A Theory of Outsourcing
and Wage Decline[*]

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ABSTRACT _____

We develop a theory of outsourcing in which there is market power in one factor market (labor) and no market power in a second factor market (capital). There are two intermediate goods: one labor-intensive and the other capital-intensive. We show there is always outsourcing in the market allocation when a friction limiting outsourcing is not too big. The key factor underlying the result is that labor demand is more elastic, the greater the labor share. Integrated plants pay higher wages than the specialist producers of labor-intensive intermediates. We derive conditions under which there are multiple equilibria that vary in the degree of outsourcing. Across these equilibria, wages are lower the greater the degree of outsourcing. Wages fall when outsourcing increases in response to a decline in the outsourcing friction.

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1   Introduction

In the 1920s, Henry Ford famously built a factory in which the raw materials for steel went in on one end and finished automobiles went out the other. Extreme vertical integration like this is not the fashion today. Ford Motor has in recent years spun off a significant portion of its parts-making operations as a separate company, and General Motors has done the same. Within its assembly plants, General Motors is currently trying to outsource janitorial services and forklift operations to outside contractors.

This paper develops a theory of outsourcing in which the circumstances under which factors of production can grab rents play the leading role. One factor has some monopoly power (call this *labor*) while a second factor does not (call this *capital*). There are two stages of production: a labor-intensive task and a capital-intensive task. For example, auto part production (such as hand soldering of wire harnesses) tends to be labor-intensive, while final assembly of automobiles (with robots and huge machines) tends to be capital-intensive. In the model, all firms have the same abilities, so there is no motivation to specialize to exploit Ricardian comparative advantage. Furthermore, an *outsourcing friction* is incurred when the two tasks are not integrated in the same firm. So, in the absence of any monopoly power by labor, all firms are completely integrated, doing the labor-intensive and capital-intensive tasks as part of the same operation, and the outsourcing friction is avoided. However, if labor has monopoly power and if the friction is not too large, outsourcing necessarily takes place, with some firms specializing in the labor-intensive task and other firms specializing in the capital-intensive task.

The mechanism underlying our outsourcing result is simply that labor demand is more elastic, the greater the labor share of a firm's overall factor bill. In particular, the demand of an integrated firm is less elastic than the demand of a specialist labor-intensive producer, such as a janitorial services provider. Therefore, a monopolist selling labor to a labor-intensive firm will choose a lower wage than one selling to an integrated plant. This wage difference is the force driving the disintegration of production of the high and low labor-intensity tasks. (But note the wage of a specialist capital-intensive plant is higher than for an integrated plant. This is a subtlety we return to below.)

The main question that we address with the model is: How are changes in outsourcing related to changes in wage payments in general equilibrium? Our unambiguous finding is that increases in outsourcing go hand in hand with decreases in wage payments. Our result allows for two forces underlying an increase in outsourcing. The first force is a decrease over time

in the outsourcing friction. A recent series of papers, including Antràs, Garicano, and Rossi-Hansberg (2006) and Grossman and Rossi-Hansberg (2008), have emphasized the advances in information and transportation technology that have permitted the separation of tasks previously required to be part of the same operation. When components are manufactured by separate operations, information costs must be incurred to ensure that separately produced components fit together both physically and in a timely production schedule (such as in *just-in-time* production). Japanese automobile manufacturers pioneered new ways to coordinate production with suppliers (see Womack, Jones, and Roos (1990) and Mair, Florida, and Kenney (1988)), and these methods have been adopted by U.S. manufacturers. This is an example of a decrease in the outsourcing friction. In recent years, there have been innovations in organizational forms and administrative structures, such as professional employer organizations that facilitate the segmentation of factors of production across different firm boundary lines. We broadly interpret these innovations as decreases in the friction.

The second force potentially underlying an increase in outsourcing is a positive feedback between lower wages and more outsourcing. This makes it possible to observe different outcomes over time, even when technological fundamentals do not change. That is, for certain parameters in our model, there can be multiple equilibria with different levels of outsourcing. When this happens, we show the equilibria can be ordered so that more outsourcing is associated with lower average wages. Put in another way, an increase in outsourcing can depress wages in general equilibrium which, in turn, can change incentives so that firms want to do more outsourcing, making the original increase in outsourcing self-reinforcing. We find that the possibility of multiple equilibria is contingent on an assumption that rent-seeking activity (for union rents) absorbs a portion of the labor supply, putting upward pressure on wages. Our finding that for the same model parameters both an outsourcing and a vertical integration equilibrium might exist is reminiscent of a finding by McLaren (2000). But the mechanism and context are completely different here.

A premise of this paper is that labor has market power while capital does not. There are good reasons to accept this premise. Workers can go on strike, and various job market protections in labor law can enhance labor bargaining power. Perhaps there will come a day when robots go on strike, but for now, business managers need not worry about capital walking off the job. In the formal analysis, we model market power narrowly as taking the form of a union with a monopoly on labor at the plant level. Our idea applies more broadly to include other sources of market power for workers, including job search frictions and potentially even social norms. Finally, while we call factor one labor and factor two

capital, we can just as easily think of factor one as unionized unskilled labor and factor two as nonunion skilled labor.

Whether to integrate or outsource—the "make or buy" decision—is a classic topic in the theory of the firm. Much of the literature has focused on the role of incomplete contracts (Williamson (1979), Grossman and Hart (1986)).[1] In these models, economic agents cannot contractually commit to future behavior. Firm boundaries are drawn to optimally influence incentives, given the constraints of incomplete contracts. This kind of timing and commitment issue arises in our model. We make a crucial assumption that firms make long-run decisions about integration status before they engage in wage negotiations; the anticipation of lower wages on the labor-intensive task is precisely what induces firms to outsource. While our paper follows this literature in that timing and commitment play a key role, the model we consider is very different from those in the existing literature. This is especially true in the way we highlight and incorporate differences in factor composition across tasks Also, the main question we address, How does outsourcing impact wages in general equilibrium? is different from the standard topic in the incomplete contracts literature, which is: When and how does outsourcing occur?

In its general equilibrium approach and its focus on what happens when trade frictions decline, the paper is close in spirit to the trade literature on offshoring (e.g., Grossman and Helpman (2005), Antràs, Garicano, and Rossi-Hansberg (2006)) and assignment theories of foreign direct investment (FDI) about who should specialize in which task (Nocke and Yeaple (2008)).[2] Our paper is particularly related to Grossman and Rossi-Hansberg (2008). That paper highlights how a reduction in an outsourcing friction can potentially raise low-skill wages in a high-skill country through what they call a *productivity effect*. This effect operates through a fixed amount of outsourcing proceeding more efficiently on account of the technical change. This channel operates in our model as well. *Fixing* the extent of outsourcing at some positive level, a reduction in the outsourcing friction raises wages as workers share the benefit of technological improvements. The negative impact on wages from technical change in our model comes entirely from its impact on *increasing* the extent of outsourcing.

The tensions that exist in our model are related to issues that arise in the analysis of price discrimination. With integration, there is a uniform wage for labor. With outsourcing, there

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[1]A more recent literature examines how information flows affect integration decisions (Alonso, Dessein, and Matouschek (2008), Friebel and Raith (2006)).

[2]See also Liao (2009) for the analogous impact in an urban economics model.

is one wage for workers who do the labor-intensive task and a second, higher wage for workers who do the capital-intensive task. The uniform wage under outsourcing lies between the two task-specific, discriminatory wages. In the standard decision-theoretic analysis of monopoly price discrimination, a monopolist is always better off when price discrimination is feasible. However, as shown in Holmes (1989) and Corts (1998), the ability to price discriminate in a competitive context can potentially reduce equilibrium returns to price discriminators. In an analogous way, unions in our model are worse off in equilibrium when workers doing different tasks receive different wages because of outsourcing. As noted parenthetically above, wage levels and total wage payments to workers doing the capital-intensive task actually do increase with outsourcing compared to integration. But we show that these gains are more than offset by losses suffered by workers doing the labor-intensive task.

Existing empirical research provides evidence that wage reductions play a role in motivating outsourcing decisions. The building service industry (i.e., janitorial services) is obviously a labor-intensive industry. Abraham (1990) and Dube and Kaplan (2008) show that building service employees employed in the business service sector (e.g., contract cleaning firms) receive substantially lower wages and benefits than employees doing the same jobs and with similar characteristics employed by manufacturing firms. And Abraham and Taylor (1996) show that it is the higher-wage firms that are more likely to contract out cleaning services. Forbes and Lederman (forthcoming) discuss how airlines spin off short routes to regional airlines because the pilots of these airlines are then less able to extract rents. This fits our model if short routes with small planes are less capital intensive than long-haul routes with large planes. Doellgast and Greer (2007) provide a case study of the German automobile industry to show how outsourcing parts has cut rents. We do not know of such a study for the U.S. automobile industry, but as mentioned in the first paragraph, there is anecdotal evidence that the same has happened in the United States. More specifically, General Motors (GM) spun off Delphi, its labor-intensive parts operations, and subsequently Delphi is trying to cut wages "to as little as $10 an hour from as much as $30."[3] GM spun off the parts operation American Axle in 1992, and subsequently American Axle succeeded in cutting wages by about a third.[4] As part of the plan to outsource janitorial services at GM assembly plants, the expectation is that wages to janitors would fall from $28 an hour for an in-house GM employee to around $12 an hour for an employee of a contract cleaning firm.[5]

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[3]The quote is from the *New York Times*, November 19, 2005, "For a G.M. Family, the American Dream Vanishes," by Danny Hakim.

[4]See "American Axle Contract Ratified, Strike Ends," Reuters, May 22, 2008.

[5]See "GM to UAW: Let's Cut Costs," *Workforce Management: News in Brief*, April 16, 2007.

The remainder of the paper is organized as follows. Section 2 lays out the model, and Section 3 works out what happens when wages are fixed. Section 4 is the main analysis determining the link between outsourcing and wages. Section 5 concludes.

2 Model

We describe the technology and then explain how unions operate. Next we explain timing in the model and define equilibrium.

2.1 The Technology

We model an industry in which a final good is made out of two intermediates. The first is labor-intensive (intermediate 1). The other is capital-intensive (intermediate 2). Let q_i denote a quantity of intermediate i.

A firm may be vertically integrated, producing both intermediates and combining them on its premises. Or it can be specialized in the production of a particular intermediate good. The final good results from combining the two intermediates according to a fixed-proportions technology. If the two inputs q_1 and q_2 are produced in a vertically integrated firm, then final good output equals

$$q_f = f_{VI}(q_1, q_2) = \min\{q_1, q_2\}. \tag{1}$$

If the inputs are produced by two specialists, then final good output is

$$q_f = f_S(q_1, q_2) = (1 - \tau)\min\{q_1, q_2\}.$$

The parameter $\tau \geq 0$ is the *outsourcing friction* that must be incurred when the two steps of production are undertaken by two different firms.

To illustrate the friction τ, suppose that if the two steps are undertaken by separate firms, the intermediates need to be wrapped in certain protective packaging before being shipped off to final assembly. An integrated plant making both intermediates and combining them on premises can avoid the packaging process. In this illustrative case, τ represents the physical cost of wrapping and unwrapping the intermediates as well as the cost of the packing materials. In addition to physical costs like these, as discussed in the introduction we interpret τ broadly as representing costs of coordinating production. Recent advances in

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information and transportation technology have substantially lowered such costs; i.e., τ has decreased.

The three factors of production in the economy are: managers, labor, and capital. There is a unit measure of managers who each own one firm. Given the one-to-one relationship, for simplicity, will refer to the combination of one manager and one firm as a "firm." There is a measure $\bar{L}$ of labor that is supplied inelastically to the economy. There is a perfectly elastic supply of capital to the economy at a rental price of r per unit.

Each firm has access to the same technology for converting capital and labor into intermediates. Let (k_1, l_1, k_2, l_2) denote an input vector for a particular firm, where k_i and l_i denote the amount of capital and labor the firm allocates to the production of intermediate i. Given such an input vector, the production levels for each intermediate equal

$$
\begin{aligned}
q_1 &= y_1 \left(y_1 + y_2\right)^{-(1-\gamma)} \\
q_2 &= y_2 \left(y_1 + y_2\right)^{-(1-\gamma)},
\end{aligned}
\tag{2}
$$

where y_i is the *composite input for intermediate i* defined by

$$
y_i = f_i \left(k_i, l_i\right) = \frac{1}{\alpha_i^{\alpha_i} \left(1 - \alpha_i\right)^{1-\alpha_i}} l_i^{\alpha_i} k_i^{1-\alpha_i}.
\tag{3}
$$

To understand this technology, observe that the k_i and l_i are first combined in a constant-returns Cobb-Douglas fashion through (3) to create a composite input y_i. Then y_1 and y_2 are run through (2) to determine the intermediate production levels q_1 and q_2. This last step allows for diminishing marginal product where the level of curvature is governed by the parameter γ. If $\gamma = 1$, then $q_i = y_i$, and we have constant returns to scale in the relationship between capital and labor and the intermediate outputs. We assume $\gamma \in (0, 1)$, which implies diminishing returns. Note the degree of diminishing returns in (3) is determined by the total composite input $y_1 + y_2$. An increase in production of one intermediate lowers the marginal product of the other intermediate. Finally, we emphasize that in addition to the explicit vector of inputs (k_1, l_1, k_2, l_2) in the technology above, there is also implicitly an input requirement of a unit manager.

