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A Quantitative Theory of the Credit Score*

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Abstract

What is the role of credit scores in credit markets? We argue that it is a stand in for a market assessment of a person's unobservable type (which here we take to be patience). We pose a model of persistent hidden types where observable actions shape the public assessment of a person's type via Bayesian updating. We show how dynamic reputation can incentivize repayment without monetary costs of default beyond the administrative cost of filing for bankruptcy. Importantly we show how an economy with credit scores implements the same equilibrium allocation. We estimate the model using both credit market data and the evolution of individual's credit scores. We find a 3% difference in patience in almost equally sized groups in the population with significant turnover and a shift towards becoming more patient with age. If tracking of individual credit actions is outlawed, the benefits of bankruptcy forgiveness are outweighed by the higher interest rates associated with lower incentives to repay.

Keywords: Credit Scores, Unsecured Consumer Credit, Bankruptcy, Persistent Private Information.

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1 Introduction

Credit scores are a fundamental ingredient of a borrower's access to credit. In the United States, credit bureaus and credit rating agencies serve this function for individual and business credit by creating and maintaining credit scores for individual borrowers. Similar agencies exist in many other countries. Credit scores affect borrowing terms and change with credit use and repayments. Despite the widespread use of credit scores in actual credit markets, they are conspicuously absent from standard quantitative models of consumer default which are typically more concerned with allocations than the contractual arrangements that generate them.

In this paper we provide a theory of the joint behavior of unsecured credit and credit scores, that is, both allocations and arrangements, over the lifetime that is based on hidden information about some persistent, credit-relevant individual characteristic – which we take to be patience – and where maintaining a good reputation plays a central role.

Our theory is founded on the premise that an individual's true propensity to repay — i.e., the individual's true *type* — is hidden from her creditors, and it is the presence of this *persistent hidden information* that makes an individual's history of actions relevant for lenders. Our theory is dynamic: at any point in time, lenders use a person's observable history of actions to perform a Bayesian update of her type; individuals understand this and choose actions mindful of the consequence any action has on the future beliefs of lenders. A loss of reputation, rather than stigma or exogenous exclusion from future borrowing, is the only dynamic punishment from default. Specifically, an individual's credit score falls upon default and she subsequently faces worse borrowing terms. Our theory is competitive: information available to any lender is assumed to be available to all lenders and there is free entry into the business of lending. Finally, our theory respects a key feature of the institutional arrangement under which unsecured consumer credit is extended in the United States: at some monetary cost, individuals can choose to have their debts discharged via Chapter 7 bankruptcy.

We make several contributions. First, we extend the theory of unsecured credit to accommodate persistent hidden information about individual types. Our model environment is rich enough to cover four of the five characteristics lenders use to assess creditworthiness: character (reflected in credit history), capacity (reflected in debt-to-income ratio), capital (wealth), and conditions (amount of the loan).[1] Competition drives lending contracts to be indexed by all observable borrower and loan characteristics relevant for predicting the probability of default on a loan. When there is hidden information, a new individual characteristic becomes relevant: a Bayesian update of the borrower's type probability — in

[1] The fifth, collateral, is not relevant for unsecured credit. See, for instance, Investopedia.

the terminology of this paper, the borrower's *type score* — indicating the probability that a person is of each of the different types existing in the economy. The update conditions on all relevant observables: the individual's current type score, their current net wealth, all the relevant information to forecast future earnings, and, of course, their current action, be that to save, borrow or default. One way to interpret the large number of conditioning variables is that the lender is using "big data." Our framework easily encompasses "small data" cases in which lenders observe only some strict subset of actions.

