International Reserves and Rollover Risk

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Abstract

We study the optimal accumulation of international reserves in a quantitative model of
sovereign default with long-term debt and a risk-free asset. Keeping higher levels of re-
serves provides a hedge against rollover risk, but this is costly because using reserves to pay
down debt allows the government to reduce sovereign spreads. Our model, parameterized
to mimic salient features of a typical emerging economy, can account for a significant frac-
tion of the holdings of international reserves, and the larger accumulation of both debt and
reserves in periods of low spreads and high income. We also show that income windfalls,
improved policy frameworks, larger contingent liabilities, and an increase in the importance
of rollover risk imply increases in the optimal holdings of reserves that are consistent with
the upward trend in reserves in emerging economies. It is essential for our results that debt
maturity exceeds one period.

Keywords: Sovereign default, international reserves, rollover risk, safe assets
JEL Codes: F32, F34, F41

1 Introduction

The large accumulation of international reserves in emerging economies has reinvigorated debates about the adequate levels of reserves.[1] A popular view suggests that emerging markets should build a large stock of reserves as a buffer against disruptions in international financial markets.[2] On the other hand, resources used to accumulate reserves could be used instead to lower debt levels, which could in turn help reduce sovereign risk and spreads. What constitutes an adequate level of reserves? Specifically, what is the optimal level of reserves for an indebted government facing default risk and the possibility of surges in borrowing costs? Furthermore, what explains the secular increase in reserves?

This paper studies the optimal accumulation of international reserves in a canonical model of sovereign default (Eaton and Gersovitz, 1981; Aguiar and Gopinath, 2006; Arellano, 2008) with long-term debt, a risk-free asset (i.e., reserves), and risk-averse foreign lenders.[3] In this setup, shocks to domestic fundamentals or to the risk premia required by bondholders exposes the government to rollover risk, that is, the risk of having to roll over its debt obligations at times when its borrowing opportunities deteriorate. We show that keeping higher levels of reserves provides a hedge against rollover risk, but this is costly because using reserves to pay down debt allows the government to reduce spreads. This is the key trade-off the government faces in our model.

In our quantitative investigation, we find that it is optimal for an indebted government paying a significant sovereign spread to hold large amounts of reserves. We calibrate the model by targeting salient features of a typical emerging economy paying a significant spread. Model simulations generate an optimal level of reserves of 6 percent of annual income on average, and reserve holdings can reach values as high as 40 percent in some simulation periods. We find that in the simulations, the government accumulates both reserves and debt in periods of low spread and high income (i.e., in good times), and conversely, the government reduces debt and reserves in periods of high spread and low income (i.e., in bad times). We show that this pattern is consistent with the behavior of debt and reserves in a majority of emerging economies.

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[1]International reserves, defined by the IMF (2001) as "official public sector foreign assets that are readily available," reached $8 trillion in emerging markets in 2015. Section 2 presents empirical regularities of international reserves for emerging economies.

[2]Building a buffer for liquidity needs is the most frequently cited reason for reserve accumulation in the IMF Survey of Reserve Managers (80 percent of respondents; IMF, 2011). There is also extensive empirical evidence that supports the precautionary role of reserves (e.g., Aizenman and Lee, 2007, Calvo et al., 2012, Bussiere et al., 2013, Dominguez et al., 2012, Frankel and Saravelos, 2012, and Gourinchas and Obstfeld, 2012).

[3]In important early work, Alfaro and Kanczuk (2009) studied optimal reserve accumulation in a sovereign default model with *one-period debt*. They found that the optimal policy is not to hold reserves. As we explain below, the insurance motive for reserves that we study in this paper does not arise in their study.

We also show that the secular increase in reserves is consistent with four recent developments in emerging economies: (i) exceptional increases in real income arising, to a large extent, from the boom in commodity prices; (ii) improvements in policy frameworks that reduced political myopia (iii) increases in the size of the public sector's contingent liabilities driven, for example, by the growth of the banking sector and nonfinancial corporate debt; and (iv) increases in the severity of global disruptions to international financial markets. These developments also imply lower optimal debt levels, which are consistent with the data.

Mechanism. In our model, an indebted government facing default risk finds it optimal to accumulate reserves. When the government faces a negative shock, this leads to increases in spreads, which makes it costly for the government to roll over its debt. Having reserves in those states has the benefit of reducing borrowing needs and mitigating the drop in consumption, but so does having lower debt levels. Why, then, would the government choose to issue debt to finance the accumulation of reserves? Next-period payoffs from issuing debt today to accumulate reserves are given by the stock of the reserves accumulated minus the next-period value of the debt issued. Given that with long-term debt, the next-period value of the debt issued decreases with the next-period borrowing cost, issuing debt to buy reserves allows the government to transfer resources from next-period states with low borrowing costs to next-period states with high borrowing costs. Because consumption is lower in periods with high borrowing cost, such operation provides a hedging value to the government.[4] In a nutshell, issuing long-term debt to accumulate reserves provides insurance against rollover risk.

Issuing debt to accumulate reserves provides insurance against rollover risk only if debt maturity exceeds one period. With *one-period* debt, issuing one extra bond implies that the government has to allocate one extra unit of resources to service that bond in the next period, if it decides to repay. The benefit of accumulating one extra unit of reserves is that there is one extra unit of resources available in the next period, regardless of the cost of borrowing and the repayment decision. Thus, by issuing one-period debt to accumulate reserves, the government can only transfer resources from next-period repayment states to next-period default states (in contrast, issuing long-term debt allows the government to transfer resources *across* repayment states with different borrowing costs). We show that this channel is not quantitatively important. When we assume one-period bonds and recalibrate the model to match the same moments as in our benchmark calibration, we find that the level of reserves is close to zero, in line with

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[4]See eq. (13) and the discussion therein for a clear underpinning of the hedging benefit of accumulating debt and reserves and the role of debt maturity.

the results obtained by Alfaro and Kanczuk (2009).

Accumulating reserves is costly. The government pays a higher spread when choosing portfolios with higher debt and reserves positions. Although a higher spread reflects that the government defaults in more future states, this benefit from lower repayments is exactly offset by the extra default cost the government incurs. As a result, the higher spread implied by higher debt and reserves positions represents a cost for the government. When solving the optimal portfolio problem, the government trades off this cost against the insurance benefits of reserves.

We find that it is relatively more costly to accumulate reserves in bad times, when lenders are more risk averse or income is lower. In those states, the borrowing cost is more sensitive to increases in gross positions. Consequently, the government buys more insurance against adverse future shocks by choosing portfolios with higher gross positions in good times. Conversely, in bad times the government tends to deplete reserves and reduce debt. This is consistent with the cyclical behavior of debt and reserves in the data, presented in Section 2.

Related Literature. We build on the quantitative sovereign default literature that follows Aguiar and Gopinath (2006) and Arellano (2008).[5] They show that predictions of the sovereign default model are consistent with several features of emerging markets, including countercyclical spreads and procyclical borrowing. These papers, however, do not allow indebted governments to accumulate assets. We allow governments to simultaneously accumulate assets and liabilities and show that the model's key predictions are still consistent with features of emerging markets. Our model is also able to account for the accumulation of reserves by indebted governments.

Alfaro and Kanczuk (2009) study the joint accumulation of international reserves and sovereign defaultable debt using a benchmark model with *one-period bonds*. Adding a risk-free asset to the sovereign debt model increases spanning because by accumulating assets and a defaultable bond, the government shifts resources from repayment states to default states. They find, however, that the sovereign should not accumulate reserves at all. In contrast, we study reserves as a hedge against rollover risk, an insurance role that, as we show, arises only when maturity exceeds one period. In our model, by accumulating assets and long-duration debt, the government transfers resources *across* repayment states. In particular, it transfers resources from states with low borrowing costs to states with high borrowing costs. Quantitatively, we find that this hedging motive against rollover risk makes the canonical sovereign debt model consistent with the pattern of reserves in the data.

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[5]Aguiar and Amador (2013a) and Aguiar et al. (2016b) analyze, respectively, theoretical and quantitative issues in this literature.

Related studies on sovereign debt address debt maturity management. Arellano and Rama-narayanan (2012) study a quantitative model with default risk in which the government balances the incentive benefits of short-term debt and the hedging benefits of long-term debt. A similar trade-off is present in our model: reserves provide an insurance value whereas paying down debt allows the reduction of spreads by reducing future incentives to default. Trade-offs between short-term debt and long-term debt are also analyzed in an active growing literature, which includes Niepelt (2014), Hatchondo, Martinez, and Sosa Padilla (2015), Broner, Lorenzoni, and Schmukler (2013b), Dovis (2013), and Aguiar et al. (2016a). None of these studies allow the government to accumulate assets for insurance purposes.

Our work is also related to papers that study the optimal maturity structure of government debt in the presence of distortionary taxation. Angeletos (2002) and Buera and Nicolini (2004) study a closed-economy model in which the government can issue non-state-contingent bonds of different maturities under perfect commitment. They present examples in which the government can replicate complete market allocations by issuing nondefaultable long-term debt and accumulating short-term assets. In their model, changes in the term structure of interest rates, which contribute to offseting shocks to the government budget constraint, arise as a result of fluctuations in the marginal rate of substitution of domestic consumers. Quantitatively, sustaining the complete market allocations requires gross positions that are on the order of a few hundred times output (Buera and Nicolini, 2004).[6] In contrast, in our model, fluctuations in the interest rate reflect changes in the premium that foreign investors demand in order to be compensated for the possibility of a government default. This not only provides a different reason for the asset-spanning motive—without default risk, our model would have an indeterminate portfolio position—but also leads to empirically plausible levels of government assets and liabilities. In addition, gross positions affect incentives for debt repayment, a channel absent in this literature. Overall, our paper provides an alternative theory of debt management based on limited commitment, shifting the focus from managing deadweight losses of taxation to managing default risk.

Our paper is also related to the literature on reserves, precautionary savings, and sudden stops. Durdu, Mendoza, and Terrones (2009) study a dynamic precautionary savings model in which a higher *net* foreign asset position reduces the frequency and the severity of binding credit constraints. In contrast, we study a setup with endogenous borrowing constraints resulting from

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[6]Faraglia, Marcet, and Scott (2010) argue that the qualitative predictions of the complete market approach to debt management are sensitive to the type of shocks considered, and Debortoli, Nunes, and Yared (2016) argue that the predictions for gross positions are sensitive to the assumption that the government can commit to fiscal policy.

default risk and analyze *gross* portfolio positions. Jeanne and Ranciere (2011) present a simple analytical formula to quantify the optimal amount of reserves. They model reserves as an Arrow-Debreu security that pays off in a sudden stop. In a similar vein, Caballero and Panageas (2008) propose a quantitative setup in which the government issues nondefaultable debt that is indexed to the income-growth shock. They find that there are significant gains from introducing financial instruments that provide insurance against both the occurrence of sudden stops and changes in the sudden-stop probability. Aizenman and Lee (2007) and Hur and Kondo (2014) study reserve accumulation in open economy versions of Diamond and Dybvig (1983), generating endogenous sudden stops. In particular, Hur and Kondo consider a multicountry model with learning about the volatility of liquidity shocks to shed light on the upward trend in the reserves-to-debt ratio. Our paper complements this literature by considering endogenous borrowing costs that are due to default risk, by considering the role of debt maturity, and by focusing on non-state-contingent assets rather than insurance arrangements. Overall, a key contribution of our paper is to present a unified framework for studying the joint dynamics of reserves, debt, and sovereign spreads.

Other studies emphasize other benefits of reserve accumulation. Korinek and Servén (2011) and Benigno and Fornaro (2012) investigate the "mercantilist motive." They present models in which learning-by-doing externalities in the tradable sector lead the government to accumulate reserves to depreciate the real exchange rate. In Aguiar and Amador (2011), the accumulation of net foreign assets allows the government to credibly commit to not expropriating capital. These studies, however, do not present endogenous gross debt positions and hence do not address whether governments should accumulate reserves or lower their debt level.

Telyukova (2013) and Telyukova and Wright (2008) address the "credit card puzzle," that is, the fact that households pay high interest rates on credit cards while earning low rates on bank accounts. In their models, the demand for liquid assets arises because of a transaction motive, since credit cards cannot be used to buy some goods. Although we also study savings decisions by an indebted agent, we offer a distinct mechanism for the demand of liquid assets based on rollover risk. This mechanism could be relevant for understanding the financial decisions of households and corporate borrowers.

The rest of the article proceeds as follows. Section 2 documents stylized facts about reserves, public debt, and sovereign spreads. Section 3 presents the model. Section 4 presents the quantitative analysis. Section 5 concludes.

2 Facts on Reserves, Debt, and Spreads

We next present three basic facts on reserves, public debt, and sovereign spreads in emerging markets.[7]

1. *Indebted governments that hold reserves pay significant and volatile spreads on their debt.* This fact, also highlighted by Rodrik (2006) and Alfaro and Kanczuk (2009) among others, is illustrated in Figure 1: panel (a) shows debt levels, panel (b) shows reserve levels, panel (c) shows spread levels, and panel (d) shows the spread volatility. All panels are sorted according to the reserve level for each country. Across countries, the median values for the average levels of debt, reserves, and spread, and the standard deviation of the spread are 42 percent, 16 percent, 224 basis points, and 155 basis points, respectively.

2. *There has been a secular increase in reserves.* Emerging markets have substantially increased their holdings of international reserves, a fact much noted in discussions on global imbalances. Figure 2 (panel a) presents the trend in reserves for the median level of reserves, as well as the interquantile range, for the sample of countries considered. The figure also shows the evolution of the levels of debt (panel b) and spreads (panel c).

3. *Reserves and debt tend to increase when the sovereign spread is low or income is high.* Table 1 presents correlations between the growth rate of either reserves or debt ($\%\Delta a$ and $\%\Delta b$, respectively) with either the spread or real GDP growth.[8] That is, in good times, the government issues debt to finance the accumulation of reserves, and in bad times, the government uses reserves to pay back debt.[9] In other words, during good times, emerging economies receive capital inflows and increase their capital outflows, and during bad times, capital inflows retrench

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[7]We focus on countries classified as emerging markets and developing economies by the IMF's World Economic Outlook and not classified as low-income countries by the World Bank. Reserves data are from the International Financial Statistics. Debt data are from the World Economic Outlook Database when we look at the more recent 2000-2014 period (Figure 1) and from the IMF Fiscal Affairs Department Historical Public Debt Database when we look at the 1980-2014 period (Figure 2). Spread data is from the Emerging Markets Bond Index Plus (EMBI+ blended). We exclude from our sample, data for sovereign default episodes (Asonuma and Trebesch, 2016). The appendix presents data for all available data for countries classified as emerging markets and developing economies by the IMF's World Economic Outlook.

[8]Aguiar et al. (2016b) document a negative correlation between the growth rate of international debt and spreads for a majority of emerging economies.

[9]Governments tend to deplete reserves during episodes of low income and high spreads, as recently experienced in the global financial crises (e.g., Frankel and Saravelos, 2012). It is not surprising that we find a higher number (although still a minority) of countries in which the growth rate of public debt is positively correlated with the spread or negative correlated with real GDP growth. An increase of public debt in bad times is consistent with emerging economies receiving loans from the official sector during crises (for instance, Boz, 2011, documents the countercyclicality of loans from International Financial Institutions).

and the there is a reduction in capital outflows.[10]

To summarize, these three facts present important regularities about the levels, trend, and cycle of debt and reserve positions. Next, we present a model of the optimal level of reserves as insurance against rollover risk that is consistent with these empirical regularities: (i) the government simultaneously holds large gross debt and asset positions while paying significant and volatile spreads for its debt, (ii) recent developments in emerging markets are consistent with significant increases of reserve holdings, and (iii) the government accumulates more reserves and debt when spreads are low and aggregate income is high.
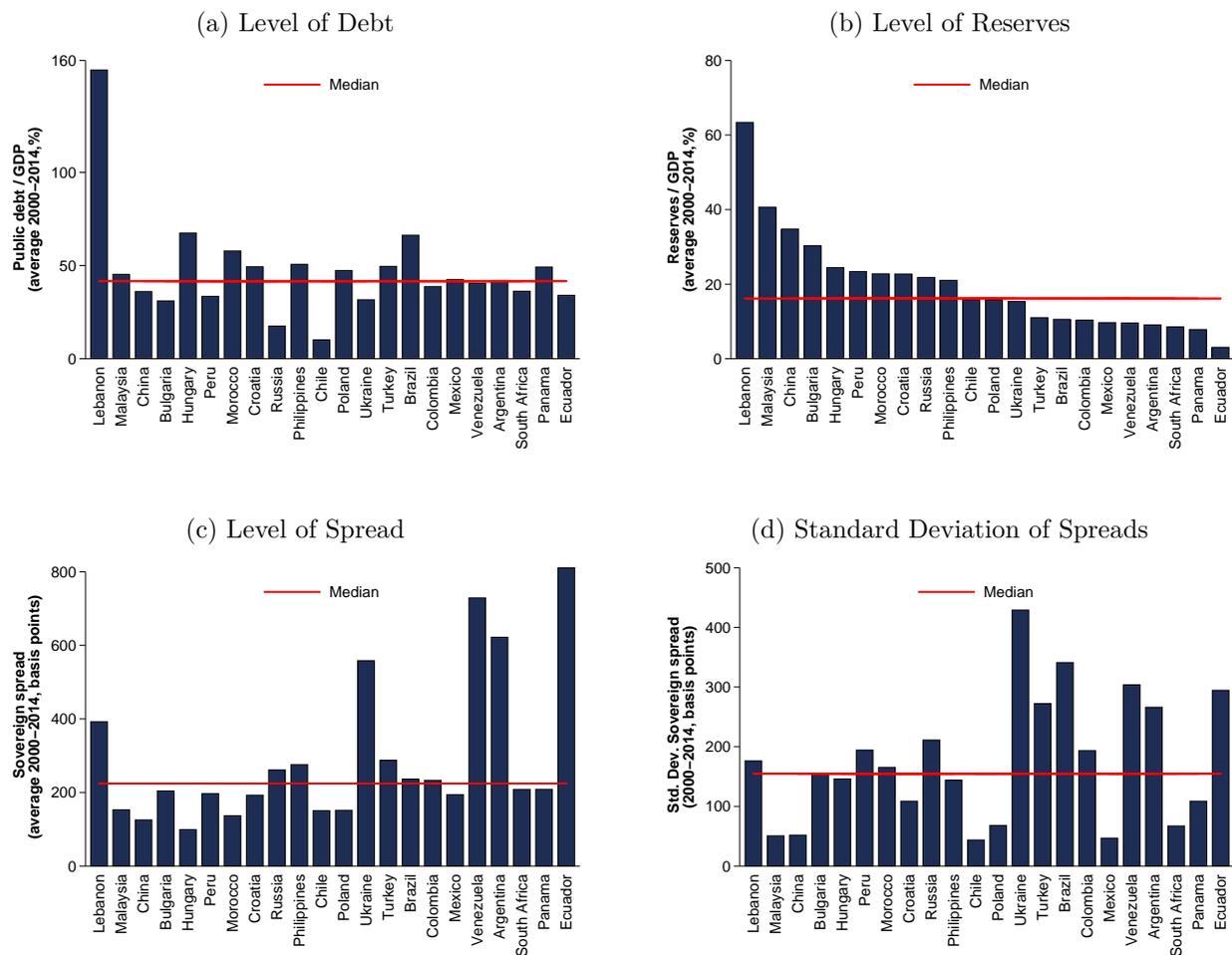
Figure 1: International Reserves, Debt, and Spreads in Emerging Economies.