The parameter $\alpha_i \in [0, 1]$ is the labor share coefficient for intermediate i. In keeping with the notion that intermediate 1 is labor-intensive and intermediate 2 is capital-intensive, we

assume

$$\alpha_1 > \frac{1}{2},$$
$$\alpha_2 < \frac{1}{2}.$$

To simplify calculations, we assume symmetry,

$$\alpha_1 = 1 - \alpha_2.$$

For some of the analysis, it is convenient to focus on the limiting case where $\alpha_1 = 1$ and $\alpha_2 = 0$, so intermediate 1 uses only labor and intermediate 2 uses only capital. In the limiting case, the Cobb-Douglas composite inputs (3) reduce to

$$y_1 = l_1, \tag{4}$$
$$y_2 = k_2.$$

A firm can choose one of three types of vertical structure. The first possibility is an intermediate 1 specialist. In this case, the firm sets $y_1 > 0$ and $y_2 = 0$, and the production levels from (2) reduce to

$$q_1 = y_1^\gamma \tag{5}$$
$$q_2 = 0.$$

(Again, in addition to the composite input y_1, there is implicitly an input requirement here of a unit manager.[6]) The second possibility is the analogous case of an intermediate 2 specialist. The third possibility is a fully vertically integrated plant. Given the fixed-proportions technology for final goods, an integrated plant sets $y_1 = y_2 = y_{VI}$ and the output of each intermediate q_i—and final good output q_f —equals

$$q_i = q_f = y_{VI}(y_{VI} + y_{VI})^{-(1-\gamma)} = 2^{-(1-\gamma)}y_{VI}^\gamma. \tag{6}$$

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[6]We could be explicit about the management input by including it in the production function with a share coefficient of $1 - \gamma$.

By design, the vertical structure is indeterminate when factor markets are perfectly competitive and the outsourcing friction τ is zero. To illustrate, consider the limiting case where $\alpha_1 = 1$, $\alpha_2 = 0$, and keep things simple by supposing the wage and the rental rate are identical and equal to one, $w = r = 1$. Then given (4), the per unit costs of making the composite inputs y_1 and y_2 both equal one. The symmetry between labor and capital, together with $\tau = 0$, implies that the competitive equilibrium price of each intermediate is half the price of the final good, $p_1 = p_2 = \frac{1}{2}p_f$. Using (5), a specialist producer of intermediate 1 operating at a composite input level y_1 collects a profit of

$$
\begin{aligned}
\pi_1 &= p_1 y_1^\gamma - y_1 \\
&= \frac{1}{2}p_f y_1^\gamma - y_1.
\end{aligned}
$$

Using (6), a vertically integrated plant doing both intermediates at half this level, $y_{VI} = \frac{1}{2}y_1$, collects a profit of

$$
\begin{aligned}
\pi_{VI} &= p_f 2^{-(1-\gamma)} y_{VI}^\gamma - y_{VI} - y_{VI} \\
&= p_f 2^{-(1-\gamma)} \left(\frac{1}{2}y_1\right)^\gamma - \frac{1}{2}y_1 - \frac{1}{2}y_1 \\
&= \frac{1}{2}p_f y_1^\gamma - y_1.
\end{aligned}
$$

Since this is the same as the profit π_1 to being an intermediate 1 specialist, firms are indifferent to the choice of vertical structure. Once we add a positive outsourcing friction $\tau > 0$, obviously firms will strictly prefer to be vertically integrated. But again, we have presumed competitive factor markets. We will see that when we add monopoly power to the labor market, an incentive for outsourcing emerges that can potentially overcome a positive friction τ.

2.2 Unions

Each firm has a union that acts as a monopolist over the supply of labor to the firm. The union at a particular firm buys labor on the open market at a competitive wage w° and resells it to the firm at wage w, pocketing the difference $w - w^\circ$. The firm then makes its input choices, taking w as given. This setup—where the union picks the wage and the firm

picks the employment level—is called the "right-to-manage" model in the labor literature. The union operates at the plant level rather than the economy level. So the union at firm j sets a wage that is specific to firm j.

We assume that the existence of monopoly rents attracts resources, following a long tradition in the economics literature. (See Posner (1975) for an early treatment and Cole and Ohanian (2004) for a recent treatment.) That is, resources get dissipated through rent seeking behavior. Specifically, of the $\bar{L}$ units of labor that are inelastically supplied to the economy, a portion L^R goes toward rent seeking and the remaining portion L° goes to the competitive spot market to work in production,

$$\bar{L} = L^R + L^\circ.$$

We assume there is some matching process between the unit measure of firms and measure L^R of rent seekers that operates in a fashion such that the union rents in the economy are divided equally (in expectation) across rent seekers. As an example matching process, suppose for simplicity there are a finite number of firms and a finite number of rent seekers. Begin by taking one firm and randomly draw a rent seeker from the pool of L^R rent seekers to which to assign the monopoly. Next, take a second firm and again randomly assign it to a draw from the original pool of L^R rent seekers. That is, use a "sampling with replacement" process. With this process, any one rent seeker might end up with zero, one, or two or more monopolies. Given our assumption of risk neutrality, the particulars of the distribution across rent seekers will not matter, only the mean number of monopolies per rent seeker.

In summary, the abstraction employed here captures two main elements. First, there is market power in the provision of the labor factor, but no analogous market power in the provision of capital. In the formal setup, the rents go to the union and the production workers receive the competitive wage. However, it is straightforward to reinterpret the structure so that the people doing the production work are also engaging in rent seeking, so that the actual wage received includes the competitive wage as well as a rent component. Second, rent seeking consumes labor resources. This can be best understood as workers engaging in search behavior to obtain union jobs. Finally, while we treat the rent-seeking assumption as our baseline case, we also determine what happens without the rent-seeking assumption.

2.3 Timing and Equilibrium

Timing is in three stages. In stage 1, each firm commits to its vertical structure choice: intermediate 1 specialist, intermediate 2 specialist, or fully integrated. Let z_k be the share of firms choosing vertical structure k (with $k = 1$ or $k = 2$ for specialists and $k = VI$ for a vertically integrated producer). Also in stage 1, a measure L^R of workers choose to seek the position of labor monopolist available at each firm. In stage 2, the labor monopolist at each particular firm j sets the wage w_j for firm j. In stage 3, inputs are procured and production takes place.

An equilibrium is a list $\left(z_1, z_2, z_{VI}, L^R, R, w_1, w_2, w_{VI}, w^\circ, p_1, p_2, p_f\right)$ that satisfies six conditions. First, each firm's choice of vertical structure $k \in \{1, 2, VI\}$ must be optimal, taking as given how its choice k impacts its wage w_k and taking as given the output prices. Second, the return to directing a unit of labor to the competitive spot market must equal the return to rent seeking, that is,

$$w^\circ = \frac{R}{L^R}, \tag{7}$$

where the right-hand-side term averages the aggregate union rent R over the L^R labor units that seek it. Third, the choice of wage offered by the union of a type k firm must maximize the union's profit, given the anticipated demand behavior of the firm. Fourth, each firm of type k chooses inputs to maximize profits. Fifth, there must be market clearing in the spot market for labor. Sixth, the intermediate prices p_1 and p_2 faced by specialist producers must satisfy the arbitrage condition,

$$p_1 + p_2 = (1 - \tau) p_f, \tag{8}$$

since combining one unit of each of the specialist produced goods yields $1 - \tau$ units of final good.[7]

3 Results for a Fixed Open-Market Wage

We split the analysis of the model into two sections. In this section, we derive an initial set of results on the workings of the model for a fixed open-market wage w°. In the next section, we take the open-market wage as endogenous and determine how it is impacted by

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[7]Since we have assumed the supply of capital is perfectly elastic at r, and since the rental rate is the numeraire, $r = 1$, the final good price p_f in terms of the rental rate is an exogenous parameter.

outsourcing.

3.1 How Union Wages and Production Costs Vary with Vertical Structure

It turns out that it is possible to derive optimal union wages without taking into account equilibrium conditions in the output markets. This subsection derives the optimal union wages and determines how production costs depend upon vertical structure.

We begin with an analysis of an intermediate 1 specialist. Suppose we are at stage 3 and an intermediate 1 specialist faces a wage of w_1. To study the firm's choice problem, it is useful to first determine the cost function of constructing y_1 units of the composite input. Given the constant returns Cobb-Douglas production function (3), it is a standard result that the cost function equals

$$c_1(w_1, y_1) = w_1^{\alpha_1} y_1. \tag{9}$$

(Since we are setting the rental rate on capital as the numeraire, $r = 1$, the term $r^{1-\alpha}$ that would normally appear drops out.) Furthermore, the cost-minimizing labor input choice is

$$l_1(w_1, y_1) = \alpha_1 w_1^{-(1-\alpha_1)} y_1. \tag{10}$$

From (5), a specialist producer with composite input level y_1 produces y_1^{γ} units of intermediate 1. Hence, profit at composite input y_1 equals

$$\pi_1(y_1) = p_1 y_1^{\gamma} - w_1^{\alpha_1} y_1. \tag{11}$$

Setting y_1 to maximize profit yields the optimal choice of composite input y_1

$$y_1 = (p_1 \gamma)^{\frac{1}{1-\gamma}} w_1^{-\frac{\alpha}{1-\gamma}}. \tag{12}$$

Plugging this into (10) yields the firm's labor demand

$$l_1^D(w) = \alpha_1 (p_1 \gamma)^{\frac{1}{1-\gamma}} w_1^{-\varepsilon_1} \tag{13}$$

with constant elasticity

$$\varepsilon_1 \equiv \frac{1 - (1 - \alpha_1)\gamma}{1 - \gamma}. \tag{14}$$

A key point is that elasticity depends on the labor share coefficient α_1. The higher α_1, the more elastic the labor demand.

With firm behavior at stage 3 pinned down, we go backward and examine the problem of the labor monopolist in stage 2 picking w_1. The labor monopolist obtains labor at the open-market rate w° and posts a wage w_1 to maximize the union rent

$$(w_1 - w^\circ)\, l_1^D(w).$$

Note that the monopoly operates at the firm level, not the economy level. If the monopoly were at the economy level, it would take into account the general equilibrium impact of the posted wage. By operating at the firm level, the labor monopolist takes output prices and the open-market wage w° as fixed. Since labor demand is constant elasticity, the rent-maximizing wage satisfies the standard monopoly markup over marginal cost condition

$$w_1 = \frac{1}{1 - \frac{1}{\varepsilon_1}} w^\circ \tag{15}$$

$$= \frac{1 - (1 - \alpha_1)\gamma}{\alpha_1 \gamma} w^\circ. \tag{16}$$

The analysis for an intermediate 2 specialist is the same as above with a change in subscripts. Since $\alpha_2 < \alpha_1$, labor demand for an intermediate 2 firm is less elastic than for an intermediate 1 firm, $\varepsilon_2 < \varepsilon_1$, and the wage is greater, $w_2 > w_1$.

We turn next to the case of a vertically integrated firm. Given the fixed coefficient technology for the final good, an integrated firm produces an equal amount of both intermediate inputs. Let y_{VI} be the composite input level of each intermediate. Given w, the cost of producing both intermediates at this level is

$$c_{VI}(w, y_{VI}) = w^{\alpha_1} y_{VI} + w^{\alpha_2} y_{VI} = (w^{\alpha_1} + w^{\alpha_2})\, y_{VI}. \tag{17}$$

Using the production function of the vertically integrated firm given by (6), we can write profit as

$$\pi_{VI} = p_f 2^{-(1-\gamma)} y_{VI}^{\gamma} - (w^{\alpha_1} + w^{\alpha_2})\, y_{VI}. \tag{18}$$

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From (10), the firm's labor demand added up over both intermediate goods equals

$$l_{VI} = \alpha_1 w^{-(1-\alpha_1)} y_{VI} + \alpha_2 w^{-(1-\alpha_2)} y_{VI}. \tag{19}$$

Maximizing (18) with respect to the choice of y and then plugging y into (19) yields the labor demand function faced by the union,

$$l_{VI}^D(w) = \left(p_f 2^{-(1-\gamma)} \gamma\right)^{\frac{1}{1-\gamma}} \left(\alpha_1 w^{-(1-\alpha_1)} + \alpha_2 w^{-(1-\alpha_2)}\right) \left(w^{\alpha_1} + w^{\alpha_2}\right)^{-\frac{1}{1-\gamma}}. \tag{20}$$

The union picks w to maximize

$$\left(w - w^\circ\right) l_{VI}^D(w).$$

Let w_{VI} solve this problem. In general, no analytic solution exists. However, we can obtain an expression for the limiting case where α_1 goes to 1 and α_2 goes to zero,

$$\lim_{\alpha_1 \to 1} w_{VI} = \frac{1 - \gamma + w^\circ}{\gamma}.$$

By way of comparison, the wages set to specialist firms from (15) at this limit are

$$\lim_{\alpha_1 \to 1} w_1 = \frac{w^\circ}{\gamma} \tag{21}$$
$$\lim_{\alpha_1 \to 1} w_2 = \infty.$$

This follows because the elasticity for a specialist producing intermediate 1 goes to $\frac{1}{1-\gamma}$ while it goes to 1 for intermediate 2.

