Wealth, Job Search, and the Personal Distribution of Income

John P. Danforth

October 1976

Working Paper #: 69

Rsch. File #: 332.1

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Wealth, Job Search, and the Personal Distribution of Income

by

John P. Danforth*
University of Minnesota and
the Federal Reserve Bank of Minneapolis

In his 1953 article, "Choice, Chance and the Personal Distribution of Income" [5], Milton Friedman argues convincingly that "the link between differences in natural endowment or inherited wealth and the realized distribution of wealth or income is less direct and simple than is generally supposed...". In particular, he claims that the personal distribution of income is to a large extent the result of individual propensities to take risks and that these differences depend crucially on initial endowments. So plausible are his arguments that it is quite remarkable that considerations of initial wealth endowments are absent from the existing theoretical literature on labor income determination under uncertainty.

In this paper we present a model of one aspect of labor income determination, job search. Initial or current wealth endowments assume a role in this model similar to that suggested by Friedman. Our analysis differs slightly from that found in earlier models of job search in that the rate at which the unemployed individual depletes his limited asset holdings, i.e. consumes, is sequentially codetermined with his search strategy.[1/]

In the first section of the paper we present our assumptions and deduce the form of the optimal job search-cum-consumption allocation strategy. This strategy specifies current consumption, next period's asset holdings, and a set of acceptable wage offers as a function of past wage observations, age, and current asset endowments. Two interesting aspects of the unemployed individual's behavior can be inferred directly from our general solution. First, his strategy cannot in general be decomposed into separate income maximization and consumption allocation problems. Second, his decisions relating to job acceptability are sensitive to his current financial endowments.

In the second half of the paper the precise nature of this sensitivity is examined. We find that the individual's attitudes towards sequences of risks provide a link between search strategy and wealth. Three important theorems are obtained under the assumption that the individual's aversion to financial risks diminishes as his wealth increases. The first of these theorems states that a ceteris paribus addition to an unemployed individual's savings will cause a reduction in the number of potential job offers he would accept if received in the current period. The second asserts that the individual's consumption-savings response to an increase in asset holdings will lead to a persistent increase in job selectivity, and hence an increase in the expected duration of unemployment. The third theorem indicates that the present dollar value of the expected returns from job search are directly related to initial or current financial asset holdings.

The Model

Our first task is to characterize the economic actor whose behavior is to be analyzed below. We assume that his preferences are representable by an increasing, continuously differentiable, and strictly concave utility function, $V: R_{+}^{N+1} \to R^{1}$, the arguments of which are the N+1 single-period consumption levels $c(0), \ldots, c(N)$. The individual's preferences are also assumed to conform to the expected utility hypothesis for V and to be intertemporally separable, i.e.,

$$V(c(0),\ldots,c(N)) = \sum_{i=0}^{N} u_i(c(i)). \tag{1}$$

In addition to preferences, this individual has a memory and expectations. We shall only be concerned with his memories of past wage offers and his expectations concerning future wage offers. In particular, we assume that sequences of wage-offer observations, $\{y(0),\ldots,y(N)\}$, are well-defined measurable random vectors on the probability space $(\Omega, \mathcal{F}, Q)$ with range, $\mathcal{Y}(0)\times\ldots\times\mathcal{Y}(N)$. Letting $\gamma(t) = (y^0(0),\ldots,y^0(t-1))$, the particular t-tuple of wage offers observed as of date t, the individual's conditional expectations regarding the wage offer he will observe during period t if he continues to search are represented by:

$$G^t(\gamma(t),B) \equiv Q\{w \in \Omega\colon y(t,w) \in B|\gamma(t)\} \tag{2}$$

and

$$dG^t(\gamma(t),b) \equiv Q\{w \in \Omega\colon y(t,w) \in [b, b + db)|\gamma(t)\}. \tag{3}$$