Note: The figure focuses on the 22 countries for which we can construct a balanced panel of data for reserves, debt, and spreads from 2000 (when data on sovereign spreads become more widely available) to 2014.

[10]Broner, Didier, Erce, and Schmukler (2013a) report a high volatility of international reserves and document that broad measures of capital inflows and capital outflows are procyclical.

| (a) Evolution of Reserves | (b) Evolution of Debt | (c) Evolution of Spreads |
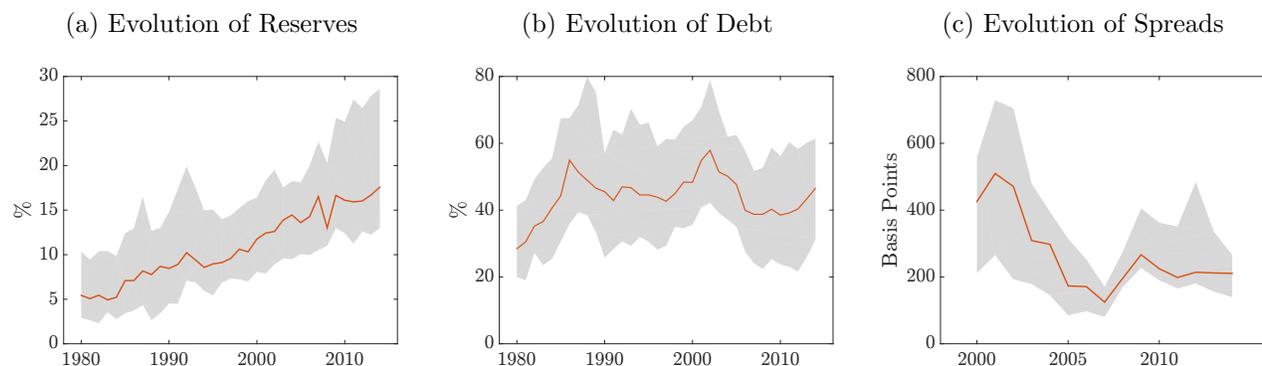
Figure 2: Trends in Reserves, Debt, and Spreads.

Note: The figure presents the median level and interquartile range for reserves, public debt, and sovereign spreads in emerging economies. Panels (a) and (b) use the 51 emerging economies for which we have data on both public debt and reserves between 1980 and 2014. Panel (c) uses the 22 countries in Figure 1.

Table 1: Correlations between the accumulation of reserves or debt with spreads or GDP growth.

|	Spread, $\%\Delta a$	Spread, $\%\Delta b$	Growth, $\%\Delta a$	Growth, $\%\Delta b$
Argentina	−0.67	0.47	0.36	−0.08
Brazil	−0.28	−0.37	0.27	0.71
Bulgaria	−0.20	−0.35	0.52	−0.28
Chile	0.14	0.41	0.05	0.02
China	−0.17	0.14	0.65	0.36
Colombia	−0.30	−0.41	0.41	0.34
Croatia	−0.27	−0.44	0.42	0.40
Ecuador	−0.36	−0.40	0.03	0.08
Hungary	−0.27	−0.65	−0.22	0.49
Lebanon	0.49	0.37	0.26	0.12
Malaysia	−0.25	0.32	0.13	0.08
Mexico	0.31	−0.32	0.09	0.54
Morocco	0.44	−0.25	0.57	−0.01
Panama	−0.26	−0.21	−0.05	0.07
Peru	−0.43	0.28	0.67	0.22
Philippines	−0.40	−0.17	0.18	0.34
Poland	−0.13	−0.45	0.38	0.25
Russia	0.04	−0.07	0.63	−0.15
South Africa	−0.61	−0.23	0.39	−0.14
Turkey	−0.12	0.52	0.47	0.15
Ukraine	−0.04	0.09	0.65	−0.19
Venezuela	−0.53	0.29	0.09	−0.15
Median	**−0.26**	**−0.19**	**0.37**	**0.10**

8

3 Model

This section presents a dynamic small open economy model with a stochastic endowment stream in which the government issues non-state-contingent defaultable debt and buys a reserve asset that pays the risk-free interest rate.[11]

3.1 Environment

Endowments. Time is discrete and indexed by $t \in 0, 1....$ The economy's endowment of the single tradable good is denoted by $y \in Y \subset \mathbb{R}_{++}$. The endowment process follows:

$$\log(y_t) = (1 - \rho)\,\mu + \rho \log(y_{t-1}) + \varepsilon_t,$$

with $|\rho| < 1$, and $\varepsilon_t \sim N\left(0, \sigma_\varepsilon^2\right)$.

Preferences.   Preferences of the government over private consumption are given by

$$\mathbb{E}_t \sum_{j=t}^{\infty} \beta^{j-t} u\left(c_j\right), \tag{1}$$

where $\mathbb{E}$ denotes the expectation operator, β denotes the discount factor, and c represents private consumption. The utility function is strictly increasing and strictly concave.

Asset/Debt Structure.   As in Arellano and Ramanarayanan (2012) and Hatchondo and Martinez (2009), we assume that a bond issued in period t promises a deterministic infinite stream of coupons that decreases at an exogenous constant rate δ. In particular, a bond issued in period t promises to pay $\delta(1 - \delta)^{j-1}$ units of the tradable good in period $t + j$, for all $j \geq 1$. Hence, debt dynamics can be represented by the following law of motion:

$$b_{t+1} = (1 - \delta)b_t + i_t, \tag{2}$$

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[11]We abstract from possible conflicts of interest between different branches of the government and treat it as a consolidated entity. In practice, reserves are often held by the monetary authority, while borrowing is conducted by the fiscal authority. However, independence of the monetary authority is often limited. As anecdotal evidence from Argentina, the *New York Times* reported that "President Cristina Fernandez fired Argentina's central bank chief Thursday after he refused to step down in a dispute over whether the country's international reserves should be used to pay debt." Similar debates are also present in developed economies. For instance, the Swedish National Debt Office and the Riksbank discussed whether the debt office should borrow to strengthen liquidity buffers and whether buffers should be at the fiscal authority or the Riksbank's disposal (Riksgalden, 2013). Such debates are beyond the scope of this paper. Liquidity buffers are also sometimes under the direct control of fiscal authorities. For example, Uruguay's Debt Management Unit reports that as of September 2015, the central government's liquid assets represent 133 percent of debt services for the next 12 months, while contingent credit lines represent an additional 122 percent.

where b_t is the number of bonds due at the beginning of period t, and i_t is the amount of bonds issued in period t. The government issues these bonds at a price q_t, which in equilibrium will depend on the government's portfolio decisions and the exogenous shocks.

The government has access to a one-period risk-free reserve asset that pays one unit of the consumption good in the next period and is traded at a constant price q_a. Let $a_t \geq 0$ denote the government's reserve holdings at the beginning of period t. The government faces the following budget constraint during each period in which it has access to debt markets:

$$c_t = y_t - \delta b_t + a_t + i_t q_t - q_a a_{t+1} - g, \tag{3}$$

where g denotes a time-invariant expenditure in a public good, which captures rigidities in the government budget constraint.[12]

Default. When the government defaults, it does so on all current and future debt obligations. This is consistent with the observed behavior of defaulting governments, and it is a standard assumption in the literature.[13] As in most previous studies, we also assume that the recovery rate for debt in default (i.e., the fraction of the loan that lenders recover after a default) is zero. In the default period, the government cannot borrow and suffers a one-time utility loss $U^D(y)$, which is increasing in income.[14] We think of this utility loss as a form of capturing various default costs related to reputation, sanctions, or misallocation of resources.

Upon default, the government retains control of its reserves and access to savings. Hence, the budget constraint becomes

$$c_t = y_t + a_t - q_a a_{t+1} - g. \tag{4}$$

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[12]Rigidities in the government budget constraint play an important role in standard debt sustainability analysis. The IMF, for example, assumes that the government cannot adjust spending for two years in response to macro-fiscal stress arising from shocks to GDP or contingent liabilities (IMF, 2013). With a similar motivation, Bocola and Dovis (2015) introduce a minimum government expenditure. Recalibrating our model to match the same targets with $g = 0$ generates half the average level of reserves in our benchmark simulations (illustrating the importance of budget rigidities).

[13]Sovereign debt contracts often contain an acceleration clause and a cross-default clause. The first clause allows creditors to call the debt they hold in case the government defaults on a debt payment. The cross-default clause states that a default in any government obligation constitutes a default in the contract containing that clause. These clauses imply that after a default event, future debt obligations become current.

[14]In the calibration, a period in the model is a year and thus the exclusion from debt markets lasts for a year, which is consistent with the literature and empirical estimates (Gelos et al., 2011). Representing default costs with the utility loss enables us to calibrate the model with one-period debt matching the same targets used in our benchmark calibration with long-term debt (as discussed by Chatterjee and Eyigungor, 2012, this is not possible with an income cost of defaulting). Comparing the versions of the model with one-period and long-term debt allows us to gauge the quantitative role that debt duration plays in the optimal accumulation of reserves.

Foreign Lenders and Risk Premium Shocks. Foreign lenders value a stochastic future stream of payments x_{t+n} using a one-period-ahead stochastic discount factor $m_{t,t+1}$, to be described below. The market value of a stochastic payment stream $\{x_{t+n}\}_{n=1}^{n=\infty}$ for foreign lenders is given by

$$\mathbb{E}_t \sum_{n=1}^{\infty} \tilde{m}_t^n x_{t+n}, \tag{5}$$

where $\tilde{m}_t^n = \prod_{j=1}^n m_{t+j-1,t+j}$.

To capture dislocations to international credit markets that are exogenous to local conditions, we assume a global shock that increases lenders' risk aversion. Several studies find that investors' risk aversion is an important driver of global liquidity (Rey, 2013) and that a significant fraction of the sovereign spread volatility in the data can be accounted for by the volatility of the risk premium (Borri and Verdelhan, 2015; Broner et al., 2013b; Longstaff et al., 2011; González-Rozada and Levy Yeyati, 2008). A vast empirical literature shows that extreme capital flow episodes are typically driven by global factors (Calvo et al., 1993; Uribe and Yue, 2006; Forbes and Warnock, 2012). Aguiar et al. (2016b) show that sovereign defaults are not tightly connected to poor fundamentals and that risk premia are an important component of sovereign spreads.

To introduce risk premium shocks, we assume that foreign lenders price bonds' payoffs using the following stochastic discount factor:

$$m_{t,t+1} = e^{-r-(\kappa_t \varepsilon_{t+1} + 0.5\kappa_t^2 \sigma_\varepsilon^2)}, \quad \text{with} \quad \kappa \geq 0. \tag{6}$$

This formulation introduces a positive risk premium because bond payoffs are more valuable to lenders in states in which the government defaults (i.e., in states in which income shocks ε are low). Here, r is the discount rate, and κ_t is the parameter governing the risk premium shock. The risk premium shock follows a two-state Markov process with values $\kappa_L = 0, \kappa_H > 0$ and transition probabilities π_{LH}, π_{HL}. We assume that in normal times $\kappa_t = \kappa_L = 0$ and lenders are risk neutral. When $\kappa_t = \kappa_H$, lenders become more risk averse and require a higher expected return to buy government bonds. A higher value of κ_H can be seen as capturing how correlated the small open economy is with respect to the lenders' income process, or alternatively, the degree of diversification of foreign lenders.[15]

This specification of the lenders' stochastic discount factor (SDF) is a special case of the discrete-time version of the Vasicek (1977) one-factor model of the term structure, and it has been used in models of sovereign default (e.g., Arellano and Ramanarayanan, 2012). For our

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[15]Aguiar et al. (2016b) explicitly model the lenders' portfolio problem featuring random finite wealth and limited investment opportunities. In their model, shocks to the foreign lenders' wealth shift the menu of borrowing opportunities.

purpose, this specification is conveniently tractable and delivers a risk premium of bonds relative to reserves. Notice that this risk premium will be endogenous to the gross portfolio positions chosen by the government, which determine default risk. Without default, the risk premium would disappear, and the government's portfolio would become indeterminate. While not crucial for the core mechanism of the model, this shock plays an important role in our simulations (we study the importance of this shock in Section 4.7.2). In states in which lenders demand a higher premium for government bonds, the government uses the reserves accumulated in earlier periods to avoid rolling over debt at high rates.

Discussion on Asset/Debt Management. The government in our model has access to a saving instrument, a one-period risk-free asset, and a debt instrument, which is long-duration debt. This asset/debt structure deserves some discussion. On the asset side, a key assumption for the mechanism in the paper is that reserves can be adjusted freely every period, which is consistent with reserves being liquid assets (e.g., US Treasury bills). Because reserves are a perfectly liquid risk-free asset that pays a constant interest rate each period, the assumed duration of reserves is irrelevant. We assume a duration of one period without loss of generality.

On the debt side, the fact that we take δ as a primitive of the model prevents us from addressing the management of the maturity structure. Notice that while choosing a longer maturity would mitigate the need of reserves to insure against rollover risk, a longer maturity has larger costs in terms of debt dilution and risk premium.[16] Given these costs from long-term debt, the government would remain exposed to rollover risk and reserves would remain valuable in the government's portfolio. Hence, the key forces in our model would also be present in a framework with both active maturity and asset management.[17]

3.2 Recursive Government Problem

We now describe the recursive formulation of the government's optimization problem. The government cannot commit to future (default, borrowing, and saving) decisions. Thus, one may interpret this environment as a game in which the government making decisions in period t is a player who takes as given the (default, borrowing, and saving) strategies of other players (governments) who decide after t. We focus on Markov perfect equilibrium. That is, we assume that

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[16] Arellano and Ramanarayanan (2012), Hatchondo et al. (2015), and Aguiar et al. (2016a), analyze debt dilution. Broner et al. (2013b) study the effect of the risk premium on the government's maturity choice.

[17] A joint analysis of reserve and maturity management would be interesting but is beyond the scope of this paper. Computationally, this would require introducing a third endogenous state variable (e.g., adding a short-term bond in addition to the long-duration bond).

in each period, the government's equilibrium default, borrowing, and saving strategies depend only on payoff-relevant state variables.

Let $s = \{y, \kappa\}$ denote the current exogenous state of the world and $V(a, b, s)$ denote the optimal value for the government. For any bond price function q, the function V satisfies the following functional equation:

$$V(a, b, s) = \max \left\{ V^R(a, b, s), V^D(a, s) \right\},$$ (V)

where the government's value of repaying is given by

$$V^R(a, b, s) = \max_{a' \geq 0, b' \geq 0} \left\{ u(c) + \beta \mathbb{E}_{s'|s} V(b', a', s') \right\},$$ (VR)

subject to

$$c = y - \delta b + a + q(b', a', s)[b' - (1 - \delta)b] - q_a a' - g.$$

The value of defaulting is given by

$$V^D(a, s) = \max_{a' \geq 0} \left\{ u(c) - U^D(y) + \beta \mathbb{E}_{s'|s} V(0, a', s'), \right\}$$ (VD)

subject to

$$c = y + a - q_a a' - g.$$

The solution to the government's problem yields decision rules for default $\hat{d}(a, b, s)$, debt $\hat{b}(a, b, s)$, reserves in default $\hat{a}^D(a, s)$, reserves when not in default $\hat{a}^R(a, b, s)$, consumption in default $\hat{c}^D(a, s)$, and consumption when not in default $\hat{c}^R(a, b, s)$. The default rule $\hat{d}$ is equal to 1 if the government defaults and is equal to 0 otherwise. In a rational expectations equilibrium (defined below), lenders use these decision rules to price debt contracts.