A key result that plays an important role in the subsequent analysis is that the total cost of constructing a composite input for each intermediate is less under specialization than under vertical integration. Let $c_{i,VI}$ and $c_{i,S}$ be the cost of producing one unit of composite input i in the vertically integrated and specialist cases. Note that if $\alpha_1 = \alpha_2 = \frac{1}{2}$, the two intermediates are the same in elasticity, and there is no difference between the specialization structure and vertical integration in terms of union wage setting behavior; i.e., $c_{i,VI} = c_{i,S}$. But when $\alpha_1 > \frac{1}{2}$, there is a difference. We begin with an analytic result at the limit where α_1 goes to one. Let $c_{VI} = c_{1,VI} + c_{2,VI}$ be the combined cost to the vertically integrated firm of one unit of each composite input.

Lemma 1.

$$\lim_{\alpha_1 \to 1} [c_{1,S} + c_{2,S}] < \lim_{\alpha_1 \to 1} c_{VI}$$

Proof. In the appendix we show that

$$
\begin{aligned}
\lim_{\alpha_1 \to 1} c_{1,S} &= \frac{w^\circ}{\gamma} \\
\lim_{\alpha_1 \to 1} c_{2,S} &= 1 \\
\lim_{\alpha_1 \to 1} c_{VI} &= \frac{1 + w^\circ}{\gamma},
\end{aligned}
$$

which immediately implies the result. ■

One thing to note about the limit for an intermediate 2 specialist is that even though the wage is going to infinity, the spending share on labor is going to zero. In the limit, the intermediate 2 specialist needs only to worry about expenditures on capital. At the limit, it requires one unit of capital to make one composite input. Since $r = 1$, it follows that $\lim_{\alpha_1 \to 1} c_{2,S} = 1$.

Understanding the limit where intermediate 1 is pure labor and intermediate 2 pure capital is easy. The demand for labor by a vertically integrated firm, for which labor represents only part of the factor bill, is obviously less elastic than for an intermediate 1 specialist, for which labor is the only input. So the union sets a higher wage to an integrated plant. By outsourcing the labor task, there is a clear savings in the wage bill, while the task using capital pays the market rate for capital either way.

Things are more subtle in the general case, $\alpha_1 < 1$, $\alpha_2 > 0$, where both intermediates use both factors. Here, there is a trade-off. By splitting up production, an intermediate 1 specialist pays a lower wage compared to an integrated plant. But an intermediate 2 specialist pays a higher wage. Intuitively, since the former uses labor intensively while the latter does not, the cost savings on the former should outweigh the cost increases on the latter. We can show with numerical methods that this is true.[8] By scanning over a fine grid of $\alpha_1 \in (\frac{1}{2}, 1)$, $\gamma \in (0, 1)$, and w° (and $\alpha_2 = 1 - \alpha_1$), we find that

$$[c_{1,S} + c_{2,S}] < c_{VI}. \tag{22}$$

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[8]We verify this claim over a fine grid across the ranges $\alpha_1 \in [.51, .99]$, $\gamma \in [.03, .99]$, and $w^\circ \in [.01, 500]$. We stay away from extreme values to avoid numerical inaccuracies.

3.2   Choice of Vertical Structure

In this subsection we examine a firm's choice of vertical structure. For a fixed open-market wage $w°$, we determine how the vertical structure choice depends on the outsourcing friction τ.

Proposition 1. For any open-market wage $w°$, there exists a unique $\hat{\tau}(w°) > 0$, such that if $\tau < \hat{\tau}(w°)$, all firms are specialized while if $\tau > \hat{\tau}(w°)$, all firms are vertically integrated.

Proof. Given an output price p_i and a unit composite input cost $c_{i,S}$ of an intermediate i specialist, it is straightforward to derive its maximized profit,

$$\pi_i = \left[\gamma^{\frac{\gamma}{1-\gamma}} - \gamma^{\frac{1}{1-\gamma}} \right] p_i^{\frac{1}{1-\gamma}} (c_{i,S})^{-\frac{\gamma}{1-\gamma}}. \tag{23}$$

Analogously, using (18), the profit of a vertically integrated union firm equals

$$\pi_{VI} = \frac{1}{2} \left[\gamma^{\frac{\gamma}{1-\gamma}} - \gamma^{\frac{1}{1-\gamma}} \right] p_f^{\frac{1}{1-\gamma}} (c_{VI})^{-\frac{\gamma}{1-\gamma}}. \tag{24}$$

If there is an equilibrium with specialization, both intermediate goods must be produced, and so firms must be indifferent between producing both, i.e., $\pi_1 = \pi_2$. By arbitrage, we also have $p_1 + p_2 = (1 - \tau) p_f$. Using these two equations, we can solve out for p_1 as

$$p_1 = \frac{(c_{2,S})^{-\gamma}}{(c_{1,S})^{-\gamma} + (c_{2,S})^{-\gamma}} (1 - \tau) p_f. \tag{25}$$

Plugging this into (23) and using straightforward algebra, a firm prefers specialization to integration if and only if

$$\frac{(c_{2,S})^{-\gamma} (c_{1,S})^{-\gamma}}{(c_{1,S})^{-\gamma} + (c_{2,S})^{-\gamma}} (1 - \tau) > \left(\frac{1}{2} \right)^{1-\gamma} (c_{VI})^{-\gamma}. \tag{26}$$

Recall from the previous section that $c_{1,S} + c_{2,S} < c_{VI}$. Lemma A1 in the appendix shows that when $c_{1,S} + c_{2,S} < c_{VI}$ holds, (26) holds at $\tau = 0$. Define $\hat{\tau}$ to be the unique τ where (26) holds with equality. ■

That outsourcing does not take place when τ is large is obvious. What is interesting about Proposition 1 is that outsourcing necessarily *does* take place when the friction τ is small but still positive. Recall that, in this environment, vertical structure would be indeterminate

if both factor markets were competitive and if the friction τ were zero. If factor markets were competitive and there were any positive friction, integration would necessarily prevail, as there is no technological benefit from specialization. But with monopoly in the labor market, there is an incentive to vertically disintegrate to limit rent extraction by the unions. So specialization prevails if the outsourcing friction is small enough.

The shape of the relationship between the cutoff $\hat{\tau}$ and the open-market wage w° plays a role in the subsequent analysis. We call this the "$\hat{\tau}$ curve" and we plot it in Figure 1 for several different values of α_1, $\alpha_1 \in \{0.6, 0.8, 1\}$, fixing $\gamma = 0.5$.[9] We put the wage w° on the vertical axis and $\hat{\tau}$ on the horizontal, because this will be convenient later when τ is exogenous and w° is endogenous. As illustrated, for a given value of α_1, there exists a $w^\circ_{\min}$ such that $\hat{\tau}(w^\circ)$ strictly decreases in w° for $w^\circ < w^\circ_{\min}$, and strictly increases in w° for $w^\circ > w^\circ_{\min}$. We can prove analytically that this property holds at $\alpha_1 = 1$, and we verify numerically that the property holds for general α_1.[10] Note that in the opposite limiting case of $\alpha_1 = \alpha_2 = \frac{1}{2}$, vertical structure has no impact on wages, since both intermediates have the same labor intensity. In this extreme case, $\hat{\tau}(w^\circ) = 0$ for all w°. So the case of high α_1 is the interesting case.

4 Outsourcing and Wage Decline

We now solve for equilibrium in the labor market and determine how both the open-market wage w° and the level of union rents vary with the outsourcing friction τ.

It is convenient to split up the analysis. First, we impose an artificial restriction that vertical integration is infeasible so all firms must be specialists and determine what the equilibrium would look like in this case. Next, we flip the restriction so only integration is feasible. In the end, we allow both to be feasible and determine what happens.

4.1 Only Specialization

This subsection determines equilibrium when integration is not an option. We solve for how the demand for labor depends upon a given w° and then solve for market clearing.

Given w° and the associated union markups over w° (see (15)), let $\tilde{p}_1(w^\circ)$ and $\tilde{p}_2(w^\circ)$

---

[9]We fix $\gamma = .5$ to construct all the figures. Also, the wage on the vertical axis is in log scale.
[10]The analytic proof for the $\alpha_1 = 1$ case is available in separate notes. Our numerical result for $\alpha_1 < 1$ uses the same grid as in footnote 8.

be the prices that equate the profit of an intermediate 1 specialist with an intermediate 2 specialist (see (25)) and that solve the arbitrage condition $p_1 + p_2 = (1 - \tau) p_f$. Let $\tilde{l}_i(w^\circ)$ be the derived demand for labor for specialist i taking into account the impact of w° on price. That is, using (13),

$$\tilde{l}_i(w^\circ) = \alpha_i \left(\tilde{p}_i(w^\circ)\gamma\right)^{\frac{1}{1-\gamma}} w_1(w^\circ)^{-\varepsilon_i}.$$

Analogously, let $\tilde{q}_i(w^\circ)$ be the derived supply of intermediate i by a specialist i. Let $\tilde{z}_i(w^\circ)$ be the share of firms producing intermediate i consistent with market clearing of intermediates, i.e.,

$$\tilde{z}_1(w^\circ)\tilde{q}_1(w^\circ) = \tilde{z}_2(w^\circ)\tilde{q}_2(w^\circ),$$

since the left and right sides are the total productions of intermediates 1 and 2 and these must be equalized because of fixed coefficients. Using $z_1 = 1 - z_2$, we have

$$\tilde{z}_1(w^\circ) = \frac{\tilde{q}_2(w^\circ)}{\tilde{q}_1(w^\circ) + \tilde{q}_2(w^\circ)}.$$

Putting all of this together, the total demand in the economy for production labor at an open-market wage w° across the two kinds of intermediates equals

$$\tilde{L}_S^\circ(w^\circ) = \sum_{i=1}^{2} \tilde{z}_i(w^\circ)\tilde{l}_i(w^\circ),$$

where the subscript S signifies we are in the *Only Specialization* case.

We turn now to the labor allocated to rent seeking. When the open-market wage is w°, total union rents are

$$\tilde{R}_S(w^\circ) = \sum_{i=1}^{2} \left[\tilde{w}_i(w^\circ) - w^\circ\right] \tilde{z}_i(w^\circ)\tilde{l}_i(w^\circ).$$

Plugging this into the equilibrium condition (7) that equalizes the return to rent seeking and w°, we can solve for the demand for rent-seeking labor,

$$\tilde{L}_S^R(w^\circ) = \frac{\tilde{R}_S(w^\circ)}{w^\circ}.$$

Adding this to the demand for production labor yields the overall demand for labor in the economy,

$$\tilde{L}_S^D(w^\circ, \tau) = \tilde{L}_S^\circ(w^\circ, \tau) + \tilde{L}_S^R(w^\circ, \tau), \tag{27}$$

17

where we note the explicit dependence of demand on the outsourcing friction, as this is useful at this point. Market clearing requires that demand $\tilde{L}_S^D(w^\circ, \tau)$ equal the exogenous supply $\bar{L}$. We obtain the following result for the limiting case.

Lemma 2. Assume the limiting case of $\alpha_1 = 1$ and $\alpha_2 = 0$. (i) Overall demand $\tilde{L}_S^D(w^\circ, \tau)$ strictly decreases in w° and τ. (ii) For a given τ, there exists a unique $w_S^{\circ*}(\tau)$ that clears the open market, $\tilde{L}_S^D(w_S^{\circ*}(\tau), \tau) \equiv \bar{L}$, and $w_S^{\circ*}(\tau)$ is strictly decreasing in τ. (iii) A single-crossing property holds between the $w_S^{\circ*}$ curve and the $\hat{\tau}$ curve defined earlier. Formally, for any τ' where $\tau' > \hat{\tau}(w_S^{\circ*}(\tau'))$, then $\tau > \hat{\tau}(w_S^{\circ*}(\tau))$ for all $\tau > \tau'$.

