

Historical

Modeling the CD Market: Hypotheses and Tests

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The views expressed herein do not necessarily represent the views of the Federal Reserve Bank of Minneapolis or the Federal Reserve System. For his many criticisms and suggestions, I am indebted to Neil Wallace. Any errors, however, are my own.

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Introduction

Since their introduction in 1961, large-denominated negotiable certificates of deposits (CDs) have been a significant source of bank funds. Banks have been better able to manage their liabilities by offering attractive rates of return on CDs, something they cannot do explicitly with demand deposits. And because these certificates are negotiable in a secondary market, banks have easily attracted large amounts of funds whenever they were needed. As a result, CDs have grown dramatically over the years and now account for over 10 percent of all bank deposits.

But while CDs may have made it easier for banks to manage their liabilities, they have made it harder for the Federal Reserve to conduct monetary policy. For example, when the Fed attempts to slow the economy by reducing bank liabilities, banks can partially offset the action by raising funds through the CD market.

Although the question of how much regulation is needed has not been answered, the Fed has tried in two ways to reduce the influence of the CD market. The first was with an interest rate ceiling (Regulation Q). But while Regulation Q may have restricted banks from expanding their liabilities, it created another problem. As market rates rose above the Q ceiling, CDs declined sharply causing great uncertainty in financial markets. Responding to market pressures and the outflow of funds from commercial banks, the Fed raised the ceiling frequently and finally abandoned it in 1973. So the monetary authorities have come to rely solely on their second method of reducing the influence of CDs, reserve requirements. But exactly what level is best still eludes the Fed as requirements have been changed frequently since 1973.

The difficulty the policy maker has had in regulating CDs stems largely from our inability to understand and correctly model this market. With the increasing importance of the CD market both to commercial banks and to policy makers, it's not surprising that it's a part of most large scale econometric models. The approach used to model the demand side of this market first assumes that new and secondary CD issues are perfect substitutes. Then based on restrictions implied by a standard portfolio theory, a single demand equation is estimated using Ordinary Least Squares (OLS), but not using all the available data. Observations during periods when market rates were above the Q ceiling are dropped from the estimation period because, it is argued, agents were off their demand curves during such times. On the supply side of this market, banks are either assumed to offer whatever is demanded at the going rate, with the rate determined in a term-structure equation, or the quantity supplied is exogenous. The CD rate then is determined by the demand equation.

While this type of demand-oriented model of the CD market fits the data period fairly well, such an approach can be criticized on several grounds.

The most serious criticism comes from two facts which contradict the assumption that new and old issues are perfect substitutes. The secondary market rate has always been somewhat higher than the new issue rate even in nonceiling periods; and during Q-ceiling periods (when the new issue rate is at the ceiling and market rates are well above the new issue rate), a significant number of new issues are still sold. Modeling new and old issues as perfect substitutes, therefore, would appear to be a serious misspecification of the true structure.

Other criticisms of the current approach deal with the estimation techniques used. Even if new and old issues are perfect substitutes, the portfolio theory underlying the CD equation implies that OLS yields inconsistent and biased estimates. Furthermore, if new and old issues are perfect substitutes, Q-ceiling observations shouldn't be dropped. Since agents can go to the secondary market and buy the same product at a lower price, the rate restriction should affect supply, not demand.

In this paper we attempt to take a closer look at the demand side of the CD market, examining the current approach that treats new and old issues as identical assets and then an alternative approach that treats new and old issues as competing, but different assets. When estimated correctly, the current approach performs reasonably well, yet is still inconsistent with certain facts. The alternative approach has much richer implications for the data, and the empirical results are generally quite favorable.

1.

The Current Approach: New and Old CDs are Perfect Substitutes

Financial sectors of large scale econometric models of the U.S. economy usually contain linear asset demand equations which are based on a standard portfolio theory.^{1/} (Such a theory is developed in detail in the appendix.) The theory assumes agents are maximizing a nonlinear utility function where utility is an increasing function of the rate of return on financial wealth. Financial wealth is known at the time of the portfolio decision. Interest rates, however, are assumed to be stochastic, with agents only knowing the probability distributions and maximizing expected utility subject to their wealth constraint. The first and second order maximization conditions yield linear asset demand equations with expected (mathematical expectation) interest rates as the independent variables and with these properties:

1. Equations are homogeneous of degree one in wealth.
2. Expected own-rates enter their equation with a positive sign.
3. Across all asset demand equations, the constant terms sum to one and the coefficients on each expected interest rate sum to zero.
4. Cross elasticities are equal.
5. The parameters of these equations are all functions of the known probability distribution of interest rates.

