: variations in labor effort, variations in the rate of capital accumulation, and variations in the sectoral allocation of factors of production.

The impulse response functions displayed in Figure 4 document how agents in the $b > 0$ economy exploit these opportunities relative to agents in the $b = 0$ economy. The figures display the log deviation of a variable from a steady-state growth path in response to a single, one standard deviation innovation in θ_t. The results have been multiplied by 100 to convert them into percent terms. In measuring quantities in the model, we follow the National Income and Product Accounts practice; we measure quantities in base-year prices, which we set to unity.

Figure 4a shows that the impact on consumption of a shock is greatly reduced in the $b > 0$ economy (see the solid line), compared with the $b = 0$ economy (dashed line). Eventually the two responses converge, but this takes about 12 model periods (three years). At the aggregate level, this smoothing of consumption is accomplished, in part, by a relatively strong investment response (Figure 4b) and a relatively weak employment response (Figure

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4c).

For further diagnosis of the factors underlying the smoothed consumption response, we find it useful to adopt the two-sector interpretation of our one-sector model. Figures 4d and 4e show that with $b > 0$ there is a strong shift of labor resources (and capital resources, which are not shown) out of the consumption goods sector and into the investment goods sector. It is well-known that, with $b = 0$, there is some shift in resources out of the consumption sector and into the investment goods sector after a positive technology shock. The prediction of the standard real business-cycle model, that employment in the consumption sector is countercyclical, is counterfactual; the adoption of habit-persistent preferences evidently exacerbates that problem.

The amplified positive aggregate investment response with $b > 0$ masks much larger effects at the sectoral level. There is a substantial increase in the already positive response of investment in capital for the investment goods industry. The production of capital goods for the consumption goods industry experiences a greater *decline* with $b > 0$ than with $b = 0$. This reflects efforts to shift resources out of the consumption sector dynamically, one of the ways that households bring about the slow rise in consumption evident in Figure 4a.

These findings are reflected in the second moment statistics reported in Table 3. For example, those statistics show a substantial reduction in the standard deviation of consumption with $b > 0$. At the same time, there is a sharp increase in the variability of investment. Although there is an absolute fall in the volatility of employment, the reduction is less than the reduction in output.

The preceding observations are consistent with the hypothesis that the absence of an equity premium in the one-sector production economy reflects the smoothing of consumption across states of nature. According to this hypothesis, the general equilibrium smoothing of consumption prevents a rise in the equity premium that would have occurred otherwise.

Taken literally, this hypothesis is incorrect. In particular, suppose there had been no general equilibrium impact on consumption or on the rate of return on capital, in response to the change in preferences from $(b = h = 0)$ to $(b = 0.8, h = 0)$. Then, according to the covariance formula (15), the equity premium would have gone from 1.0000028 to 1.000057, which is only a trivial increase. That is, if no consumption smoothing or change in the

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rate of return on capital had occured, the equity premium would have jumped to a mere $100(1.000057^4 - 1) = 0.02$, or, two hundredths of a percent per year.[12] The failure of habit persistence to generate an equity premium in the one-sector production economy is not a consequence of the fact that households choose a smooth consumption sequence.

There is nevertheless a sense in which the absence of an equity premium and the smoothing of consumption are related. In a one-sector economy the marginal rate of transformation between investment and consumption goods is unity; that is to say, the supply of capital goods is infinitely elastic. While this is the reason it is feasible for households to smooth consumption, it also has the implication that the equilibrium price of capital is constant. The latter fact essentially eliminates the fluctuations across states of nature in the rate of return on equity, and is the reason why there is no equity premium. Once again, this illustrates the central role played by the capital gains channel. In the next section we put this intuition to work.

5 Two-Sector Economies

Here we consider three modifications of the model economy analyzed in the previous section. In our first modification, we assume that consumption and investment goods are produced in distinct sectors, and that the sectoral allocation of the date t capital stock is a function of the previous period's state of nature. In our second modification, we assume that households also decide their date t allocation of sectoral employment prior to the realization of the date t state of nature. We then further modify the model to allow firms to issue risk-free debt, in addition to equity, to finance their purchases of capital. The model embodying all three modifications implies a risk-free rate and a return on equity that lies within a 5 percent confidence ellipse around the corresponding empirical estimates. We also examine the business-cycle implications of this model.

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[12]The 0.02 result is not strictly comparable to what is in Table 1a, because of Jensen's inequality. The 0.02 value is the annualized average equity premium, while what is in the table is the average of annualized equity premiums. The latter can be expected to be larger.

5.1 A Two-Sector Economy with Full Labor Mobility

The quantity allocations in a sequence-of-markets competitive equilibrium solve the programming problem:

$$(25) \quad \max \ \mathcal{E}_0 \left\{ \sum_{t=0}^{\infty} \beta^t \frac{[(C_t - X_t)\,(1 - h_{i,t} - h_{c,t})^\nu]^{1-\phi} - 1}{1 - \phi} \right\}$$

subject to:

$$(26) \quad K_{c,t}^\alpha (Z_t h_{c,t})^{1-\alpha} \geq C_t,$$

$$(27) \quad h_{c,t}, h_{i,t}, h_{c,t} + h_{i,t} \in [0,1], \quad \forall t \geq 0,$$

and

$$(28) \quad K_{i,t}^\alpha (Z_t h_{i,t})^{1-\alpha} + (1-\delta)(K_{c,t} + K_{i,t}) \geq K_{c,t+1} + K_{i,t+1}.$$

Here the subscripts c and i denote the consumption and investment sectors, respectively. As before, we require that the sum $K_{c,t+1} + K_{i,t+1}$ be chosen as a function of the date t state of nature. However, we now also require that the individual terms $K_{c,t+1}$ and $K_{i,t+1}$ be chosen as a function of the date t state of nature. Finally, we assume that the state of technology Z_t is drawn from the time series representation described in (23). Let $\Lambda_{c,t}$ and $\Lambda_{i,t}$ denote the Lagrange multipliers on (26) and (28).

We assume the same financing arrangements that we have used up to now: firms have a one-period planning horizon, and, to operate in period $t+1$, they must issue enough equity in period t to finance their purchase of whatever quantity of capital they plan to use. Different equity is used to finance consumption and investment goods firms. It is readily verified that the rate of return on equity in market x is $r_{x,t}^e$, $x = i, c$, where

$$(29) \quad r_{c,t+1}^e = \frac{\alpha \left[\frac{Z_{t+1} h_{c,t+1}}{K_{c,t+1}} \right]^{1-\alpha} + P_{k,t+1}(1-\delta)}{P_{k,t}} - 1$$

for $x = c$ and

$$(30) \quad r_{i,t+1}^e = \frac{P_{k,t+1}\alpha \left[\frac{Z_{t+1} h_{i,t+1}}{K_{i,t+1}} \right]^{1-\alpha} + P_{k,t+1}(1-\delta)}{P_{k,t}} - 1$$

for $x = i$. The rate of return on the market portfolio, r_t^e, is:

$$(31) \quad r_{t+1}^e = \frac{K_{c,t+1}}{K_{t+1}} r_{c,t+1}^e + \frac{K_{i,t+1}}{K_{t+1}} r_{i,t+1}^e,$$

where $K_{t+1} = K_{c,t+1} + K_{i,t+1}$.

To find the objects in competitive equilibrium, first get the quantities and multipliers by solving the planning problem stated above. Second, find $P_{k,t}$ using the relation[13]

$$(32) \quad P_{k,t} = \Lambda_{i,t}/\Lambda_{c,t}.$$

The price, $P_{k,t}$, varies with the realization of the date t state of nature, because of the diminishing productivity of labor as labor is reallocated between sectors; the distribution of capital is fixed. The rate of return, r_t^f, on the risk-free asset is obtained as before, using (12)-(13). The various equity rates of return are then obtained from (29)-(31).

The model is parameterized in exactly the same way as the one-sector model in the previous section. We adopt these parameter values because they meet the criteria we set out when we estimated them for the one-sector production economy. In particular, they allow the model's implication for mean hours worked, and the mean growth rate and variance of the Solow residual, to coincide with their sample analogs.

Values for the risk-free rate and equity premium implied by feasible (b, h) combinations are displayed in Figure 5. Note that the model—though now a little closer to the data—is still not close even to the data's 1 percent confidence interval. The parameterization that is closest to the data, according to the metric defined in Section 3, is one with $b = 0.9, h = 0$. The financial properties of this model are reported in Table 1a. The business-cycle properties are reported in Table 3.

Figure 7 reports the impulse response functions for the $b = h = 0$ ("Full No Habit," indicated by long dashes) and $b = 0.9, h = 0$ ("Full Habit," shown by short dashes) versions of the model. The quantity effects of habit persistence resemble very closely the ones seen in Figure 4 for the one-sector economy. For example, consumption is considerably smoother

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[13]There is a slight inconsistency here relative to the endowment economy. There, $P_{k,t}$ is a variable that grows in equilibrium, while here it is not. The difference is that there the stock of capital is constant and the payoff per unit of capital is growing, while here the stock of capital is growing and the payoff per unit of capital is stationary.

and investment more volatile with the introduction of habit persistence. The success that households have in smoothing consumption reflects the fact that, although capital supply is not infinitely elastic, it still exhibits considerable short-term elasticity in this model. This high elasticity also underlies the fact that the volatility of P_k, though positive, is still very small. This in turn accounts, via the capital gains channel, for the fact that there is no substantial equity premium in this model.