We assume that there exists a nonnegative integer s such that wage offers received in periods t, t-1,..., t-s may be accepted during period t. If the individual should choose to accept a wage offer, $\bar{y}$, during period t, he will have a labor income of $\bar{y}$ per period commencing at date t+1 and ceasing at the date T+1, the mandatory retirement date for all jobs. Thus, after observing a wage offer during period t, $y^0(t)$, the most lucrative offer available to the individual will have a present value as of date t+1 (discounted at the market interest rate r) equal to:

$$(4) \qquad Y^s_{t+1}(\gamma(t),\, y^0(t)) \equiv \sum_{i=0}^{T-t-1} [\max\{y^0(t-s),\ldots,\, y^0(t)\}](1+r)^{-i}.$$

A certain market price, $p(i)$, is assumed to exist for each composite consumption good, $c(i)$, $i=0,\ldots, N$. We assume that lenders act so as to insure that the individual's outstanding debts at the end of period N do not exceed some fixed upper bound, $-B_{N+1}$. The necessity of satisfying this terminal constraint imposes limitations on the individual's consumption and savings which depend on his current employment status. Thus asset holdings at date t, $A(t)$, must satisfy

$$(5) \qquad A(t) = [A(t-1) - p(t-1)c(t-1) - s(t-1)](1+r), \text{ and}$$

$$(5') \qquad A(t) \geq \sum_{i=t}^{T} s(i)(1+r)^{t-i} + B_{N+1}(1+r)^{-(N+1-t)} \equiv B_t$$

if the individual has not accepted employment as of date $t-1 \leq T$, where $s(t-1)$ is the net cost of search in period $t-1$. Alternatively,

$$(6) \qquad A(t) = [A(t-1) - p(t-1)c(t-1) + \bar{y}](1+r), \text{ and}$$

$$(6') \qquad A(t) \geq \sum_{i=t}^{T} \bar{y}(1+r)^{t-i} + B_{N+1}(1+r)^{-(N+1-t)}$$

if the individual is employed at wage $\bar{y}$ as of date $t-1 \leq T$. Once the individual has retired, $T+1 < t \leq N+1$, asset holdings must satisfy

$$(7) \qquad A(t) = [A(t-1) - p(t-1)c(t-1)](1+r), \text{ and}$$

$$(7') \qquad A(t) \geq B_{N+1}(1+r)^{-(N+1-t)}$$

Finally, we assume that if the individual has not accepted a job prior to date $t=0, 1,\ldots, T$, then $c(t)$ is chosen before the value of the random variable $y(t, \circ)$ is observed. The ordering of these two events is arbitrary in that reversing it leaves all of the implications of our analysis intact.

Given these assumptions and definitions, a dynamic programming backward solution to the individual's sequential expected utility maximization may be obtained. This is accomplished by first considering the individual's utility-maximizing strategy upon entering one of the absorbing states of the model, retirement and employment. Then a sequential decision rule is derived which specifies consumption as well as conditions under which an absorbing state will be entered.

The individual's utility-maximizing strategy after retiring is easily characterized since his only decisions relate to the allocation of his asset holdings among his remaining years. The utility he obtains from date T+1 forward thus depends solely on A(T+1), and is representable by a function, R_{T+1}, with

$$R_{T+1}(A(T+1)) = \max_{c(T+1),\ldots,c(N)} \sum_{i=T+1}^{N} u_i(c(i))$$

subject to (7), (7'). Our assumptions on the u_i imply that $R_{T+1}(\cdot)$ is a continuously differentiable, increasing, and strictly concave function of A(T+1) on $[B_{T+1}, \infty)$.