To illustrate, these are the demand curves for the three-asset case:

$$\begin{aligned}
 A_1/W &= a_{10} + a_{11}\hat{R}_1 + a_{12}\hat{R}_2 + a_{13}\hat{R}_3 \\
 (1) \quad A_2/W &= a_{20} + a_{21}\hat{R}_1 + a_{22}\hat{R}_2 + a_{23}\hat{R}_3 \\
 A_3/W &= a_{30} + a_{31}\hat{R}_1 + a_{32}\hat{R}_2 + a_{33}\hat{R}_3 \\
 W &= A_1 + A_2 + A_3
 \end{aligned}$$

The A_i s are the assets and the \hat{R}_i s the corresponding expected own-rates of return. The parameters a_{ij} s have the following properties:

$$a_{ij} > 0 \quad (i=j)$$

$$a_{1j} + a_{2j} + a_{3j} = \begin{cases} 0 & j=1, 2, 3 \\ 1 & j=0 \end{cases}$$

$$a_{ij} = a_{ji} \quad (j \neq 0)$$

$$a_{ij} = f^{ij}(\Sigma)$$

Σ is the known 3x3 variance-covariance matrix of interest rates.

Some of the constraints across equations are a result of the balance sheet identity. Any change in wealth must be divided among the existing assets. And any change in the demand for one asset must be exactly offset by an opposite change in demand for at least one other asset. The other constraints are due to symmetry conditions that are analogous to the symmetry conditions of consumer theory. For example, a decrease in demand for A_1 due to a small increase in \hat{R}_2 is equal to a decrease in demand for A_2 due to a small increase in \hat{R}_1 .

It's important to note that balance sheet constraints together with symmetry constraints imply that the interest-rate coefficients within an equation sum to zero. Thus, by estimating only one equation and testing for the zero-sum constraint, we can jointly test both symmetry and balance sheet constraints for the entire model.

To see how this theory is used in practice, we have reproduced the CD equations from two prominent econometric models: the Wharton and the MIT-PENN-SSRC.^{2/}

Wharton Equation

$$CD_t/Y = .0160 + .0121R_s - .0121R_{tb} - .0024D66.3 - .0013D66.4 + .0006D68.2$$

(10.4) (-4.2) (1.3) (.60) (.30)

$$\bar{R}^2 = .43 \quad S.E.E. = .0021 \quad D.W. = 1.3$$

PERIOD FIT: 1963.4-1968.3.

MIT-PENN-SSRC Equation

$$CD_t/Y = .0164 + .0223R_s - .0140R_{tb} - .0083R_{cp}$$

(12.5) (-5.3) (-1.7)

$$\bar{R}^2 = .62 \quad S.E.E. = .0019 \quad D.W. = .97$$

PERIOD FIT: 1963.3-1968.3 EXCLUDING: 1966.3, 66.4, 68.2

where CD_t = Total outstanding stock of CDs,

Y = Gross National Product

R_{tb} = Treasury bill rate (3-month)

R_{cp} = Commercial paper rate (4-6-month)

R_s = Secondary market rate on CDs (3-month)

D66.3, D66.4, D68.2 = Dummy variables for quarters 1966.3, 1966.4, 1968.2

These equations are similar in form and yield roughly the same statistical results. Both equations are linear, both define the dependent variable as the ratio of CDs to income, both assume Treasury bills and CDs are substitutes, and both constrain the sum of the interest-rate coefficients to zero.^{3/} Except for one extra observation, the equations are estimated over the same data period and exclude Q-ceiling observations.

Coefficients on variables appearing in each equation are statistically close. And the standard errors of estimates are almost identical.

Critique of Current Approach

Because the parameter estimates appear consistent with the theory and because the equations fit the data reasonably well, these equations have been accepted as good estimates of the underlying behavioral relationships between the demand for CDs, the CD rate, and competing rates. But for several reasons, they have been accepted too readily.

The first is related to the assumption that new and old CDs are perfect substitutes, that is, in any given period, agents are indifferent between holding new and old issues. This must imply that the new and secondary rates will be equal or at least be equal on average. And it also implies that during Q-ceiling periods, when the new rate can't be raised and is significantly lower than the secondary rate, no new issues will be sold.

But this is not what we observe. The new rate has been consistently lower than the secondary rate and during Q-ceiling periods, when the difference has become quite substantial, a large number of new issues are still sold. During 1967-1975, not including periods when market rates were above Regulation Q ceilings, the three-month secondary CD rate averaged 30 basis points higher than the two-to-three-month new issue rate (New York), 15 basis points higher than the three-to-six-month rate (New York), and the secondary rate rarely fell below a new issue rate. And when market rates were well above the Q ceiling in 1969, (the average difference between the secondary rate and the Q ceiling was 200 basis points) new issues were still being sold at an average of close to \$3 billion a month, down only 40 percent from the previous nonceiling year.^{4/}

So the assumption that new and old CDs are perfect substitutes is obviously suspect. This implies that the current approach is probably misspecified and that treating new and old CDs as competing but not identical assets is more consistent with the facts. In the next section of this paper such a strategy is pursued. But before going to this analysis, there are other things wrong with the current approach.