Second, after proving an equilibrium with type scores exists, we show that a market arrangement which uses credit scores as in modern societies replicates the same equilibrium allocation without any reference to type scores. Specifically, we use the type score to define a *credit score* – an object that yields a ranking of individuals with regard to their probability of default on a particular contract. Such an ordinal ranking is widely used by credit bureaus. We provide a sufficient condition such that the equilibrium of the arrangement that uses credit scores to index contracts has the same allocation as the equilibrium of our baseline economy with type scores. Just as agents take prices as given in standard competitive equilibrium models, in our equilibrium with credit scores individuals and lenders take credit-score-dependent prices and the distribution of future credit scores conditional on their actions as given; they do not need to know what is behind such functions, just that they exist. In doing so, we provide a theory of the credit score itself and of how it evolves over time in response to fundamentals of the economy. In this context, we take to heart that the actual market arrangement is a form of data and our equivalence result allows for the use of such data for empirical purposes.

Third, we take our model to the data, estimating preference parameters from the joint behavior of credit scores over an individual's lifetime and aggregate credit market moments. It is here that our decision to model age variation in the evolution of earnings and hidden characteristics pays off. For these estimates, we verify that the sufficient condition which guarantees equivalence between the type score economy and credit score economy holds. We find what we believe are important properties of the U.S. population with regard to (hidden) patience as revealed by the properties of the credit market: the difference between patient and impatient people are 3% annually, slightly less than three quarters of people are born impatient while slowly tilting towards patience (by age 60 slightly less than half of them have become patient), and patience is highly persistent. Nevertheless, these infrequent changes in type, along with the estimated small variation in extreme value shocks, prevent excessively fast learning about an individual's type.[2]

Fourth, we use our estimates to explore the role of hidden information in the U.S. unsecured credit

[2]An empirical literature that finds evidence of adverse selection in credit markets includes Ausubel (1999), Agarwal et al. (2010), Hertzberg et al. (2018), and Einav et al. (2013).

market. We start by considering a policy counterfactual in which lenders are prohibited from keeping track of the history of an individual's asset market actions but can condition on the observable length of an individual's credit history (effectively their age). In this case, impatient types are pooled with patient types without having to bear the costs of imitating them in order to obtain better borrowing terms. Since young adults wish to borrow against their higher expected future income, and most start their adult life impatient, the policy has the possibility of improving the welfare of those young adults. However, the policy removes the incentives to maintain a good reputation which leads to individuals facing higher interest rate offerings. We find the negative incentive effects offset the potential pooling benefits for impatient young adults such that all young adults are made worse off.

Our second counterfactual considers an economy where one's type is perfectly observable. The findings are intuitive. Since the impatient are now known, they face a more adverse situation; their interest rates are higher and they borrow less. The contrary is true for the patient. As people age, this knowledge becomes less relevant because people accumulate precautionary balances and rarely borrow. We also find that whatever changes do occur at the individual level roughly offset each other at the aggregate level so that the aggregate differences between the hidden and full information worlds are minor.

Fifth, we make a couple of methodological contributions. First, we combine both screening and dynamic signalling where these screening and signalling opportunities are constrained by noise which we introduce via extreme value shocks.[3] The shocks ensure that beliefs held by lenders following any feasible action are determined in equilibrium (reminiscent of Selten (1975) and Myerson (1978)).[4] Second, we extend quantitative theory models of default to include hidden information which requires us to index

[3]The microeconomic literature classifies hidden knowledge/adverse selection models as "screening" or "signaling" models (Riley (2001)). As in screening models, in our paper lenders offer loans distinguished by loan characteristics (size of the loan) and observable personal characteristics (income, previous history) that give ample scope for separation (if such separation is desirable from an individual point of view and can be sustained in equilibrium). And, as in signaling models, there are actions that an individual can take that have no effect on the payoff to any creditor but which convey valuable information to them. For instance, when an individual who has no debt (and therefore is not interacting with any creditors) takes an action that reveals information about her true type (such as increasing her savings), that action has signaling value for creditors in the event she were to borrow from them in some future period. Thus there is also ample scope for individuals to signal their type for use in some future contract via an update on their type score.