Notice that the form of the utility function renders that which has transpired prior to (T+1) only important through its effect on A(T+1) at this point in time. Similarly, the individual's strategy subsequent to his acceptance of employment is simply an optimal allocation of his certain assets and guaranteed income. The maximum utility attainable from date t forward given current asset holdings, A(t), and having accepted the most lucrative wage offer available in the preceding s periods is represented by the function $E_t(\cdot)$,

$$(8) \qquad E_t(A(t)+Y_t^s(\gamma(t))) = \max_{c(t),\ldots,c(T),A(T+1)} \sum_{i=t}^{T} u_i(c(i))+R_{T+1}(A(T+1))$$

subject to

$$A(t)+Y_t^s(\gamma(t)) \geq \sum_{i=t}^{T} p(i)c(i)(1+r)^{i-1} + A(T+1)(1+r)^{t-(T+1)}$$

$$c(i) \geq 0 \quad i=t,\ldots,T, \quad A(T+1) \geq B_{T+1}.$$

The above-noted properties of the u_i and R_{T+1} functions imply that $E_t(\cdot)$ is a continuously differentiable, increasing, and concave function on $[B_{N+1}(1+r)^{t-(N+1)}, \infty)$.

We may now consider the more interesting problem of the individual's strategy prior to date T+1 when he has not entered the absorbing employment state. At date T the individual once again has no options regarding becoming employed since he must retire before any job taken in this period could be commenced. The function S_T represents the maximum utility attainable if period T is entered unemployed with assets A(T) and past wage observations $\gamma(T)$.

$$S_T(A(T), \gamma(T)) = \max_{c(T),A(T+1)} u_T(C(T)) + R_{T+1}(A(T+1))$$

subject to (5) and (5').[2/] The function S_T, is clearly continuously differentiable, increasing, and concave in A(T) on $[B_T, \infty)$.

During periods prior to T the individual may accept employment if he has not already done so. Joint decisions on consumption and job acceptance are called for under these circumstances. In order to make these decisions optimally, the individual requires knowledge of the expected utility consequences of staying unemployed as well as the previously considered utility of accepting employment.

The function S_t: $[B_t, \infty) \times \mathcal{Y}(0) \times \ldots \times \mathcal{Y}(t-1) \rightarrow R^1$ maps an unemployed individual's current assets, A(t), and past wage stream observations, $\gamma(t)$, into his maximum attainable utility. By induction arguments, one can show that S_t, t=0, 1,..., T-1, is given by

(9) $$S_t(A(t),\gamma(t)) = \max_{c(t),A(t+1)} u_t(c(t))$$

$$+ \int_{Y(t)} \max[E_{t+1}(A(t+1) + Y^s_{t+1}(\gamma(t),v)) \text{ or }$$

$$S_{t+1}(A(t+1),(\gamma(t),v))]dG^t(\gamma(t),v)$$

subject to (5) and (5').

It can be shown, again by induction, that S_t is continuous and increasing in its first argument.

The two maximization operations appearing on the right-hand side of (9) occur first with respect to the choice of an optimal consumption-savings pair, c(t)-A(t+1), and second with respect to the decision to continue or to terminate job search.[3/] This second maximization, carried on under the integral in (9), reflects the usual type of optimal stopping rule derived in sequential models of search. In particular, there is a set of wage offers which would induce the individual to accept employment in t, $\mathcal{J}_t$, defined by:

(10) $$\mathcal{J}_t(A(t+1),\gamma(t)) \equiv \{y(t): E_{t+1}(A(t+1) + Y^s_{t+1}(\gamma(t),y(t))) \geq$$

$$S_{t+1}(A(t+1),(\gamma(t),y(t)))\}.$$

Thus if $y(t) \in (\notin) \mathcal{J}_t(A(t+1),\gamma(t))$ employment will (will not) be accepted, and expected utility as of date t+1 is given by $E_{t+1}(S_{t+1})$.

One may easily verify that no alternative stopping rule can yield expected utility greater than this one.

A particular y(t) may be an element of J_t for some values of A(t+1) and not others. For such a value of y(t) it is clearly doubtful that the integrand of the integral in (9) will be a strictly concave function of A(t+1). Indeed, one can easily construct examples where the integral in (9) is not concave. The underlying reason nonconcavities can arise here even though the individual's utility function is concave is that the constraint set is nonconvex. That is, the individual can choose to accept employment or to remain unemployed but not a convex combination of the two.