These considerations suggest that a version of the model with a smaller short-term elasticity of capital supply is required. One avenue for obtaining this is suggested by the results in Table 3, which indicate that gross investment in capital for the investment sector is negative 9.4 percent of the time. A nonnegativity constraint on gross investment would therefore be binding in this model. Imposing this constraint would have the effect of making capital supply more inelastic, at least in the range of low investment. As expected, when we experimented with such modifications, the equity premium rose, but only by a modest amount.

5.2   A Two-Sector Economy with Limited Labor Mobility

In this section, we investigate the impact of requiring that the household's date t labor supply decisions be made prior to the realization of the current state of nature. After the state of technology is realized, the sectoral labor markets meet and clear (subject to the predetermined labor supply) at a competitive wage. In effect, this makes the short-term supply of capital completely inelastic, as in the exchange economy.

The formulas for the risky and risk-free rates of return are not altered by this modification, so they are not repeated here. For the same reasons cited in the previous subsection, the model parameter values for the one-sector model are adopted here. Values of the risk free rate and equity premium associated with feasible (b, h) combinations are reported in Figure 6. Note that now the model is noticeably closer to the data, although it is still not within the 1 percent confidence ellipse. Note also that the volatility of the price of capital has now increased by a factor of 10, moving this model implication closer to the empirical results reported in Table 1b. This result is not surprising, in view of the short-term inelasticity of capital supply. Interestingly, the model correlation between the price of capital and output is now 0.25, which is very close to the data.

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To gain further insight into the reasons for these results, consider again Figure 7. In calculations not documented here, we found that impulse response functions for the no habit, full, and limited labor mobility models are roughly identical. Figure 7 then indicates that, in the limited labor mobility model, the impulse response functions of quantities are essentially invariant to the introduction of habit persistence preferences. This reflects the fact that the inelasticity of capital supply in effect prevents households from smoothing consumption. The consequent sharp response in the price of capital is indicated in Figure 7d.

5.3 The Effect of Financial Leverage

Consider now the model economy in the previous subsection, with the modification that we also allow firms to issue risk-free debt. This debt is identical, in maturity and rate of return, to the privately issued bonds considered until now. Unlike the risk-free security of the previous sections, the firm-issued bond will be traded in equilibrium.

As is well-known, profit-maximizing firms are indifferent to the debt-equity composition of their liabilities in an environment such as ours. Moreover, the quantity allocations in general equilibrium are invariant to the pattern, across dates and states of nature, of the debt-to-equity ratio in firm liabilities (the Modigliani-Miller theorem). What is not invariant to the debt-to-equity ratio is the mean and variance of the return on equity. The premium of the return on equity over debt is strictly increasing in the debt-to-equity ratio. This simply reflects that equity must bear the full degree of uncertainty in firm cash flow across states of nature. In the experiments analyzed below, we consider the equity premium for an economy with a debt-to-equity ratio similar in magnitude to that reported for the U.S. economy.

With leverage, the finance constraint for a firm in industry x is:

$$(33) \quad P_{k,t} K_{x,t+1} \leq S_{x,t} + B_{x,t},$$

for $x = i, c$. In period $t + 1$, the firm in sector x hires labor and carries on production. Let $\pi_{x,t+1}$ denote its revenues from sales (output, $Y_{x,t+1}$, plus the undepreciated stock of capital), net of expenses on labor and on its financial obligations. All terms in $\pi_{x,t+1}$ are denominated

in consumption units. We require that the firm's expenses not exceed its receipts:

$$(34) \quad \pi_{x,t+1} = Y_{x,t+1} + (1-\delta)K_{x,t+1}P_{k,t+1}$$
$$-W_{x,t+1}h_{x,t+1} - (1+r^e_{x,t+1})S_{x,t} - (1+r^f_t)B_{x,t} \geq 0,$$

where $r^e_{x,t+1}$ and r^f_t denote the rates of return on equity and debt, respectively. Also, $W_{x,t+1}$ denotes the wage paid to workers in sector x, expressed in period $t+1$ consumption units. The firm takes prices and rates of return, $P_{k,t}, r^e_{x,t+1}, r^f_t, P_{k,t+1}, W_{x,t+1}$, as given. At date t, the objective of the firm is:

$$(35) \quad \max_{S_{x,t}, K_{x,t+1}, B_{x,t}} \mathcal{E}_t p_{c,t+1} \max_{h_{x,t+1}} \pi_{x,t+1},$$

subject to (12), (33) and (34). The firm is assumed to take $p_{c,t+1}$ as given. Assuming an interior solution to (35), the first-order condition for $h_{x,t+1}$ is:

$$(36) \quad mpl_{x,t+1} = W_{x,t+1},$$

where $mpl_{x,t+1}$ is the marginal product of labor in industry x, expressed in consumption units. The first-order condition associated with $S_{x,t}$ is:

$$(37) \quad \mathcal{E}_t p_{c,t+1} \left[\frac{mpk_{x,t+1} + (1-\delta)P_{k,t+1}}{P_{k,t}} - (1+r^e_{t+1}) \right] = 0,$$

where $mpk_{x,t+1}$ is the marginal productivity of capital in sector x, measured in consumption units. The first-order condition for $B_{x,t}$ is (37), with r^e_{t+1} replaced by r^f_t.

Linear homogeneity guarantees that the only equilibrium is one in which profits, (35), are zero. Given the weak inequality in (34), this implies $\pi_{x,t+1} = 0$ for all t and states of nature. This, in conjunction with the facts: (i) profit maximization causes (33) to be satisfied as an equality; (ii) linear homogeneity implies $Y_{x,t+1} = mpk_{x,t+1}K_{x,t+1} + mpl_{x,t+1}h_{x,t+1}$; and (iii) the first-order condition for labor, (36), allows us to derive the following expression for the rate of return on equity in sector x:

$$(38) \quad 1 + r^{e*}_{x,t+1} = \frac{mpk_{x,t+1} + (1-\delta)P_{k,t+1}}{P_{k,t}}(1+\gamma_{x,t}) - (1+r^f_t)\gamma_{x,t},$$

where $\gamma_{x,t} = B_{x,t}/S_{x,t}$ is the debt-to-equity ratio and $r^{e*}_{x,t+1}$ is the leveraged rate of return on equity. We impose $\gamma_{x,t}$ exogenously, given that it is not determined in equilibrium.

Rearranging (38) and taking into account the formula for the unleveraged rate of return on equity, $r_{x,t}^e$, in (29) and (30), we get the following expression for the equity premium:

$$(39) \quad r_{x,t+1}^{e*} - r_t^f = (r_{x,t+1}^e - r_t^f)(1 + \gamma_{x,t}).$$

From this it is evident that leverage raises the equity premium proportionally. Obviously, if the equity premium were small in the unleveraged economy, then leveraging would have only a small impact on the equity premium. For example, (39) indicates that to convert an unleveraged equity premium of 1 percent into a leveraged equity premium of 5 percent requires a debt-to-equity ratio of 4. The actual debt-to-equity ratio is closer to 2/3.[14]

In our computations, we study equilibria in which $\gamma_{i,t} = \gamma_{c,t} = \gamma = 2/3$. In other respects, the model parameterization conforms to what was used in previous subsections. Table 2 shows that the introduction of leverage in this way raises the equity premium by roughly a factor of two in practically every model we have considered.[15] Figure 8 shows that we are now well within the 5 percent confidence interval for the equity premium and risk-free rate.

5.4 Business-Cycle Implications

The business-cycle implications of the model just analyzed, what we refer to as "our model," are identical to those of the model with no leverage. In Table 3 we see that its predictions conform closely to those of the standard real business-cycle model, namely the one-sector economy, with no habit. The performance of our model is poor with respect to both the volatility of aggregate hours worked and the cyclical behavior of hours worked in the consumption sector. But on these dimensions it does no worse than the standard model.

There are two other dimensions in which our model actually does better than the standard model. First, the standard model implies that the first-order autocorrelation of equilibrium output growth essentially coincides with the assumed autocorrelation of the growth rate of technology, which is zero here.[16] The coincidence of these two autocorrelations reflects the

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[14] See Benninga and Protopapadakis (1990) and literature cited there.

[15] The impact of leveraging is not precisely what is predicted by (39), since in the table we compute the equity premium with the underlying rates compounded at an annual rate, while the objects in (39) are denominated at a quarterly rate.