Proof. See appendix.∎

We use numerical methods to verify that the three properties listed hold for general α_1.[11]

The relationship $w_S^{\circ*}(\tau)$ is what the equilibrium open-market wage would be if specialization were the only feasible vertical structure. We call this the "$w_S^{\circ*}$ curve" and illustrate it in Figure 2 for two cases: one where labor supply $\bar{L}$ is small, so $w_S^{\circ*}$ is high, and the other where $\bar{L}$ is large, so $w_S^{\circ*}$ is low. The figure illustrates that $w_S^{\circ*}$ strictly decreases in τ (part (ii) of the lemma). Moreover, it crosses the $\hat{\tau}$ curve only once (part (iii)). Let τ_S^+ be defined as the τ where the intersection occurs,

$$\bar{L} \equiv \tilde{L}_S^D(\hat{\tau}^{-1}(\tau_S^+), \tau_S^+). \tag{28}$$

4.2 Only Integration

Now consider the case where only vertical integration is feasible. Analogous to (27) above, we can construct labor demand $\tilde{L}_{VI}^D(w^\circ, \tau)$ and equate this to supply $\bar{L}$ to obtain the equilibrium open-market wage $w_{VI}^{\circ*}(\tau)$. We can show the analog of Lemma 2 holds for this case, with one difference.[12] In the integration case, the outsourcing friction τ is irrelevant, so $w_{VI}^{\circ*}(\tau)$ and $\tilde{L}_{VI}^D(w^\circ, \tau)$ are constant as functions of τ. (This makes the single crossing property trivial in the integration case.) Figure 3 illustrates two example $w_{VI}^{\circ*}$ curves: one with $\bar{L}$ small and the other with $\bar{L}$ large. Analogous to (28), define τ_{VI}^+ to be the point where $w_{VI}^{\circ*}$ intersects $\hat{\tau}$.

---

[11] We verify (i) holds over a grid across the ranges $\alpha_1 \in [.55, .9]$, $\gamma = [.1, .9]$, $p_f \in [.5, 10]$, $w^\circ \in [.2, 100]$ and $\tau \in [0, .95]$. For (ii) and (iii), we consider $\bar{L} \in [.1, .5]$ and narrow the range of τ to $\tau \in [0, .6]$ to maintain numerical accuracy.

[12] Beyond analytic results for the $\alpha_1 = 1$ case, we also confirm that the analog of Lemma 2 holds for general $\alpha_1 \in \left(\frac{1}{2}, 1\right)$. Parts (ii) and (iii) are analytically straightforward. We verify part (i) (that $\tilde{L}_{VI}^D(w^\circ, \tau)$ is strictly decreasing in w°) numerically using the same grid as in footnote 11.

4.3 A Key Step

The key step underlying our main result is what happens to labor demand at the $\hat{\tau}$ boundary of indifference between specialization and integration. Our result is

Lemma 3. Take a point $(\tau', w^{\circ\prime})$ on the $\hat{\tau}$ curve so that $\tau' = \hat{\tau}(w^{\circ\prime})$. Assume the limiting case, $\alpha_1 = 1$, $\alpha_2 = 0$. Then $\tilde{L}_S^D(w^{\circ\prime}, \tau') < \tilde{L}_{VI}^D(w^{\circ\prime}, \tau')$. That is, labor demand in the only-specialization case is less than in the only-integration case.

Proof. See appendix.∎

As before, we use numerical analysis to verify that this result holds for general α_1.[13]

To shed some light on the mechanics of the result, we note that if we only look at the demand for production labor, there is an ambiguous relationship between the two cases, i.e., $\tilde{L}_S^{\circ}(w^{\circ\prime}, \tau')$ could be bigger or smaller than $\tilde{L}_{VI}^{\circ}(w^{\circ\prime}, \tau')$. But for rent-seeking labor, there is an unambiguous comparison. Rents are strictly lower in the S case compared to VI, so fewer workers are attracted to rent seeking in the S case. This effect is large enough to more than offset any countervailing effect on the demand for production labor.

4.4 Putting It All Together

We now put this all together and completely characterize how the equilibrium in the economy depends upon the degree of outsourcing.

We start by defining a critical level for the exogenous labor supply,

$$\bar{L}_{\min} \equiv \tilde{L}_S^D(w_{\min}^{\circ}, \hat{\tau}_{\min}),$$

where as defined earlier $w_{\min}^{\circ}$ minimizes $\hat{\tau}(w^{\circ})$ and $\hat{\tau}_{\min} = \hat{\tau}(w_{\min}^{\circ})$. When $\bar{L} < \bar{L}_{\min}$, the $w_S^{\circ*}$ curve intersects $\hat{\tau}$ on the upward-sloping portion of $\hat{\tau}$ (as in the small $\bar{L}$ case in Figure 2). When $\bar{L} > \bar{L}_{\min}$, $w_S^{\circ*}$ intersects $\hat{\tau}$ on the downward-sloping portion (as in the large $\bar{L}$ case).

We next provide our formal characterization. On a first reading, it might be helpful to skip the formal statement and jump to the discussion below about the graphical illustration of the result.

Proposition 2. Assume the limiting case $\alpha_1 = 1$, $\alpha_2 = 0$.

Case 1: Suppose $\bar{L} \leq \bar{L}_{\min}$. Define $\underline{\tau} = \tau_S^+$ and $\bar{\tau} = \tau_{VI}^+$. Then $\underline{\tau} < \bar{\tau}$. There is a unique equilibrium that depends upon τ in the following way:

---

[13]We use the same grid as in footnote 11.

Subcase 1(i) For $\tau < \underline{\tau}$, no firms are integrated, i.e., $z_{VI} = 0$, and the equilibrium open-market wage is

$$w^{\circ*}(\tau) = w_S^{\circ*}(\tau).$$

Subcase 1(ii) For $\tau > \bar{\tau}$, all firms are vertically integrated, $z_{VI} = 1$, and the equilibrium open-market wage is

$$w^{\circ*}(\tau) = w_{VI}^{\circ*}(\tau).$$

Subcase 1(iii) For $\tau \in (\underline{\tau}, \bar{\tau})$, there is *partial* integration, $0 < z_{VI} < 1$. The wage is determined from the $\hat{\tau}$ curve,

$$w^{\circ*}(\tau) = \hat{\tau}^{-1}(\tau).$$

The share of firms that are vertically integrated is the unique z_{VI} satisfying

$$z_{VI}\tilde{L}_{VI}^D(\hat{\tau}^{-1}(\tau), \tau) + (1 - z_{VI})\tilde{L}_S^D(\hat{\tau}^{-1}(\tau), \tau) = \bar{L}$$

and z_{VI} strictly increases in τ. Also, the equilibrium wage $w^{\circ*}(\tau)$ strictly increases in τ, so more outsourcing co-moves with lower wages.

Case 2: Suppose $\bar{L} > \bar{L}_{\min}$. If $w_{VI}^{\circ*} \leq w_{\min}^{\circ}$ (meaning τ_{VI}^+ is on the downward-sloping portion of the $\hat{\tau}$ curve as illustrated in the large-$\bar{L}$ case in Figure 4), then define $\underline{\tau} \equiv \tau_{VI}^+$ and $\bar{\tau} \equiv \tau_S^+$. If $w_{VI}^{\circ*} > w_{\min}^{\circ}$, then define $\underline{\tau} \equiv \hat{\tau}_{\min}$ and $\bar{\tau} \equiv \tau_S^+$. Note $\underline{\tau} < \bar{\tau}$.

For $\tau < \underline{\tau}$ and $\tau > \max\left\{\bar{\tau}, \tau_{VI}^+\right\}$, there is a unique equilibrium that follows the pattern of Subcases 1(i) and 1(ii) above. If $\bar{\tau} < \tau_{VI}^+$ and $\tau \in (\bar{\tau}, \tau_{VI}^+)$, there is a unique equilibrium with partial integration following Subcase 1(iii) above.

For $\tau \in (\underline{\tau}, \bar{\tau})$, a nonempty open interval, there are three equilibria labeled a, b, and c. Equilibrium a has full specialization, $z_{VI}^a = 0$. Equilibria b has partial integration, $z_{VI}^b \in (0, 1)$. Equilibrium c has partial or full integration, $z_{VI}^c \in (0, 1]$, and $z_{VI}^c > z_{VI}^b$. Across the three equilibria, there is a strict ordering that less integration is associated with a lower open-market wage, $w^{\circ a} < w^{\circ b} < w^{\circ c}$.

Proof. See appendix.∎

The result is best understood with a graph. Figure 4 combines Figure 2 (keeping the $w_S^{\circ*}$ to the left of $\hat{\tau}$ because specialization is preferred there) with Figure 3 (keeping $w_{VI}^{\circ*}$ to the right of $\hat{\tau}$ because integration is preferred there). Consider first the small-$\bar{L}$ case on the top portion of Figure 4. The set of (τ, w°) pairs that are equilibria are highlighted by the solid dark line. Notice point A where the $w_S^{\circ*}$ curve intersects the $\hat{\tau}$ curve. There is an equilibrium

here where all firms are specialized because (1) being on the $\hat{\tau}$ curve means firms are just as happy to be specialized as integrated and (2) being on the $w_S^{\circ*}$ curve means supply equals demand in the labor market. For τ to the left of point A, complete specialization is the only possibility, and we follow along the $w_S^{\circ*}$ curve to pin down the wage. Notice that as τ is decreased in this range, wages rise. This is an example of the *productivity effect* highlighted in Grossman and Rossi-Hansberg (2008). Over this region, all activity is outsourced, and the impact of reducing the friction τ is that this *given* level of outsourcing takes place more efficiently. Some of the gain is passed along in terms of higher wages.

Next, notice point B where $w_S^{\circ*}$ intersects the $\hat{\tau}$ curve. Observe point B is at a higher wage than A. This necessarily follows from Lemma 3. At point A, if firms were to switch over to vertical integration from specialization (which they are indifferent to doing), total labor demand would increase. To get market clearing, the wage needs to increase, and that is what happens at point B. For τ in between points A and B, the wage is set on the $\hat{\tau}$ curve to make firms indifferent to vertical structure. A fraction z_{VI} are integrated, and this share is chosen to clear the labor market. The key thing to note is that as we move from B to A and lower the outsourcing friction, the extent of outsourcing increases at the same time that the wage falls. This is the main result of the paper.

Next, consider the large $\bar{L}$ case illustrated at the bottom of Figure 4 where the action is on the downward-sloping portion of the $\hat{\tau}$ curve. Again, the equilibria are highlighted by a solid dark line. Consider what happens for τ between points C and D. For any such τ, we see there are three equilibria: (1) a pure specialization equilibrium ($z_{VI} = 0$) on the $w_S^{\circ*}$ curve with a low wage, (2) a pure integration equilibrium ($z_{VI} = 1$) on the $w_{VI}^{\circ*}$ curve with a high wage, and (3) a partial integration equilibrium with an intermediate wage. Again, an increase in outsourcing goes together with a decline in wages. But this time we are moving across equilibria with no change in economic fundamentals (i.e., τ is fixed).

We have been focusing on the open-market wage, but we now turn to the average wage paid by firms. This is the wage bill added up across all firms divided by the number of production workers. It equals

$$
\begin{aligned}
w^{mean} &= \frac{z_1 l_1 w_1 + z_2 l_2 w_2 + z_{VI} l_{VI} w_{VI}}{z_1 l_1 + z_2 l_2 + z_{VI} l_{VI}} \\
&= \frac{w^\circ L^\circ + R}{L^\circ} = w^\circ + \frac{R}{L^\circ},
\end{aligned}
$$

which is the open-market wage plus the average union markup. Above we show that the

open-market wage declines with outsourcing. Our next result shows that the mean firm wage also declines with outsourcing.

Proposition 3. Suppose $\alpha_1 = 1$, $\alpha_2 = 0$.

(i) Assume Case 1 where $L \leq \bar{L}_{\min}$ so there is a unique equilibrium. In the region of partial integration, $\tau \in (\underline{\tau}, \bar{\tau})$, the mean firm wage is lower the lower is τ.

(ii) Assume Case 2 where $L > \bar{L}_{\min}$ so there is a range of $\tau \in (\underline{\tau}, \bar{\tau})$ with multiple equilibria. Across the multiple equilibria, less integration is associated with a lower mean firm wage.

Sketch of Proof. Calculations in the appendix show that R/L° decreases with outsourcing. Since w° goes down, both effects work together to lower the mean firm wage. ∎

4.5 The Role of Rent Seeking

The above analysis assumes that rent seeking absorbs part of the labor supply according to the equilibrium condition (7). In this subsection, we consider an alternative formulation where the rents are distributed across workers, without absorbing labor resources.