The behavioral implications of this type of nonconcavity are twofold. First, there may be a multiplicity of expected utility-maximizing search strategies. Though each strategy yields the same expected utility, the consumption, savings, and job acceptance decisions may differ among them. As we shall see in the next section, no fundamental analytical difficulties result from such nonuniqueness.

Second, if the maximum expected utility of search is a nonconcave function of assets, the unemployed individual will be a risk preferer for some levels of wealth holdings. Thus, while the individual would never think of accepting an unfair gamble when employed or retired, he might be anxious to do so when unemployed. This testable implication is contrary to the commonly accepted notion that a person's propensity to take financial risks increases with his financial security.

It should be noted that financial security is here being gauged by employment status rather than wealth. In the next section we consider the relationship between attitudes toward risks and the financial security afforded by asset holdings. We find that if individual propensities to take risks increase as this type of financial security increases, then individual job selectivity, expected duration of search, and expected labor earnings will be positively related to initial asset endowments.

Attitudes Toward Risk and the Search Strategy

One may view the job search process discussed in the previous section as a type of lottery. Each outcome is a particular income stream. The lottery evolves over time as various unacceptable job offers are encountered and terminates when a specific job is accepted. Given any level of asset holdings, $A(t)$, and past job offer observations, $\gamma(t)$, there is some dollar amount which the individual would be just willing to accept rather than continue the job search lottery. This amount, $CV_t(A(t),\gamma(t))$, is defined implicitly by $S_t(A(t),\gamma(t)) - E_t(A(t)+CV_t(A(t),\gamma(t))) = 0$. Of course, since S_t and E_t are continuous and increasing, $CV_t(\circ,\gamma(t))$ is continuous.

There exists empirical evidence (see for example [3] and [6]) which suggests that consumers generally place higher dollar values on risky ventures as their wealth holdings are increased. Arrow [1] and Pratt [5] have shown that when the domain of an individual's utility function is the real line, i.e. $V: R^1 \to R^1$, such a positive correlation between wealth and risk value will obtain if and only if

$$r_V(x) = -V''(x)/V'(x)$$

is a decreasing function of x. When this condition, referred to as decreasing absolute risk aversion, is satisfied, risks are said to be normal goods.

In [4] we have considered conditions which insure that multidimensional risks increase in value as wealth increases. When an individual has a separable utility function of the sort dealt with here we have demonstrated that if

$$r_{u_i}(c(i)) = \frac{-u_i''(c(i))}{u_i'(c(i))} \text{ is a decreasing}$$

function of $c(i)$ on $[0,\infty)$, $i=0, 1,\ldots, N$, then multidimensional risks will be normal goods. This result (see Theorem 2, page 17 of [4]) implies that $CV_t(\cdot,\gamma(t))$ is an increasing function of $A(t)$ for any feasible $\gamma(t)$ if $r_{u_i}(c(i))$ is a decreasing function of $c(i)$ on $[0,\infty)$, $i=0, 1,\ldots, N$. In words, if each of the one-period utility functions displays decreasing absolute risk aversion, then, given any previously observed wage offers, the certain dollar value of job search is an increasing function of asset holdings.

<u>We assume throughout the remainder of the paper that all of the r_{u_i}'s are strictly decreasing functions on $[0, \infty)$.</u> Given this assumption, Theorem 1 is a simple consequence of the monotonicity of $CV_t(\cdot,\gamma(t))$.

<u>Theorem 1</u>: For any $\delta > 0$, and given $\gamma(t-1)$ and $A(t)$,

$$\mathcal{J}_{t-1}(A(t),\gamma(t-1)) \supseteq \mathcal{J}_{t-1}(A(t) + \delta,\gamma(t-1))$$

<u>Proof</u>: Assume by way of contradiction that there exists a $\bar{y}(t-1)$ such that

$$\bar{y}(t-1) \in \mathcal{J}_{t-1}(A(t) + \delta,\gamma(t-1)) \text{ and } \bar{y}(t-1) \notin \mathcal{J}_{t-1}(A(t),\gamma(t-1)).$$