[4]Even if there was no hidden information and no Bayesian updating of beliefs, a continuous support of shocks would be needed to ensure the existence of a *pure* strategy equilibrium; otherwise existence would require that people be allowed to play mixed strategies. Despite individuals playing pure strategies, the shocks ensure different types may choose the same action analogous to a "semi-separating" equilibrium. Further, the assumption that the shocks are drawn from a Type 1 extreme value distribution delivers the tractability as in McFadden (1974) and Rust (1987).

the pricing of credit to the market assessment of individual types.[5,6,7] While previous quantitative theory models imposed exogenous punishment, we incorporate dynamic reputation as a means of disciplining borrowers.[8]

The paper is organized as follows. Section 2 describes our baseline economy with private information. Section 3 describes the equilibrium problems faced by our agents. Section 4 studies the properties of the baseline model. Section 5 describes how we map the model to the data. Section 6 compares our baseline economy to alternative economies with different information structures. Section 7 concludes. There is an accompanying online appendix where we provide additional theoretical, computational, and data results.[9]

2 Environment

Time is discrete and infinite. At each point in time, there is a unit measure of individuals. An individual dies with probability $1 - \rho$ at the end of the period and individuals who die are replaced by newborns. An individual's earnings class, denoted $e_t \in \mathcal{E} = \{e_1, e_2, ..., e_E\} \subset \mathbb{R}_{++}$, is exogenously drawn from a stationary finite state Markov process $Q^e(e_{t+1}|e_t)$. In addition, there is a purely transitory component to an individual's earnings, denoted $z_t \in \mathcal{Z} = \{z_1, z_2, \dots, z_Z\} \subset \mathbb{R}_{++}$, which is exogenously drawn from a stationary probability distribution $H(z_t)$. All earnings draws are independent across individuals.

[5] Gale (1992) and Dubey and Geanokoplos (2002) prove existence of competitive equilibrium in environments with hidden information. In contrast to us, they adopt the *anonymous markets assumption* of classical GE theory which does not permit prices to depend on personal characteristics of buyers or sellers (such as a credit score). Prescott and Townsend (1984) characterize constrained efficient allocation in an adverse selection environment but show that there is no natural decentralization of it via a price system. Guerreri et al. (2010) prove existence and uniqueness of separating equilibria in static adverse selection models by expanding the contract space to include competitive search over submarkets which helps sustain separation. Our framework expands the contract space to include dynamic type scores which are used to help separate borrowers.

[6] The observation that the competitive pricing of defaultable debt requires indexing the price of the loan to some observable characteristics like its size appeared in a clear form in Jaffee and Russell (1976) and Eaton and Gersovitz (1981). In full information environments, a natural extension of this approach is to index the loan price by characteristics of the *borrower* as in Livshits et al. (2007b) and Chatterjee et al. (2007). A large literature on quantitative models of defaultable consumer and sovereign debt now exists (see Exler and Tertilt (2020) and Aguiar et al. (2016) for recent surveys).

[7] Other related papers which include an information problem include D'erasmo (2011), Narajabad (2012), Athreya et al. (2012), Livshits et al. (2016), Drozd and Serrano-Padial (2017), Luo (2017), Sanchez (2018), and Nelson (2020).

In the use of history to condition prices, our extension shares a strong connection to the developing microeconomic literature that studies the conditioning of prices on customers' purchase histories (see, for instance, Acquisti and Varian (2005)).

[8] Our reputational environment, where everyone optimizes but people have hidden knowledge about their preferences is closely linked to the literature on repeated games with incomplete information (see Peski (2014) for a discussion of this literature). Reputation in debt markets in which one player is a commitment type have been studied by Diamond (1989), Elul and Gottardi (2015), and Amador and Phelan (2018). The fact that reputation in one market may discipline behavior in another market has been considered in Cole and Kehoe (1998), Chatterjee et al. (2008), Corbae and Glover (2018), and Carter et al. (2019).