It is straightforward to modify the equilibrium conditions for this alternative formulation. In particular, we need to change the definition of labor demand (27) to only include production labor $\tilde{L}_S^\circ(w^\circ, \tau)$ (and not any rent-seeking labor). Analogous to the above, define $\bar{L}_{\min}^{alternative}$ as the exogenous labor supply at which the $w_S^{\circ*}$ curve intersects the minimum point on the $\hat{\tau}$ curve,

$$\bar{L}_{\min}^{alternative} \equiv \tilde{L}_S^\circ(w_{\min}^\circ, \hat{\tau}_{\min}),$$

i.e., at $\bar{L}_{\min}^{alternative}$ and $\hat{\tau}_{\min}$, the labor market clears at $w_{\min}^\circ$ with only specialization. Our result is

Proposition 4. Consider the limiting case $\alpha_1 = 1$, $\alpha_2 = 0$ in the alternative model where rent seeking does not absorb resources. There is a unique equilibrium for all parameters. Define $\underline{\tau} \equiv \tau_S^+$ and $\bar{\tau} \equiv \tau_{VI}^+$. For $\bar{L} \neq \bar{L}_{\min}^{alternative}$, $\underline{\tau} < \bar{\tau}$. There are two cases:

(i) For Case 1, $\bar{L} \leq \bar{L}_{\min}^{alternative}$, the characterization is the same as Case 1 of Proposition 2. In particular, for τ over the range $(\underline{\tau}, \bar{\tau})$, the vertically integrated share $z_{VI}^*(\tau)$, the open-market wage $w^{\circ*}(\tau)$, and the mean firm wage $w^{mean}(\tau)$ all strictly increase in τ.

(ii) For Case 2, $\bar{L} > \bar{L}_{\min}^{alternative}$, for $\tau < \underline{\tau}$ and $\tau > \bar{\tau}$, the characterization is the same as in Case 1. For $\tau \in (\underline{\tau}, \bar{\tau})$, the integrated share $z_{VI}^*(\tau)$ increases in τ, but the open-market wage $w^{\circ*}(\tau)$ actually decreases with τ. The mean-firm wage $w^{mean*}(\tau)$ increases in τ over

this range.

Proof. The appendix sketches the proof. We emphasize that the final claim that $w^{mean*}(\tau)$ increases in τ in Case 2 and $\tau \in (\underline{\tau}, \bar{\tau})$ is based on numerical analysis. The calculation involves a nonlinear equation that is difficult to characterize analytically.∎

When $\bar{L}$ is low so that the $w_S^{\circ*}$ intersects $\hat{\tau}$ on the upward-sloping portion, the analysis in the alternative model is identical to the baseline. But things are different when the intersection occurs on the downward-sloping portion, as illustrated in Figure 5. Here, the result in Lemma 3 flips sign, so that the $w_{VI}^{\circ*}$ curve actually cuts the $\hat{\tau}$ curve below where it intersects $w_S^{\circ*}$, as illustrated. Readily apparent from the figure is that the equilibrium is unique for each τ; the region of multiplicity in Figure 4 is gone. Note the equilibrium open-market wage $w^{\circ*}$ actually decreases in τ from point E to point F, so in this way the analysis is different from the baseline. But the mean wage w^{mean} paid by firms is the relevant wage measure to be looking at. This increases with τ from point E to point F, analogous to Proposition 3. So our main result from the baseline model that increases in outsourcing go together with declines in wages is preserved in the alternative model.

It is interesting that there are no multiple equilibria in the alternative model. To get a sense of what is different here, observe first that the emergence of outsourcing depresses labor rents. In the baseline model, the reduction in rents attracts less labor to outsourcing, leaving more labor for production and further depressing wages. It is possible for these lower wages to spur additional outsourcing, creating a positive feedback loop and reinforcing the original outsourcing at the start of the story. When the rent-seeking element is taken out, a piece of the loop is gone, and the equilibrium is now unique.

4.6   General Input Shares

Our formal results assume the limiting case $\alpha_1 = 1$, $\alpha_2 = 0$. We illustrate what happens for general $\alpha_1 < 1$, $\alpha_2 = 1 - \alpha_1$ with numerical analysis of Case 1 (the small-$\bar{L}$ case with a unique equilibrium). Fixing all of the parameters other than τ, we determine τ_{VI}^+ (the lowest τ with complete vertical integration at point B in Figure 4) and τ_S^+ (the highest τ with complete specialization at point A). We decrease the friction from τ_{VI}^+ to τ_S^+ and determine how equilibrium variables change as the economy moves from complete integration to complete specialization. We report the results in Table 1 for both the baseline case with rent seeking

and the non-rent-seeking case.[14]

Panel A reports the limiting case of $\alpha_1 = 1$ that is covered in Propositions 3 and 4. We know from these results that w° and w^{mean} both decrease as τ decreases from τ_{VI}^+ to τ_S^+, and we can see this in the table. Note that wages are significantly higher in the rent-seeking case because rent seeking absorbs labor driving up wages. We also report the total wage bill, total firm profits, and total surplus (the sum of firm profits plus the wage bill). In terms of effects on total surplus, there are three considerations. First, the switch from integration to outsourcing weakens the labor monopoly, which reduces wasteful rent-seeking behavior. Second, firms' decisions are less distorted. Offsetting these positive impacts is a third consideration that outsourcing entails a friction (approximately 13 percent of output in Panel A) that is avoided with integration. In the rent-seeking case, the net effect of outsourcing on surplus is strictly positive.[15] With no rent seeking, the first consideration is eliminated, and the net effect on total surplus is negative; the cost of the friction outweighs the benefits of reduced distortions. Despite the friction, firms outsource because profits are higher.

The remaining panels in Table 1 report the results for values of α_1 below one. The *qualitative* results are identical to what we get for the $\alpha_1 = 1$ case in Panel A. *Quantitatively,* as α_1 is decreased toward its lower bound of $\frac{1}{2}$, the effects become smaller and go to zero. At this lower bound, there is no difference in factor intensity between the two intermediates and the incentive for outsourcing disappears ($\hat{\tau}$ goes to zero). One interesting thing in these remaining panels is what happens to the wage bill for intermediate 2, the capital-intensive task. This is positive in the remaining panels, since $\alpha_2 > 0$. Note that this wage bill strictly increases with outsourcing throughout all the cases. So from labor's perspective, the process of vertical disintegration has offsetting effects: the wage bill for intermediate 2 goes up, the wage bill for intermediate 1 goes down. The key finding of the paper is that the combined impact is negative.

5 Concluding Remarks

We have developed a new general equilibrium model of outsourcing and have put it to work analyzing the connection between outsourcing and wages. We believe this structure

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[14]Throughout the table, $\gamma = .5$ and $\bar{L} = .16$ are held fixed. We have rescaled the total wage bill, profits, and total surplus by multiplying through by 100.

[15]The net positive impact is negligible for $\alpha_1 = 1$ but is significant in the other cases.

is potentially useful for examining other issues related to outsourcing. For example, in our analysis, firms are identical in the extent to which they face a union monopoly. A natural extension is to allow firms to differ in this dimension. We have done some preliminary analysis of what happens when some firms are union and others nonunion and can show that if nonunion firms ever specialize, they do the labor-intensive task. We could use the structure to look at how changes in the extent of unionism impact the degree of outsourcing and also how outsourcing feeds back and impacts the incentive to organize unions. As another application, we can put this structure in an international context, and study how exposure to international trade impacts the incentive for *domestic* outsourcing. We expect interesting effects to emerge here because wages impact the incentive for domestic outsourcing (Figure 1) and trade impacts wages.

Appendix

A.1 Background Calculations for Section 3

We begin by deriving the integrated firm's labor demand. The cost function can be written as

$$
\begin{aligned}
c_{VI}(w,y) &= w^{\alpha_1}y + w^{\alpha_2}y = (w^{\alpha_1} + w^{\alpha_2})\, y && (29) \\
&= c_{VI}y.
\end{aligned}
$$

Now if the firm produces y of each input, it obtains final good production of

$$
\begin{aligned}
q_f &= y\,(2y)^{-(1-\gamma)} && (30) \\
&= 2^{-(1-\gamma)}y^{\gamma}. && (31)
\end{aligned}
$$

So the firm's problem is

$$
\max p_f 2^{-(1-\gamma)}y^{\gamma} - c_{VI}y.
$$

The FONC is

$$
p_f 2^{-(1-\gamma)}\gamma y^{\gamma-1} - c_{VI} = 0.
$$

Or

$$
y = \left(p_f 2^{-(1-\gamma)}\gamma\right)^{\frac{1}{1-\gamma}} c_{VI}^{-\frac{1}{1-\gamma}}. \tag{32}
$$

We can plug this into the firm's labor demand to obtain the demand faced by the labor union:

$$
\begin{aligned}
l_{VI}^{D}(w) &= \alpha_1 w^{-(1-\alpha_1)}y + \alpha_2 w^{-(1-\alpha_2)}y \\
&= \left(\alpha_1 w^{-(1-\alpha_1)} + \alpha_2 w^{-(1-\alpha_2)}\right)\left(p_f 2^{-(1-\gamma)}\gamma\right)^{\frac{1}{1-\gamma}} c_{VI}^{-\frac{1}{1-\gamma}} \\
&= \left(p_f 2^{-(1-\gamma)}\gamma\right)^{\frac{1}{1-\gamma}}\left(\alpha_1 w^{-(1-\alpha_1)} + \alpha_2 w^{-(1-\alpha_2)}\right)\left(w^{\alpha_1} + w^{\alpha_2}\right)^{-\frac{1}{1-\gamma}} \\
&= \xi\left(\alpha_1 w^{-(1-\alpha_1)} + \alpha_2 w^{-(1-\alpha_2)}\right)\left(w^{\alpha_1} + w^{\alpha_2}\right)^{-\frac{1}{1-\gamma}}.
\end{aligned}
$$

We use (29) to substitute in for c_{VI} in the third line. The term ξ in the fourth line is a multiplicative constant independent of w.

Thus, the labor union facing a vertically integrated firm sets w to maximize:

$$(w - w^\circ)\xi \left(\alpha_1 w^{-(1-\alpha_1)} + \alpha_2 w^{-(1-\alpha_2)}\right) \left(w^{\alpha_1} + w^{\alpha_2}\right)^{-\frac{1}{1-\gamma}}. \tag{33}$$

Straightforward manipulations of the FONC of problem (33) yields

$$\begin{aligned}
0 &= 1 - \alpha (1 - \alpha) (w - w^\circ)\frac{\left(w^{-(1-\alpha)} + w^{-\alpha}\right)}{\left(\alpha w^\alpha + (1 - \alpha) w^{1-\alpha}\right)} \\
&\quad - \left(\frac{1}{1 - \gamma}\right)(w - w^\circ)\frac{\left(\alpha w^{-(1-\alpha)} + (1 - \alpha) w^{-\alpha}\right)}{w^\alpha + w^{(1-\alpha)}},
\end{aligned}$$

where to simplify notation, we set $\alpha_1 = \alpha$ and $\alpha_2 = 1 - \alpha$. In the limiting case of $\alpha = 1$, we can solve for the optimal union wage w_{VI} as

$$\lim_{\alpha \to 1} w_{VI} = \frac{1 - \gamma + w^\circ}{\gamma}.$$

We next look at costs at the limit under specialization. Using $\alpha_2 = 1 - \alpha$, and equation (15) for the optimal union wage, we can solve for the limiting cost of an intermediate 2 specialist as

$$\begin{aligned}
\lim_{\alpha \to 1} c_{2,S} &= \lim_{\alpha \to 1} \left[w_2^{1-\alpha}\right] \\
&= \lim_{\alpha \to 1} \left[\frac{1 - \alpha\gamma}{(1 - \alpha)\gamma}w^\circ\right]^{1-\alpha} \\
&= 1,
\end{aligned}$$

where the last line follows from straightforward limit analysis. For an intermediate 1 specialist, using (21), we have

$$\lim_{\alpha \to 1} c_{1,S} = w_1^\alpha = \frac{w^\circ}{\gamma}.$$

Next we prove Lemma A1 used in Proposition 1.