[16] We computed the first-order autocorrelation of output growth in 500 artificial data sets of 120 obser-

well-known absence of internal propagation in that model (see Christiano 1988 and Cogley and Nason 1995). This absence of internal propagation is a problem, in view of the fact that a standard measure of technology growth indicates little first-order autocorrelation, while the estimated first-order autocorrelation of output growth is 0.37, with a two-standard deviation confidence interval of $[0.23, 0.51]$ (see Table 4.) In our model, by contrast, the autocorrelation of equilibrium output growth is 0.20, which is very close to this confidence interval. The model exhibits persistence in output growth, despite the fact that the state of technology is a random walk.

Our model also dominates the standard model in relation to the excess sensitivity puzzle. To define this puzzle, consider the following relation:

$$(40) \quad \Delta C_t = \mu + \lambda \Delta Y_t + \theta r_t + \varepsilon_t.$$

Here, $\Delta x_t \equiv \log(x_t) - \log(x_{t-1})$. Campbell and Mankiw (1989, 1991) estimate the parameters in this relation by a two-step instrumental variables procedure: in the first step they replace the left and right variables by their regression forecasts, based on a set of instruments; in the second step they run an ordinary, least squares regression to estimate μ, λ, and θ. A potential pitfall, particularly for estimating λ, is the possibility that the instruments are not correlated with ΔY_t.[17] To help guard against this in practice, it is useful to obtain a measure of $R^2_{\Delta y}$, the $R-$bar squared in the regression of ΔY_t on the instruments. In Christiano (1989), Monte Carlo evidence is presented which suggests that the $R^2_{\Delta y}$ reported in Campbell and Mankiw (1989), 0.047, is sufficiently large that the resulting estimate of λ is not simply an artifact of poor instrument quality. In our model, there is enough persistence in equilibrium output growth that the implied $R^2_{\Delta y}$ is in the acceptable range (see Table 4).

Campbell and Mankiw (1989) interpret their empirical estimates of (40), reported in Table 4, as reflecting that the forecastable component of consumption growth is an increasing function of the forecastable components of output growth and the interest rate and, moreover, that the latter plays a smaller role than the former. It is interesting to note from Table 4 that our model's implication for λ overshoots Campbell and Mankiw's (1989) empirical estimate,

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vations each, generated using the standard model. The average across the 500 autocorrelations was 0.0009, and the associated Monte Carlo standard error was 0.004.

[17]For a formal analysis of these issues, see Nelson and Startz (1990a,b).

though it is within the range of λ's found using data for several European countries (see Campbell and Mankiw 1991). Also, the value predicted by our model for the instrumental variable estimate of θ is even smaller than Campbell and Mankiw's (1989) estimates. Thus our model can account for the evidence that the forecastable part of consumption growth is an increasing function of the forecastable part of output growth, and that it is more closely related to this than it is to the forecastable part of the interest rate.[18] That a version of the standard model is incompatible with these observations is documented in Christiano (1989).

6   Concluding Remarks

Macroeconomists have long been interested in understanding precisely where and when the complete markets, representative-agent paradigm breaks down, if it does at all. A widely-held view is that the equity premium is a prime example of where this happens. An important lesson from our analysis is that this is not so obvious. A production economy with complete markets and reasonable risk aversion was constructed that accounts moderately well for both the business cycle and for key features of asset returns. Other asset-pricing lessons we take away from this analysis are that both (a) habit persistence preferences and (b) multisector technologies with limitations on the intersectoral mobility of factors of production are likely to be important elements in a successful model of the business cycle.

To understand how these elements contribute to an equity premium, recall that the key to generating an equity premium in the general equilibrium models considered here is to produce the "right" dynamic behavior in the price of capital. In particular, innovations in the price of capital must be large, and negatively correlated with the marginal utility of consumption. Under these circumstances, equity is a bad hedge against risk, and thus requires a large premium to induce households to hold it. As we pointed out in Section 2, in order to get the appropriate movements in the price of capital, we require that (i) households have a strong incentive to buy assets when the marginal utility of consumption is low, and to sell when the marginal utility of consumption is high, and (ii) a technology that frustrates

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[18]Baxter and Jermann (1994) document that a model with home production can also account for the excess sensitivity puzzle.

these desires. Ingredient (a) above contributes to (i), and ingredient (b) contributes to (ii).[19]

We now briefly discuss some of the limitations of the analysis. First, consistent with the intuition in the previous paragraph, we find (in results not reported here) that our model implies a high correlation between consumption growth and the rate of return—higher than in the data.[20] This is a long-standing puzzle for the type of equilibrium model used here. One possible resolution, which deserves formal investigation, is that the discrepancy reflects measurement error in consumption data, or in the price data used to convert nominal returns into real returns.[21] Alternatively, the resolution to the puzzle may lie in a discrepancy between the marginal utility of consumption and consumption itself. There is such a discrepancy in the model presented in this paper; however, quantitatively it is not sufficiently large to resolve the puzzle. A second shortcoming of the model lies in its prediction for the cyclical behavior of the price of new investment goods. Our model has the implication that this coincides with the value of capital to the firm, which we associate with the price of stock. This is inconsistent with observations such as those documented in Greenwood, Hercowitz and Krusell (1992) that the prices of many types of new investment goods are countercyclical, and, more generally, with the observation that Tobin's q is not constant. We are currently investigating ways to confront these limitations of our model.

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[19]Recently, Rouwenhorst (1995) has explored the asset pricing implications of a general equilibrium business-cycle model which resembles ours in the sense that aggregate factor supplies are determined prior to the realization of the shocks. His model nevertheless fails to generate a sizable equity premium because it does not have ingredients (a) and (b). He adopts a one-sector formulation, and a power utility function with low curvature. The one-sector formulation is equivalent to a two-sector model with identical production functions and with complete mobility of factors of production between sectors.

[20]Our model's predicted correlation between consumption growth and the return on equity is over 0.90, while the corresponding object in the data is closer to a range of 0.0 to 0.3, depending on which measure of the return on equity one uses (see Christiano 1989). That the model is also consistent with the small instrumental variables regression relationship between consumption and interest rates reflects the distinction between a regression coefficient and a correlation.

[21]For a discussion of measurement error in consumption data, see Wilcox (1992). Gibbons (1989) cites the measurement error in consumption data as a reason not to use these data at all in evaluating theories of asset pricing. A quantitative analysis of the impact of measurement error in prices appears in Christiano (1989). For a formal, maximum likelihood approach to estimation and testing when there is measurement error in the data, see Sargent (1989).

A   Some Habit-Persistence Algebra

In this appendix, we derive the coefficient of relative risk aversion (RRA) and the elasticity of intertemporal substitution for habit persistence preferences. The presentation adapts the discussion in Constantinides (1990) to a discrete-time context. A similar discussion appears in Ferson and Constantinides (1991), but we need to generalize their results slightly in order to accommodate the case $h \neq 0$.

A.1   Policy and Value Functions

Here we derive the policy function and value function for a household with habit persistence preferences and, potentially, $\phi \neq 1$. We do so under the assumption that the household faces no uncertainty and a fixed rate of interest, $1 + r_t^f = 1 + r_t^e = \frac{\gamma^\phi}{\beta}$.

Let $\Lambda_{c,t}$ denote the derivative of (1) with respect to C_t:

$$(41) \quad \Lambda_{c,t} = (C_t - X_t)^{-\phi} - \frac{b}{h} \sum_{j=1}^{\infty} (\beta h)^j \left(C_{t+j} - X_{t+j} \right)^{-\phi}.$$

The Euler equation for the household's problem is $\Lambda_{c,t} = \beta(1+r^e)\Lambda_{c,t+1}$, for $t = 0, 1, \dots$. This is satisfied by the following class of policies, indexed by the undetermined constant Q:

$$(C_t - X_t) = Q\gamma^t.$$

This and equation (2) imply, for $t = 1, 2, 3, \dots$

$$C_t = (h+b)c_{t-1} + Q(1 - \frac{h}{\gamma})\gamma^t,$$

or

$$(42) \quad C_t = (h+b)^t C_0 + [\gamma^t - \psi^t]Q\frac{1 - h/(\gamma)}{1 - \psi/(\gamma)},$$

for $t = 0, 1, 2, \dots$. We assume

$$0 < h + b < \gamma,$$

a condition satisfied in all the cases considered in the text. The parameter Q is found by requiring that the intertemporal budget equation be satisfied: $\sum_{t=0}^{\infty} \left(\frac{1}{1+r^e}\right)^t C_t = (1+r^e)W_0$, where $W_0 = B_0 + S_0$ is the initial stock of wealth. Substituting (42) into this and solving for Q yields

$$(43) \quad Q(W_0, X_0) = \frac{(\frac{\gamma^\phi}{\beta} - \gamma)\left\{\left[\frac{\gamma^\phi}{\beta} - (h+b)\right] W_0 - X_0\right\}}{(\frac{\gamma^\phi}{\beta} - h)},$$

where X_0 is the initial habit stock. Then, assuming $\beta\gamma^{1-\phi} < 1$, the value function is

$$(44) \quad v(W_0, X_0) = \frac{Q^{(1-\phi)}/(1-\phi)}{1 - \beta\gamma^{(1-\phi)}}.$$

Equation (42) for $t = 0$, with Q defined in (43), is the policy function for consumption. Note that

$$\frac{dC_0}{dW_0} = \gamma \left(\frac{1}{\beta\gamma^{(1-\phi)}} - 1\right) \left(\frac{\frac{\gamma^\phi}{\beta} - (h+b)}{\frac{\gamma^\phi}{\beta} - h}\right).$$

There are two interesting features of this expression. First, when $b = 0$ and $\gamma = 1$, so that $\beta(1 + r^e) = 1$, then $dC_0/dW_0 = r^e$, which is exactly the prediction of standard permanent income theory. Second, dC_0/dW_0 is decreasing in b, and also decreasing in h, when $b > 0$. This is as expected. With habit persistence, the optimal response to a decrease in wealth is to use financial markets to bring down consumption slowly so that the stock of habit has a chance to fall.