By the definition of $\mathcal{J}_{t-1}$ we have:

$$E_t(A(t) + \delta+Y_t^S(\gamma(t-1),\bar{y}(t-1))) \geq S_t(A(t) + \delta,(\gamma(t-1),\bar{y}(t-1)))$$

and

$$E_t(A(t) + Y_t^s(\gamma(t-1),\overline{y}(t-1))) < S_t(A(t),(\gamma(t-1),\overline{y}(t-1))).$$

These inequalities imply that $CV_t(A(t) + \delta,(\gamma(t-1),\overline{y}(t-1))) \leq Y_t^s(\gamma(t-1),\overline{y}(t-1)) < CV_t(A(t),(\gamma(t-1),\overline{y}(t-1)))$. This is impossible since $CV_t(\cdot,\gamma(t))$ is monotone increasing//

The probability that an individual will no longer be unemployed in the next period is, of course, directly related to the size of his current wage stream acceptance set. Theorem 1 therefore provides a link between asset holdings and state transition probabilities. One might suspect that this link could be easily extended to establish an empirically testable relationship between the expected duration of unemployment and wealth. The possible multiplicity of expected utility-maximizing strategies mentioned in the previous section, however, means that the expected duration of unemployment may depend on expected utility-maximizing strategy which is chosen. While this complicates our analysis to some extent, we are able to demonstrate that the expected duration of unemployment associated with <u>any</u> expected utility-maximizing strategy for asset holdings $A(t)$ is no greater than the expected duration of unemployment associated with <u>any</u> expected utility-maximizing strategy for asset holdings $A(t) + \delta$.

First let $\sum(A(t),\gamma(t))$ represent the nonempty set of expected utility maximizing completely specified strategies for any feasible $(A(t),\gamma(t))$ pair. Any $\sigma \in \sum(A(t),\gamma(t))$ specifies a particular feasible sequence of consumption, asset stocks, and acceptance sets for each possible sequence of wage offers from periods t through T. Thus, if the individual decides at date t to employ strategy $\sigma \in \sum(A(t),\gamma(t))$, there is a uniquely determined level of wealth, denoted $A^{\sigma}_{t+i}(\gamma(t+i-1))$, which will be held at date (t+i), $i \geq 1$ if wage offers $\gamma(t+i-1) = (\bar{\gamma}(t),\bar{y}(t),\ldots,\bar{y}(t+i-2))$ are realized in the intervening periods.4/ Notice that since c(t) is determined prior to the observation of a y(t), the level of wealth held at date (t+1) for strategy σ does not depend on wage stream offers other than $\gamma(t)$.

The set of wage offers which would lead the individual to accept employment in period (t+i), given he is employing strategy $\sigma \in \sum(\bar{A}(t),\bar{\gamma}(t))$ and has observed wage stream offers $(\bar{y}(t),\ldots,\bar{y}(t+i-1))$, is

$$J_{t+i}(A^{\sigma}_{t+i+1}(\gamma(t+i)),\gamma(t+i))$$

given $\gamma(t+i) = (\bar{\gamma}(t),\bar{y}(t),\ldots,\bar{y}(t+i-1))$. The probability that this individual will be unemployed and searching at date (t+n) may, therefore, be expressed as,

$$P^{\sigma}_{t+n} = Q\{\omega \in \Omega\colon y(t,\omega) \notin J_t(A^{\sigma}_{t+1}(\bar{\gamma}(t)),\bar{\gamma}(t)), \ldots, y(t+n-1) \in J_{t+n-1}(A^{\sigma}_{t+n}(\gamma(t+n-1)),\gamma(t+n-1))\}. \tag{10}$$

Using this precise notion of the probability of being unemployed at any date for a particular expected utility-maximizing strategy, we define the expected duration of search unemployment as of date t for an unemployed individual employing strategy $\sigma \in \sum(A(t),\gamma(t))$, $E(DS|\sigma)$, as:

$$E(DS|\sigma) \equiv \sum_{i=t}^{T} (1+i-t)[P^{\sigma}_{i} - P^{\sigma}_{i+1}]. \tag{11}$$

Theorem 2. If $\sigma^1 \in \sum(A^1(t),\bar{\gamma}(t))$ and $\sigma^2 \in \sum(A^2(t),\bar{\gamma}(t))$ and $A^2(t) > A^1(t)$, then

$$E(DS|\sigma^1) \leq E(DS|\sigma^2) \ .$$

This theorem states that, all other things being equal, the expected duration of search unemployment is nonnegatively related to wealth holdings.