Lemma A1. Suppose for $x > 0$, $y > 0$ there is a $z \geq x + y$. Then

$$\frac{x^{-\gamma}y^{-\gamma}}{x^{-\gamma} + y^{-\gamma}} > \left(\frac{1}{2}\right)^{1-\gamma} z^{-\gamma}.$$

27

Proof. We can rescale things so that $x + y = 1$ and show the results holds for $z = 1$. Equivalently, that

$$\frac{x^{-\gamma}(1-x)^{-\gamma}}{x^{-\gamma}+(1-x)^{-\gamma}} > \left(\frac{1}{2}\right)^{1-\gamma}. \tag{34}$$

Define $H(x)$ by

$$H(x) \equiv 2x^{-\gamma}(1-x)^{-\gamma} - 2^{\gamma}x^{-\gamma} - 2^{\gamma}(1-x)^{-\gamma}.$$

Straightforward manipulation of (34) shows that it holds if $H(x) > 0$ for $x \in [0, \frac{1}{2})$. Now it is easy to verify that $H(\frac{1}{2}) = 0$. So it is sufficient to show that $H'(x) < 0$ for $x \in [0, \frac{1}{2})$. We prove this in separate notes. ∎

Finally, we prove the claims made in Section 3 about the shape of the $\hat{\tau}$ function in the limit where $\alpha_1 = 1$, $\alpha_2 = 0$. From (26), we can write $\hat{\tau}$ as

$$\hat{\tau} = 1 - \left(\frac{1}{2}^{1-\gamma} c_{VI}^{-\gamma}\right) \frac{\left(c_1^{-\gamma} + c_2^{-\gamma}\right)}{c_1^{-\gamma}c_2^{-\gamma}}.$$

Substituting in the limiting values for c_{VI}, c_1 and c_2 from above, we have

$$\hat{\tau}(w^{\circ}) = 1 - \frac{1}{2}^{1-\gamma}\left(\frac{1+w^{\circ}}{\gamma}\right)^{-\gamma}\left(1 + \left(\frac{w^{\circ}}{\gamma}\right)^{\gamma}\right).$$

Straightforward differentiation shows that the minimum of this function is obtained at a level

$$w_{\min}^{\circ} \equiv \gamma^{-\frac{\gamma}{1-\gamma}} \tag{35}$$

and that $\hat{\tau}(w^{\circ})$ is strictly decreasing or increasing as $w^{\circ} < w_{\min}^{\circ}$ or $w^{\circ} > w_{\min}^{\circ}$.

A.2 Proofs of Results in Section 4

Lemma 2. Assume the limiting case of $\alpha_1 = 1$ and $\alpha_2 = 0$. (i) Overall demand $\tilde{L}_S^D(w^{\circ}, \tau)$ strictly decreases in w° and τ. (ii) For a given τ, there exists a unique $w_S^{\circ*}(\tau)$ that clears the open market, $\tilde{L}_S^D(w_S^{\circ*}(\tau), \tau) \equiv \bar{L}$, and $w_S^{\circ*}(\tau)$ is strictly decreasing in τ. (iii) For any τ' where $\tau' > \hat{\tau}(w_S^{\circ*}(\tau'))$, then $\tau > \hat{\tau}(w_S^{\circ*}(\tau))$ for all $\tau > \tau'$.

Proof.

We begin by calculating the production labor demand $\tilde{L}_S^\circ(w^\circ, \tau)$. In the limit where $\alpha_1 = 1$ and $\alpha_2 = 0$, prices equal

$$p_1 = \frac{1}{\gamma^\gamma w^{\circ -\gamma} + 1} (1 - \tau) p_f \tag{36}$$

$$p_2 = \frac{\gamma^\gamma w^{\circ -\gamma}}{\gamma^\gamma w^{\circ -\gamma} + 1} (1 - \tau) p_f,$$

where we use (25) and we substitute in $c_{1,S}$ and $c_{2,S}$ from above. Now in the limit, labor demand simply equals composite input, $l_1 = y_1$. Using formula (12) for the optimal choice of y_1 and substituting in the optimal union wage $w_1 = w^\circ/\gamma$ at the limit, we have

$$l_1 = \left(\frac{p_1 \gamma^2}{w^\circ} \right)^{\frac{1}{1-\gamma}} \tag{37}$$

for p_1 given above.

Next we determine the share of firms z_1 choosing to be intermediate 1 specialists. From above

$$\frac{p_2}{p_1} = \gamma^\gamma w^{\circ -\gamma}.$$

Using the optimal choice of y_i from (12), we have

$$\frac{y_2}{y_1} = \frac{(p_2 \gamma/c_2)^{\frac{1}{1-\gamma}}}{(p_1 \gamma/c_1)^{\frac{1}{1-\gamma}}} = \gamma^{\frac{\gamma}{1-\gamma}} w^{\circ -\frac{\gamma}{1-\gamma}} \left(\frac{w^\circ}{\gamma} \right)^{\frac{1}{1-\gamma}} = \frac{w^\circ}{\gamma}.$$

Since $q_i = y_i^\gamma$,

$$\frac{q_2}{q_1} = \left(\frac{w^\circ}{\gamma} \right)^\gamma.$$

Now given fixed proportions,

$$z_1 q_1 = z_2 q_2 = (1 - z_1) q_2.$$

Thus

$$z_1 = \frac{1}{1 + \frac{q_1}{q_2}} = \frac{1}{1 + \gamma^\gamma w^{\circ -\gamma}}. \tag{38}$$

29

Combining (36), (37), and (38) yields

$$\tilde{L}_S^\circ(w^\circ, \tau) = z_1 l_1 = \left[\frac{1}{1 + \gamma^\gamma w^{\circ - \gamma}} \right]^{\frac{2-\gamma}{1-\gamma}} \left(\frac{1}{w^\circ} \right)^{\frac{1}{1-\gamma}} \left((1-\tau) \, p_f \gamma^2 \right)^{\frac{1}{1-\gamma}}. \tag{39}$$

The union rent is

$$\tilde{R}_S(w^\circ, \tau) = (w_1 - w^\circ) \, z_1 l_1 = \frac{1-\gamma}{\gamma} w^\circ \tilde{L}_S^\circ(w^\circ, \tau).$$

Total labor demand equals

$$\begin{aligned}
\tilde{L}_S^D(w^\circ, \tau) &= \tilde{L}_S^\circ(w^\circ, \tau) + \tilde{L}_S^R(w^\circ, \tau) \tag{40} \\
&= \tilde{L}_S^\circ(w^\circ, \tau) + \frac{\tilde{R}_S(w^\circ, \tau)}{w^\circ} \\
&= \tilde{L}_S^\circ(w^\circ, \tau) + \frac{(w_1 - w^\circ) \, \tilde{L}_S^\circ(w^\circ, \tau)}{w^\circ} \\
&= \frac{1}{\gamma} \tilde{L}_S^\circ(w^\circ, \tau).
\end{aligned}$$

Thus total demand $\tilde{L}_S^D(w^\circ, \tau)$ is proportional to production work demand $\tilde{L}_S^\circ(w^\circ, \tau)$. It is immediate that $\tilde{L}_S^\circ$ strictly decreases in τ. Straightforward calculations in separate notes show $\tilde{L}_S^\circ$ strictly decreases in w°. Claims (i) and (ii) then follow. To prove (iii), suppose we have a $\tau_a < \tau_b$, and define $w_a^\circ \equiv w^{\circ*}(\tau_a)$, $w_b^\circ \equiv w^{\circ*}(\tau_b)$, where $w_a^\circ > w_b^\circ$ from (ii). Suppose that $\tau_a = \hat{\tau}(w_a^\circ)$, i.e.

$$1 - \tau_a = \frac{1}{2}^{1-\gamma} \left(\frac{1 + w_a^\circ}{\gamma} \right)^{-\gamma} \left(1 + \left(\frac{w_a^\circ}{\gamma} \right)^\gamma \right). \tag{41}$$

We need to show $\tau_b > \hat{\tau}(w_b^\circ)$, or

$$1 - \tau_b < \frac{1}{2}^{1-\gamma} \left(\frac{1 + w_b^\circ}{\gamma} \right)^{-\gamma} \left(1 + \left(\frac{w_b^\circ}{\gamma} \right)^\gamma \right) \tag{42}$$

Now by definition of market clearing $\gamma \bar{L} = \tilde{L}_S^\circ(w_a^\circ, \tau_a) = \tilde{L}_S^\circ(w_b^\circ, \tau_b)$. Taking the formula (39) for $\tilde{L}_S^\circ$ to the $(1-\gamma)$ power and canceling terms yields

$$\left[\frac{1}{1+\gamma^\gamma w_a^{\circ-\gamma}}\right]^{(2-\gamma)} \left(\frac{1}{w_a^\circ}\right)(1-\tau_a) \tag{43}$$

$$= \left[\frac{1}{1+\gamma^\gamma w_b^{\circ-\gamma}}\right]^{(2-\gamma)} \left(\frac{1}{w_b^\circ}\right)(1-\tau_b).$$

Dividing both sides of (42) by $(1-\tau_a)$ and using (43) to substitute in for $(1-\tau_a)/(1-\tau_b)$ on the left-hand side and (41) to substitute in for $(1-\tau_a)$ on the right-hand side, it follows that we need to show

$$\frac{w_b^\circ \left[1+\gamma^\gamma (w_b^\circ)^{-\gamma}\right]^{2-\gamma}}{w_a^\circ \left[1+\gamma^\gamma (w_a^\circ)^{-\gamma}\right]^{2-\gamma}} < \frac{\left(\frac{1+w_b^\circ}{\gamma}\right)^{-\gamma}\left(1+\left(\frac{w_b^\circ}{\gamma}\right)^\gamma\right)}{\left(\frac{1+w_a^\circ}{\gamma}\right)^{-\gamma}\left(1+\left(\frac{w_a^\circ}{\gamma}\right)^\gamma\right)},$$

or equivalently, that

$$\frac{w_a^\circ \left[1+\gamma^\gamma (w_a^\circ)^{-\gamma}\right]^{2-\gamma}}{w_b^\circ \left[1+\gamma^\gamma (w_b^\circ)^{-\gamma}\right]^{2-\gamma}} > \frac{\left(\frac{1+w_a^\circ}{\gamma}\right)^{-\gamma}\left(1+\left(\frac{w_a^\circ}{\gamma}\right)^\gamma\right)}{\left(\frac{1+w_b^\circ}{\gamma}\right)^{-\gamma}\left(1+\left(\frac{w_b^\circ}{\gamma}\right)^\gamma\right)}.$$

Since $w_a^\circ > w_b^\circ$, it is sufficient to show

$$G(w^\circ) \equiv \frac{\left(1+\left(\frac{w^\circ}{\gamma}\right)^\gamma\right)}{\left[1+\gamma^\gamma (w^\circ)^{-\gamma}\right]^{2-\gamma}} \left(\frac{1}{w^\circ}\right) \left(\frac{1+w^\circ}{\gamma}\right)^{-\gamma}$$

$$= \frac{\left(\frac{1+w^\circ}{\gamma}\right)^{-\gamma}\gamma^{-\gamma}}{\left[w^\circ + \gamma^\gamma (w^\circ)^{1-\gamma}\right]^{1-\gamma}}$$

is strictly decreasing in w° which is immediate. This proves (iii). ∎

As background for the proof of the next Lemma, we derive demand and rents for the only-integration case in the limiting case. Using (32), that $c_{VI} = \frac{1+w^\circ}{\gamma}$, that $l_{VI} = y_{VI}$ in the limiting case, and that $L_{VI}^\circ = l_{VI}$ since $z_{VI} = 1$, production labor demand equals

$$\tilde{L}_{VI}^\circ(w^\circ, \tau) = \frac{1}{2}\left(\frac{p_f \gamma^2}{1+w^\circ}\right)^{\frac{1}{1-\gamma}} \tag{44}$$

31

Total labor demand equals

$$
\begin{aligned}
\tilde{L}^D_{VI}(w^\circ, \tau) &= \tilde{L}^\circ_{VI}(w^\circ, \tau) + \tilde{L}^R_{VI}(w^\circ, \tau) = \tilde{L}^\circ_{VI}(w^\circ, \tau) + \frac{\tilde{R}_{VI}(w^\circ, \tau)}{w^\circ} \quad (45) \\
&= \tilde{L}^\circ_{VI}(w^\circ, \tau) + \frac{1}{w^\circ}\left(\frac{1-\gamma+w^\circ}{\gamma} - w^\circ\right)\tilde{L}^\circ_{VI}(w^\circ, \tau) \\
&= \left(\frac{1-\gamma+w^\circ}{\gamma w^\circ}\right)\tilde{L}^\circ_{VI}(w^\circ, \tau).
\end{aligned}
$$

This is strictly decreasing in w°, so $w^{\circ*}_{VI}(\tau)$ solving $\bar{L} = \tilde{L}^D_{VI}(w^\circ, \tau)$ is unique. Since $\tilde{L}^\circ_{VI}(w^\circ, \tau)$ does not depend on τ, it follows that $w^{\circ*}_{VI}(\tau)$ is independent of τ.