A.2 Risk Aversion

The discrete-time analog of the steady-state formula for RRA provided in Constantinides (1990) is:

$$(45) \quad RRA \equiv -\frac{W v_{WW}}{v_W} = \frac{\phi}{1 - \frac{x/w}{\frac{\gamma^\phi}{\beta} - (h+b)}}, \quad \gamma = \exp(\bar{\theta}),$$

where x/w is the steady-state habit stock to wealth ratio, and $\bar{\theta}$ is the growth rate of consumption. The expressions v_W and v_{WW} are the first and second derivatives of the value function (44) with respect to W.

It is readily verified from (2) and the facts $C_t + W_{t+1} = (1+r)W_t, W_{t+1}/W_t = \gamma$ in steady state, that

$$\frac{x}{c} = \frac{6}{\gamma - h}, \frac{c}{w} = \frac{\gamma^\phi}{\beta} - \gamma.$$

Note that, for $\phi = 1, c/w \to 0$ as $\beta \to 1$. It then follows from (45) that, for $\phi = 1$,

$$RRA \to 1 \text{ as } \beta \to 1.$$

We briefly repeat here the standard, textbook interpretation of RRA. Consider the following fair bet on wealth: the household receives $W_0(1 + \mu)$ or $W_0(1 - \mu)$, each with probability $1/2$. Let ν denote the largest fraction of the household's wealth that it would be willing to sacrifice to avoid this bet:

$$v\left(W_0(1 - \nu), X_0\right) = \frac{1}{2}\left[v\left(W_0(1 + \mu), X_0\right) + v\left(W_0(1 - \mu), X_0\right)\right].$$

Take a first-order Taylor series expansion of the expression on the left of the equality about W_0, and a second-order Taylor series expansion of the expression on the right, and then solve for ν:

$$\nu = -\frac{W_0 v_{WW}}{v_W}\frac{1}{2}\mu^2 = RRA\frac{1}{2}\mu^2.$$

Now if $\mu = 0.1414$, then $\frac{1}{2}\mu^2 = .01$. Thus a habit persistence household faced with a 50-50 chance of losing or increasing its wealth by 14 percent would be willing to pay RRA ($= \nu \times 100$) percent of its wealth to avoid the gamble.

A.3   Intertemporal Substitution

We exploit the fact that

$$\frac{d\log(C_{t+1}/C_t)}{d\log(1 + r_{t+1}^e)} = -\frac{\Lambda_{c,t}}{C_t \Lambda_{cc,t}},$$

where the object on the left of the equality is the definition of the intertemporal elasticity of substitution. We derive the object on the right of the equality, the inverse of our measure of curvature, along a balanced growth path.

Note from (41) that, along a nonstochastic steady-state growth path, where $C_t = \gamma C_{t-1}$,

$$\Lambda_{c,t} = C_t^{-\phi}Q_c, \ \Lambda_{cc,t} = -\phi C_t^{-(\phi+1)}Q_{cc}.$$

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Here

$$Q_c = s^{-\phi} \left[1 - \frac{b\beta/\gamma^\phi}{1 - \beta h/\gamma^\phi} \right]$$

and

$$Q_{cc} = s^{-(1+\phi)} \left[1 + \frac{b^2\beta/\gamma^{\phi+1}}{1 - h^2\beta/\gamma^{\phi+1}} \right].$$

Also, $s = 1 - x$ denotes the steady-state value of $(C_t - X_t)/C_t$. Note that Q_{cc} is increasing, and Q_c and s are decreasing, in b and h. Consequently, intertemporal substitution in the steady state,

$$-\frac{\Lambda_{c,t}}{C_t \Lambda_{cc,t}} = \frac{Q_c}{\phi Q_{cc}},$$

is decreasing in b and h.

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Table 1a. Financial Statistics without Leverage

| Statistic | Data | Endowment Economy | | One-Sector Economy | | Two-Sector Economy | | | |
| | | | | | | Full Labor Mobility | | Limited Labor Mobility | |
		No Habit	Habit	No Habit	Habit	No Habit	Habit	No Habit	Habit
r^e	7.82	1.84	8.54	1.60	1.60	1.62	1.76	1.63	4.79
		(0.01)	(0.05)	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)	(0.03)
r^f	1.19	1.81	1.68	1.60	1.60	1.60	1.63	1.60	2.70
	(0.81)	(4e-3)	(0.12)	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)	(0.06)
$r^e - r^f$	6.63	0.02	6.86	0.001	0.001	0.02	0.13	0.03	2.10
	(1.78)	(0.01)	(0.14)	(1e-3)	(6e-4)	(0.01)	(0.02)	(0.01)	(0.07)
σ_{r^e}	19.53	2.32	40.0	0.40	0.37	1.84	5.76	2.52	26.4
		(0.01)	(0.2)	(0.01)	(4e-3)	(0.01)	(0.02)	(0.01)	(0.1)
σ_{r^f}	5.27	0.72	15.0	0.39	0.36	1.23	4.03	1.72	18.2
	(0.74)	(2e-3)	(0.1)	(5e-3)	(4e-3)	(4e-3)	(0.01)	(5e-3)	(0.1)
$\sigma_{r^e - r^f}$	19.02	2.23	36.5	0.14	0.14	1.39	4.13	1.87	18.8
	(1.73)	(0.01)	(0.1)	(4e-4)	(4e-3)	(4e-3)	(0.01)	(0.01)	(0.1)
σ_{P_k}		0.91	9.56	na	na	0.30	0.97	0.42	4.27
		(0.01)	(0.04)			(1e-3)	(1e-3)	(1e-3)	(0.01)
$\rho(Y, P_k)$		1.00	0.54	na	na	0.31	0.07	0.25	0.25
		(1e-6)	(2e-3)			(2e-3)	(1e-3)	(2e-3)	(1e-3)

Notes: (i) The "Data" column contains estimates of the mean and standard deviation of the risk-free return and the equity premium, with standard errors in parentheses, over the period 1892–1987 for U.S. data. These estimates are taken from Cecchetti, Lam, and Mark (1993). These authors do not supply standard errors for the return to equity. (ii) All statistics are annualized and in percent terms. (iii) Results for the models are based on 500 replications of sample size 120; Monte Carlo standard errors are reported in parentheses. The latter are the standard deviation, across replications, of the associated statistics, divided by $\sqrt{500}$. (iv) Data results for σ_{P_k} and $\rho(Y, P_k)$ are reported in Table 1b. (v) "No Habit" columns correspond to power utility, with $\phi = 1$; "Habit" columns correspond to habit persistence utility, with $\phi = 1$ and estimated b, h.

Table 1b. Business-Cycle Properties of Stock Prices

Industry	Second Moment Statistics S&P 500		Dow Jones		NYSE	
	$\frac{\sigma_p}{\sigma_y}$	$corr(p,y)$	$\frac{\sigma_p}{\sigma_y}$	$corr(p,y)$	$\frac{\sigma_p}{\sigma_y}$	$corr(p,y)$
Composite	5.14	0.36	5.22	0.30	5.20	0.37
	(0.53)	(0.09)	(0.55)	(0.10)	(0.53)	(0.09)
Capital Goods	6.06	0.39			na	na
	(0.56)	(0.09)				
Utilities	4.66	0.21			6.82	0.30
	(0.64)	(0.11)			(1.40)	(0.09)
Finance	6.42	0.30			6.73	0.38
	(1.03)	(0.12)			(0.93)	(0.10)
Industrial	5.33	0.35			5.62	0.35
	(0.55)	(0.09)			(0.75)	(0.12)
Transportation	5.82	0.19			7.45	0.36
	(0.79)	(0.15)			(1.08)	(0.10)

Notes: (i) Data source: CITIBASE. The sample period is 1947I–1995I for NYSE Composite and Dow Jones, and 1966I–1995I for the other NYSE variables. The sample period is 1947I–1995I for all but two of the S&P 500 data series. It is 1970I–1995I for S&P 500 finance and transportation. Data on gross domestic product (GDP) cover the period 1947I–1995I. Stock price data are deflated by an implicit price deflator for consumption of nondurables and services.