Proof: First notice that since the individual is unemployed at date t, $P_t^\sigma = 1$. Also, since retirement commences at T+1, $P_{T+1}^\sigma=0$. This allows us to rewrite our equation (11) as

$$E(DS|\sigma) = 1 + \sum_{i=t+1}^{T} P_i^\sigma .$$

Next we observe that for j=0,...,T-1-t,

$$\bar{y}(t+j) \notin J_{t+j}(A_{t+j+1}^{\sigma^1}(\gamma(t+j)),\gamma(t+j)) \tag{12}$$
$$\Rightarrow \bar{y}(t+j) \notin J_{t+j}(A_{t+j+1}^{\sigma^2}(\gamma(t+j)),\gamma(t+j)).$$

This implication follows from Theorem 1 and the fact that savings can be shown to be a normal good in our model in the sense that if $\sigma^1 \in \sum(A^1(t),\bar{\gamma}(t))$ and $\sigma^2 \in \sum(A^2(t),\bar{\gamma}(t))$ and $A^2(t) > A^1(t)$, then either

$$A_{j+1}^{\sigma^2}(\gamma(j)) > A_{j+1}^{\sigma^1}(\gamma(j)) \text{ or } A_{j+1}^{\sigma^2}(\gamma(j)) = A_{j+1}^{\sigma^1}(\gamma(j)) =$$

$$B_{j+1}, \ j=t,\ldots,T-1.$$

Referring to the expression for P_{t+i}^σ given above, it is apparent that (12) implies $P_{t+i}^{\sigma^1} \leq P_{t+i}^{\sigma^2}$, i=1,...,T-t. Thus

$$\sum_{i=t+1}^{T} P_i^{\sigma^1} \leq \sum_{i=t+1}^{T} P_i^{\sigma^2}$$ and the proof is complete//

We shall now consider another type of expectation held by the individual. In particular, his expected noninterest income. When net search costs equal zero, i.e., physical search costs and unemployment benefits just equal one another in every period, expected noninterest income is simply expected wage earnings. In general, the expected present value of noninterest income for an unemployed individual at any date t+i, i=0,..., T-t, denoted PVY_{t+i}, depends on $A(t+i) = A^{\sigma}_{t+i}(\gamma(t+i-1)),\gamma(t+i))$ and, of course, that maximizing strategy which has been chosen $\sigma \in \sum(A(t),\gamma(t))$.

PVY_t given $A(t),\gamma(t)$ and $\sigma \in \sum(A(t),\gamma(t))$ is obtained using the inductive definition

$$
\begin{aligned}
&(PVY_{t+1}|A^{\sigma}_{t+i}(\gamma(t+i-1)),\gamma(t+i),\sigma) \\
&= -s(t+i) \\
&\quad + (1+r)^{-1}\int Y^{s}_{t+i+1}(\gamma(t+i),v)dG^{t+i}(\gamma(t+i),v) \\
&\quad \mathcal{J}_{t+i}(A^{\sigma}_{t+i+1}(\gamma(t+i)),\gamma(t+i)) \\
(13)\quad &\quad + (1+r)^{-1}\int(PVY_{t+i+1}|A^{\sigma}_{t+i+1}(\gamma(t+i)),(\gamma(t+i),v,\sigma)dG^{t+i}(\gamma(t+i),v), \\
&\quad \mathcal{Y}(t+i) - \mathcal{J}_{t+i}(A^{\sigma}_{t+i+1}(\gamma(t+i)),\gamma(t+i)) \\
&\quad \text{and} \quad (PVY_T|A^{\sigma}_{T}(\gamma(T-1)),\gamma(T),\sigma) = -s(T).
\end{aligned}
$$