Even if new and old issues could be considered perfect substitutes, the CD demand equations have not been estimated properly. Recall that the portfolio theory yielded linear asset demand equations with expected interest rates. Expected rates, not being observed, were replaced by actual rates in estimation. In general this implies that the independent variables will be correlated with the residual so that OLS will give inconsistent and biased estimates of the true parameters.^{5/} Thus a simultaneous estimator is more appropriate, yet both the Wharton and MIT-PENN-SSRC model builders used OLS.

Another estimation problem that has not been handled properly is the use or nonuse of Q-ceiling observations.

The standard argument for excluding Q-ceiling observations is that investors are off their normal demand curve when banks are unable to offer a competitive rate. An example of this reasoning is found in the early developmental stages of the Federal Reserve Board of Governors' money market model.

The CD market presents some unique problems in econometric analysis because of the role played by the Federal Reserve's use of the Regulation Q policy tool. It is not reasonable, for example, to estimate the public's demand over all time periods. When Q is effective (i.e., banks would pay more in the absence of the ceiling), it is not possible for the public in aggregate to rid itself of all the CDs they might wish. CDs are time deposits and banks don't refund until a specific future date. The coefficient of the offering rate in a demand equation estimated over all time periods would consequently be distorted.^{6/}

But if old and new CD issues are identical assets, this reasoning is faulty. When banks' offering rates hit the Q-ceiling, investors are free to go to the secondary market and purchase old CDs at higher rates. The demand for CDs should not be affected by the ability of banks to offer competitive rates. The effect should be on supply. During periods when the secondary rate significantly exceeds the new issue rate, no new issues should be sold; the supply of CDs should be completely exogenous.

Thus, under the assumption of perfect substitutes, a more efficient and consistent estimation procedure exists. The demand equation can be estimated with the CD rate as the dependent variable and the quantity as an independent variable. Then an instrumental variable estimator can be used to reestimate the equation, with the quantity of CDs as its own instrument during Q-ceiling periods.^{7/}

Reestimating the Aggregate CD Equation

Using monthly data over the period 1967-1975 and the procedure described above, we estimated and tested a CD demand equation. Besides providing more degrees of freedom, the advantage of this data set over those previously used is that it contains longer and more significant periods when Q-ceilings were effective.^{8/} The tests include an F-test for differences between ceiling and nonceiling observations and a t-test for the zero-sum restriction on the interest-rate coefficients. The variables used in the monthly regressions are those in the quarterly regressions reported above except for income, where we used personal income instead of GNP.^{9/}

Table 1 shows the results of these tests. Unrestricted estimation results are presented for the nonceiling period, the ceiling period, and

Table 1

AGGREGATE EQUATION: DEPENDENT VARIABLE R_S

	Constant	CD_T/Y	R_{CP}	R_{TB}	$1-\sum R$	\bar{R}^2	D.W.	Rho
Nonceiling Period (7/67-11/68, 6/70-12/75)	-.637 (-5.0)*	3.17 (1.2)	.999 (21.1)*	.101 (1.5)	-.101 (-3.4)*	.99	2.3	.38
Ceiling Period (11/68-5/70)	-2.22 (-1.11)	19.8 (0.6)	1.05 (5.0)*	.202 (1.4)	-.257 (-1.3)	.99	1.9	.56
TOTAL PERIOD (7/67-12/75)	-.680 (-5.4)*	5.48 (3.4)*	.983 (22.6)*	.106 (1.9)*	-.089 (-3.8)*	.99	1.9	.41

RENORMALIZED EQUATION: DEPENDENT VARIABLE CD_T/Y

	Constant	R_S	R_{CP}	R_{TB}	$R_S+R_{CP}+R_{TB}$
Nonceiling Period	.201	.315	-.315	-.032	-.032
Ceiling Period	.112	.050	-.165	-.013	-.128
TOTAL PERIOD	.124	.183	-.180	-.018	-.015

Test for structural change: $F_{4,93} = 1.13$

*Significant at 95 percent level of confidence

the total sample period. Equations are corrected for first-order serial correlation; a correlation coefficient (ρ) and a Durbin Watson (D.W.) statistic are reported. Other statistics shown are the adjusted R^2 , the regression coefficients, one minus the sum of interest-rate coefficients, and the corresponding t-statistics.^{10/} In order to examine these equations in a more familiar way, they are renormalized with the quantity as the dependent variable. And below the renormalized equation data is an F-statistic which tests the null hypothesis that Q-ceiling observations are structurally equivalent to nonceiling observations.^{11/}