Equilibrium Bond Prices. To be consistent with lenders' portfolio conditions, the bond price schedule needs to satisfy

$$q(a', b', s) = \mathbb{E}_{s'|s} \left[m(s', s) \left[1 - \hat{d}(a', b', s') \right] \left[\delta + (1 - \delta) q(a'', b'', s') \right] \right],$$ (7)

where

$$b'' = \hat{b}(a', b', s')$$
$$a'' = \hat{a}^R(a', b', s').$$

Equation (7) indicates that, in equilibrium, an investor has to be indifferent between selling a government bond today and keeping the bond and selling it in the next period. If the investor keeps the bond and the government does not default in the next period, he first receives a coupon payment of δ units and then sells the bond at the market price, which is equal to $(1 - \delta)$ times the price of a bond issued in the next period.

Using (6), lenders' portfolio condition for the risk-free assets yields

$$e^{-r} = q_a.$$

3.3 Recursive Equilibrium

Definition 1 (Equilibrium). *A Markov-perfect equilibrium is defined by*

 1. *a set of value functions V, V^R, and V^D,*

 2. *rules for default $\hat{d}$, borrowing $\hat{b}$, reserves $\left\{\hat{a}^R, \hat{a}^D\right\}$, and consumption $\left\{\hat{c}^R, \hat{c}^D\right\}$,*

 3. *and a bond price function q*

such that

 i. *given a bond price function q, the policy functions $\hat{d}$, $\hat{b}$, $\hat{a}^R$, $\hat{c}^R$, $\hat{a}^D$, $\hat{c}^D$ and the value functions V, V^R, V^D solve the Bellman equations (V), (VR), and (VD).*

 ii. *given government policies, the bond price function q satisfies condition (7).*

4 Quantitative Analysis

In this section we present the quantitative analysis of the model. Section 4.1 describes the computation of the model. Section 4.2 presents the calibration. Section 4.3 presents key statistics from the benchmark simulations. Sections 4.4 and 4.5 analyzes rollover risk, and portfolio choices. Section 4.6 inspects the key trade-offs of the model. Section 4.7 examines the importance of debt maturity and risk premium shocks. Finally, Section 4.8 shows that the model can rationalize the upward trend in reserves observed in emerging economies.

4.1 Computation

The recursive problem is solved using value function iteration. As in Hatchondo et al. (2010), we solve for the equilibrium by computing the limit of the finite-horizon version of our economy. That is, the approximated value and bond price functions correspond to the ones in the first period of a finite-horizon economy with a number of periods large enough that the maximum

deviation between the value and bond price functions in the first and second period is no larger than 10^{-6}. We solve the optimal portfolio allocation in each state by searching over a grid of debt and reserve levels and then using the best portfolio on that grid as an initial guess in a nonlinear optimization routine. The value functions V^D and V^R and the function that indicates the equilibrium bond price function conditional on repayment $q\left(\hat{b}(\cdot), \hat{a}^R(\cdot), \cdot, \cdot\right)$ are approximated using linear interpolation over y and cubic spline interpolation over debt and reserves positions. We use 40 grid points for reserves, 40 grid points for debt, and 30 grid points for income realizations. Expectations are calculated using 50 quadrature points for the income shock.

4.2 Calibration

The calibration has two elements. First, we use a set of parameters values that can be directly pinned down from the data. Second, we choose a second set of parameter values that allow the model to match a set of key aspects of the data. We proceed by specifying the functional forms, and then we address these two steps in the calibration.

Functional Forms. The utility function displays a constant coefficient of relative risk aversion, that is,

$$u\left(c\right) = \frac{c^{1-\gamma} - 1}{1 - \gamma}, \text{ with } \gamma \neq 1.$$

The utility cost of defaulting is given by $U^D(y) = \alpha_0 + \alpha_1 log(y)$. As in Chatterjee and Eyigungor (2012), having two parameters in the cost of defaulting gives us the flexibility to match the behavior of the spread in the data.

Parameter Values. Table 2 presents the benchmark values given to all parameters in the model. A period in the model refers to a year. The values of the risk-free interest rate and the domestic discount factor ($r = 0.04$ and $\beta = 0.92$) are standard in quantitative business cycle and sovereign default studies.

We use Mexico as a reference for choosing the parameters that govern the endowment process, the level and duration of debt, and the mean spread. Mexico is a common reference for studies on emerging economies because its business cycle displays the same properties that are observed in other emerging economies (Aguiar and Gopinath, 2007; Neumeyer and Perri, 2005; and Uribe and Yue, 2006), and it is often used in the quantitative sovereign default literature (Mendoza and Yue, 2009; Aguiar et al., 2016b). Mexico also gives us calibration targets for the average levels

of debt and spread that are close to the median value of these levels for emerging economies (Figure 1). Unless specified otherwise, we use use data from 1993 to 2014.

The parameter values that govern the endowment process are chosen so as to mimic the behavior of logged and linearly detrended GDP in Mexico during the sample period. The estimation of the AR(1) process for the cyclical component of GDP yields $\rho = 0.66$ and $\sigma_\varepsilon = 0.034$. The level of public goods g is set to 12 percent to match the average level of public consumption to GDP in Mexico. We set $\delta = 0.2845$. With this value and the targeted level of sovereign spread, sovereign debt has an average duration of 3 years in the simulations, which is roughly the average duration of public debt in Mexico.[18]

We use the average EMBI+ spread to parameterize the shock process to lenders' risk aversion. We assume that a period with high lenders' risk aversion is one in which the global EMBI+ without countries in default is one standard deviation above the median over the sample period (we use quarterly data from 1993 to 2014). With this procedure, we obtain three episodes of a high risk premium every 20 years with an average duration of each episode equal to 1.25 years, which implies $\pi_{LH} = 0.15$ and $\pi_{HL} = 0.8$. The high risk-premium episodes are observed in 1994-1995 (Tequila crisis), 1998 (Russian default), and 2008 (global financial crisis). On average, the global EMBI+ was 2 percentage points higher in those episodes than in normal periods.

Targeted Moments. We need to calibrate the value of four other parameters: the two parameters of the utility cost of defaulting α_0 and α_1, the parameter κ_H determining the increase in lenders' risk aversion in periods of high risk premium, and the government's risk aversion γ. We choose to make the domestic risk aversion part of the calibration because it is a key parameter determining the government's willingness to tolerate rollover risk.

We use these four parameters $\{\alpha_0, \alpha_1, \kappa_H, \gamma\}$ to match four targets in the data: (i) a public debt-to-income ratio of 43.5 percent, (ii) a mean level of spreads of 240 basis points, (iii) an increase in the spread during high risk-premium periods of 200 basis points, which is the average increase in the sovereign spread observed in Mexico during the three high risk-premium periods we identify in the data, and (iv) a volatility of consumption relative to output equal to one.

To compute the sovereign spread that is implicit in a bond price, we first compute the yield i_b, defined as the return an investor would earn if he holds the bond to maturity (forever) and

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[18]We use data from the central bank of Mexico for debt duration and the Macaulay definition of duration that, with the coupon structure in this paper, is given by $D = \frac{1+i_b}{\delta+i_b}$, where i_b denotes the constant per-period yield delivered by the bond.

Table 2: Parameter Values

Parameter	Description	Value
r	Risk free rate	0.04
β	Domestic discount factor	0.92
π_{LH}	Probability of transiting to high risk-premium	0.15
π_{HL}	Probability of transiting to low risk-premium	0.8
σ_ε	std. dev of innovation to y	0.034
ρ	Autocorrelation of y	0.66
g	Government consumption	0.12
δ	Coupon decaying rate	0.2845
	Parameters set by simulation	
α_0	Default cost parameter	2.45
α_1	Default cost parameter	19
κ_H	pricing kernel parameter	23
γ	Coeff. of relative risk aversion	3.3

no default is declared. This yield satisfies

$$q_t = \sum_{j=1}^{\infty} \delta(1-\delta)^{j-1} e^{-j i_b}.$$

The sovereign spread, r_t^s, is then computed as the difference between the yield i_b and the risk-free rate r. Debt levels in the simulations are calculated as the present value of future payment obligations discounted at the risk-free rate, that is, $\frac{\delta}{1-(1-\delta)e^{-r}} b_t$.

The values for the default cost α_0 and α_1, listed in the bottom panel of Table 2, mainly determine the average debt and spread levels, while κ_H mainly determines the average increase in spreads in periods of high lenders' risk aversion. The choice of the value for the risk aversion parameter is determined mainly by the consumption-volatility target. We choose to target a volatility of consumption equal to the volatility of income, in line with the findings of Alvarez et al. (2013).[19] The value of the risk aversion parameter that results from the calibration ($\gamma = 3.3$) is within the range of values used for macro models of precautionary savings.

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[19] Alvarez et al. (2013) show that in emerging economies (including Mexico), the volatility of total consumption is higher than the volatility of aggregate income, but the volatility of the consumption of nondurable goods is lower than the volatility of income. Since our model does not differentiate between total and nondurable consumption, we choose to target a relative volatility of 1.

4.3   Key Statistics: Model and Data

Table 3 reports long-run moments in the data and in the model simulations. The first panel of this table shows that the simulations match the calibration targets. The second panel shows that the model also does a good job in mimicking nontargeted moments. In particular, the simulations generate a volatile and countercyclical spread, and a high correlation between consumption and income.[20] This is in line with previous studies that have shown that the sovereign default model without reserve accumulation can account for these features of the data (Arellano, 2008; Aguiar and Gopinath, 2006). We show that this is still the case when we extend the baseline model to allow for the empirically relevant case in which indebted governments can hold reserves and choose to do so.

Table 3: Basic Statistics: Model and Data

	Data	Model
Targeted		
$\sigma(c)/\sigma(y)$	1.0	1.0
mean debt (b/y)	43.0	43.5
mean r_s	2.4	2.4
Δr_s with risk-prem. shock	2.0	2.0
Nontargeted		
$\sigma(r_s)$	0.9	2.0
$\rho(r_s, y)$	-0.5	-0.7
$\rho(y, c)$	0.8	0.9
mean reserves (a/y)	8.5	6.0

Note: Moments are computed by generating 1,000 simulation samples of 300 periods each and taking the last 35 observations of samples in which the last default was observed at least 25 periods before the beginning of the sample. The standard deviation of x is denoted by $\sigma(x)$. The coefficient of correlation between x and z is denoted by $\rho(x, z)$.

Reserves.   Model simulations generate an average reserve-to-income ratio of 6 percent, which is close to the average ratio observed in Mexico in the post-Tequila period (1996-2014), 8.5

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[20]The spread volatility in the model is higher than in Mexico but is close to the median for emerging economies (Figure 1). The spread volatility in Mexico is also higher when computed using the stripped EMBI (throughout the paper we use the blended EMBI instead).

percent. Figure 3 shows that the simulations feature periods with reserve levels that are much higher than the average, of up to 40 percent of annual income. (In Section 4.8, we show how recent developments in emerging markets can account for significant increases in reserves.)
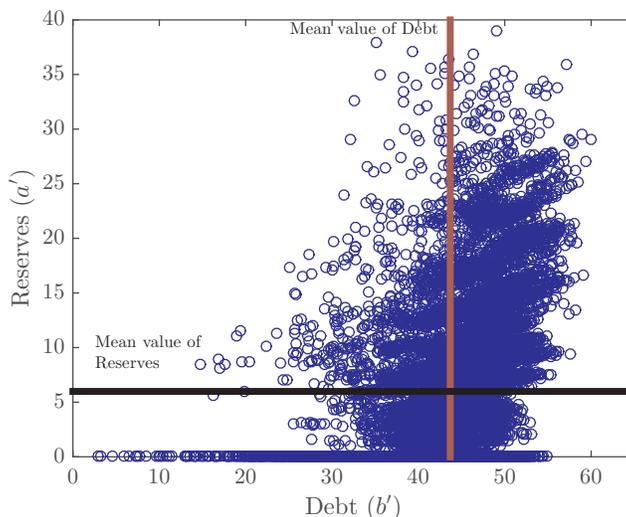

Figure 3: Portfolio of Debt and Reserves in the Simulations

4.4   Rollover Risk

We now analyze the two sources of rollover risk in the model, which are crucial for understanding the optimal portfolio of the government. Figure 4 presents the spread the government is asked to pay as a function of its debt level for different income shocks (panel a) and risk premium shocks (panel b) when the government chooses a value of reserves equal to the mean. That is, we plot $r^s(b', \bar{a}, s)$ as a function of b' for different values of s.

Panel (a) shows that for the same level of debt, investors demand higher spreads when income is low. This occurs because it is more attractive to default when income is low and income shocks are serially correlated. As emphasized in Arellano (2008) and Aguiar and Gopinath (2006), this feature enables the model to generate countercyclical spreads, as observed in the data.

Panel (b) shows that the government also faces higher spreads when lenders are more risk averse (and thus demand a larger compensation for default risk). Note that the effect of the exogenous risk premium shock on the endogenous spread is an increasing function of the debt level. This illustrates how even though the risk-premium shock process is exogenous, the incidence of this shock on the domestic economy is a function of the government's portfolio choices (in particular, if the government were to commit to a portfolio that eliminates default risk, there would be no premium on government bonds).
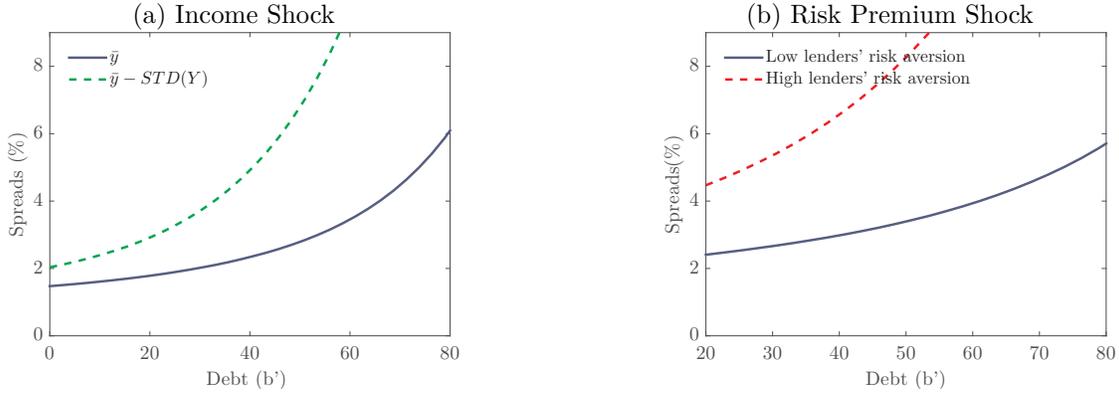
19

Figure 4: Spread Schedule

Note: The figure presents the spread asked by lenders as a function of the debt level when the government chooses the average level of reserves in the simulations. In panel (a), the lenders' risk aversion takes the low value. In panel (b), income takes the mean value. Debt levels are expressed as a percentage of average annual income.

4.5 Portfolio Policies

Figure 5 illustrates the government's optimal debt and reserves policies as functions of the lenders' risk aversion and the level of income (the two sources of rollover risk in the model). Panels (a) and (b) show respectively the increase in reserves and debt given initial values of debt and reserves equal to the mean, for the two values of the lenders' risk aversion, and for income levels between -10 and 10 percent. (Levels of income below -10 percent would lead the government to default.)

As panel (a) shows, when the risk aversion is low (blue straight line), the government increases reserve holdings for high income values and decreases reserve holdings for low income values. When the risk aversion is high (red broken line), the government depletes the initial stock of reserves (6 percent of income) in one year. This may look like a drastic response to the risk premium shock but is consistent with governments sharply reducing reserve holdings during times of stress as they run down the stock of reserves accumulated during good times.[21]

Panel (b) shows that when the lenders' risk aversion is low, the government accumulates debt when income is high and reduces debt when income is low. (Given the initial states considered, the government reduces debt when income is below the mean value and vice versa.) Notice that the increase in debt slows down for high levels of income as consumption-smoothing motives become stronger than the borrowing cost effect arising from the fact that high income improves credit market access. In fact, the debt-to-income ratio actually decreases with income for levels

[21]While it is natural in our model that governments hit the lower bound on the stock of reserves, in reality there may be reasons for governments to not fully deplete reserves.