Lemma 3. Take a point $(\tau', w^{\circ\prime})$ on the $\hat{\tau}$ curve so that $\tau' = \hat{\tau}(w^{\circ\prime})$. Assume the limiting case, $\alpha_1 = 1$, $\alpha_2 = 0$. Then $\tilde{L}^D_S(w^{\circ\prime}, \tau') < \tilde{L}^D_{VI}(w^{\circ\prime}, \tau')$. That is, labor demand in the only-specialization case is less than in the only-integration case.

Proof. Using the formulas (37) for l_1 and (44) for l_{VI} (and keeping in mind that $L^\circ_{VI} = l_{VI}$ since $z_{VI} = 1$), we have

$$
\frac{l_1}{l_{VI}} = \frac{\left(\frac{p_1\gamma^2}{w^\circ}\right)^{\frac{1}{1-\gamma}}}{\frac{1}{2}\left(\frac{p_f\gamma^2}{1+w^\circ}\right)^{\frac{1}{1-\gamma}}}.
$$

On the $\hat{\tau}$ curve, $\pi_1 = \pi_{VI}$. Hence, using (23) and (24), and substituting in $c_1 = w^\circ/\gamma$, $c_{VI} = (1+w^\circ)/\gamma$, we have

$$
p_1^{\frac{1}{1-\gamma}}\left(\frac{w^\circ}{\gamma}\right)^{-\frac{\gamma}{1-\gamma}} = \frac{1}{2}p_f^{\frac{1}{1-\gamma}}\left(\frac{1+w^\circ}{\gamma}\right)^{-\frac{\gamma}{1-\gamma}}.
$$

Therefore,

$$
\frac{l_1}{l_{VI}} = \frac{1+w^\circ}{w^\circ}.
$$

Recalling that $\tilde{L}^D_S = \frac{1}{\gamma}\tilde{L}^\circ_S$, that $\tilde{L}^\circ_S = z_1 l_1$, substituting in for z_1 from (38) and using $\tilde{L}^\circ_{VI} = l_{VI}$, it follows that

$$
\begin{aligned}
\tilde{L}^D_S &= \frac{1}{\gamma}z_1 l_1 = \frac{1}{\gamma}z_1\frac{1+w^\circ}{w^\circ}\tilde{L}^\circ_{VI} \quad (46) \\
&= \frac{1}{\gamma}\frac{1}{1+\gamma^\gamma w^{\circ-\gamma}}\frac{1+w^\circ}{w^\circ}\tilde{L}^\circ_{VI}.
\end{aligned}
$$

Using (45) above, $\tilde{L}_S^D < \tilde{L}_{VI}^D$ if and only if

$$\frac{1}{\gamma}\frac{1}{1+\gamma^\gamma w^{\circ-\gamma}}\frac{1+w^\circ}{w^\circ} < \frac{1-\gamma+w^\circ}{\gamma w^\circ}$$

or

$$(1+w^\circ) < (1-\gamma+w^\circ)\left(1+\gamma^\gamma w^{\circ-\gamma}\right).$$

We use straightforward calculations to show this is true for $w^\circ > 0$ in separate notes. ∎

Proof of Proposition 2

Case 1. Since $\underline{\tau}= \tau_S^+$ and $\bar{\tau} = \tau_{VI}^+$ in the statement of the proposition, Lemma 3 immediately implies $\underline{\tau}< \bar{\tau}$. It is immediate that the equilibrium has to be on the $w_S^{\circ*}$ curve for $\tau <\underline{\tau}$, the $\hat{\tau}$ curve for $\tau \in (\underline{\tau},\bar{\tau})$, and the $w_{VI}^{\circ*}$ curve, as claimed. The only thing left to prove for Case 1 is that for $\tau \in (\underline{\tau},\bar{\tau})$, there exists a unique z_{VI} defined by

$$z_{VI}\tilde{L}_{VI}^D(\hat{\tau}^{-1}(\tau),\tau) + (1-z_{VI})\tilde{L}_S^D(\hat{\tau}^{-1}(\tau),\tau) = \bar{L} \tag{47}$$

that strictly increases in τ.

Note that by the definition of τ_S^+, for $\tau > \tau_S^+ =\underline{\tau}$, $\tilde{L}_S^D(\hat{\tau}^{-1}(\tau),\tau) < \bar{L}$, analogously, for $\tau < \tau_{VI}^+ = \bar{\tau}$, $\tilde{L}_S^D(\hat{\tau}^{-1}(\tau),\tau) > \bar{L}$. Hence for $\tau \in (\underline{\tau},\bar{\tau})$, a unique $z_{VI} \in (0,1)$ solves the equation (47) as claimed. For a given $\tau' \in (\underline{\tau},\bar{\tau})$, let z'_{VI} be the share solving (47). Consider a higher $\tau'' > \tau'$, but still $\tau'' < \bar{\tau}$. We note first that

$$z'_{VI}\tilde{L}_{VI}^D(\hat{\tau}^{-1}(\tau''),\tau'') + (1-z'_{VI})\tilde{L}_S^D(\hat{\tau}^{-1}(\tau''),\tau'') < \bar{L},$$

i.e. holding the VI share fixed, but increasing τ along that $\hat{\tau}$ curve, lowers demand. To show this, note that with Case 1 we are on the upward-sloping portion of the $\hat{\tau}$ curve so $\hat{\tau}^{-1}(\tau'') > \hat{\tau}^{-1}(\tau')$. Since $\tilde{L}_S^D$ decreases in w° and τ, and since $\tilde{L}_{VI}^D$ decreases in τ, $\tilde{L}_{VI}^D(\hat{\tau}^{-1}(\tau''),\tau'') < \tilde{L}_{VI}^D(\hat{\tau}^{-1}(\tau'),\tau)$ and $\tilde{L}_S^D(\hat{\tau}^{-1}(\tau''),\tau'') < \tilde{L}_S^D(\hat{\tau}^{-1}(\tau'),\tau')$, proving the above inequality. Hence the the equilibrium share z''_{VI} for τ'' must satisfy $z''_{VI} > z'_{VI}$.

Case 2.

For τ outside the interval $(\underline{\tau},\tau_S^+)$, the arguments are the same as Case 1.

Given $\bar{L} > \bar{L}_{\min}$, the point $\underline{\tau}$ defined in the statement of the proposition satisfies $\underline{\tau} < \tau_S^+$. For all $\tau \in (\underline{\tau},\tau_S^+)$, all the points on the $w_S^{\circ*}$ curve are to the left of the $\hat{\tau}$ curve, $\tau < \hat{\tau}(w_S^{\circ*}(\tau))$. For all these points, firms strictly prefer specialization over integration and we have labor-

market clearing, so each pair $(\tau, w_S^{\circ*}(\tau))$ is equilibrium with $z_{VI} = 0$. This is the type a equilibrium.

To construct the other two equilibria, we need to consider three different subcases.

The first subcase is $w_{VI}^{\circ*} \leq w_{\min}^{\circ}$. Then $\tau_{VI}^+ \in [\hat{\tau}_{\min}, \tau_S^+)$ and $w_{VI}^{\circ*}(\tau) < w_{\min}^{\circ}$, for $\tau \in (\underline{\tau}, \tau_S^+)$. Note that for all such τ, all the points on $w_{VI}^{\circ*}$ are to the right of the $\hat{\tau}$ curve, $\tau > \hat{\tau}(w_{VI}^{\circ*})$, so firms strictly prefer vertical integration. The points in this region are type c equilibria, with $z_{VI} = 1$. Next, for this range of τ define $w^{\circ b} = \hat{\tau}^{-1}(\tau)$, where $w_S^{\circ*}(\tau) < w^{\circ b} < w_{VI}^{\circ*}$. Letting z_{VI}^b solve (47), this is a partial integration equilibrium. That $w^{\circ a} < w^{\circ b} < w^{\circ c}$ is immediate.

Next suppose $w_{VI}^{\circ*} > w_{\min}^{\circ}$, but $\tau_{VI}^+ < \tau_S^+$. For $\tau \in (\tau_{VI}^+, \tau_S^+)$, the b and c equilibria are constructed the same in the first subcase. For $\tau \in (\tau_{\min}, \tau_{VI}^+)$, there are two points on the $\hat{\tau}$ curve between $w_S^{\circ*}$ and $w_{VI}^{\circ*}$. Formally, for such τ there exists a $w^{\circ b}$ and a $w^{\circ c}$ such that $\tau = \hat{\tau}(w^{\circ b}) = \hat{\tau}(w^{\circ c})$ and $w_S^{\circ*}(\tau) < w^{\circ b} < w^{\circ c} < w_{VI}^{\circ*}$. Pick z_{VI}^b and z_{VI}^c to satisfy (47).

Finally suppose $w_{VI}^{\circ*} > w_{\min}^{\circ}$ but $\tau_{VI}^+ \geq \tau_S^+$. Then for $\tau \in (\tau_{\min}, \tau_S^+)$ there are two partial equilibria analogous to that described in the previous case. For $\tau > \tau_S^+$, we are back to the case of a unique equilibrium.■

Proof of Proposition 3.

Using (7), we can write rents divided by production workers as:

$$
\begin{aligned}
\frac{R}{L^\circ} &= w^\circ \frac{L^R}{L^\circ} \\
&= w^\circ \frac{z_{VI} L_{VI}^R + (1 - z_{VI}) L_S^R}{z_{VI} L_{VI}^\circ + (1 - z_{VI}) L_S^\circ}.
\end{aligned}
$$

Note that in constructing the equilibrium, we are taking a convex combination of the equilibria of the only-specialization and only-integration regimes. For each extreme case the return to rent-seeking is w° and so this is also the return to rent-seeking in the convex combination. To simplify, we express L_{VI}^R, L_S^R, and L_S° in formulas proportional to L_{VI}°. In particular, we use line 2 of (45) to substitute in for L_{VI}^R, and (40) and (46) to substitute in for L_S^R and L_S°.

We also simplify using $w_1 = w°/\gamma$. This yields

$$
\begin{aligned}
\frac{R}{L°} &= w° \frac{z_{VI}\frac{1}{w°}\left(\frac{1-\gamma+w°}{\gamma} - w°\right)L°_{VI} + (1-z_{VI})\frac{(1-\gamma)}{\gamma}\frac{1}{1+\gamma^\gamma w°-\gamma}\frac{1+w°}{w°}L°_{VI}}{z_{VI}L°_{VI} + (1-z_{VI})\frac{1}{1+\gamma^\gamma w°-\gamma}\frac{1+w°}{w°}L°_{VI}} \\
&= w° \frac{(1-\gamma)}{\gamma}\frac{z_{VI} + (1-z_{VI})\frac{1}{1+\gamma^\gamma w°-\gamma}}{z_{VI}\frac{w°}{1+w°} + (1-z_{VI})\frac{1}{1+\gamma^\gamma w°-\gamma}} \\
&= \frac{(1-\gamma)}{\gamma}\frac{z_{VI} + (1-z_{VI})\frac{1}{1+\gamma^\gamma w°-\gamma}}{z_{VI}\frac{1}{1+w°} + (1-z_{VI})\frac{1}{w°+\gamma^\gamma w°1-\gamma}}.
\end{aligned}
$$

Inspection of the bottom line shows that this strictly increases in $w°$ for fixed z_{VI}. Furthermore, it strictly increases in z_{VI} for fixed $w°$. In case 1, z_{VI} and $w°$ both strictly increase with τ in the partial integration region, so $R/L°$ also strictly increases with τ. In case 2, in the region of multiple equilibria, for fixed τ, equilibria with higher z_{VI} have higher $w°$ and thus higher $R/L°$. ■

Proof of Proposition 4.