Overall the results indicate that when estimated properly the approach, which uses a standard portfolio model and treats new and old CDs as perfect substitutes, fits the data reasonably well. The F-test result was consistent with the assumption that old and new issues are perfect substitutes. The insignificant F-statistic implies that agents were not off their demand curves during ceiling periods; to a large degree they simply went to the secondary markets where rates were higher. Many of the coefficient restrictions implied by the portfolio theory held. In each subperiod, the coefficient on the CD rate is positive and on each competing rate it's negative. Moreover, all coefficients are statistically significant in the total sample period regression. The interest-rate coefficients sum to zero, however, only in the ceiling period.

2.

An Alternative Hypothesis: New and Old CDs are Different Assets

Although results reported in Table 1 are consistent with the assumption that new and old CDs are perfect substitutes, that model cannot explain why new issues are sold during Q-ceiling periods or why secondary rates are generally higher than new issue rates. We now attempt to develop and test a model consistent with these observations.

We posit two types of investors: the general public--those who treat new and secondary issues as perfect substitutes--and the hometowners--those who, other things equal, prefer new issues of hometown banks to secondary issues.

Hometown investors exist for two reasons. First, bank customers usually buy more than one product from their banks. By supporting a local bank's CD sale today, for example, customers may expect better financing privileges in the future. Second, state and local governments which invest some idle funds in CDs may feel obliged to do business in their home territory. In fact, some states legally require this.^{12/}

The existence of two types of investors implies that the CD new issue rate will be lower than the secondary rate and that new issues will be sold during Q-ceiling periods. Suppose the new issue rate were higher than the secondary rate. Since the general public view new and old CDs as the same product, they would clearly buy only new. And so would hometowners because even if the new rate were equal to the secondary rate, they would still prefer new. To explain the secondary market, therefore, in our model the new rate must be lower than the secondary rate. And because of hometowners' preference for new issues, some will still be demanded in Q-ceiling periods.

To test this hypothesis, we postulate the following set of asset demand equations

Hometowners

$$\begin{aligned}
 X_s^h/W^h &= a_{10} + a_{11}\hat{R}_s + a_{12}\hat{R}_n + a_{13}\hat{R}_o \\
 (2) \quad X_n^h/W^h &= a_{20} + a_{21}\hat{R}_s + a_{22}\hat{R}_n + a_{23}\hat{R}_o \\
 X_o^h/W^h &= a_{30} + a_{31}\hat{R}_s + a_{32}\hat{R}_n + a_{33}\hat{R}_o \\
 W^h &= X_s^h + X_n^h + X_o^h
 \end{aligned}$$

where $a_{ij} > 0 \quad i=j$

$$a_{1j} + a_{2j} + a_{3j} = \begin{cases} 0 & j=1,2,3, \text{ and} \\ 1 & j=0 \end{cases}$$

$$a_{ij} = a_{ji} \quad j \neq 0.$$

It follows that $a_{i1} + a_{i2} + a_{i3} = 0 \quad (i=1,2,3)$.

General Public ($\hat{R}_s > \hat{R}_n$)

$$\begin{aligned}
 X_s^g/W^g &= b_{10} + b_{11}\hat{R}_s + b_{13}\hat{R}_o \\
 (3) \quad X_n^g/W^g &= 0 \\
 X_o^g/W^g &= b_{30} + b_{31}\hat{R}_s + b_{33}\hat{R}_o \\
 W^g &= X_s^g + X_n^g + X_o^g
 \end{aligned}$$

where $b_{ij} \geq 0 \quad i=j$

$$b_{1j} + b_{3j} = \begin{cases} 0 & i=1,3, \text{ and} \\ 1 & j=0 \end{cases}$$

$$b_{ij} = b_{ji} \quad j \neq 0.$$

It follows that $b_{i1} + b_{i3} = 0 \quad (i=1,3)$.

The hometowners divide their wealth (W^h) among three assets: secondary CD issues (X_s^h), new CD issues (X_n^h), and others (X_o^h). The

properties of the asset demand equations (2) are the same as discussed above for equations (1). The demand for an asset is a positive function of its own expected rate and a negative function of expected competing rates, and it is homogeneous of degree one in wealth. Coefficients across equations sum to zero except the constants, which sum to one. And balance sheet and symmetry constraints imply that interest-rate coefficients within an equation sum to zero.

Similarly, the general public divide their wealth among the three assets (X_s^g , X_n^g , and X_o^g) and their asset demand equations (3) have the properties listed above. But here, since the secondary rate is always greater than the new issue rate, the general public never purchase new issues.