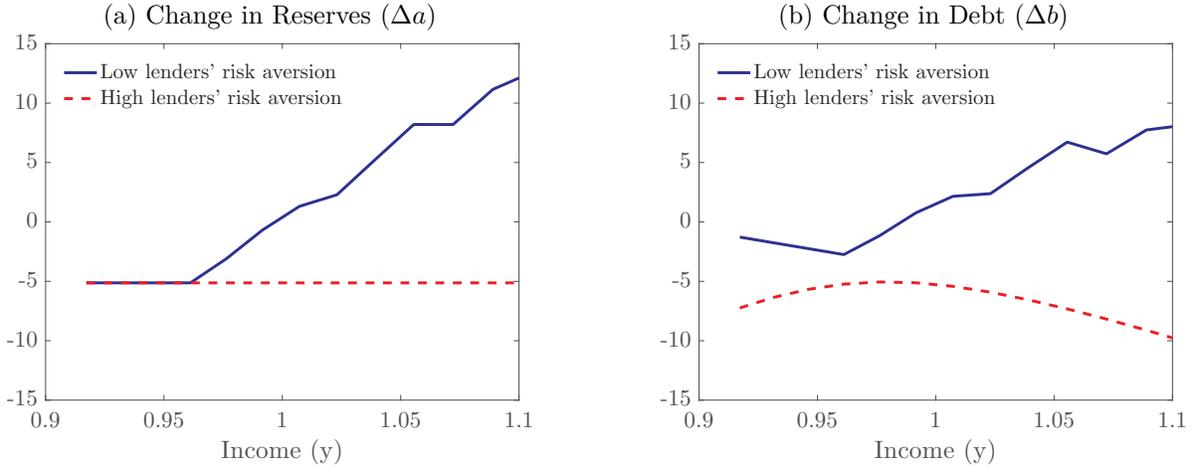of income larger than approximately 1.05.

Figure 5: Optimal accumulation of reserves and debt.

Note: The figure shows the increase in reserves and debt for initial values of debt and reserves equal to the mean levels in the simulations. The figure displays income levels for which the government chooses to pay its debt. Changes in reserves and debt are expressed as a percentage of mean annual income.

Overall, Figure 5 shows that the government increases both reserves and debt when the lenders' risk aversion is low and income is sufficiently high. In the simulations, the correlations of debt and reserves with income are 0.27 and 0.38, respectively, and the correlations of debt and reserves with spread are -0.15 and -0.32, respectively. This is consistent with the properties of the accumulation of reserves and debt in a majority of emerging economies, as illustrated in Table 1. This is also consistent with the broader movement of capital flows. In particular, capital inflows (sovereign debt accumulation) and capital outflows (reserve accumulation) are both procyclical in the model and in the data, as documented by Broner et al. (2013a).

4.6   Inspecting the Mechanism

We now analyze the key forces that shape the optimal portfolio and the fundamental trade-off faced by the government. We will show that keeping higher levels of reserves provides a hedge against adverse shocks that increase the cost of borrowing. This is costly because using reserves to pay down debt, however, allows the government to reduce spreads because it weakens its incentives to default in the future.

Optimality conditions.   For expositional purposes, we present the optimality conditions for a period in which the government finds it optimal to repay $(\hat{d}(a, b, s) = 0)$ and to accumulate reserves $(\hat{a}^R(a, b, s) > 0)$. We also assume that the price function q and the value functions

21

are differentiable (our numerical solution method does not rely on this). Applying the envelope theorem on (VR) and (VD), we obtain the following Euler equations for debt and reserves:

$$b' :: \qquad u'(c) \underbrace{\left[q + \frac{\partial q(b', a', s)}{\partial b'} i \right]}_{\text{Increase of this-period consumption}} = \beta \mathbb{E}_{s'|s} \left[u'(c') \underbrace{[\delta + q'(1 - \delta)](1 - d')}_{\text{Decline of next-period consumption}} \right], \quad (8)$$

$$a' :: \qquad u'(c) \underbrace{\left[q_a - \frac{\partial q(b', a', s)}{\partial a'} i \right]}_{\text{Decline of this-period consumption}} = \beta \mathbb{E}_{s'|s} u'(c') \underbrace{\mathbb{1}}_{\text{Increase of next-period consumption}}. \quad (9)$$

Equation (8) equates the benefits from issuing one more unit of bonds in the current period to the expected cost of repaying it in the next period. The government issues one bond in exchange for q units of consumption but the bond issuance also lowers the price of all issuances i, reducing revenue by $\frac{\partial q(b', a', s)}{\partial b'} i$. The marginal cost from borrowing is given by the costs from paying the coupon that matures in the next period and retiring the remaining $1 - \delta$ units of next-period debt at the market price $q' = q \left(\hat{a}^R(a', b', s'), \hat{b}(a', b', s'), s' \right)$.

Equation (9) equates the costs from cutting current consumption to buy one extra reserve asset to the benefits of consuming the proceeds from selling that extra reserve asset in the next period. The marginal cost of buying an extra reserve asset differs from q_a because reserve purchases affect the price at which the government issues debt.

Combining equations (8) and (9) shows that the government's optimal portfolio choice equates the marginal cost and benefit of issuing debt to finance the accumulation of reserves (while keeping the level of current consumption constant). To gain further insights on the trade-offs the government faces when it decides its current gross portfolio positions, we next compare across portfolios that generate a current consumption level equal to the optimal level.[22] Formally, we compare across portfolios (a', b') that satisfy

$$y - \delta b + a + q(a', b', s) \left[b' - (1 - \delta)b \right] - q_a a' - g = \hat{c}^R(a, b, s). \quad (10)$$

Let $\tilde{a}(\cdot)$ denote the reserve choice that is consistent with debt choice b', initial values for (a, b, s) and equation (10). Applying the implicit function theorem to equation (10) implies that issuing

---

[22]This approach is similar to the zero-cost trades studied by Aguiar and Amador (2013b). In their analysis of maturity management, they perturb short-term debt and long-term debt around the optimal portfolio while keeping continuation values constant. They establish that issuing or repurchasing long-term debt shrinks the budget set, which is undesirable because of incentive reasons.

an extra bond in the current period enables the government to purchase

$$\frac{\partial \tilde{a}(a,b,s,b')}{\partial b'} = \frac{q(\tilde{a},b',s) + \frac{\partial q(\tilde{a},b',s)}{\partial b'}[b' - (1-\delta)b]}{q_a - \frac{\partial q(\tilde{a},b',s)}{\partial a'}[b' - (1-\delta)b]} \tag{11}$$

additional reserves without deviating current consumption from its equilibrium level. (We omit the arguments of $\tilde{a}$ in the right-hand side of equation (11) to simplify notation.)

Let us consider how different portfolio choices affect lifetime utility. Because any portfolio in the set $(\tilde{a}, b')$ delivers the same current utility level, different portfolio choices affect lifetime utility through different continuation values. Thus, the issuance of an additional bond to finance the accumulation of reserves while keeping current consumption constant has the following effects on lifetime utility:

$$\frac{d\mathbb{E}_{s'|s} V(\tilde{a},b',s')}{db'} = \overbrace{\underbrace{\frac{\partial \tilde{a}}{\partial b'}}_{\text{Reserves bought}}}^{\text{Mg. benefit of buying reserves}} \mathbb{E}_{s'|s}[u'(c')] - \overbrace{\mathbb{E}_{s'|s}[u'(c')[\delta + (1-\delta)q'](1-d')]}^{\text{Mg. cost of issuing debt}}.^{23} \tag{12}$$

Equation (12) is a key expression in our model. The first term in (12) indicates the marginal utility benefit of starting next period with $\frac{\partial \tilde{a}}{\partial b'}$ additional reserves. The second term indicates the marginal utility cost of starting next period with an additional unit of debt. Notice that using (8)-(9), we obtain that the right-hand side of equation (12) equals zero. That is, at the optimum $\frac{d\mathbb{E}_{s'|s}V(\tilde{a},b',s')}{db'} = 0$ (the marginal cost and benefit of issuing debt to finance the accumulation of reserves—while keeping the level of current consumption constant—are equated). We inspect this condition below and analyze the costs and benefits of larger gross positions.

Insurance benefits. We now show that issuing debt to finance the accumulation of reserves allows the government to transfer resources to future states with low consumption, thereby providing insurance. For that, it is convenient to rearrange equation (12) as

$$\frac{d\mathbb{E}_{s'|s} V(\tilde{a},b',s')}{db'} = \underbrace{\overbrace{\frac{\partial \tilde{a}}{\partial b'}}^{\substack{\text{Transfer to} \\ \text{default states}}} \mathbb{E}_{s'|s}[u'(c')d']}_{\text{Net mg. benefits in default states}} + \underbrace{\mathbb{E}_{s'|s}\left[u'(c')\left[\overbrace{\frac{\partial \tilde{a}}{\partial b'} - \delta - (1-\delta)q'}^{\text{Transfer to repayment states}}\right](1-d')\right]}_{\text{Net mg. benefits in repayment states}}. \tag{13}$$

---

[23]To derive this, we use the chain rule together with envelope conditions $V_a(a,b,s) = u'(c)$ and $V_b(a,b,s) = -u'(c)(\delta + q(b',a',s)(1-\delta))$ for states in which the government repays and $V_b(a,b,s) = 0$ for states in which the government defaults. All future variables are evaluated at their equilibrium levels in equation (12). That is, $d' = \hat{d}(\hat{a}^R(a,b,s), \hat{b}(a,b,s), s')$, $c' = d'\hat{c}^D(\hat{a}^R(a,b,s), \hat{b}(a,b,s), s') + (1-d')\hat{c}^R(\hat{a}^R(a,b,s), \hat{b}(a,b,s), s')$, and $q' = q(\hat{a}^R(\hat{a}^R(a,b,s), \hat{b}(a,b,s), s'), \hat{b}(\hat{a}^R(a,b,s), \hat{b}(a,b,s), s'), s')$.

Equation (13) shows that issuing debt to finance the accumulation of reserves allows the government to transfer $\frac{\partial \tilde{a}}{\partial b'}$ resources to default states (first term) and $\frac{\partial \tilde{a}}{\partial b'} - \delta - (1-\delta)q'$ resources to repayment states (second term). In the second term, notice that the resources transferred to repayment states are decreasing in q', as long as $\delta < 1$ (with $\delta = 1$ transfers are constant across repayment states). This implies that issuing long-term debt to accumulate reserves allows the government to transfer more resources to next-period states with a higher borrowing cost. Thus, issuing debt to accumulate reserves is an instrument to hedge against rollover risk.
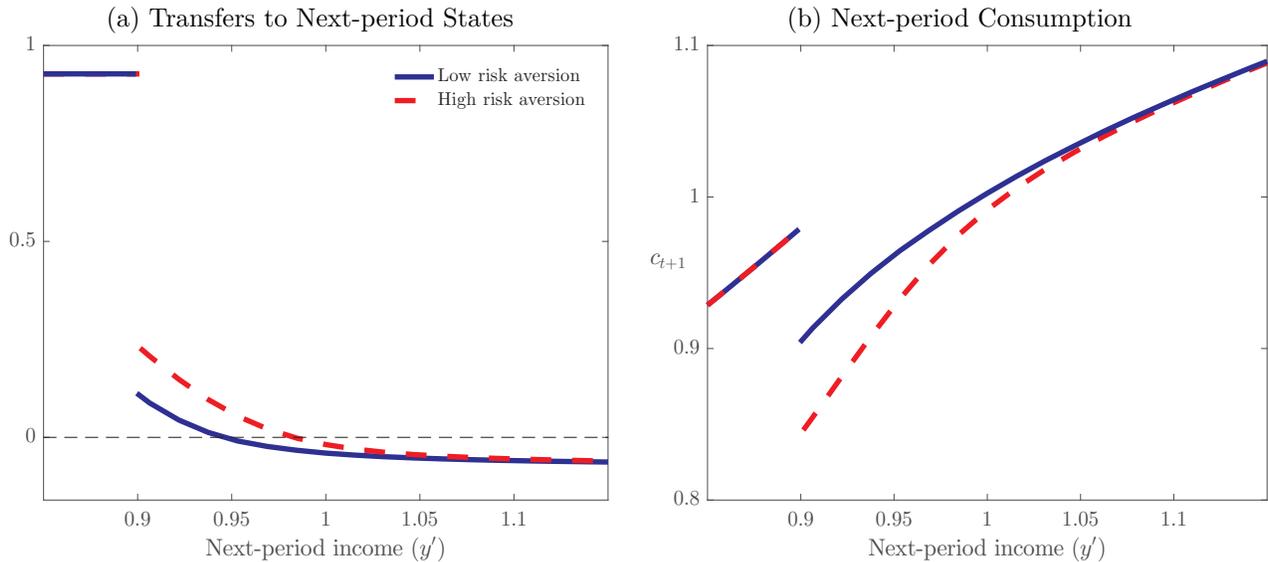

Figure 6: Insurance Benefits of Issuing Debt to Accumulate Reserves

Note: The figure assumes that in the current period, the level of income and the initial levels of debt and reserves are equal to the mean level in the simulations, the lender's risk aversion is low ($\kappa_t = 0$), and the government chooses the optimal portfolio. Panel (a) presents the resources transferred to the next period by issuing an additional bond to buy reserves, which are given by $\frac{\partial \tilde{a}}{\partial b'}$ for default states and by $\frac{\partial \tilde{a}}{\partial b'} - \delta - (1-\delta)q'$ for repayment states. Panel (b) presents the next-period consumption level. The panels present a discontinuity at the level of next-period income that triggers a default (this income threshold is higher when the lenders' risk aversion is higher).

Figure 6 shows that, indeed, issuing debt to accumulate reserves allows the government to transfer resources to low-consumption states. Panels (a) and (b) show respectively how transfers and consumption vary with next-period income for different values of the lenders' risk aversion. Notice that for the lower range of values of y' such that the government repays, and for a given lenders' risk aversion, the government experiences a positive return from issuing debt to

accumulate reserves (i.e., $\frac{\partial \tilde{a}}{\partial b'} - \delta - (1-\delta)q' \geq 0$). The range of shocks such that this happens is approximately $0.95 \geq y' \geq 0.89$ for low lenders' risk aversion and $0.98 \geq y' \geq 0.90$ for high lenders' risk aversion (returns are higher when the lenders' risk aversion is higher). In contrast, for higher values of income, the government experiences a negative return. Since consumption is lower when the lenders' risk aversion is higher or income is lower across repayment states, issuing debt to buy reserves transfers more resources to repayment states with higher marginal utility.

Higher borrowing cost. We next establish that issuing debt to accumulate reserves is costly. To allow for a sharper characterization, we assume the government is risk neutral, and thus, shut-down the insurance benefits from reserve accumulation described above. Formally, we evaluate equation (13) but assuming a constant marginal utility normalized to one. In this case, equation (13) simplifies to

$$
\begin{aligned}
\frac{d\mathbb{E}_{s'|s} V\left(\tilde{a}, b', s'\right)}{\partial b'} &= \frac{\partial \tilde{a}}{\partial b'} - \mathbb{E}_{s'|s}\left[\left[\delta + (1-\delta)q'\right](1-d')\right]. \\
&\leq \frac{\partial \tilde{a}}{\partial b'} - \frac{q}{q_a}.
\end{aligned}
\tag{14}
$$

where the second line follows from the non-negative risk premium required by foreign lenders. Let us define $\tilde{q}(b', a, b, s) = q(\tilde{a}'(a, b, s, b'), b', s)$. We can combine (11) and (14) with the derivative of $\tilde{q}$ with respect to b' to obtain

$$
\frac{d\mathbb{E}_{s'|s} V\left(\tilde{a}, b', s'\right)}{db'} \leq \frac{i}{q_a} \frac{d\tilde{q}}{db'}.^{24}
\tag{15}
$$

Equation (15) shows that the effect on welfare of issuing debt to accumulate reserves depends on the effect of this operation on the bond price and on the level of debt issuances. If the bond price decreases when the government issues debt to buy reserves ($\frac{d\tilde{q}}{db'} < 0$), a risk-neutral government issuing debt, $i > 0$, prefers strictly lower reserves.

Equation (15) shows that the effect of changing gross financial positions on the borrowing cost is a key component of the government's decision to issue debt in order to finance the accumulation of reserves. As is standard in endogenous default models, the marginal gain from increasing the set of states in which the government does not pay its debt is compensated exactly by the marginal cost of increasing the set of states in which the government pays the default cost. Therefore, only consumption-smoothing and the effect of financial positions on the bond

---

[24]Notice that $\frac{\partial \tilde{a}}{\partial b'} - \frac{q}{q_a} = \frac{i\left(\frac{\partial q}{\partial b'}q_a - \frac{\partial q}{\partial a'}a\right)}{\left(q_a - \frac{\partial q}{\partial a'}i\right)q_a}$ and $\frac{d\tilde{q}}{db'} = \frac{q\frac{\partial q}{\partial a'} - \frac{dq}{db'}q_a}{q_a - \frac{\partial q}{\partial a'}i}$. For $\frac{d\tilde{q}}{db'}$ to be negative, it has to be that $\frac{\partial q}{\partial b'} < -\frac{\partial q}{\partial a'}\frac{\partial \tilde{a}}{\partial b'}$ (i.e, the reduction in the bond price that is due to the accumulation of debt is larger than the increase in the bond price implied by the increase in reserves financed with the accumulation of debt).

price appear in optimality conditions (as shown in equations 8 and 9).