(ii) Statistics: all data were logged and then Hodrick-Prescott filtered prior to analysis. σ_p denotes the standard deviation of the (detrended) stock price; σ_y denotes the standard deviation of output; and $corr(p,y)$ denotes the correlation between p and y. Numbers in parentheses denote the standard errors of σ_p/σ_y and $corr(p,y)$, computed as in Christiano and Eichenbaum (1992). For estimation of the relevant zero-frequency spectral density, a Bartlett window, truncated at lag six, was used.

Table 2. Financial Statistics in the Production Economies, $\gamma = 2/3$

Statistic	Data	One-Sector Economy		Two-Sector Economy			
				Full Labor Mobility		Limited Labor Mobility	
		No Habit	Habit	No Habit	Habit	No Habit	Habit
r^e	7.82	1.65	1.60	1.64	1.92	1.67	7.56
		(0.01)	(0.01)	(0.02)	(0.02)	(0.02)	(0.07)
$r^e - r^f$	6.63	0.002	0.002	0.02	0.29	0.07	4.86
	(1.78)	(1e-3)	(1e-3)	(0.01)	(0.03)	(0.01)	(0.13)
σ_{r^e}	19.53	0.44	0.41	2.60	7.97	3.55	37.3
		(4e-3)	(4e-3)	(0.01)	(0.03)	(0.01)	(0.2)
$\sigma_{r^e - r^f}$	19.02	0.24	0.23	2.31	6.89	3.12	32.2
	(1.73)	(1e-3)	(7e-4)	(0.01)	(0.02)	(0.01)	(0.12)

Notes: See table 1a.

Table 3: Business-Cycle Statistics in the Production Economies

| Statistic | Data | One-Sector Economy | | Two-Sector Economy | | | |
| | | | | Full Labor Mobility | | Limited Labor Mobility | |
		No Habit	Habit	No Habit	Habit	No Habit	Habit
σ_Y	2.00	1.77	1.72	1.71	1.49	1.69	1.68
	(0.25)	(1e-4)	(0.01)	(0.01)	(0.01)	(0.01)	(0.001)
σ_C/σ_Y	0.38	0.58	0.40	0.66	0.34	0.70	0.68
	(0.04)	(4e-4)	(2e-3)	(1e-3)	(2e-3)	(1e-3)	(1e-3)
σ_I/σ_Y	2.43	1.78	2.38	1.67	2.46	1.70	1.74
	(0.06)	(2e-3)	(4e-3)	(2e-3)	(4e-3)	(2e-3)	(2e-3)
σ_H/σ_Y	0.83	0.29	0.23	0.26	0.46	0.29	0.29
	(0.07)	(2e-3)	(2e-4)	(2e-4)	(3e-3)	(3e-4)	(3e-4)
$\rho(Y,C)$	0.79	0.99	0.66	0.97	0.62	0.93	0.92
	(0.06)	(1e-3)	(1e-3)	(2e-4)	(1e-3)	(5e-4)	(6e-4)
$\rho(Y,I)$	0.96	1.00	0.97	0.98	0.98	0.96	0.96
	(0.01)	(4e-5)	(2e-4)	(2e-4)	(2e-4)	(5e-4)	(5e-4)
$\rho(Y,H)$	0.82	0.99	0.99	0.93	0.23	0.81	0.80
	(0.05)	(2e-3)	(2e-4)	(2e-4)	(3e-3)	(2e-3)	(2e-3)
$\rho(Y,H_C)$	0.72	-0.97	-0.88	-0.99	-0.66	-0.80	-0.80
	(0.08)	(2e-3)	(1e-3)	(2e-4)	(1e-3)	(2e-3)	(2e-3)
$\rho(Y,H_K)$	0.86	0.98	0.94	0.95	0.95	0.81	0.80
	(0.04)	(2e-3)	(6e-4)	(1e-3)	(1e-3)	(2e-3)	(2e-3)
FREQ($I_C \leq 0$)		0.02	2.08	0	0.85	0	0
		(0.01)	(0.06)		(0.04)		
FREQ($I_K \leq 0$)		1.65	11.3	0.98	9.40	1.07	1.71
		(0.07)	(0.2)	(0.05)	(0.14)	(0.05)	(0.07)

Notes to Table 3: (i) In the "Data" column are reported estimates of the relevant statistic, and in parentheses are standard errors of the estimates from U.S. data 1964:1–1988:2. The standard errors are based on the GMM procedure described in Christiano and Eichenbaum (1992). With the exception of the sectoral hours data, all the data used for these estimates are from an updated version of the Christiano (1988) database, compiled by Fisher (1994). The sectoral hours data are from Citibase. For the consumption sector we used two alternative measures: an index of hours worked in the service sector (Citibase series LWHPX) and an index of hours worked in the nondurable manufacturing sector (LWHNX). The point estimate and standard error for the correlation of consumption sector hours worked with output based on LWHPX are reported in the table. The analogous point estimate and standard error based on LWHNX are 0.83 (0.05); (ii) With the exception of the correlations and the relative volatilities, all the statistics are reported in percent terms; (iii) Results for the model are based on 500 replications of sample size 120, and Monte Carlo standard errors are reported in parentheses.

Table 4: Other Business-Cycle Statistics for the Limited Labor Model with Habit

	ρ	Instrumental Variables Estimates $\Delta C_t = \mu + \lambda \Delta Y_t + \theta r_t + \varepsilon_t$		
		λ	θ	$R^2_{\Delta y}$
U.S. data	0.37	0.47	0.089	0.046
	(0.07)			
Model Results	0.20	0.98	0.034	0.036
	(4e-3)	(0.01)	(1e-4)	(2e-3)

Notes:

(i) $\Delta x_t \equiv \log(x_t) - \log(x_{i-1})$. U.S. data estimates for $\lambda, \theta, R^2_{\Delta y}$ are taken from Campbell and Mankiw (1989). The estimate of ρ, the first order autocorrelation of ΔY_t, is based on logged U.S. real GDP growth covering the period 1947:I–1995:I. The standard error for this statistic is reported in parentheses, and is computed using the procedure described in the note to Table 1b. "Model Results" entries represent the indicated statistic based on 120 simulated observations from the limited labor model with habit, averaged across 500 replications. Numbers in parentheses are the associated Monte Carlo standard errors.

(ii) $\rho \sim$ autocorrelation of consumption growth, $R^2_{\Delta y} \sim R-$bar squared of regression of Δy_t on instruments.

(iii) Instruments: U.S. data—$\{\Delta c_{t-2}, \ldots, \Delta c_{t-4}, r_{t-2}, \ldots, r_{t-4}\}$; Model simulations—$\{\Delta c_{t-1}, \ldots, \Delta c_{t-3}, r_{t-1}, \ldots, r_{t-3}\}$.

(iv) Simulation results are based on $r_t = r^f_{t-1}$. We also performed the simulations with $r_t = r^e_t$, and obtained results essentially the same as those reported here.