Estimating these two sets of equations over a sample period that includes Q-ceiling observations will let us test the general portfolio theory as well as the assumption that new and old issues are not perfect substitutes. The interest rate restrictions provide a direct test of the portfolio theory. A test of the different-asset hypothesis comes from the Q-ceiling period. Since the structural parameters are functions of the joint probability distribution of interest rates, the joint distribution which includes the new issue rate will change during the Q-ceiling period. Thus, parameters in the hometown demand equations will change, but parameters in the general public demand equations will be stable. Furthermore, the collinearity between the new rate and the secondary rate is much lower during Q-ceiling periods so we should be able to get sharper estimates of the hometown parameters over these periods.^{13/}

The model postulated above, however, cannot be directly estimated because the data are not disaggregated by investors, only by markets. To see how this affects the testable implication discussed above, we aggregate (2) and (3) to get market demand equations.

Secondary Issues

$$\frac{X_s}{W} = w_1 \left(\frac{X_s^h}{W^h} \right) + w_2 \left(\frac{X_s^g}{W^g} \right) =$$

$$(a_{10}w_1 + b_{10}w_2) + (a_{11}w_1 + b_{11}w_2)R_s + a_{12}w_1R_n + (a_{13}w_1 + b_{13}w_2)R_o$$

where $w_1 = \frac{W^h}{W}$, $w_2 = \frac{W^g}{W}$ and $W = W^h + W^g$.

New Issues

$$\frac{X_n}{W} = w_1 \left(\frac{X_n^h}{W^h} \right) = a_{20}w_1 + a_{21}w_1R_s + a_{22}w_1R_n + a_{23}w_1R_o$$

where $w_1 = \frac{W^h}{W}$ and $W = W^h + W^g$.

Most of the testable implications still hold. Balance sheet and symmetry constraints are intact. In particular, interest rate coefficients sum to zero, and the coefficient of the secondary rate in the new issue equation equals the coefficient of the new issue rate in the secondary equation. Since the new issue equation contains only structural parameters from the hometown equation weighted by w_1 , parameter estimates should still change during ceiling periods assuming w_1 remains constant or does not exactly offset the change in the hometown parameters. The secondary issues equation, on the other hand, should not change. Although the parameters in this equation are functions of the hometown coefficients, in most cases the general public coefficients will dominate (that is, w_2 is likely to be significantly greater than w_1), and the parameters will appear stable.

Estimation and Test Results

We estimate the new and old issue equations with a few modifications. Our proxy for wealth is personal income (Y). As a proxy for hometowners' future commitment to buying local bank CDs, we include in the new issue equation the amount of CDs maturing (Mat).^{14/} And for competing interest rates we use those in current models: the three-month Treasury bill rate (R_{tb}) and the four-to-six-month commercial paper rate (R_{cp}).

Our data period and estimation procedures are similar to those used for reestimating and testing the aggregate equation in Section 1. Observations are again monthly and cover the period 1967-1975. The instrumental variable technique discussed earlier is used again for estimating the asset demand equations; instruments are used for all endogenous explanatory variables. The quantity of secondary issues and income are the only variables assumed to be statistically exogenous.

Tables 2 and 3 show the estimation and test results for the new and secondary issue equations, respectively. These tables are similar in format to Table 1. Each shows the estimated coefficients and t-statistics for the total sample, the Q-ceiling period, and the non-ceiling period. When the secondary rate is the dependent variable, we report a set of renormalized equations, with the quantity as the dependent variable. And we report an F-statistic testing for structural change between ceiling and nonceiling periods.

Although the results are somewhat mixed, they tend to support the standard portfolio model and the hypothesis that new and old CDs are not perfect substitutes.

The new issue equation yields the most favorable results (Table 2). As predicted, it fails the Chow test with an F-statistic

Table 2

NEW ISSUE EQUATION: DEPENDENT VARIABLE $\frac{100.0 \text{ CD}_N}{Y}$

	Constant	R _N	R _S	R _{CP}	R _{TB}	Mat	ΣR	R ²	D.W.	Rho
Nonceiling Period (7/67-11/68, 6/70-12/75)	.264 (1.1)	-.051 (-0.2)	.342 (1.3)	-.164 (-0.5)	-.083 (-1.2)	.060 (17.4)*	.004 (1.2)	.95	1.9	.14
Ceiling Period (11/68-5/70)	-.246 (-0.6)	.106 (1.8)*	-.445 (-1.7)*	.403 (1.4)	.008 (0.2)	.008 (6.1)*	.072 (1.1)	.85	2.4	.14
TOTAL PERIOD (7/67-12/75)	.528 (3.2)*	.210 (5.1)*	.342 (1.4)	-.420 (-1.8)*	-.138 (2.3)*	.063 (18.4)*	-.005 (-0.2)	.96	2.0	.24

Test for structural change: F_{6,89} = 3.1*

*Significant at 95 percent level of confidence

significant at the 95 percent level of confidence. Also as expected, the substitution effect is more significant in the ceiling period. In fact, the coefficients on the new and secondary rates are appropriately signed and statistically significant in the ceiling period while statistically zero in the nonceiling period. In all periods the balance sheet constraint appears to hold as the sums of the interest rate coefficients are close to zero.