Figure 7 illustrates how, in the simulations, issuing debt to accumulate reserves increases the cost of borrowing ($\frac{d\tilde{q}}{db'} < 0$). Panel (a) presents the combination of debt (b') and reserves ($a' = \tilde{a}(a, b, s, b')$) that delivers the equilibrium level of current consumption. These "isoquants" have a positive slope: the more debt issued, the more reserves the government can buy. Panel (b) shows that issuing debt to finance the accumulation of reserves increases the spread that the government has to pay ($\frac{d\tilde{q}}{db'} < 0$). When income is at the mean value, increasing reserves from 0 to 5 percent of mean income raises spreads from 1.7 percent to 1.8 percent.[25]

Reserve accumulation is less costly when income is high. As the left panel of Figure 7 shows, the sensitivity of the spread to increases in gross positions decreases with income. This induces the government to buy more insurance by increasing its debt and reserve positions when income is higher (as shown in Figure 5).
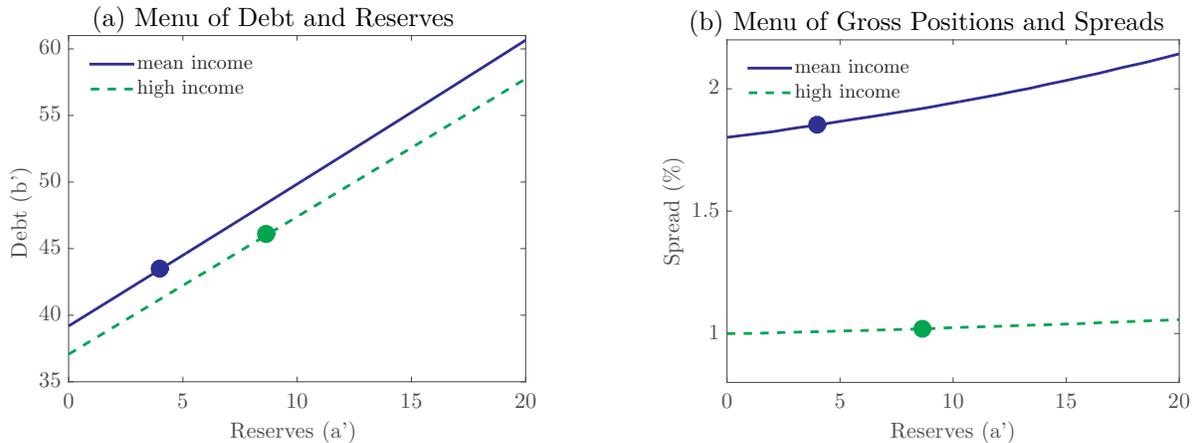

Figure 7: Issuing Debt to Accumulate Reserves

Note: Panel (a) presents combinations of reserve and debt levels that would allow the government to finance the equilibrium level of consumption. Panel (b) presents the spread the government would pay for each combination in panel (a) (with each combination identified by the level of reserves). The figure assumes that the initial level of debt and reserves are equal to the mean levels in the simulations, the lenders' risk aversion is low, and income is either equal to the mean level in the simulations or one standard deviation above this mean level. Solid dots represent optimal portfolios ($\hat{b}(a, b, s), \hat{a}^R(a, b, s)$). The levels of reserves and debt are expressed as a percentage of mean annual income.

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[25]The somewhat modest response of spreads to the accumulation of reserves financed by debt issuances is not specific to the states considered in Figure 7. If the government deviates from the optimal portfolio by increasing its debt stock in 1 percent of aggregate income and allocates the extra proceeds to purchase reserves, the mean increase in the spread in the simulations would be equal to 3.4 basis points.

4.7 Role of Maturity and Risk Premium Shocks

We next study the quantitative importance of assuming long-term debt and shocks to the lenders' risk aversion. To do so, we study versions of the model (i) with one-period bonds and (ii) without risk premium shocks.

4.7.1 One-period bonds

We now show that having bonds with maturity exceeding one period is essential for obtaining high levels of reserves in the simulations. This enables us to quantify the importance of reserves to insure against rollover risk and to contrast our results with those presented by Alfaro and Kanczuk (2009).

Equation (13) shows the importance of debt maturity. This equation shows that issuing bonds to finance the accumulation of reserves allows the government to transfer $\frac{\partial \tilde{a}}{\partial b'} - \delta - (1-\delta)q'$ resources to repayment states in the next period. Since these resources are a decreasing function of q', issuing debt to accumulate reserves allows the government to transfer more resources to states in which the cost of borrowing is higher, as long as $\delta < 1$, and thus is a hedge against rollover risk. In contrast, with one-period debt ($\delta = 1$), payoffs in repayment states are $\frac{\partial \tilde{a}}{\partial b'} - 1$, and are thus independent from the cost of borrowing next period. Therefore, issuing debt to accumulate reserves does not help the government to hedge against rollover risk. Reserves do allow the government, however, to transfer resources from repayment to default states. As we show next, this incentive to accumulate reserves is not quantitatively important, in line with the results presented by Alfaro and Kanczuk (2009).

To evaluate the quantitative performance of the one-period-bond version of the model ($\delta = 1$), we change the value of parameters to match the same targets we match in our baseline calibration. The recalibrated parameters are as follows: the parameters that affect the cost of defaulting are $\alpha_0 = 15.7$ and $\alpha_1 = 175.0$, the high level of lenders' risk aversion is $\kappa_H = 10.5$, and the risk aversion of domestic consumers is $\gamma = 4.5$. All other parameter values are the same as those assumed in the benchmark calibration. Table 4 shows that these parameter values allow the one-period-bond model to match the calibration targets well.

Table 4 shows that with one-period bonds, the average reserve ratio in the simulations falls to 0.3 percent. This is consistent with the zero reserve accumulation obtained by Alfaro and Kanczuk (2009). Again, the fact that reserves are close to zero with one-period bonds implies that the insurance value of transferring resources from repayment states to default states is not quantitatively important. In a nutshell, the insurance value of transferring resources from

Table 4: Role of Maturity and Risk-premium Shocks

	Data	Benchmark	One-period Bonds	No Risk Premium Shock
Mean duration (years)	3.0	3.0	1.0	3.0
std(cons) / std(y)	1.0	1.0	1.0	1.0
Mean debt/y	43.0	43.5	42.6	44.5
Mean spread	2.4	2.4	2.3	2.4
Spread increase for $\kappa = \kappa_H$	2.0	2.0	2.0	0
Mean (reserves/y)	**8.5**	**6.0**	**0.3**	**3.0**

repayment states to default states is outweighed by the costs of facing larger spreads.

These differences in reserve accumulation between one-period and long-duration bonds highlight the importance of debt maturity in understanding the role of reserves. Reserves provide insurance against rollover risk only if bonds' maturity exceed one period, and the model with debt maturity calibrated to the data predicts a significant level of reserve accumulation.

4.7.2 Role of Risk Premium Shocks

The goal in this section is to assess the importance of the shocks to the lenders' risk aversion. To do this, we eliminate this shock by assuming that in every period $\kappa_t = 0$. We recalibrate the parameters that affect the cost of defaulting ($\alpha_0 = 2.16$ and $\alpha_1 = 18.10$) to match the levels of sovereign debt and spread in the data, as matched in the benchmark calibration. All other parameter values are the same as those assumed in the benchmark calibration. Table 4 shows that the average holding of reserves in the simulations of the recalibrated model without shocks to the risk premium is half that of the benchmark simulations. This implies that the lack of a risk premium shock significantly reduces the need for reserves to insure against rollover risk, but reserves continue to play an important role in the model.

4.8 Understanding the Upward Trend in Reserves

In this section, we show that the model is consistent with the upward trend in reserves in emerging markets. We analyze through the lens of our model how four developments in emerging markets can deliver the upward path in reserves observed in the data since the 1990s, as presented in Figure 2. We consider the following four developments: (i) *income windfalls*; (ii) *reduction in political myopia*; (iii) *increase in contingent liabilities*; and (iv) *increase in rollover risk*. Before

providing an empirical background for these developments and describing how we map them into our model, we explain how we assess their quantitative effects.

Trend Experiments. To investigate the sources of the upward trend in reserves, we feed in shocks, or change structural parameters of the model, associated with each development and conduct transitional dynamics according to the following procedure. First, we simulate the benchmark economy 10,000 times for 25 years, starting with low lenders' risk aversion, average levels of debt, and reserves, and initial values for income weighted by its stationary distribution. Second, we average the path of endogenous variables across the 10,000 samples.[26] Because we are starting with the mean level of debt and reserves, the average paths of these variables remain close to the initial values. Third, we simulate economies with each of the four developments mentioned above, following the same steps and starting at the same state used for the simulations of the benchmark economy. Finally, we report the differences between the average paths for the simulations of the economies with each of the four developments and the average paths for the benchmark economy.

4.8.1 Developments in Emerging Markets

We now describe the empirical background for the four developments and how we map them into our model.[27]

(i) Income Windfalls. As documented by Adler and Magud (2015), the boom in commodity prices has led to large increases in real income in emerging markets. The median income windfall represented 60 percent of GDP in the last pre-boom year and lasted on average for seven years (see Table 1 in Adler and Magud, 2015).[28]

To compute the effect of income windfalls in the accumulation of reserves, we first extract simulations that satisfy the terms of trade boom criteria of Adler and Magud (2015). That is,

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[26]When averaging across samples, we exclude samples with default episodes, but results would be very similar without excluding these samples.

[27]Note that while we used Mexico for calibrating the model, we do not focus on the importance of each development for Mexico, and we discuss instead these developments for a broader set of emerging economies. As explained before, our calibration is broadly appropriate for analyzing other emerging economies.

[28]They identify terms-of-trade booms as periods in which the terms of trade increase at least 15 percent from start to peak, and the annual average increase in the terms of trade is at least 3 percent. They measure income windfalls as the cumulative difference between aggregate real income and the counterfactual aggregate real income that would have been observed if the terms of trade had remained constant at the pre-boom level. Even for economies for which Adler and Magud (2015) do not identify a terms-of-trade boom also had high income and in particular high government revenues during this period. For instance, in Mexico, roughly one-third of federal government revenues are oil related, and Levi et al. (2014) estimate that a decline in oil prices of 60 USD per barrel would imply a revenue shortfall of up to 3 percent of GDP per year.

we select among the whole set of simulations those in which real income is on average about 60 percent above trend for the initial seven years of our path. We later present results for simulations in which real income is on average between 0 and 75 percent above trend for the initial seven years.

(ii) Reduction in political myopia   Various indicators, such as the reduction in the pro-cyclicality of fiscal and monetary policy (Vegh and Vuletin, 2012), point to an improvement in government policies in emerging markets. In fact, part the apparent resilience of emerging markets during the Great Recession has been attributed to the the increase in fiscal space that was generated because of less expansionary policies in the boom periods (e.g., Frankel et al., 2013). Furthermore, between 1991 and 2014, 78 countries adopted fiscal rules and 28 countries established independent fiscal councils (Debrun and Kinda, 2014).

We model a reduction in political myopia as an increase in the time discount factor of the government, which is a common way to account for political economy aspects in emerging economies (Aguiar et al., 2009). To illustrate the effects of a reduction in political myopia, we show simulation paths for $\beta = 0.94$ (Figure 9) and later present results for a range of discount factors between 0.92, benchmark value, and 0.95 (Figure 10).

(iii) Increase in contingent liabilities.   Obstfeld et al. (2010) suggest that the rise in public contingent liabilities that is due to increased financial openness is potentially an important rationale for the accumulation of reserves. Figure 8 illustrates an increase in the size of the banking sector and of non-financial corporate debt in emerging economies. In line with the evidence provided by Reinhart and Rogoff (2011), contingent liabilities have become standard in the IMF public debt sustainability analysis. Furthermore, evidence provided by Du and Schreger (2015) shows that higher external foreign currency corporate financing is associated with a higher sovereign default risk.

We model a contingent liability shock by subtracting an amount of resources L from the budget constraint of the government.[29] This is arguably a crude way to model contingent liabilities, but it captures the problem faced by governments that suddenly need to use resources to bail

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[29]That is, when a contingent liability shock hits, the budget constraints become

$$c = y - b + a + q(b', a', y, p)(b' - (1 - \delta)b) - q_a a' - g - L$$

when the government pays and

$$c = y + a - q_a a' - g - L$$

when the government defaults.

out private institutions. We assume that the contingent liability shock is synchronized with the risk premium shock.[30] In particular, we set the probability of the occurrence of a contingent liability shock conditional on a risk premium shock to 33 percent and to zero otherwise. Given a long-run probability of a risk premium shock of 14 percent, this implies that contingent liability shocks hit on average approximately once every 20 years, which matches the empirical evidence on banking crises presented Laeven and Valencia (2008). We also set $L = 10$ percent of average aggregate income, which corresponds to the average direct fiscal cost of banking crises (IMF, 2014).
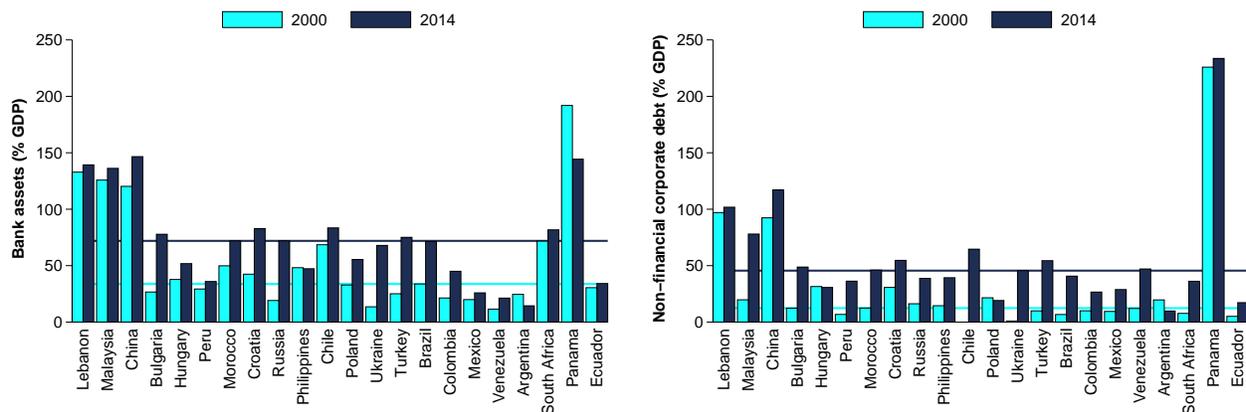
Figure 8: Assets of the Banking Sector and Non-financial Corporate Debt.

Note: Horizontal lines mark the median for each year. The source is International Financial Statistics (IMF) for banking assets and Ayala et al. (2015) for non-financial corporate debt.

(iv) Increases in rollover risk. The final development we consider is an increase in the severity of global disruptions to financial markets. As documented by Chinn and Ito (2006), capital openness in emerging economies rose in the 1990s, which coincides with the beginning of the buildup of reserves. Especially following the East Asian crisis, there was a widespread view that emerging markets needed to self-insure against financial market turmoil by accumulating reserves (Feldstein, 1999), and empirical analysis (e.g., Aizenman and Lee, 2007, Calvo et al., 2012) tends to support that this motivation indeed played an important role in policy making decisions.

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[30]Given this, our modeling of the contingent liability shock is isomorphic to assuming a negative effect of the global risk premium on aggregate income. There is ample evidence of a negative effect of global shocks to the risk premium in the real economy. A high global risk premium is often associated with credit crunches and deep recessions (Mendoza, 2010). Jeanne and Ranciere (2011) estimate the average accumulated income costs of sudden stops in capital flows above 14 percent of annual income.

To incorporate this development into the model, we consider a bigger risk premium shock. We first adjust κ_H to obtain an average increase in spreads during high risk-premium episodes 2 percentage points higher than in the benchmark (this gives us an average increase in spreads that is comparable to the increase in the 'stripped' EMBI spread around the East Asian crisis).

4.8.2 Results of Transition Experiments

Figures 9 and 10 present the results of the transition experiments. The figures present the difference between the levels of reserves, debt, and spreads in the simulations of the economy with each of the four developments described above and of the benchmark economy. Figure 9 presents the difference for each year of the simulations. Figure 10 presents the maximum difference throughout the simulations of each development for a range of shock values.

The first development we consider is the windfall. As panel (a) of Figure 9 shows, the increase in reserves is quite significant throughout the windfall phase, reaching 12 percentage points. Once the windfall is over, the increase in reserves reverts back, somewhat slowly, to zero. In addition, spreads fall in response to the windfall (panel i) and the same occurs with the debt-to-income ratio (panel e). [31]

The second development we address is the decrease in political myopia. The second column of Figure 9 shows that a more patient government accumulates more reserves, reduces debt levels, and faces lower spreads. Notice that in general the effect of the government's patience on the level of reserves is ambiguous. A rise in β implies that the government wants to transfer more resources from the present to the future, but this can be done by raising reserves or lowering debt. Moreover, lower debt levels reduce rollover risk and, therefore, the need for reserves. On the other hand, a higher β also implies that the government becomes more concerned about future fluctuations in consumption and, therefore, has stronger incentives to self-insure by accumulating reserves. In our simulations, the second effect is dominant and, in spite of a large decline in debt levels, more patient governments choose to have more reserves. A notable difference of this development relative to the other three that we consider is that the increase in β implies a change in debt positions, that is much larger than the changes in reserves. [32] Moreover, this response is not specific to the change in β considered in Figure 9 but applies to a wide range of values, as Figure 10 shows.

The third development we address is the increase in contingent liabilities, which leads to a

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[31]Notice that debt increases over the simulations but at a lower rate than income. Consistently with our discussion of optimal policy, when borrowing opportunities are more favorable, the government issues debt to finance the accumulation of reserves.