[9] `https://sites.google.com/a/wisc.edu/deancorbae/research/Appendix_theory_of_credit_scores.pdf`

At time t individuals can choose $a_{t+1} \in \mathcal{A} = \{a_1, a_2, ..., a_N\} \subset \mathbb{R}$ at discount price q_t determined in a competitive market. We assume the finite set $\mathcal{A}$ includes 0 with $a_1 < 0$ and $a_N > 0$. If an agent holds debt (i.e. $a_t < 0$), she (or he) can choose whether or not to default $d_t \in \{0, 1\}$. If she defaults (i.e. $d_t = 1$), then in the period of default *only* she cannot borrow or save (i.e. $a_{t+1} = 0$) and her earnings become $e_t + z_t - \kappa$ where $\kappa > 0$ is the static cost of default (e.g. bankruptcy filing fees). Therefore, the set of all possible choices is a discrete set of size $N + 1$.

In each period t, the individual values consumption c_t using a utility function $u(c_t) : \mathbb{R}_{++} \to \mathbb{R}$ which is is continuous, increasing, and concave. At time t, an individual discounts her future utility at rate $\beta_t \in \mathcal{B} = \{\beta_1, \beta_2, ..., \beta_B\}$ if she survives. Her discount factors varies stochastically over time drawn from a finite state Markov process $Q^{\beta}(\beta_{t+1}|\beta_t)$. The β_t are drawn independently across individuals and are unobservable to others. We will call $\beta_t \in [0, 1)$ a household's *type*.

In addition, households receive a vector

$$\epsilon_t = (\epsilon(0, a_1), ..., \epsilon(0, a_N), \epsilon(1, 0))$$

of action-specific, additively separable preference shocks each period. These preference shocks follow a generalized extreme value distribution whose cumulative is given by

$$F(\epsilon_t) = \exp\left\{-\left[\sum_{a_{t+1} \in \mathcal{A}} \exp\left(-\frac{\epsilon(0, a_{t+1}) - \mu}{\lambda\alpha}\right)\right]^{\lambda} - \exp\left(-\frac{\epsilon(1, 0) - \mu}{\alpha}\right)\right\} \tag{1}$$

where $\alpha > 0$ is a parameter governing the variance of the shocks and $\lambda \in [0, 1]$ is a parameter related to the correlation of shocks associated with non-default actions.[10] We set $\mu = -\alpha\gamma_E$ to make the shocks mean zero in expectation where $\gamma_E = 0.56767...$ is Euler's constant.

Intermediaries can observe individuals' earnings class (i.e. e_t) and asset market behavior (i.e. a_t, d_t, and a_{t+1}), but cannot observe their preferences (i.e. ϵ_t and β_t) nor the transitory component of earnings (i.e. z_t). Since ϵ_t and z_t are i.i.d. over time and individuals, there is nothing to be learned about their future values from their current values. However, since β_t is drawn from a persistent Markov process, there is something to be learned. We denote the creditor's probability assessment that an individual is of type β_i at the beginning of period t before any actions are taken as $s_t(\beta_i) = \Pr(\beta_t = \beta_i)$. We call

[10] A nested formulation allows us to address the well-known issue of independence of irrelevant alternatives (IIA) in logit models. In our context, this means that one's probability of default depends on (i) the value of defaulting and (ii) the "inclusive" value of not defaulting (as opposed to the value of each non-default action). This latter term is negligibly affected by changes in the exact location and number of points on the asset grid, a property we view as desirable in our economic context.

$s_t = (s_t(\beta_1), ..., s_t(\beta_B))$ an individual's *type score* with $\sum_{i=1}^{B} s_t(\beta_i) = 1$.[11] In what follows we assume $s_t \in \mathcal{S}$, a finite subset of $[0,1]^B$. This assumption makes it possible to apply standard methods to prove existence of equilibrium.