The results from the secondary issue equation are not quite as strong but are still consistent with the model (Table 3). As predicted, the F-statistic is not significant so we can use the total period coefficients as the "best" parameter estimates. In this equation the interest rate coefficients are all appropriately signed and all but the new issue rate's are statistically significant. The sum of the interest rate coefficients, however, is statistically different from zero.

Finally, cross coefficients seem to satisfy the symmetry conditions. Taking the ceiling period results as our "best" estimate of the new issue equation and the total period results as our "best" estimate of the secondary issue equation, the coefficient of the secondary rate in the new issue equation (-.0045) appears to be statistically equal to the coefficient of the new issue rate in the second equation (-.0054).

Table 3

SECONDARY ISSUE EQUATION: DEPENDENT VARIABLE R_S

	Constant	CD_S/Y	R_N	R_{CP}	R_{TB}	$1-\sum R$	\bar{R}^2	D.W.	Rho
Nonceiling Period (7/67-11/68, 6/70-12/75)	-.527 (-4.7)*	3.86 (1.9)*	.161 (.74)	.845 (3.8)*	.086 (1.3)	-.091 (2.7)*	.99	2.0	.38
Ceiling Period (11/68-5/70)	-1.97 (-.87)	30.7 (1.2)	-.120 (-.62)	1.10 (6.5)*	.202 (1.4)	-.182 (-.59)	.98	1.7	.46
TOTAL PERIOD (7/67-12/75)	-.683 (-4.7)*	6.87 (1.9)*	.040 (0.8)	.956 (14.4)*	.101 (1.7)*	-.100 (4.1)*	.99	1.9	.43

RENORMALIZED EQUATION: DEPENDENT VARIABLE CD_S/Y

	Constant	R_S	R_N	R_{CP}	R_{TB}	$R_S+R_N+R_{CP}+R_{TB}$
Nonceiling Period	.137	.259	-.042	-.219	-.022	-.024
Ceiling Period	.064	.033	.004	-.035	-.007	-.005
TOTAL PERIOD	.099	.146	-.005	-.139	-.014	-.012

Test for structural change: $F_{5,91} = .95$

*Significant at 95 percent level of confidence

Summary and Conclusions

As the CD market has become an important source of bank funds, it has also become an important market for policy makers and model builders to understand. But early attempts at modeling this market were deficient in several ways. They failed to consider that new and old CD issues are not perfect substitutes. By dropping Q-ceiling observations from the estimation period they discarded relevant information. And they did not use the "best" estimation procedures.

Our reexamination of this early approach shows that when Regulation Q was binding, agents were not off their demand curve. An aggregate CD demand equation estimated, with an instrumental variable technique, over both ceiling and nonceiling periods fits the data reasonably well and is consistent with a standard portfolio theory.

This approach, however, still cannot explain why the new issue rate was always lower than the secondary rate and why new issues were sold during Q-ceiling periods. Our proposed alternative approach tests whether or not new and old CDs could be modeled as different assets. Q-ceiling periods provide several testable implications along with the coefficient restrictions from the standard portfolio theory. The results are consistent both with the hypothesis that new and old issue CDs are not perfect substitutes and with the portfolio restrictions.

In general, then, these results show that a standard portfolio theory is a reasonable approach to modeling the CD market, that new and old issues are competing assets, but not perfect substitutes, and that Q-ceiling observations should be used in any serious analysis of the CD market.

Appendix^{15/}

A Theory of Portfolio Choice Under Uncertainty

The Utility Function

The individual is assumed to possess a utility function of the form

$$U = a - ce^{-b\pi/W}$$

where

U = utility per decision period; a , b , and c are parameters;

$c, b > 0$; $a \geq 0$

W = fixed portfolio to be divided among n assets $(A_1, A_2, \dots, A_m) = A'$

with corresponding interest rates $(R_1, R_2, \dots, R_m) = R'$

so that $W = i'A$ where $i' = (1, 1, \dots, 1)$ is a unit vector of size n .

π = real profit per decision period $= R'A$

The length of the decision period is assumed to be fixed and shorter than the maturity of any asset.