Figure 1: Exchange Economy, Power Utility
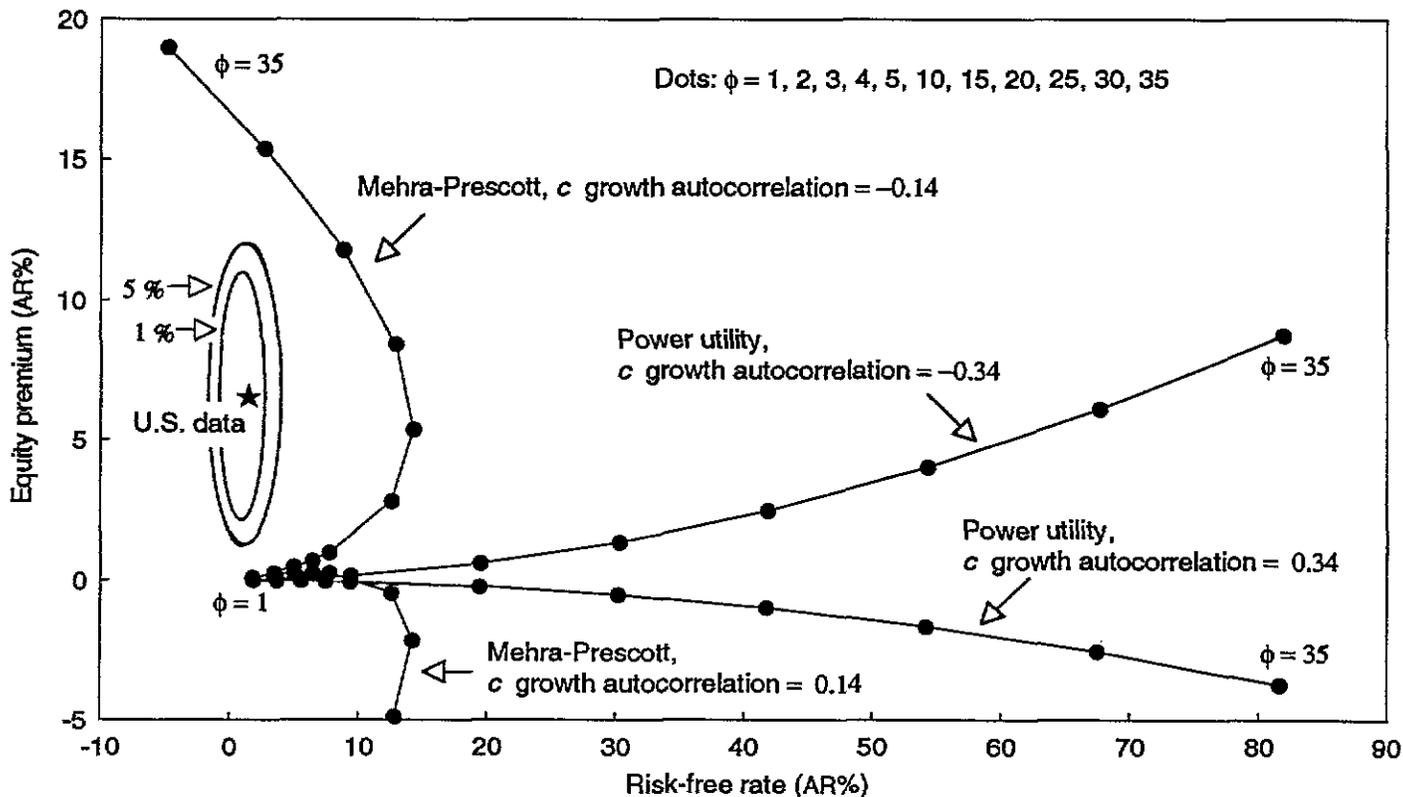
Figure 2: Exchange Economy, Habit-persistence Utility
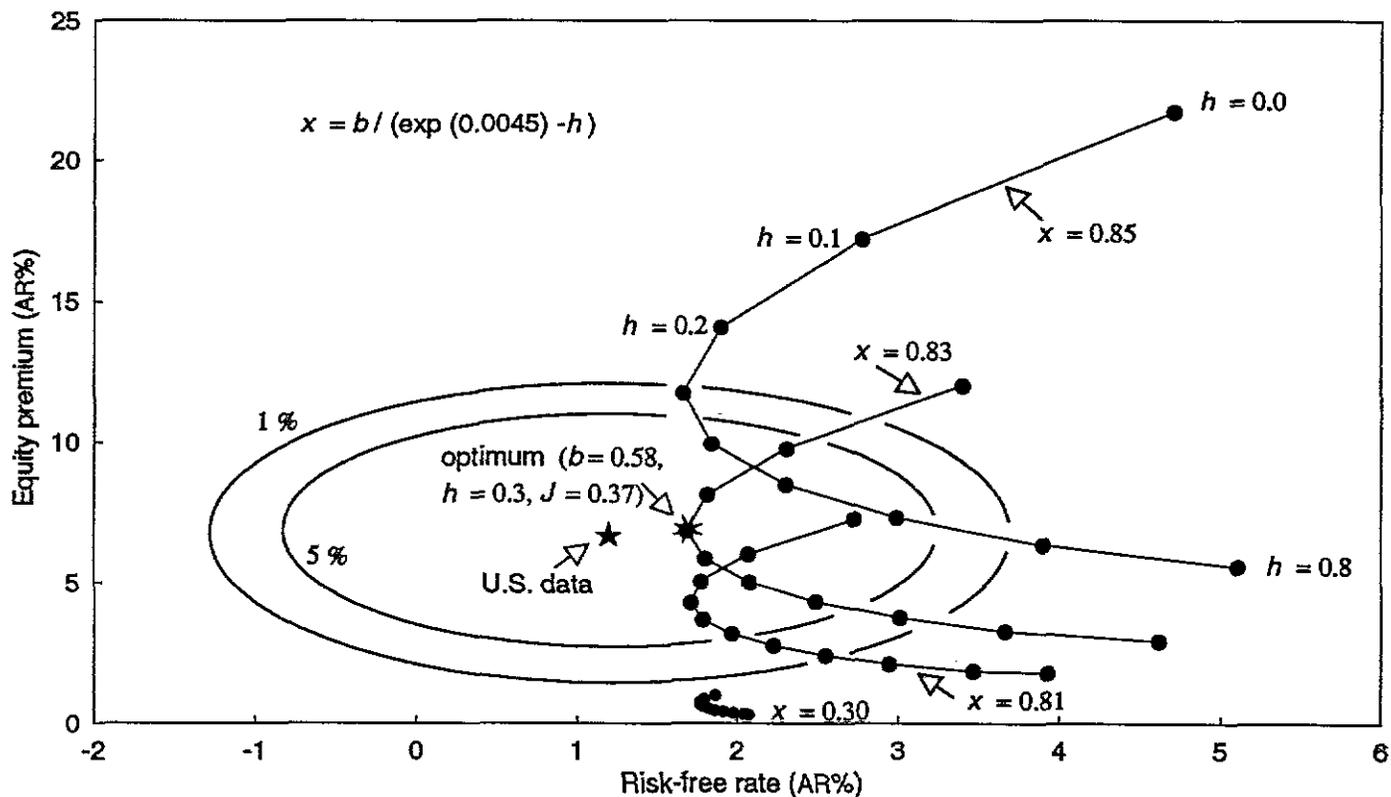
Figure 3: One-sector Production Economy
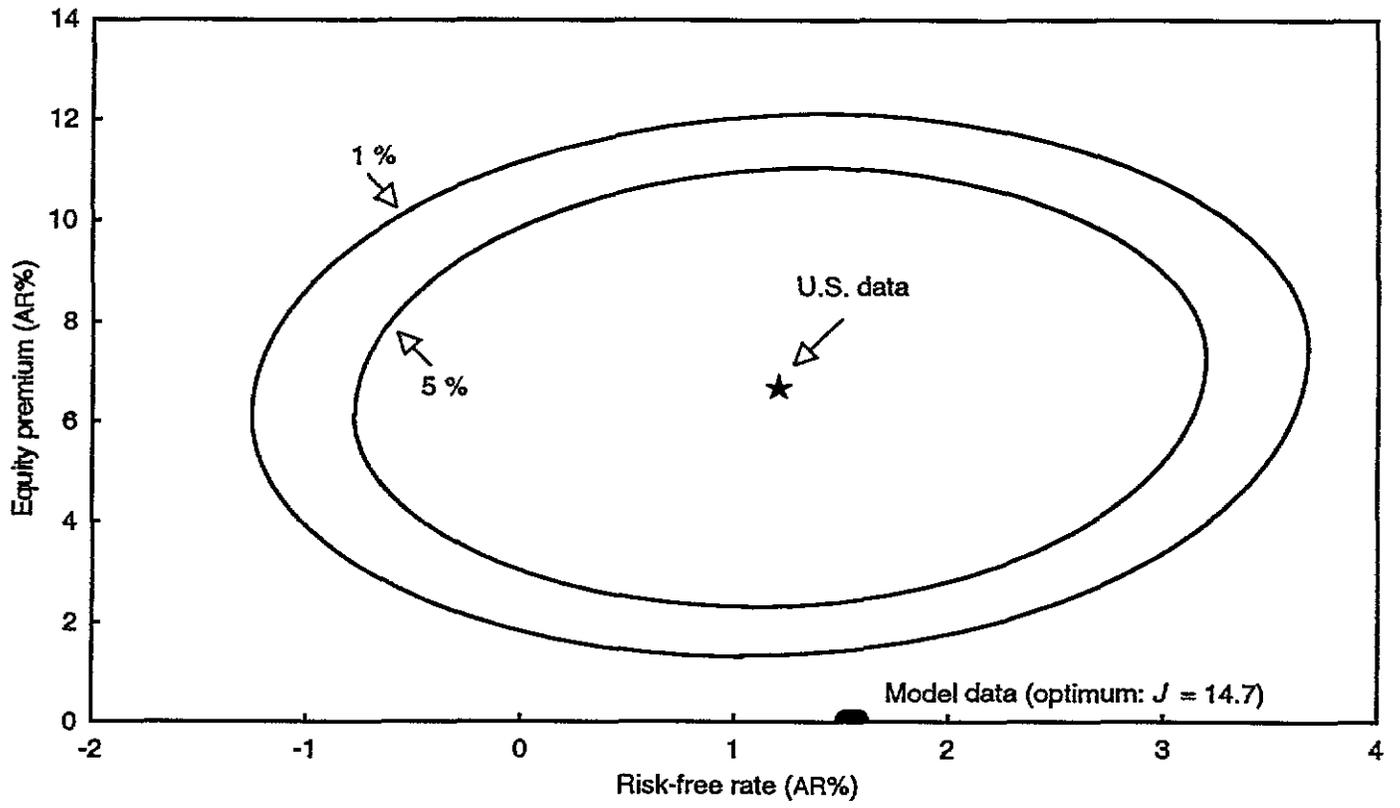
Figure 4: Impulse Response Functions for One-sector Models