[32]In the data (Figure 2), governments have adjusted relatively more their asset than their liability positions.
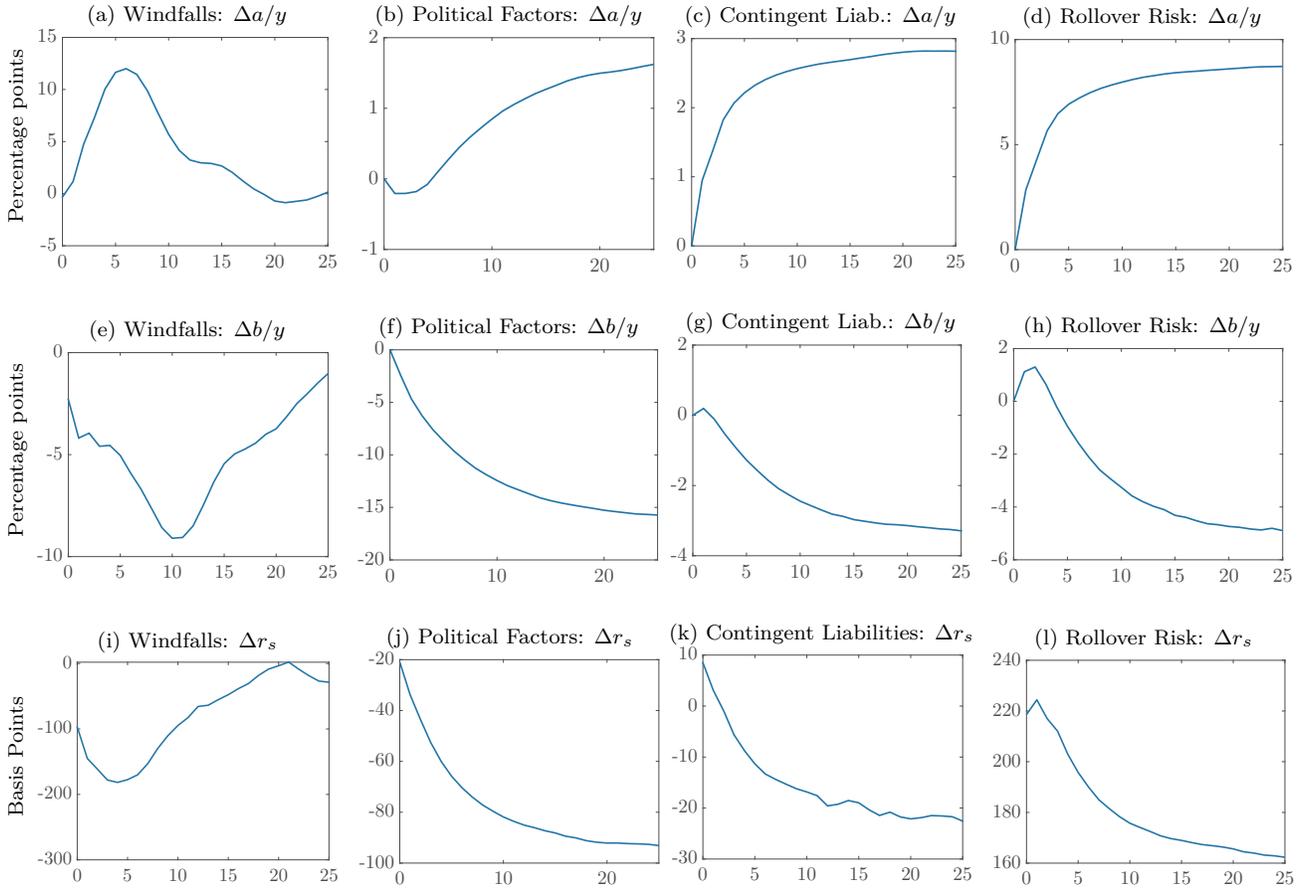
Figure 9: Transitional Dynamics.

Note: The figure presents the difference between the average value of each variable in the simulations with each development minus the average value in the benchmark simulations. Differences in the reserves and debt to income ratios are expressed in percentage points. Differences in spreads are expressed in basis points.

stronger precautionary demand for reserves. Figure 9 shows that the emergence of contingent liabilities of 10 percent of income increases the average holding of reserves by about 3 percent of income. The increases in reserves can rise sharply for larger values for contingent liabilities, as shown by Figure 10.[33] The appearance of contingent liabilities deteriorates the spread schedule as the government's ability to repay weakens, leading to an increase of spreads on impact (panel k). However, as the government reduces debt and increases reserves, spreads decline along the simulation path.

The final development we address is the increase in rollover risk. Like the increase in contingent liabilities, this leads to a stronger precautionary demand for reserves and leads qualitatively to similar patterns of debt, reserves, and spreads. In terms of magnitudes, the response of reserve

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[33]Recall that we assume that conditional on the shock occurring, the fiscal cost is always 10 percent (the average fiscal cost of a banking crisis). The nonlinearities in Figure 10 suggest that uncertainty about the magnitude of the shock may lead to larger hedging demand for reserves.
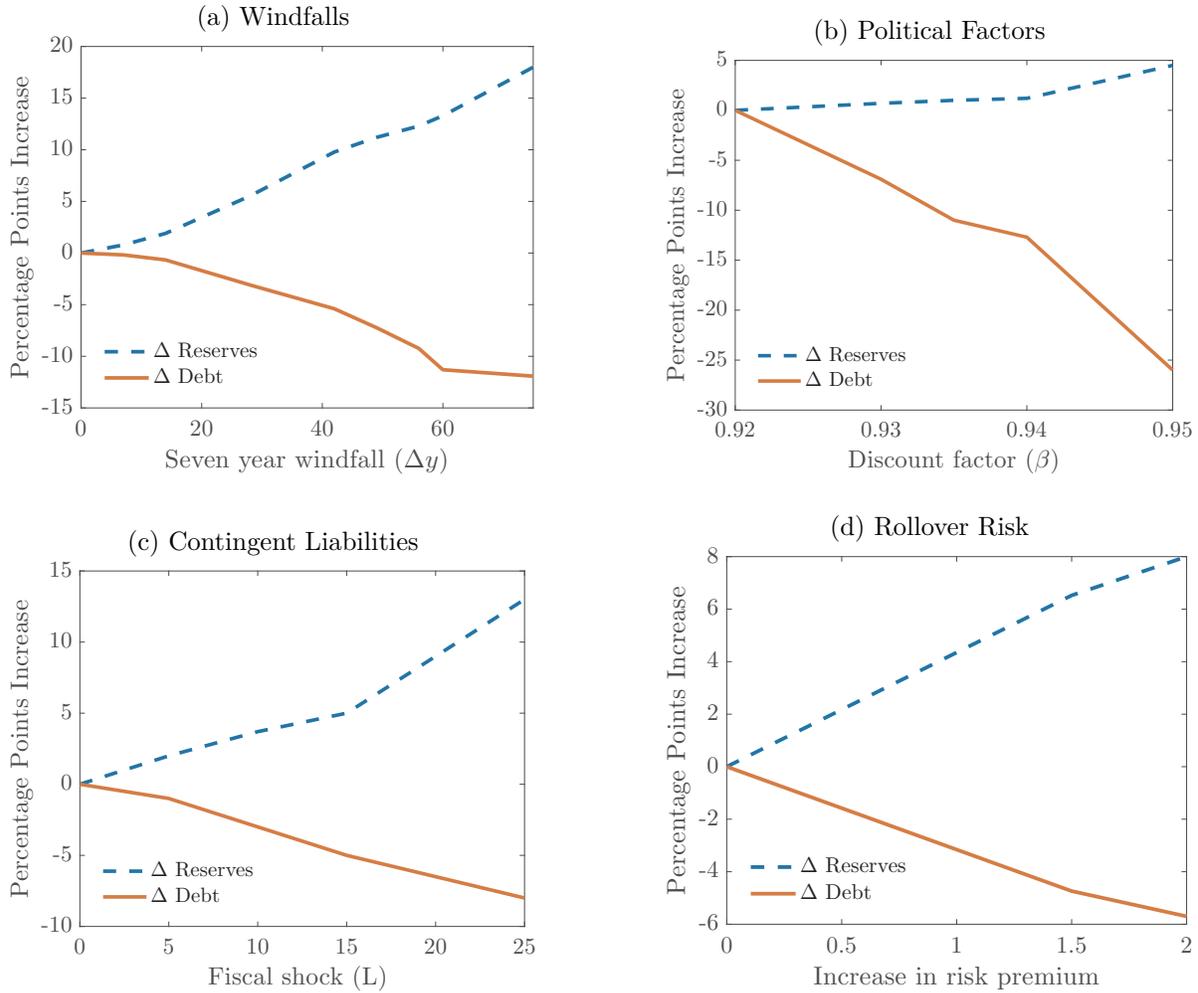
Figure 10: Sensitivity of Transitional Dynamics

Note: The figure presents the maximum difference across the simulation paths in the average levels of reserves and debt.

accumulation is much stronger, leading to an increase in reserves that is close to 10 percent of income in Figure 9.

Overall, a combination of the four developments presented in this section could easily account for the upward trend in reserve holdings in emerging economies. Moreover, these developments are also consistent with declines in debt, as observed in the data before the global financial crisis. Of course, the importance of each of these developments is likely to vary both across time and across countries.

5 Conclusions

We use a quantitative sovereign default model to study the optimal accumulation of international reserves. The model features long-term debt, risk-free assets, and shocks to foreign creditors.

We show that the optimal policy is to borrow and accumulate reserves in good times to hedge against future increases in borrowing costs. Allowing for the maturity of bonds to exceed one period is essential for understanding the role of reserves as a hedge against rollover risk.

The model is calibrated to mimic salient features of a typical emerging economy, including the average public debt and spread levels. The model simulations display an average optimal reserve ratio that is equal to 6 percent of income. The reserve ratio reaches values close to 40 percent in some simulation periods. Furthermore, the government accumulates (decumulates) both reserves and debt in periods of low (high) spread and high (low) income, which we show is consistent with the behavior of debt and reserves in a majority of emerging economies.

The paper also shows that four recent developments in emerging economies (income windfalls, improved policy frameworks, the increase in public public sector's contingent liabilities, and an increase in the severity of global shocks) imply significant increases in the optimal holding of reserves, which is consistent with the upward trend that has occurred since the 1990s. These developments also imply lower optimal debt levels, in line with what was observed in the data.

Future research could explore how self-fulfilling crises, sovereign debt composition and private capital flows affect governments' holding of reserves. The mechanisms studied in this paper are not confined to sovereign debt markets and could be relevant for understanding the financial decisions of households and corporate borrowers facing rollover risk.

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Online Apppendix to
International Reserves and Rollover Risk

Javier Bianchi

Federal Reserve Bank of Minneapolis

and NBER

Juan Carlos Hatchondo

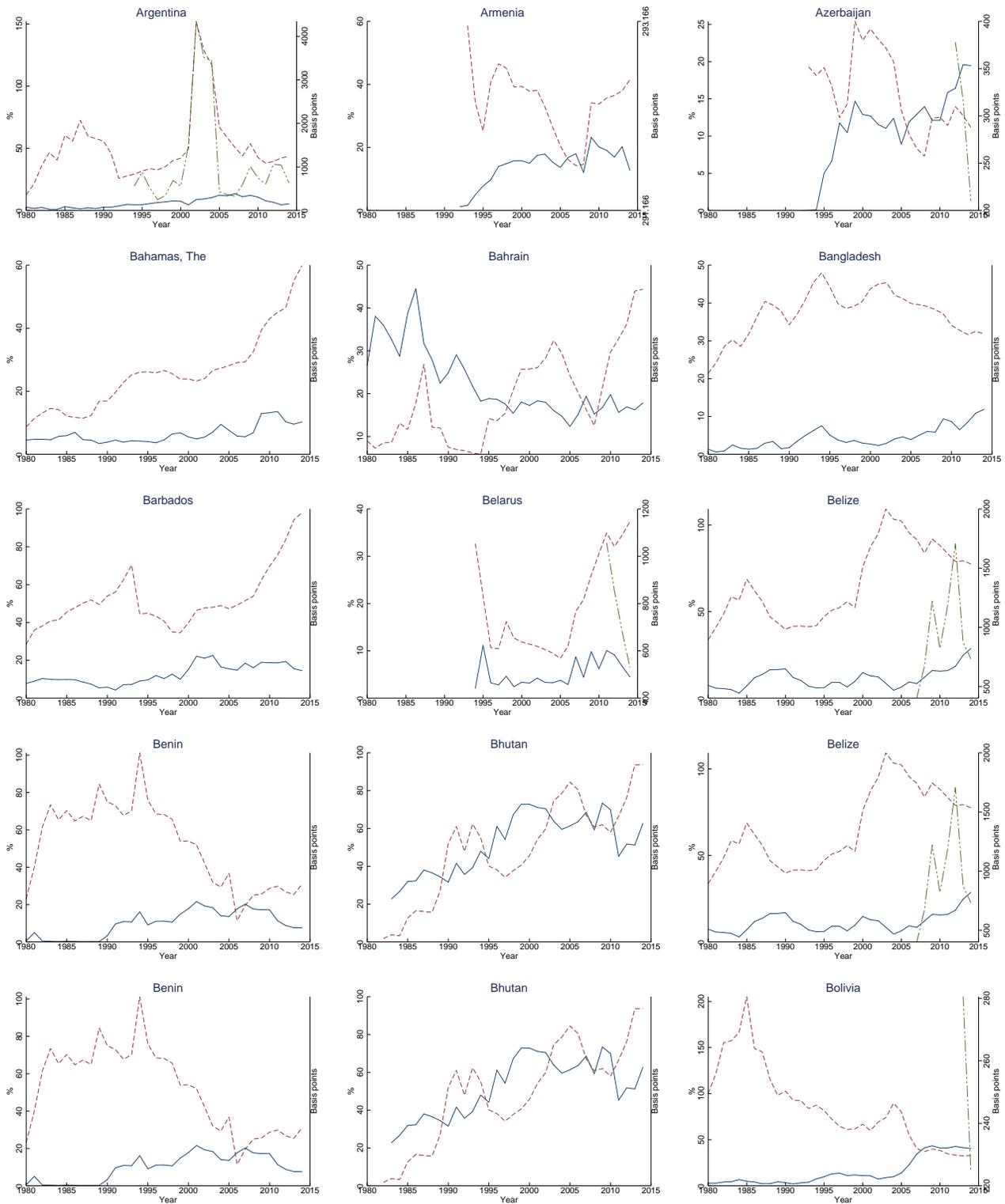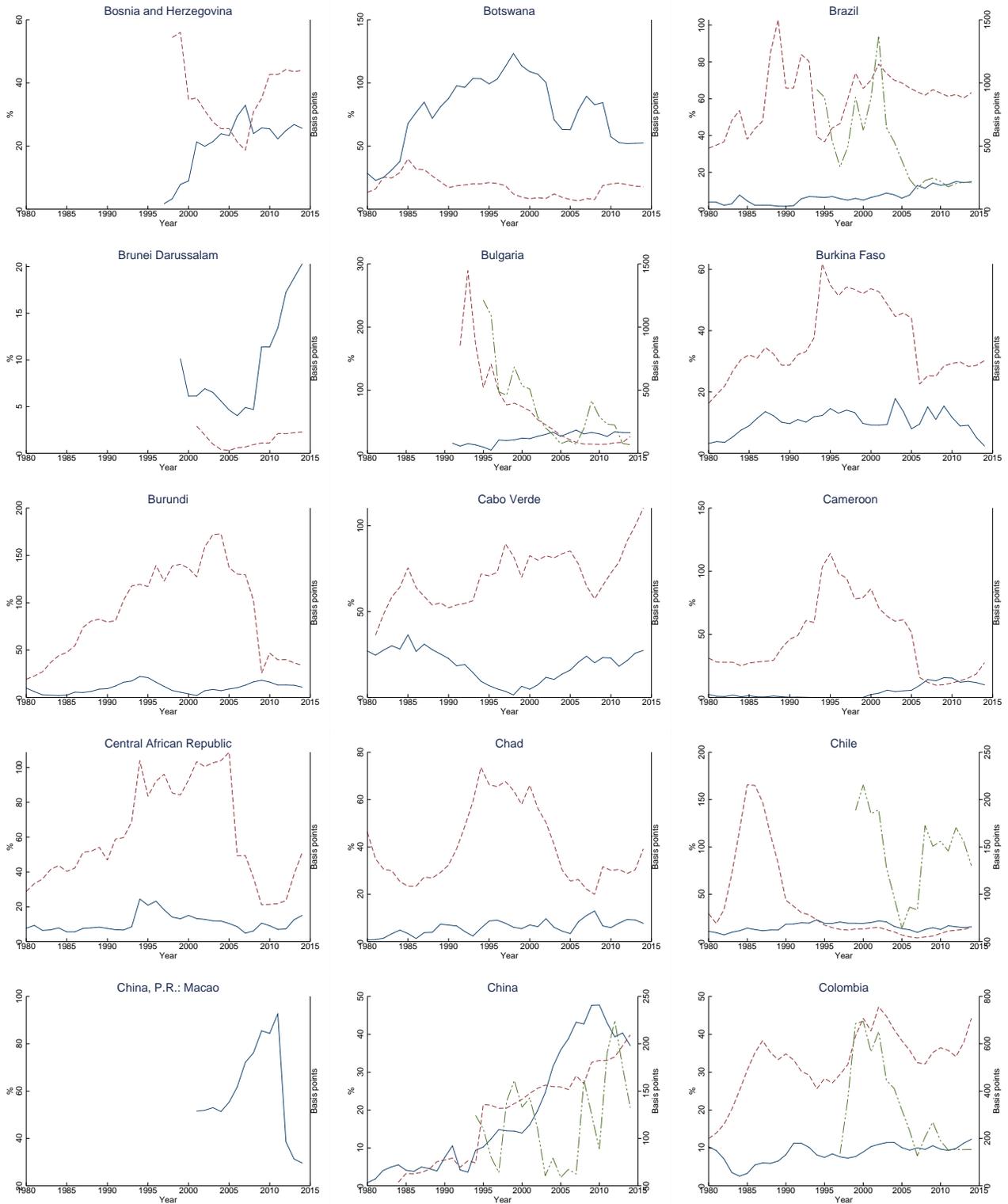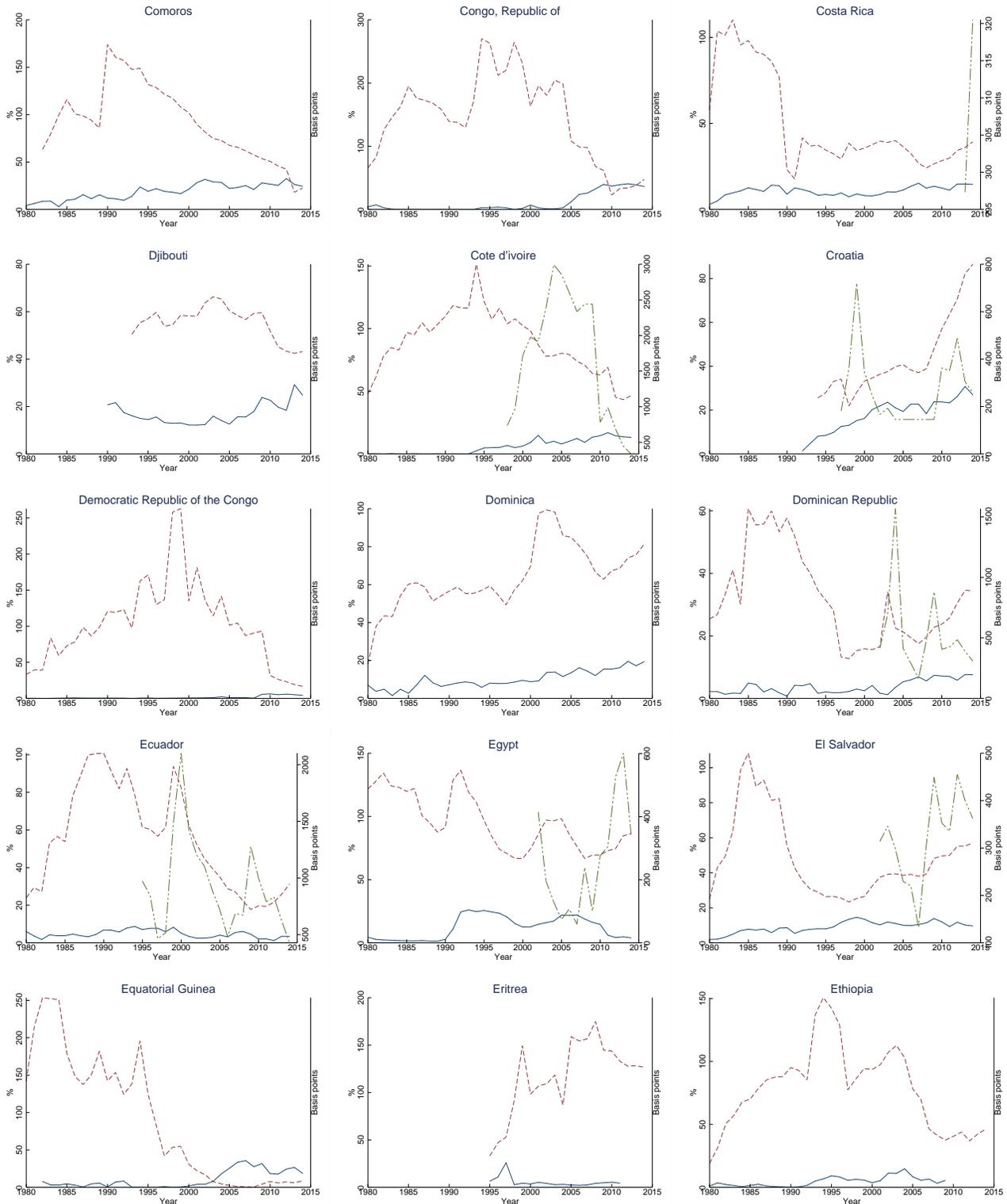Indiana University