Since each of the u_i functions is strictly concave, one can easily establish that the expected present value of noninterest income is at least as large as the certain dollar value of job search. That is, for any feasible $A(t)$, $\gamma(t)$ and $\sigma \in \sum(A(t),\gamma(t))$

$$(PVY_t|A(t),\gamma(t),\sigma) \geq CV_t(A(t),\gamma(t)),\ t=0,\ldots,\ T.$$

This observation plays an important role in proving our final theorem. This theorem, which we refer to as the "rich get richer" result, says that the expected present value of noninterest income is nonnegatively related to initial or current wealth.

<u>Theorem 3</u>: If $\sigma^1 \in \sum(A^1(t),\bar{\gamma}(t))$ and $\sigma^2 \in \sum(A^2(t),\bar{\gamma}(t))$ and $A^1(t) < A^2(t)$ then

$$(PVY_t|A^1(t),\bar{\gamma}(t),\sigma^1) \leq (PVY_t|A^2(t),\bar{\gamma}(t),\sigma^2).$$

<u>Proof</u>: Take as an induction hypothesis for date $i \leq T$

$$(PVY_i|A_i^{\sigma^1}(\gamma(i-1)),\gamma(i),\sigma^1) \leq (PVY_i|A_i^{\sigma^2}(\gamma(i-1)),\gamma(i),\sigma^2)$$

if

$$A_i^{\sigma^1}(\gamma(i-1)) < A_i^{\sigma^2}(\gamma(i-1)) .$$

We are now able to obtain the following relationships.

$$(PVY_{i-1}|A_{i-1}^{\sigma^1}(\gamma(i-2)),\gamma(i-1),\sigma^1)$$

$$= -s(i-1) + (1+r)^{-1}\left(\frac{\int Y_i^s(\gamma(i-1),v)dG^{i-1}(\gamma(i-1),v)}{J_{i-1}(A_i^{\sigma^1}(\gamma(i-1)),\gamma(i-1))} + \frac{\int(PVY_i|A_i^{\sigma^1}(\gamma(i-1)),(\gamma(i-1),v),\sigma^1)dG^{i-1}(\gamma(i-1),v)}{Y(i-1) - J_{i-1}(A_i^{\sigma^1}(\gamma(i-1)),\gamma(i-1))}\right)$$

by definition,

$$\leq -s(i-1) + (1+r)^{-1}\left(\frac{\int Y_i^s(\gamma(i-1),v)dG^{i-1}(\gamma(i-1),v)}{J_{i-1}(A_i^{\sigma^1}(\gamma(i-1)),\gamma(i-1))} + \frac{\int(PVY_i|A_i^{\sigma^2}(\gamma(i-1)),(\gamma(i-1),v),\sigma^2)dG^{i-1}(\gamma(i-1),v)}{Y(i-1) - J_{i-1}(A_i^{\sigma^1}(\gamma(i-1)),\gamma(i-1))}\right)$$

by the induction hypothesis,

$$\leq -s(i-1)$$

$$+\ (1+r)^{-1}\left(\frac{\int Y_i^s(\gamma(i-1),v)dG^{i-1}(\gamma(i-1),v)}{J_{i-1}(A_i^{\sigma^2}(\gamma(i-1)),\gamma(i-1))}\right.$$

$$+\ \frac{\int CV_i(A_i^{\sigma^2}(\gamma(i-1)),(\gamma(i-1),v))dG^{i-1}(\gamma(i-1),v)}{J_{i-1}(A_i^{\sigma^1}(\gamma(i-1)),\gamma(i-1))\ -\ J_{i-1}(A_i^{\sigma^2}(\gamma(i-1)),\gamma(i-1))}$$