Given an individual's observable characteristics $\omega_t = (e_t, a_t, s_t)$ as well as their credit market actions (d_t, a_{t+1}), the financial intermediary revises its assessment of an individual's type from s_t via Bayes' rule. We denote this update as $\psi_t^{(d_t,a_{t+1})}(\omega_t) \in [0,1]^B$. Finally, since the posterior ψ_t may not lie on one of the finite points in $\mathcal{S}$, we assign it randomly to nearby points in $\mathcal{S}$. Specifically, We denote the probability mass function implied by our random assignment rule given in equation (20) in Online Appendix A.1 as $Q^s(s_{t+1}|\psi_t)$.[12] There we prove:

Lemma 1. *There exists an assignment rule satisfying: (i)* $\mathbb{E}_{s_{t+1}\in\mathcal{S}}\left[Q^s(s_{t+1}|\psi_t)\right] = \psi_t$ *(i.e. consistency), (ii) the variance of the approximation error (i.e. of* s_{t+1} *from* ψ_t*) is arbitrarily small, and (iii)* $Q^s(s_{t+1}|\psi_t)$ *is continuous in* ψ_t*.*

Importantly, note that there is no further punishment to default except possibly loss of reputation since an individual's credit market behavior affects the intermediary's assessment of her unobservable type.

As a result of this assessment, the prices faced by an individual in the credit market will also depend on her observable state and her credit market actions. Thus, we denote the price function for an individual with observable characteristics ω_t who chooses assets a_{t+1} by $q_t^{(0,a_{t+1})}(\omega_t)$.[13] Note that in the absence of private information regarding type, the pricing function would be independent of a_t (as in fact is the case in Chatterjee et al. (2007)). These values influence prices directly because they affect the likelihood of default next period on a loan *conditional* on type (as in standard debt and default models) and they do so indirectly by revealing information about the individual's current type (this is encoded in the update $\psi_t^{(0,a_{t+1})}(\omega_t)$).

Definition 1. The *timing* in any given period is as follows:

1. Individuals begin period t with the vector (β_t, e_t, a_t, s_t) and receive a transitory earnings shock z_t.

2. An individual receives a random utility vector ϵ_t and chooses a feasible action given prices $q_t^{(0,a_{t+1})}(\omega_t)$.

[11] Of course the framework is rich enough to add more unobservables. For instance, if the persistent component of earnings are unobservable, then $s_t = (s_t(\beta_1, e_1), ..., s_t(\beta_B, e_E))$.

[12] There is nothing of substance in this lottery over contiguous elements of $\mathcal{S}$ since we use a very fine grid when we estimate the model.

[13] The "0" reminds us that the price schedule is only relevant for individuals who have not defaulted.

3. Based on each individual's actions (d_t, a_{t+1}) and observable characteristics ω_t, intermediaries revise their assessments of an individual's type via Bayes' rule, updating s_t to ψ_t.

4. (a) Individuals who survive draw beginning-of-next-period realizations of β_{t+1} and e_{t+1} from the exogenous transition functions $Q^{\beta}(\cdot|\beta_t)$ and $Q^{e}(\cdot|e_t)$. The beginning of next period type score s_{t+1} is drawn from the probability mass function $Q^{s}(\cdot|\psi_t)$.

   (b) Newborns begin life with β_{t+1} drawn from initial distribution G_β, earnings class e_{t+1} drawn from initial distribution G_e, zero assets, and a type score s_{t+1} equal to G_β for consistency. We assume $G_\beta \in \mathcal{S}$.

3 Equilibrium

3.1 Individuals' problem

Let a current value x_t be denoted x and next period's variable x_{t+1} be denoted x'. Denote the part of the state space observable to creditors by $\Omega = \{\mathcal{E} \times \mathcal{A} \times \mathcal{S}\}$ with typical element ω.