Leonardo Martinez

IMF

November 2016

Figure 1: Sovereign spreads (dashed and dotted line; RHS), and international reserves (solid line) and public debt (dashed line) to GDP ratios. Spread data is from the Emerging Markets Bond Index Plus (EMBI+ blended). Reserves data are from the International Financial Statistics. Debt data are from the IMF Fiscal Affairs Department Historical Public Debt Database.
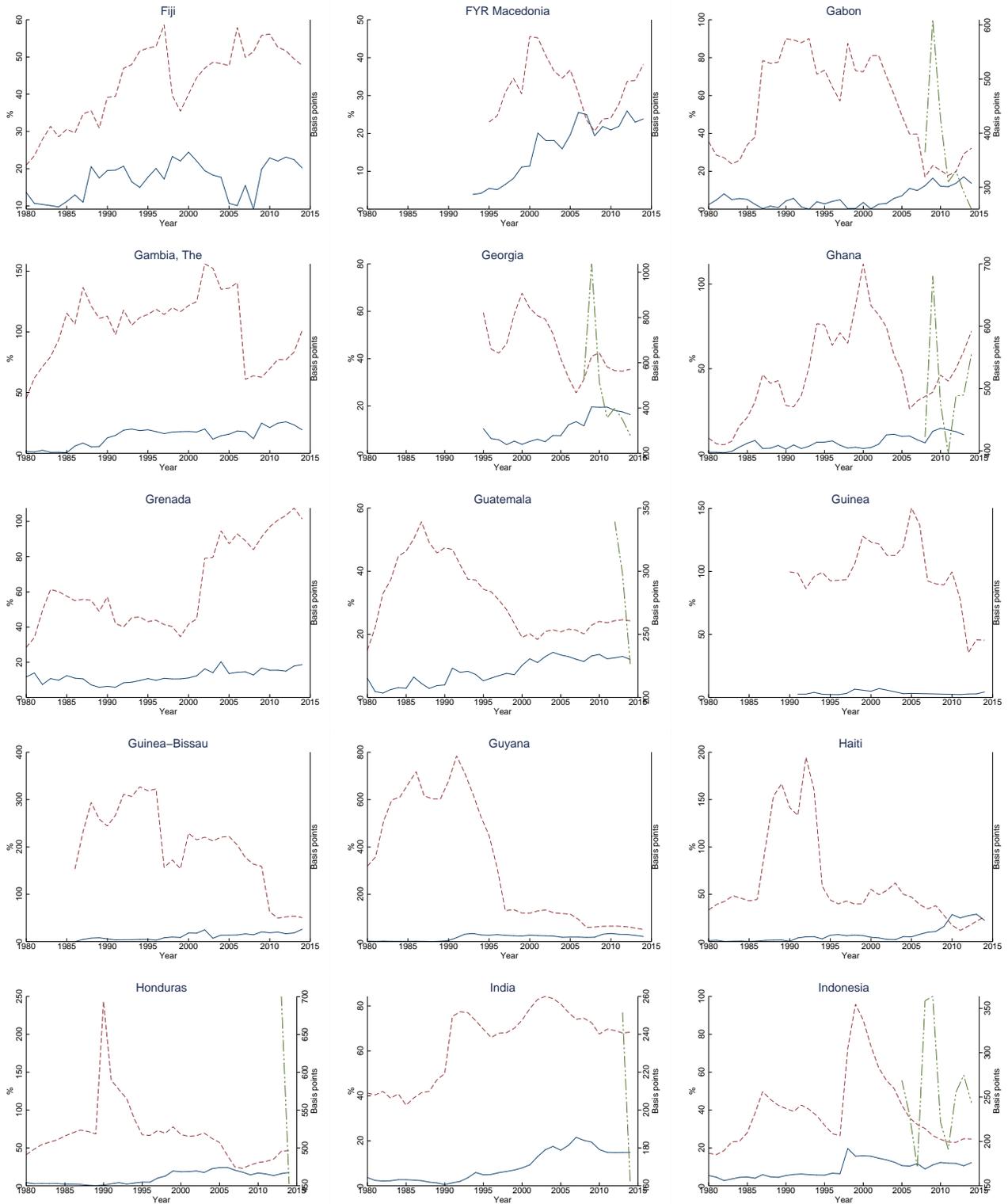
Figure 2: Sovereign spreads (dashed and dotted line; RHS), and international reserves (solid line) and public debt (dashed line) to GDP ratios. Spread data is from the Emerging Markets Bond Index Plus (EMBI+ blended). Reserves data are from the International Financial Statistics. Debt data are from the IMF Fiscal Affairs Department Historical Public Debt Database.
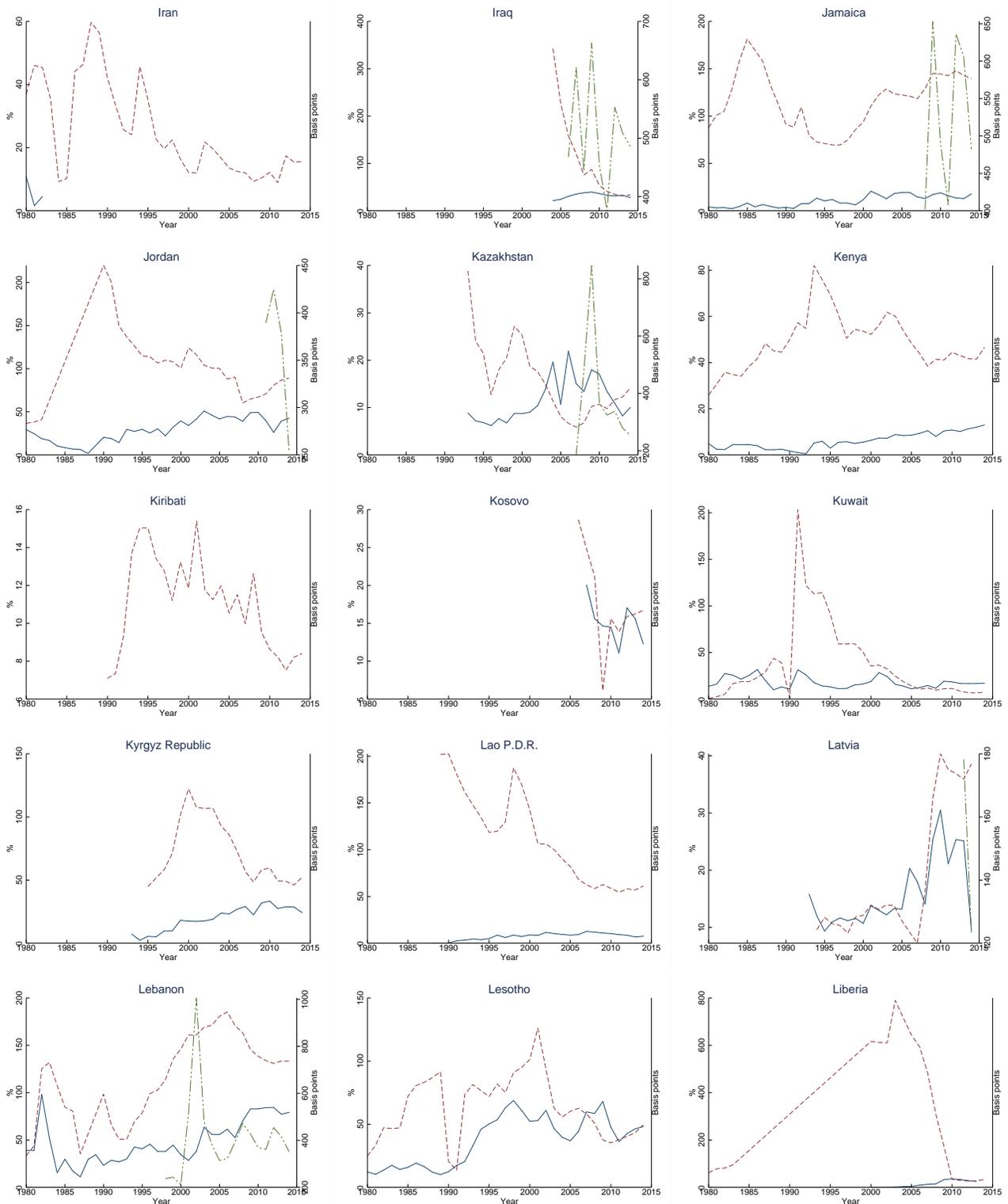
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Figure 3: Sovereign spreads (dashed and dotted line; RHS), and international reserves (solid line) and public debt (dashed line) to GDP ratios. Spread data is from the Emerging Markets Bond Index Plus (EMBI+ blended). Reserves data are from the International Financial Statistics. Debt data are from the IMF Fiscal Affairs Department Historical Public Debt Database.