$$\left.+\ \frac{\int(PVY_i|A_i^{\sigma^2}(\gamma(i-1)),(\gamma(i-1),v),\sigma^2)dG^{i-1}(\gamma(i-1),v)}{y(i-1)\ -\ J_{i-1}(A_i^{\sigma^1}(\gamma(i-1)),\gamma(i-1))}\right)$$

by the definitions of $CV_i(\cdot)$ and $J_{i-1}(\cdot)$ and Theorem 1,

$$\leq -s(i-1)$$

$$+\ (1+r)^{-1}\left(\frac{\int Y_i^s(\gamma(i-1),v)dG^{i-1}(\gamma(i-1),v)}{J_{i-1}(A_i^{\sigma^2}(\gamma(i-1)),\gamma(i-1))}\right.$$

$$\left.+\ \frac{\int(PVY_i|A_i^{\sigma^2}(\gamma(i-1)),(\gamma(i-1),v),\sigma^2)\,dG^{i-1}(\gamma(i-1),v)}{y(i-1)\ -\ J_{i-1}(A_i^{\sigma^2}(\gamma(i-1)),\gamma(i-1))}\right)$$

since $PVY_i(\cdot) \geq CV_t(\cdot)$,

$$= (PVY_{i-1}|A_{i-1}^{\sigma^2}(\gamma(i-2)),\gamma(i-1),\sigma^2).$$

Since the induction hypothesis is valid for i=T, i.e.,

$$(PVY_T|A_T^{\sigma^1}(\gamma(T-1)),\gamma(T),\sigma^1) = -s(T)$$

$$= (PVY_T|A_T^{\sigma^2}(\gamma(T-1)),\gamma(T),\sigma^2),$$

the proof is complete//

Summary and Conclusions

The optimal labor market behavior of an unemployed expected utility-maximizing individual was characterized in this paper. We demonstrated that the individual has at least one optimal job search-cum-consumption allocation strategy, and that any such strategy consists of a decision rule which specifies current consumption, savings, and acceptable wage offers as a function of previously observed wage offers, current asset holdings, and age.

The relationship between the individual's attitudes toward risk and his labor market strategy were also investigated. We concentrated on the impact of changes in attitudes toward risk resulting from variations in the individual's asset holdings. Evidence was cited suggesting a negative correlation between an individual's degree of risk aversion and his wealth, and it was shown that if such a correlation was assumed in our model, several testable hypotheses could be generated. In particular, we proved that asset holdings are negatively correlated with the size of the set of acceptable wage offers and positively correlated with the expected duration of unemployment and the expected present value of noninterest income.

These results, especially the last one, are very much in the spirit of Friedman's comments cited in the introduction. If individuals are indeed characterized by decreasing absolute risk aversion, then the job search process has been shown to translate unequal nonhuman wealth endowments into unequal lifetime labor income. Thus, with the inclusion of bequests in the utility function, our analysis suggests that the apparently typical negative wealth-risk aversion relationship characterizing individuals acts as an inequality preserving factor in the dynamic determination of the personal distribution of income.

Footnotes

*Bernt Stigum and Marcel Richter provided many helpful suggestions during the preparation of this paper. I, of course, am responsible for any errors which may remain.

1/ See, for example, Kohn and Shavell [6]. While their article contains what is probably the most comprehensive treatment of the search problem existing in the literature to date, the depletion of resources during the search process is not touched upon.

2/ It is clear that if the individual was allowed to refrain from sampling in this last period, thereby avoiding $s(T)$, he would do so if $s(T) > 0$. One may motivate the assertion that search will definitely be conducted in T if the individual is unemployed by supposing $s(T) \leq 0$ or, alternatively, one may simply interpret it as a simplifying assumption.

3/ The words "first" and "second" here refer to the order of appearance of the "max" operations. In a temporal sense the operations are carried out simultaneously.

4/ Since no wage offers are observed after employment has been accepted, $A^{\sigma}_{t+1}(\gamma(t), y(t) \ldots, y(t+i-2))$ can depend only on $\gamma(t)$ if employment has commenced as of date t.

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