Fig. 4a: Response of C

Fig. 4d: Response of h_c

Fig. 4b: Response of I

Fig. 4e: Response of h_i

Fig. 4c: Response of h

Fig. 4f: Response of I_c
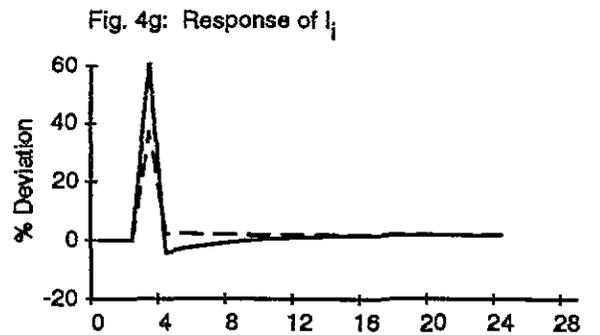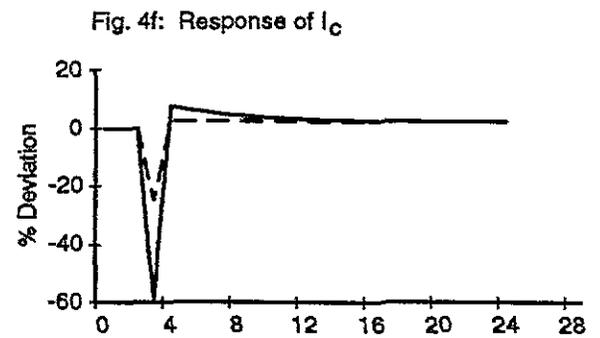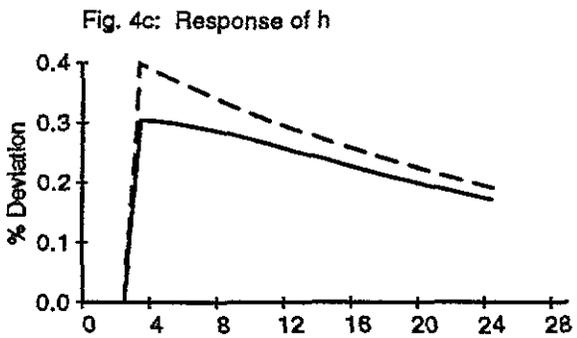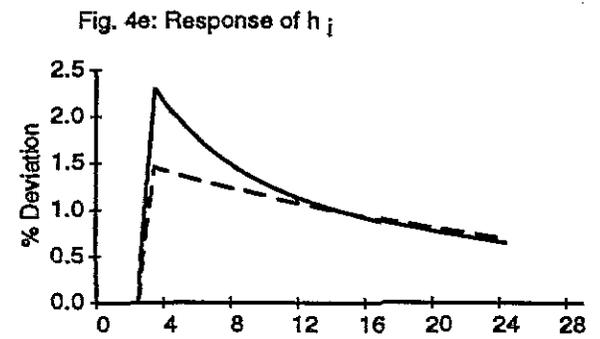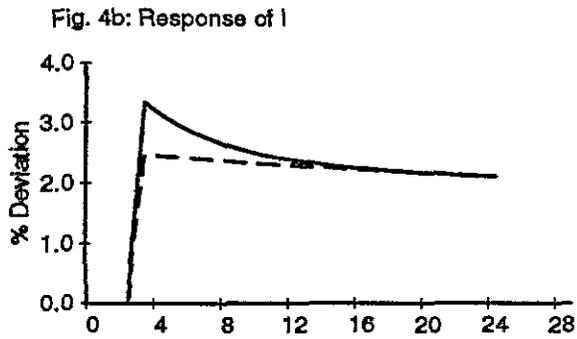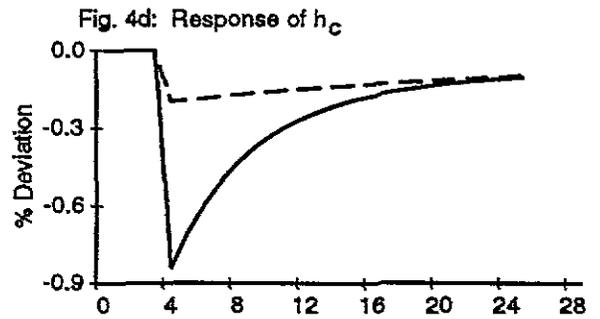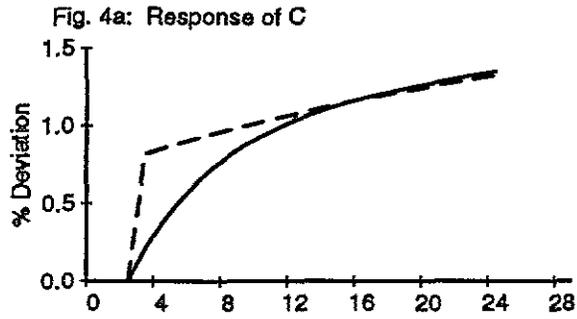
——— Habit persistence utility

– – – Power utility

Fig. 4g: Response of I_i

Figure 5: Two–sector Economy with Full Labor Mobility
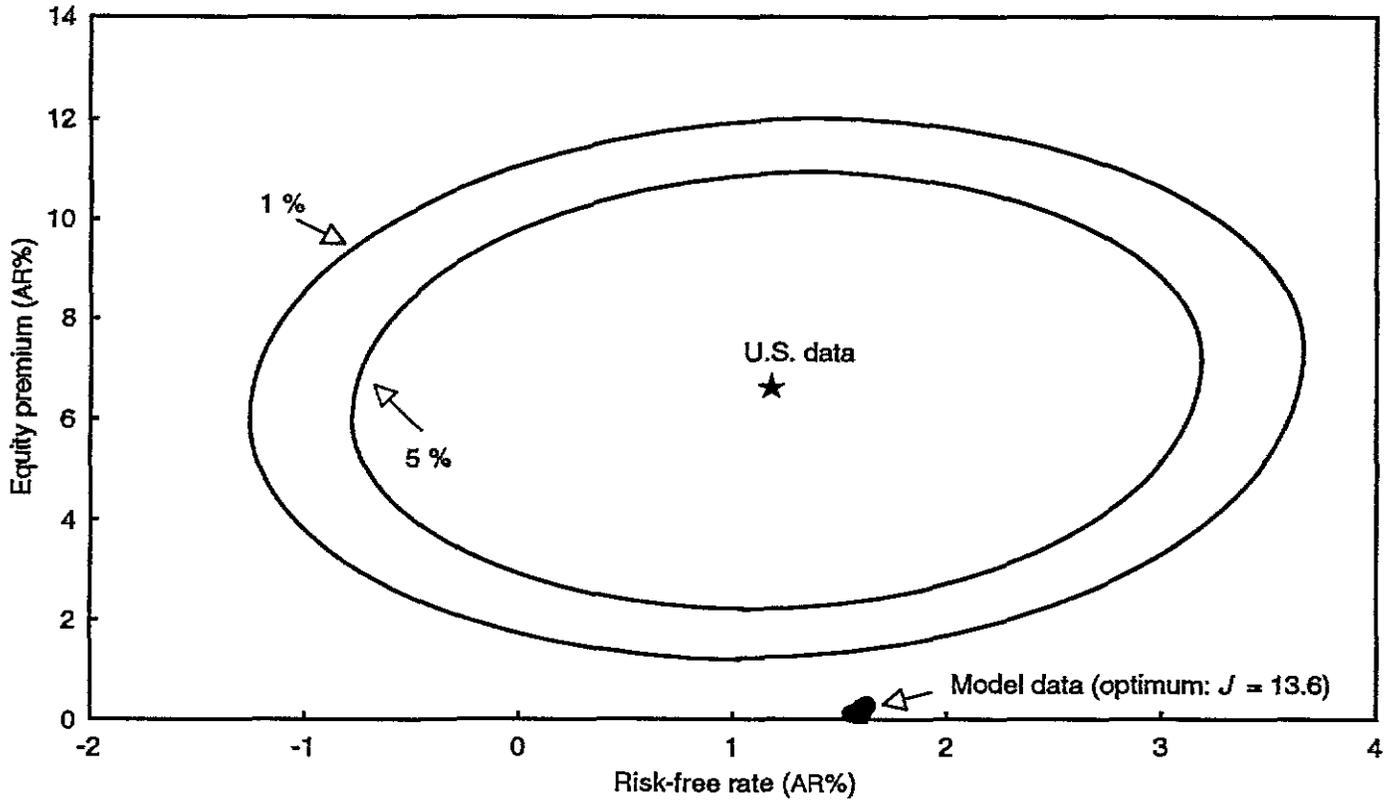
Figure 6: Two–sector Production Economy with Limited Labor Mobility
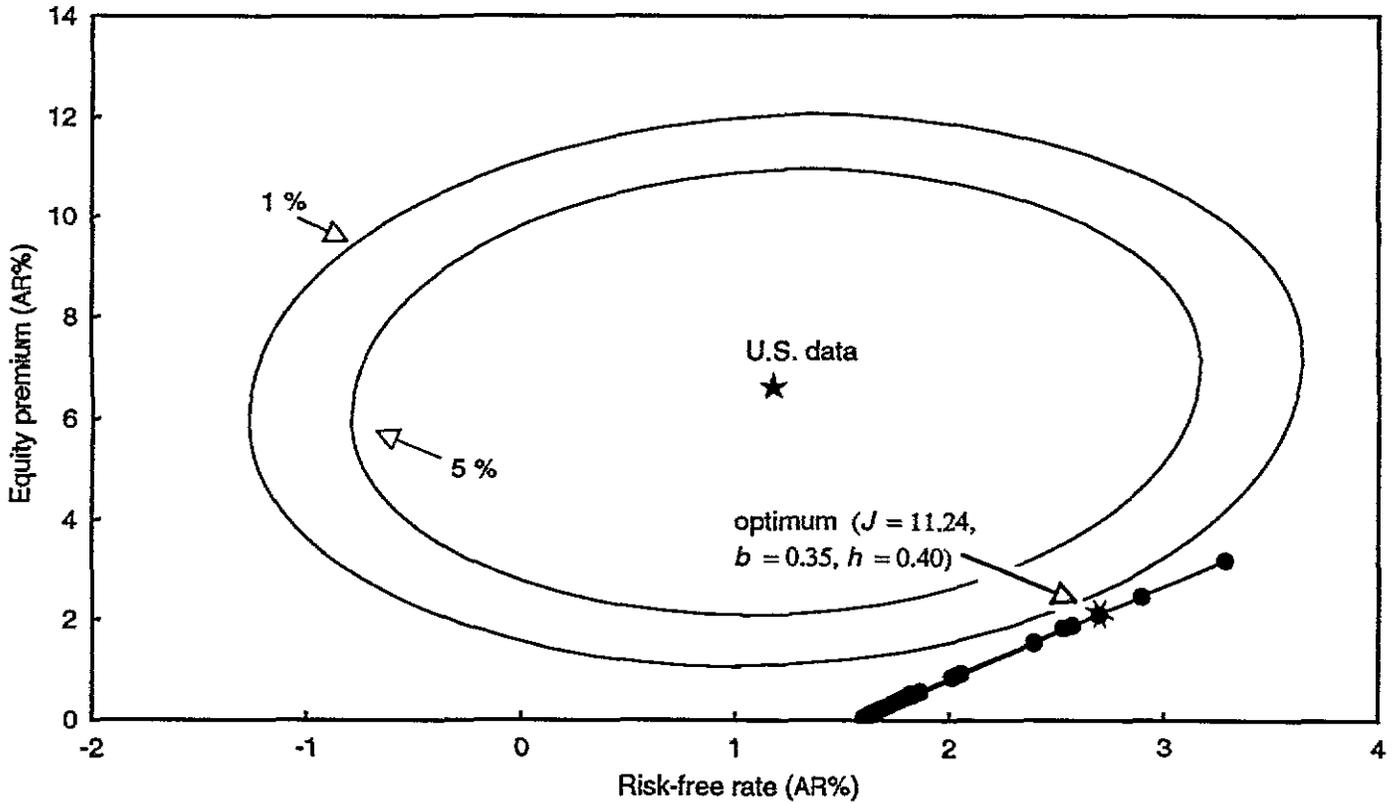
Figure 7: Impulse Response Functions for Two-sector Models