Figure 4: Sovereign spreads (dashed and dotted line; RHS), and international reserves (solid line) and public debt (dashed line) to GDP ratios. Spread data is from the Emerging Markets Bond Index Plus (EMBI+ blended). Reserves data are from the International Financial Statistics. Debt data are from the IMF Fiscal Affairs Department Historical Public Debt Database.
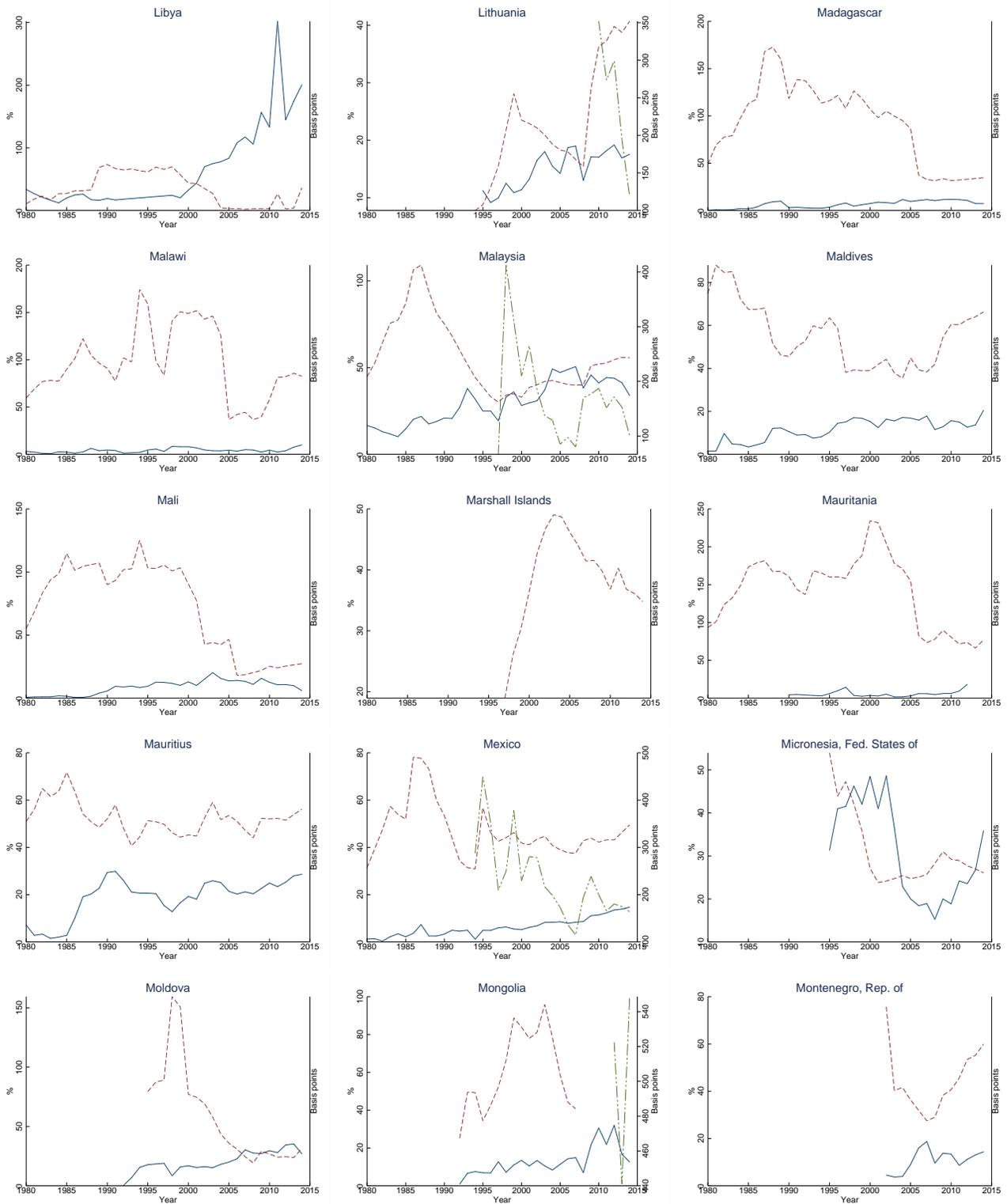
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Figure 5: Sovereign spreads (dashed and dotted line; RHS), and international reserves (solid line) and public debt (dashed line) to GDP ratios. Spread data is from the Emerging Markets Bond Index Plus (EMBI+ blended). Reserves data are from the International Financial Statistics. Debt data are from the IMF Fiscal Affairs Department Historical Public Debt Database.
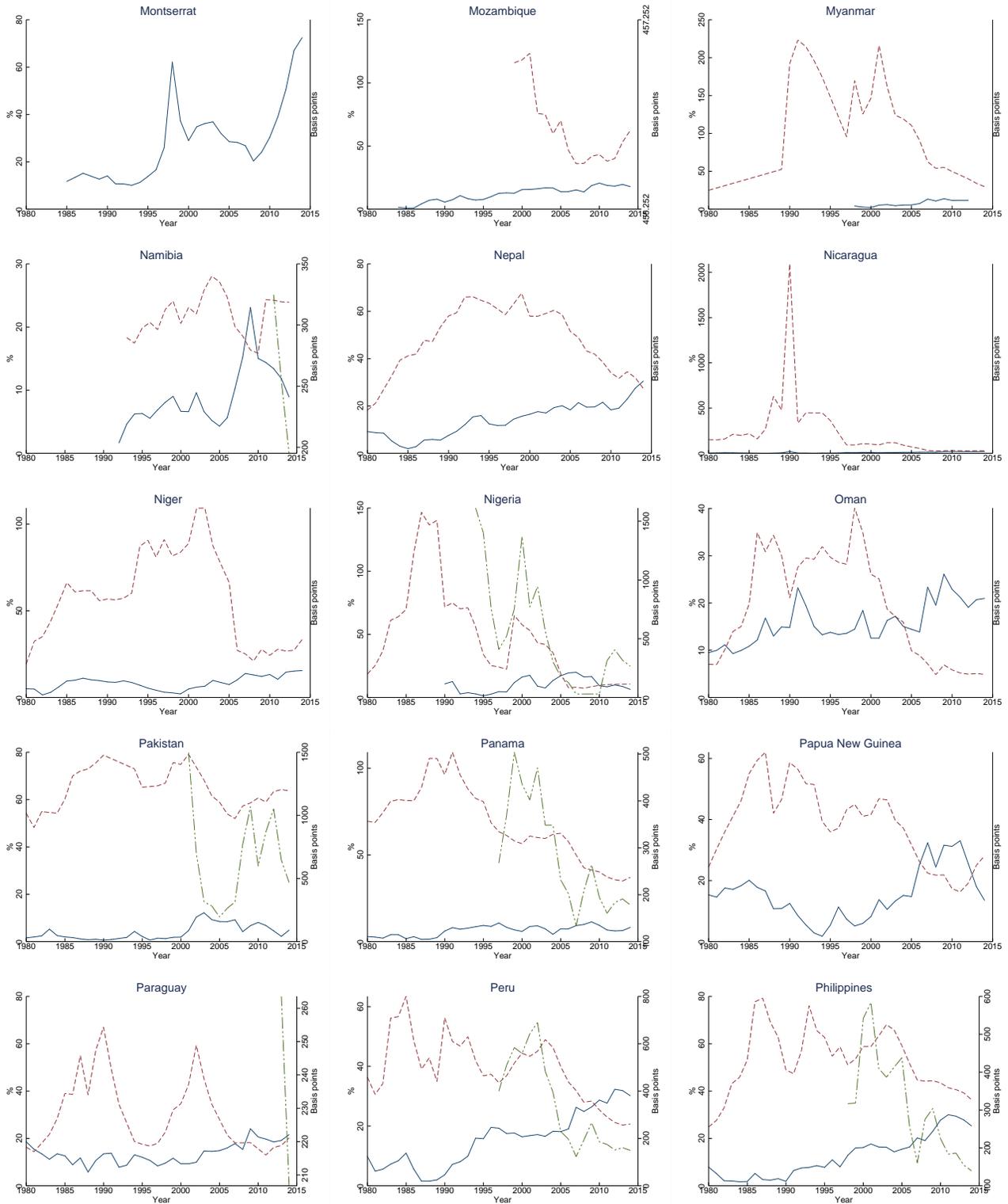
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Figure 6: Sovereign spreads (dashed and dotted line; RHS), and international reserves (solid line) and public debt (dashed line) to GDP ratios. Spread data is from the Emerging Markets Bond Index Plus (EMBI+ blended). Reserves data are from the International Financial Statistics. Debt data are from the IMF Fiscal Affairs Department Historical Public Debt Database.
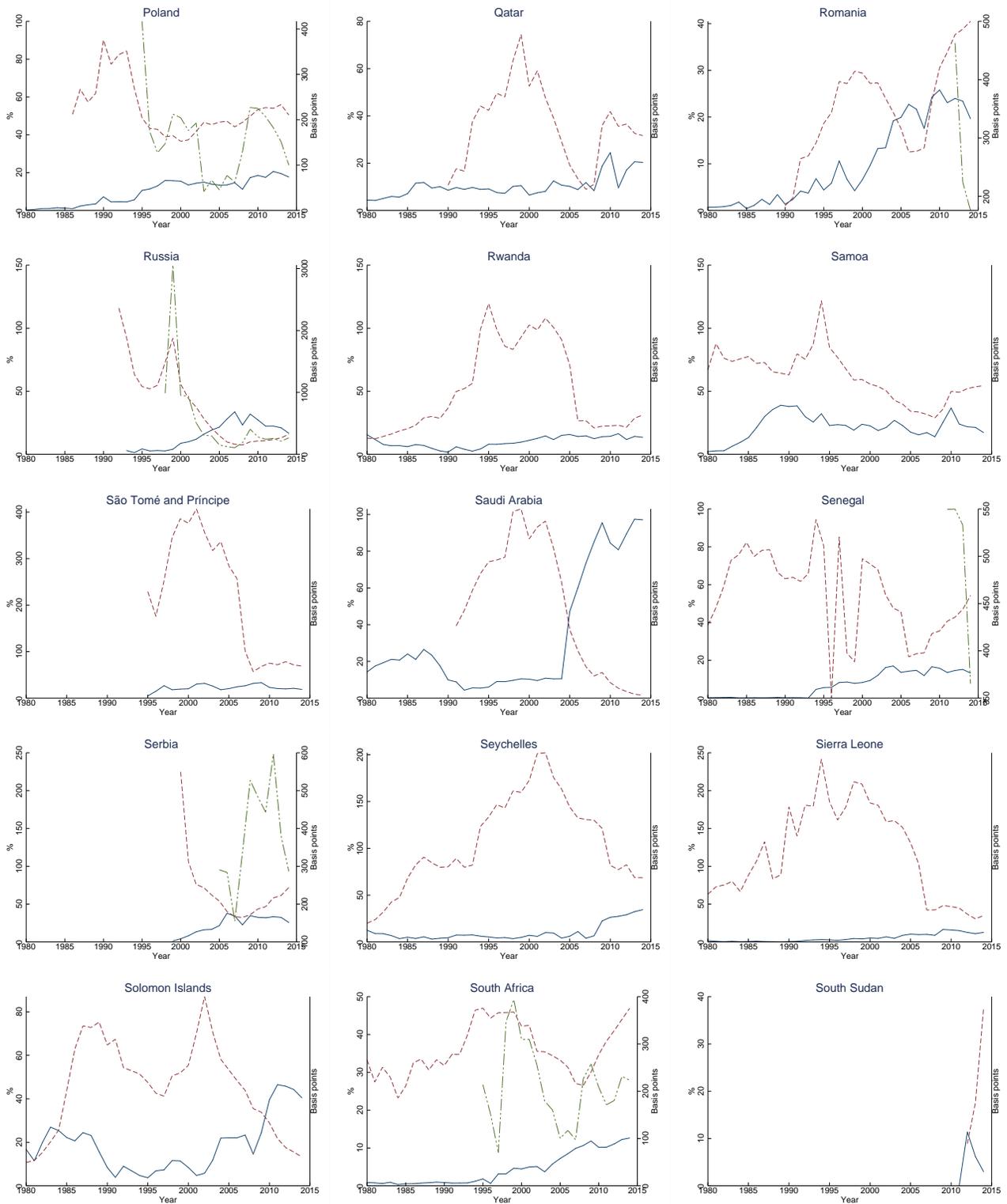
Figure 7: Sovereign spreads (dashed and dotted line; RHS), and international reserves (solid line) and public debt (dashed line) to GDP ratios. Spread data is from the Emerging Markets Bond Index Plus (EMBI+ blended). Reserves data are from the International Financial Statistics. Debt data are from the IMF Fiscal Affairs Department Historical Public Debt Database.

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Figure 8: Sovereign spreads (dashed and dotted line; RHS), and international reserves (solid line) and public debt (dashed line) to GDP ratios. Spread data is from the Emerging Markets Bond Index Plus (EMBI+ blended). Reserves data are from the International Financial Statistics. Debt data are from the IMF Fiscal Affairs Department Historical Public Debt Database.
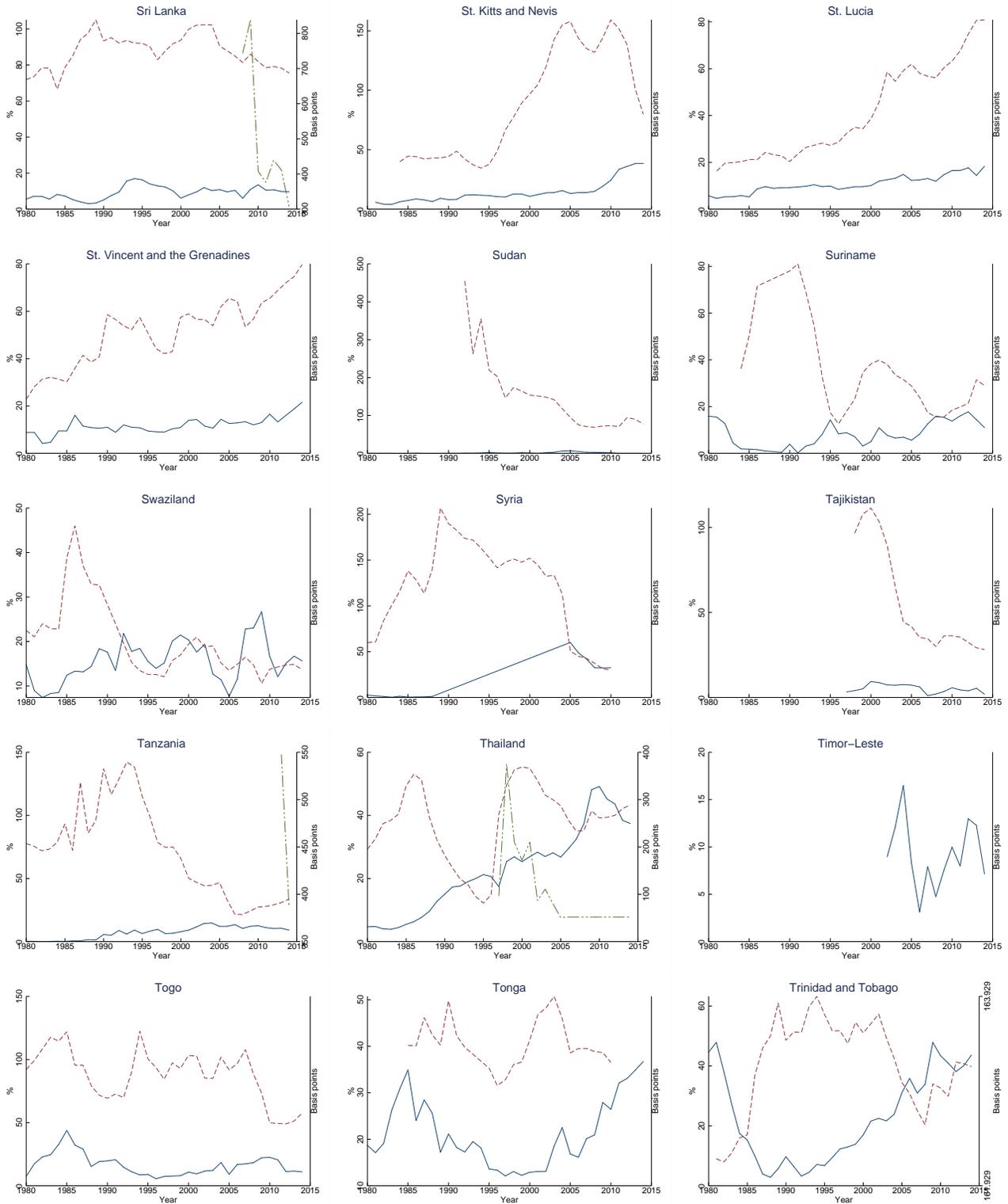
Figure 9: Sovereign spreads (dashed and dotted line; RHS), and international reserves (solid line) and public debt (dashed line) to GDP ratios. Spread data is from the Emerging Markets Bond Index Plus (EMBI+ blended). Reserves data are from the International Financial Statistics. Debt data are from the IMF Fiscal Affairs Department Historical Public Debt Database.
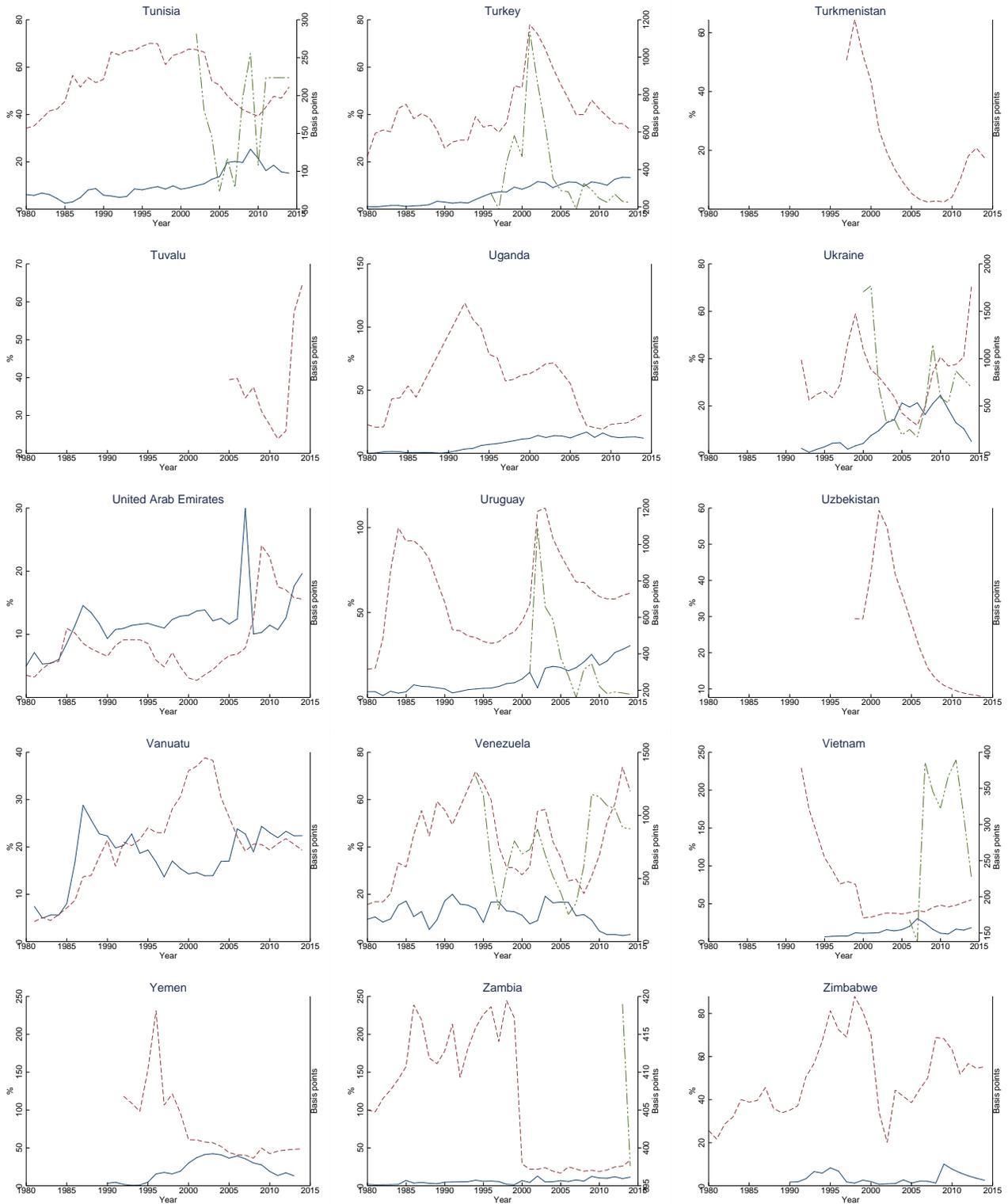
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Figure 10: Sovereign spreads (dashed and dotted line; RHS), and international reserves (solid line) and public debt (dashed line) to GDP ratios. Spread data is from the Emerging Markets Bond Index Plus (EMBI+ blended). Reserves data are from the International Financial Statistics. Debt data are from the IMF Fiscal Affairs Department Historical Public Debt Database.