Using (40) and (46), we can write production worker demand from the S case as a function of the analogous demand in the VI case,

$$
\tilde{L}°_S = \frac{1}{1+\gamma^\gamma w°-\gamma}\frac{1+w°}{w°}\tilde{L}°_{VI} = \frac{1+w°}{\gamma^\gamma w°1-\gamma + w°}\tilde{L}°_{VI}
$$

Recall from (35) that $w°_{\min} \equiv \gamma^{-\frac{\gamma}{1-\gamma}}$. Hence $\tilde{L}°_S < \tilde{L}°_{VI}$ if and only if $w° > w°_{\min}$. This implies that $\tau^+_S < \tau^+_{VI}$, if and only if $w° > w°_{\min}$. The claims made about uniqueness of equilibria and the claims made about the comparative statics properties for $w°$ and z_{VI} immediately follow. The monotonicity of w^{mean} for Case 1 follows the proof of Proposition 3. As noted in the text, we use numerical analysis as our basis for the claim that for Case 2, w^{mean} increases in τ over $\tau \in (\underline{\tau}, \bar{\tau})$. The details of our calculations are available in separate notes.■

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Table 1
How Equilibrium Variables Change When the Economy Moves from
Complete Vertical Integration to Complete Specialization
(Point B to Point A in Figure 4) for Various Values of α_1, $\alpha_2 = 1 - \alpha_1$

	Rent Seeking		No Rent Seeking	
	τ^+_{VI}	τ^+_S	τ^+_{VI}	τ^+_S
Panel A: $\alpha_1 = 1.00$, $\alpha_2 = 0.00$				
τ (in percent)	15.4	15.1	14.1	14.0
w^o	5.6	5.1	3.5	3.3
w^{mean}	12.2	10.1	8.0	6.7
Total Wage Bill	87.3	78.5	123.8	103.8
Intermediate 1 Wage Bill	87.3	78.5	123.8	103.8
Intermediate 2 Wage Bill	0.0	0.0	0.0	0.0
Profit	94.4	103.2	139.4	144.0
Total Surplus	181.7	181.8	263.2	247.8
Panel B: $\alpha_1 = 0.80$, $\alpha_2 = 0.20$				
τ (in percent)	7.6	6.3	6.3	5.8
w^o	5.5	5.0	2.9	2.8
w^{mean}	14.4	12.2	7.9	6.9
Total Wage Bill	86.2	79.5	123.2	107.9
Intermediate 1 Wage Bill	82.0	70.2	114.9	93.1
Intermediate 2 Wage Bill	4.1	9.4	8.3	14.8
Profit	123.2	134.5	185.1	190.4
Total Surplus	209.4	214.0	308.3	298.3
Panel C: $\alpha_1 = 0.60$, $\alpha_2 = 0.40$				
τ (in percent)	1.2	1.0	1.0	0.8
w^o	5.2	5.1	2.5	2.5
w^{mean}	15.4	14.8	7.5	7.3
Total Wage Bill	80.8	79.5	116.4	113.8
Intermediate 1 Wage Bill	58.3	51.6	80.5	71.9
Intermediate 2 Wage Bill	22.5	28.0	35.9	41.9
Profit	153.4	155.9	223.8	224.6
Total Surplus	234.2	235.4	340.2	338.4
Panel D: $\alpha_1 = 0.55$, $\alpha_2 = 0.45$				
τ (in percent)	0.3	0.3	0.2	0.2
w^o	5.1	5.1	2.5	2.5
w^{mean}	15.4	15.2	7.4	7.4
Total Wage Bill	80.0	79.6	115.3	114.6
Intermediate 1 Wage Bill	49.3	45.8	69.1	64.9
Intermediate 2 Wage Bill	30.7	33.8	46.2	49.7
Profit	157.8	158.5	228.3	228.5
Total Surplus	237.8	238.1	343.6	343.1

Figure 1
Illustration of $\hat{\tau}(w^o)$ for Various Values of α

w^o
(log
scale)

w^o_{min}
$(\alpha_1 = 1)$

$\alpha_1 = 1$

$\alpha_1 = 0.8$

$\alpha_1 = 0.6$

τ
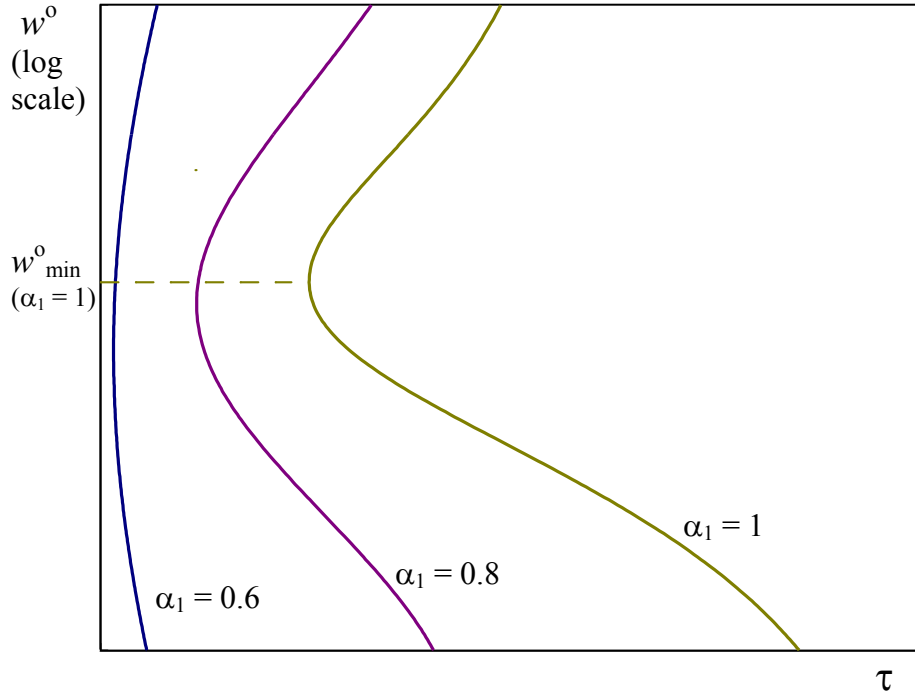
Figure 2
Illustration of $w^o_S{}^*(\tau)$ for a Small and Large Value of $\overline{L}$

w^o
(log
scale)

$w^o_S{}^*(\tau)$, Small $\overline{L}$

$w^o_S{}^*(\tau)$, Large $\overline{L}$

$\hat{\tau}(w^o)$

$\tau^+_{S,}$
Small $\overline{L}$

$\tau^+_{S,}$
Large $\overline{L}$

τ
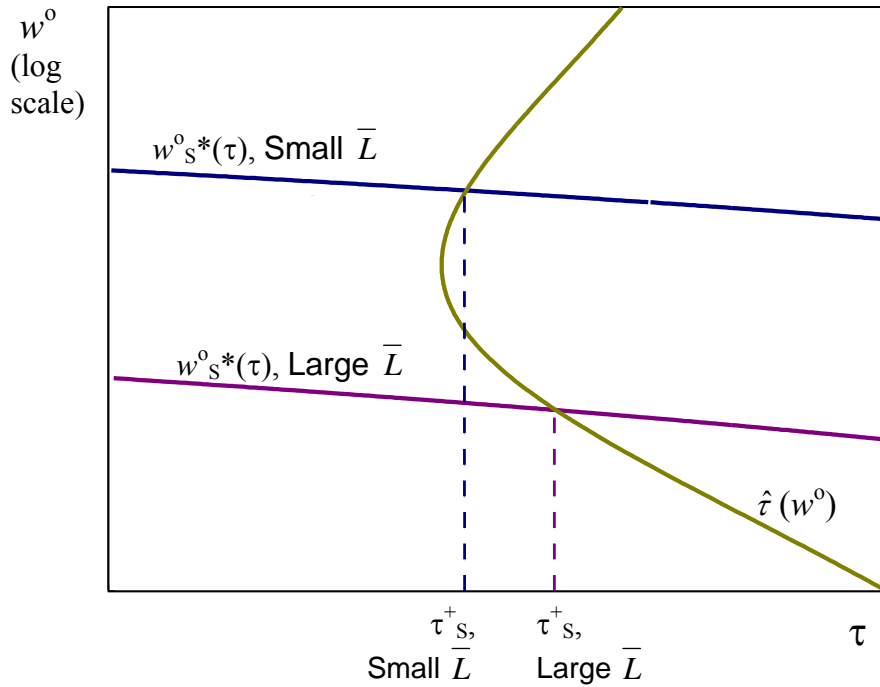
Figure 3
Illustration of $w^o_{VI}*(\tau)$ for a Small and Large Value of $\overline{L}$

w^o
(log
scale)

$w^o_{VI}*(\tau)$, Small $\overline{L}$

$w^o_{VI}*(\tau)$, Large $\overline{L}$

$\hat{\tau}(w^o)$

$\tau^+_{VI,}$ $\tau^+_{VI,}$
Small $\overline{L}$ Large $\overline{L}$

τ
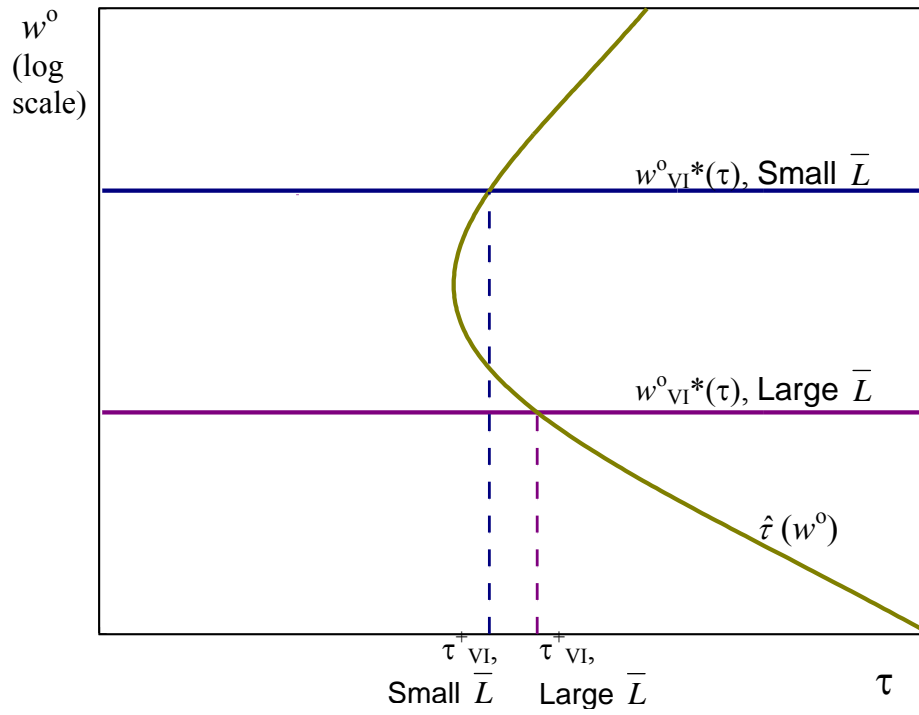
Figure 4
Equilibrium Open-Market Wages in Rent-Seeking Case for Small and Large $\overline{L}$

w^o
(log
scale)

B

$w^o*(\tau)$, Small $\overline{L}$

A

$\hat{\tau}(w^o)$

D
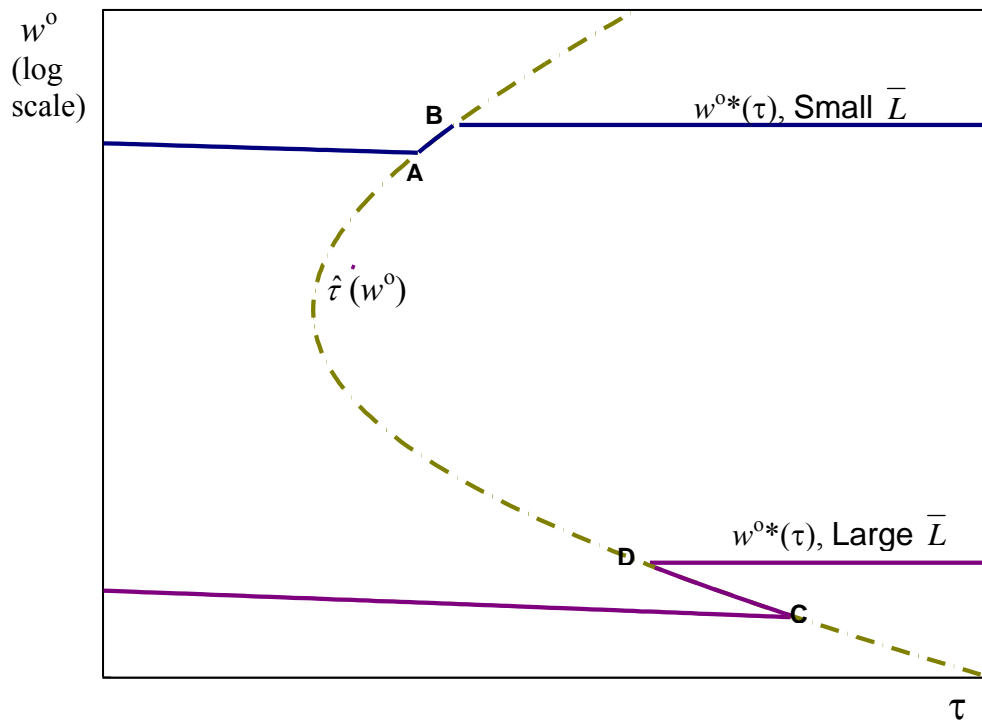
$w^o*(\tau)$, Large $\overline{L}$

C

τ

Figure 5
Case of No Rent Seeking and Large $\bar{L}$
Equilibrium Open-Market Wage and Mean Wage

w^o
(log
scale)

Mean Wage

$\hat{\tau}(w^o)$

E

Open-Market Wage

F

τ