An individual who considers whether to default d_t or choose asset a_{t+1} in state $(\epsilon, \beta, z, \omega)$ takes as given

- the price function $q^{(0,a')}(\omega) : \mathcal{A} \times \Omega \to [0,1]$
- the type scoring functions $\psi^{(0,a')}(\omega) : \mathcal{A} \times \Omega \to [0,1]^B$ and $\psi^{(1,0)}(\omega) : \Omega \to [0,1]^B$ which perform Bayesian updating of an individual's type based on all observables following asset choice and default, respectively.

For ease of notation, we will denote the triplet of functions $\{q^{(0,a')}(\omega), \psi^{(0,a')}(\omega), \psi^{(1,0)}(\omega)\}$ by $f \in F$, where $F = \{(f_1, f_2, f_3)|\ f_1 : \mathcal{A} \times \Omega \to [0,1]\,, f_2 : \mathcal{A} \times \Omega \to [0,1]^B \text{ and } f_3 : \Omega \to [0,1]^B\}$.

Definition 2. Given (z, ω) and $f \in F$, the *set of feasible actions* is a finite set $\mathcal{F}(z, \omega|f)$ that contains all actions (d, a') such that consumption $c^{(d,a')}(z, \omega|f)$ is strictly positive where:

$$c^{(d,a')}(z, \omega|f) = \begin{cases} e + z + a - q^{(0,a')}(\omega) \cdot a' & \text{if } (d, a') = (0, a'), \\ e + z - \kappa & \text{if } a < 0 \text{ and } (d, a') = (1, 0). \end{cases} \tag{2}$$

We make the following assumption:

Assumption 1. $e_1 + z_1 + \min\{-\kappa, a_1\} > 0$.

This assumption ensures that it is always feasible to default and always feasible to pay back one's debt.

Given an individual's state and the functions f, we denote by $V(\epsilon, \beta, z, \omega|f) : \mathbb{R}^{(N+1)} \times \mathcal{B} \times \mathcal{Z} \times \Omega \to \mathbb{R}$ the value function of an individual in state $(\epsilon, \beta, z, \omega)$. An individual's recursive decision problem is given by

$$V(\epsilon, \beta, z, \omega|f) = \max_{(d,a') \in \mathcal{F}(z,\omega|f)} v^{(d,a')}(\beta, z, \omega|f) + \epsilon(d, a'). \tag{3}$$

Here, for $(d, a') \in \mathcal{F}(z, \omega|f)$,

$$\begin{aligned} v^{(d,a')}(\beta, z, \omega|f) &= (1 - \beta\rho) u \left(c^{(d,a')}(z, \omega|f) \right) \\ &\quad + \beta\rho \cdot \sum_{(\beta', z', e', s')} Q^{\beta}(\beta'|\beta) Q^{e}(e'|e) H(z') Q^{s}(s'|\psi^{(d,a')}(\omega)) W(\beta', z', \omega'|f) \end{aligned} \tag{4}$$

is the *conditional value function* and the *expected value function* W integrates the value function over transitory preference shocks given by

$$W(\beta, z, \omega|f) = \int V(\epsilon, \beta, z, \omega|f) dF(\epsilon). \tag{5}$$

Let $\sigma^{(d,a')}(\beta, z, \omega|f)$ be the probability that the individual in state (β, z, ω) chooses action $(d, a') \in \mathcal{F}(z, \omega|f)$. Given the form of the extreme value distribution, the probability of default for $a < 0$ has the following well-known form (see, for instance, Rust (1987)) is

$$\sigma^{(1,0)}(\beta, z, \omega|f) = \begin{cases} \dfrac{\exp\left\{v^{(1,0)}(\beta,z,\omega|f)/\alpha\right\}}{\exp\left\{v^{(1,0)}(\beta,z,\omega|f)/\alpha\right\} + \left[\sum_{(0,a') \in \mathcal{F}(z,\omega|f)} \exp\left\{v^{(0,a')}(\beta,z,\omega|f)/\lambda\alpha\right\}\right]^{\lambda}} & \text{for } a < 0 \\ 0 \text{ for } a \geq 0. \end{cases} \tag{6}$$