The Portfolio Selection Problem

There being uncertainty about asset yields, profit (π) is stochastic. We assume the profit distribution is

$$\pi \sim N(\mu_\pi, \sigma_\pi^2).$$

And we assume the individual maximizes the expected value of utility, which, given the normality assumption, is

$$E(U) = a - c \left[\exp\left(\frac{-b}{2W}\mu_\pi + \left(\frac{b}{2W}\right)^2 \sigma_\pi^2\right) \right]$$

Thus

$$\max E(U) \Leftrightarrow \max(\mu_{\pi} - \frac{b}{2W}\sigma_{\pi}^2)$$

Actual interest rates are defined as

$$R = \hat{R} + u$$

where \hat{R} is an $n \times 1$ vector of mean or expected yields and u is an $n \times 1$ vector of random errors with $E(u) = 0$ and $E(uu') = S$. Thus, we can write

$$\begin{aligned}\pi &= R'A = (\hat{R} + u)'A \\ &= \hat{R}'A + u'A\end{aligned}$$

so that

$$\begin{aligned}\mu_{\pi} &= E(\pi) = \hat{R}'A + E(u'A) \\ &= \hat{R}'A\end{aligned}$$

and

$$\begin{aligned}\sigma_{\pi}^2 &= E[(\pi - E(\pi))^2] \\ &= E[(u'A)^2] = A'E(uu')A \\ &= A'SA\end{aligned}$$

Thus, the portfolio selection problem can be summarized as, given R , S , and W choose A such that

$$Z = \mu_{\pi} - \frac{b}{2W}\sigma_{\pi}^2$$

is maximized subject to $i'A = W$ and where

$$\begin{aligned}\mu_{\pi} &= \hat{R}'A \\ \sigma_{\pi}^2 &= A'SA.\end{aligned}$$

The Optimal Portfolio

To solve this problem form the Lagrangean

$$Z^* = \hat{R}'A - \frac{b}{2W}A'SA + \lambda(i'A - W)$$

where λ is an undetermined Lagrangean multiplier. Z^* is maximized if the first order conditions

$$\frac{\partial Z^*}{\partial A} = \hat{R} - \frac{b}{W}SA + \lambda_i = 0$$

$$\frac{\partial Z^*}{\partial \lambda} = i'A - W = 0$$

are satisfied, and the principal minors of the bordered Hessian

$$\begin{bmatrix} \frac{-b}{W}S & i \\ i' & 0 \end{bmatrix}$$

alternate in sign starting positive. It is helpful to rewrite the first-order conditions in matrix notation as

$$\begin{bmatrix} \frac{-b}{W}S & i \\ i' & 0 \end{bmatrix} \begin{bmatrix} A \\ \lambda \end{bmatrix} = \begin{bmatrix} \hat{R} \\ -W \end{bmatrix}$$

Multiplying through by -1 and solving for the unknown A and λ yields

$$\begin{bmatrix} A \\ \lambda \end{bmatrix} = \begin{bmatrix} \frac{b}{W}S & -i \\ -i' & 0 \end{bmatrix}^{-1} \begin{bmatrix} \hat{R} \\ -W \end{bmatrix}$$

By block inversion we can show that

$$\begin{bmatrix} \frac{b}{W}S & -i \\ -i' & 0 \end{bmatrix}^{-1} = \begin{bmatrix} \frac{W}{b}G & -H \\ -H' & D^{-1} \end{bmatrix}$$

where

$$D^{-1} = \frac{-W}{b}i'S^{-1}i$$

$$G = S^{-1} - \frac{S^{-1}ii'S^{-1}}{i'S^{-1}i}$$

$$H = \frac{S^{-1}i}{i'S^{-1}i}$$

Thus, solving explicitly for A yields

$$A = \frac{W}{b}\hat{G}R' + HW$$

So we can write the optimal portfolio as simply a function of the expected rates of return, that is

$$\frac{A}{W} = \frac{1}{b}\hat{G}R' + H$$

Properties of the Parameter Matrices G and H

1. From the symmetry property of the covariance matrix S, it follows that G is symmetric. Proof: $(G = S^{-1} - S^{-1}ii'S^{-1}/i'S^{-1}i)$ S symmetric implies S^{-1} symmetric. Since $i'S^{-1}i$ is a scalar, we only have to show $S^{-1}ii'S^{-1}$ is symmetric. S^{-1} symmetric implies $S^{-1}i$ is the transpose of $i'S^{-1}$. And for any matrix A, AA' is symmetric. Thus, $(S^{-1}i)(S^{-1}i)' = S^{-1}ii'S^{-1}$ is symmetric. And, therefore, G is symmetric because the sum of symmetric matrices is symmetric.

2. The columns and rows of G sum to zero. Proof: To sum the rows post multiply G by i .

$$Gi = S^{-1}i = \frac{S^{-1}iS^{-1}}{i'S^{-1}i} = S^{-1}i - S^{-1}i = 0.$$

And since G is symmetric, the column sums are also zero.