Fig. 7a: Response of C

Fig. 7e: Response of h_c

Fig. 7b: Response of I

Fig.7f: Response of h_i

Fig. 7c: Response of h

Fig. 7g: Response of I_c

Fig.7d: Response of P_k

Fig. 7h: Response of I_i
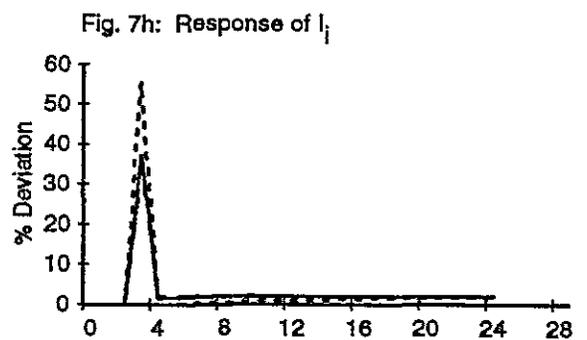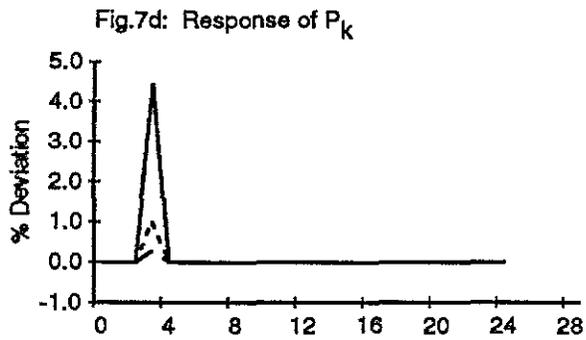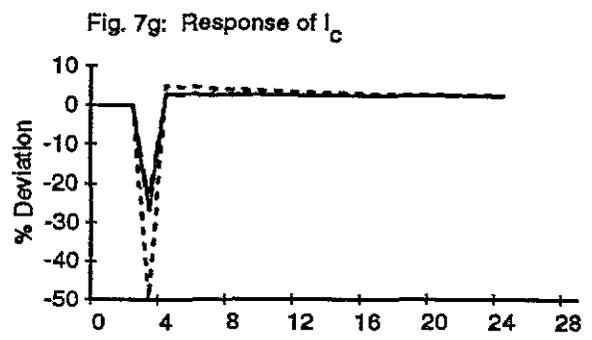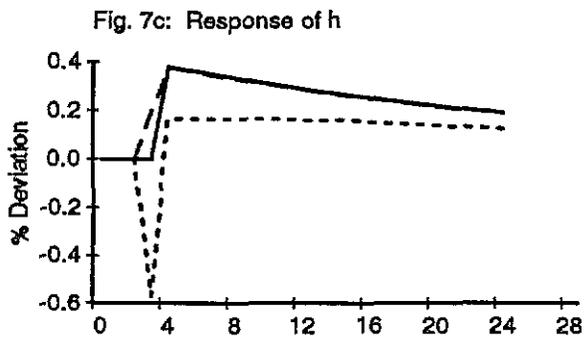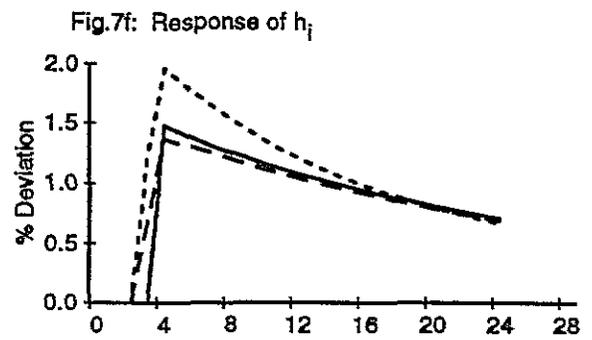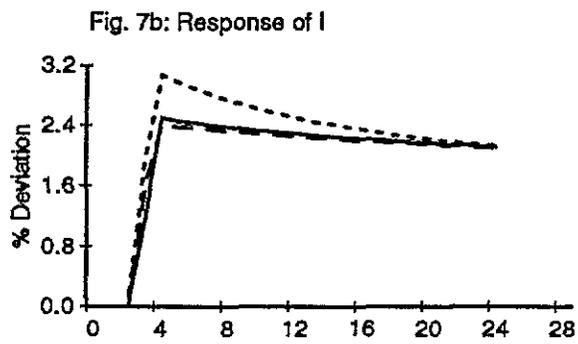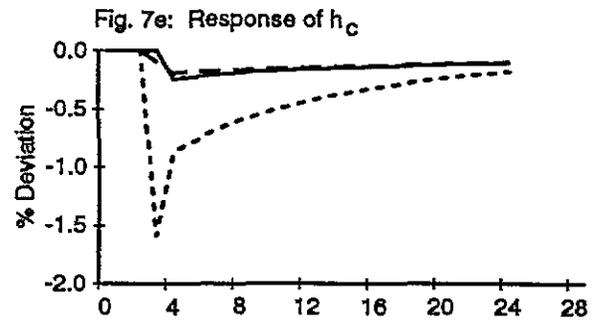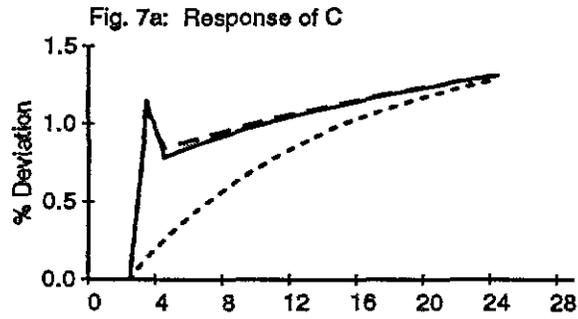
——— Limited labor mobility, habit persistence utility
- - - - Full labor mobility, habit persistence utility
– – Full labor mobility, power utility

Figure 8: Two–sector Production Economy with Limited Labor Mobility and Leverage