Conditional on not defaulting, the choice probabilities are given by

$$\tilde{\sigma}^{(0,a')}(\beta, z, \omega|f) = \begin{cases} \dfrac{\exp\left\{v^{(0,a')}(\beta,z,\omega|f)/\lambda\alpha\right\}}{\sum_{(0,\tilde{a}') \in \mathcal{F}(z,\omega|f)} \exp\left\{v^{(0,\tilde{a}')}(\beta,z,\omega|f)/\lambda\alpha\right\}} & \text{for } (0, a') \in \mathcal{F}(z, \omega|f) \\ 0 \text{ for } (0, a') \notin \mathcal{F}(z, \omega|f). \end{cases} \tag{7}$$

Note that infeasible actions are assigned zero probability. Hence, the unconditional probability of choosing $a' \in \mathcal{A}$ is

$$\sigma^{(0,a')}(\beta, z, \omega|f) = \tilde{\sigma}^{(0,a')}(\beta, z, \omega|f) \left(1 - \sigma^{(1,0)}(\beta, z, \omega|f)\right). \tag{8}$$

Given this expression, the expected value of not defaulting is given by

$$W_{ND}(\beta, z, \omega|f) = \alpha \ln\left(\sum_{(0,a')\in\mathcal{F}(z,\omega|f)} \exp\left\{v^{(d,a')}(\beta, z, \omega|f)/\lambda\alpha\right\}\right) \tag{9}$$

and the expected value function is

$$W(\beta, z, \omega|f) = \begin{cases} \alpha \ln\left(\exp\left\{v^{(1,0)}(\beta, z, \omega|f)/\alpha\right\} + \exp\left\{\lambda W_{ND}(\beta, z, \omega|f)/\alpha\right\}\right) & \text{if } a < 0 \\ \alpha \ln\left(\exp\left\{\lambda W_{ND}(\beta, z, \omega|f)/\alpha\right\}\right) & \text{if } a \geq 0. \end{cases} \tag{10}$$

In Online Appendix A.2 we prove:

Theorem 1. *Given f, there exists a unique solution $W(\beta, z, \omega|f)$ to theproblem in* (3) - (5).

3.2 Intermediaries' problem

Competitive intermediaries with deep pockets have access to an international credit market where they can borrow or lend at the risk-free interest rate $r \geq 0$.

Any given intermediary takes prices q and scoring function ψ (i.e. f) as given. We assume that both losses and gains resulting from an individual's death accrue to the financial intermediary.[14]

The profit $\pi^{(0,a')}(\omega|f)$ on contract of type $(0, a')$ with agents with observables ω is:

$$\pi^{(0,a')}(\omega|f) = \begin{cases} \rho \cdot \frac{p^{(0,a')}(\omega|f)\cdot(-a')}{1+r} - q^{(0,a')}(\omega) \cdot (-a') & \text{if } a' < 0 \\ q^{(0,a')} \cdot a' - \rho \cdot \frac{a'}{1+r} & \text{if } a' \geq 0 \end{cases} \tag{11}$$

where the probability of repayment on a financial contract of type $(0, a')$ made to individuals with observable characteristics ω is denoted $p^{(0,a')}(\omega|f) : (\mathbb{R} \cap \mathcal{A}) \times \Omega \to [0, 1]$. Given perfect competition in financial intermediation and constant returns to scale in the lending technology, if a solution to the intermediary's problem exists, then optimization by the intermediary implies zero profits for strictly positive measures of contracts issued or

$$q^{(0,a')}(\omega|f) = \begin{cases} \frac{\rho \cdot p^{(0,a')}(\omega|f)}{1+r} & \text{if } a' < 0 \\ \frac{\rho}{1+r} & \text{if } a' \geq 0. \end{cases} . \tag{12}$$

[14] That is, an individual who borrows honors her obligation only if she survives and does not declare bankruptcy and, symmetrically, an intermediary that accepts a deposit is released from its obligation if the depositor does not survive to collect.