3. The diagonal elements of G are nonnegative. Proof:

Since the principal minors of the bordered Hessian

$$\begin{bmatrix} \frac{-b}{W}S & i \\ i' & 0 \end{bmatrix}$$

alternate in sign, $\frac{-b}{W}S$ is negative definite subject to constraint. It follows that $-G$ is nonpositive definite and in particular $-G$ has nonpositive diagonal elements. Thus, G has nonnegative diagonal elements.

4. The sum of the elements in the column vector H is one. Proof: To sum the elements post multiply H by i .

$$i'H = \frac{i'S^{-1}i}{iS^{-1}i} = 1$$

Aggregation

To aggregate we assume all individuals have the same vector of expected interest rates (\hat{R}) and covariance matrix (S). They can differ, however, in their attitudes toward risk (b) and in the size of their portfolios.

For each individual i we have

$$A_i = \frac{W_i}{b_i} \hat{G}R' + W_i H$$

Summing over m individuals yields

$$\sum_{i=1}^m A = \sum_{i=1}^m \frac{W_i}{b_i} \hat{G}R' + \sum_{i=1}^m W_i H$$

or

$$\frac{A^*}{W^*} = \beta \widehat{GR}' + H$$

where $A^* = \sum A_i$, $W^* = \sum W_i$, and $\beta = \sum \frac{w_i}{b_i}$ where $w_i = \frac{W_i}{W^*}$. Notice that $\beta = \frac{1}{b}$ if $b_i = b$ for all i .

Estimation

The aggregate asset demand equations derived above cannot be estimated directly because the mean or expected interest rates (\widehat{R}) are not observed. Recall, however, that we have assumed that

$$\widehat{R} = R - u$$

and that

$$E(u) = 0.$$

So we can write

$$\begin{aligned} \frac{A^*}{W^*} &= \beta \widehat{GR} + H = \beta G(R-u) + H \\ &= \beta GR + H + v \end{aligned}$$

where $v = -\beta Gu$, $E(\beta Gu) = 0$, and $E(Rv) \neq 0$. Since the disturbance term (v) will generally be correlated with the independent variable (R), OLS will yield biased estimates of βG and H . A simultaneous estimator, therefore, is more appropriate.

Footnotes

1/ See Parkin [6], Parkin, Gray, and Barrett [7] and Gramlich and Kalchbrenner [2] for other applications of this theory.

2/ The Wharton equation comes from a recent version of the model described in McCarthy [5]. The MIT-PENN-SSRC equation comes from a recent version of the model described in [1].

3/ Although these equations are normalized on income instead of wealth, if income is a good proxy for wealth and if the income-wealth relationship is independent of the interest rates appearing in the CD equation, then the balance sheet and symmetry constraints still hold.

4/ Because foreign official institutions are exempt from Regulation Q ceilings, they explain part of the demand for new issues during Q-ceiling periods. Their purchases, however, were found to be relatively small over the data period. The new issue variable used in the empirical work in this paper is net of foreign official institution purchases. We obtained estimates of these purchases from the Board of Governors, Washington, D.C.

5/ We show that the residual is correlated with the right-hand side independent variables at the end of the appendix.

6/ See Thomson and Pierce [8].

7/ This approach was suggested to me by Neil Wallace.

8/ During quarters excluded from the Wharton and MIT-PENN-SSRC equations, the secondary rate exceeded the primary rate (on two-to-three-month issues) by only 40 basis points on average. Since the mean difference over nonceiling periods is about 30 basis points, this was not much of an experiment. The Q-ceiling period in our data period is defined to be from November 1968 to July 1970. Over this period the mean difference between the secondary and primary rate was over 200 basis points.

9/ The instrument for each right-hand side endogenous variable was obtained by regressing the endogenous variable on current and lagged values of the federal funds rate, the Baa corporate bond rate, and income and lagged values of the Treasury bill rate and the commercial paper rate.

10/ In equations where the secondary rate is the dependent variable, the interest rate coefficients must sum to one to satisfy the zero-sum restriction.

11/ This is the well-known Chow test. See Johnston [4, p.201].

12/ See Heebner [3, p.42].

13/ In fact, in the Q-ceiling period, the correlation coefficient between the primary and secondary rate is less than .30, while in the nonceiling period it is greater than .99.

14/ The idea here is that the amount of CDs maturing in the homeowners' portfolio at least partly represent a long-term commitment to local banks. An increase in maturing issues this month should increase demand for new issues. Because we do not have a breakdown by investor type, we use the total stock of maturing issues as a proxy.

15/ This appendix is based on Parken [6].

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