To assess an individual's probability $p^{(0,a')}(\omega|f)$ of repaying a debt next period given their current observable characteristics ω given unobservable $(\beta,\, \epsilon,\, z)$, takes two steps:

1. Assess the probability that an individual in state ω who takes action (d, a') will be of unobservable type β' next period via Bayes rule (the type scoring function $\psi_{\beta'}^{(d,a')}(\omega)$).
2. For each possible future unobservable type β', compute the individual's probability of future repayment conditional on being that type and transitions over observable characteristics and then compute the weighted sum over future types to obtain p.

Starting with step 1, an individual's probability of being type $(\beta'_1, \ldots, \beta'_B)$ next period is given by the type scoring function $\psi^{(d,a')}(\omega) = \left(\psi_{\beta'_1}^{(d,a')}(\omega), \ldots, \psi_{\beta'_B}^{(d,a')}(\omega)\right)$, where

$$\psi_{\beta'}^{(d,a')}(\omega|f) = \begin{cases} \sum_{\beta} Q^{\beta}(\beta'|\beta) \cdot \frac{\sum_z \sigma^{(d,a')}(\beta,z,\omega|f)\cdot H(z)\cdot s(\beta)}{\sum_{\hat{\beta},z}\sigma^{(d,a')}(\hat{\beta},z,\omega|f)\cdot H(z)\cdot s(\hat{\beta})} \text{ for } (d,a') \in \mathcal{F}(z,\omega|f) \\ \sum_{\beta} Q^{\beta}(\beta'|\beta)\cdot s(\beta) \text{ for } (d,a') \notin \mathcal{F}(z,\omega|f). \end{cases} \tag{13}$$

Note that in (13), the assessment uses Bayes' rule to assign the probability of an individual in observable state ω taking a feasible action (d, a') being of type β' next period. By (6) and (7), the probability of choosing any $(d, a') \in \mathcal{F}(z, \omega|f)$ is strictly positive for every β'. Hence, $\psi_{\beta'}^{(d,a')}(\omega|f)$ is well-defined in (13) for all feasible actions. Thus, since every feasible action is chosen with some probability due to the presence of extreme value shocks, we avoid having to assign off-the-equilibrium path beliefs for feasible actions. For completeness, without loss of generality, (13) also handles the case of infeasible actions.

Turning to step 2, given observable state ω, we obtain the probability of repayment the intermediary uses for pricing debt (i.e. for $a' < 0$) via:

$$p^{(0,a')}(\omega|f) = \sum_{\beta',z',e',s'} H(z')\cdot Q^{e}(e'|e)\cdot Q^{s}(s'(\beta')|\psi_{\beta'}^{(0,a')}(\omega|f))\cdot s'(\beta')\cdot\left(1-\sigma^{(1,0)}(\beta',z',e',a',s'|f)\right). \tag{14}$$

3.3 Evolution

Let $\mu(\beta, z, \omega|f)$ denote the beginning-of-period measure of individuals in state (β, z, ω) for a given f. Then, the cross-sectional distribution evolves according to

$$\mu'(\beta', z', \omega'|f) = \sum_{\beta,z,\omega} T(\beta', z', \omega'|\beta, z, \omega; f)\cdot\mu(\beta, z, \omega|f), \tag{15}$$

where

$$T(\beta', z', \omega'; \beta, z, \omega | f) = \tag{16}$$
$$\rho \cdot Q^{\beta}(\beta'|\beta) \cdot H(z') \cdot Q^{e}(e'|e) \cdot \sigma^{(d,a')}(\beta, z, \omega|f) \cdot Q^{s}(s'(\beta')|\psi_{\beta'}^{(d,a')}(\omega|f))$$
$$+ (1-\rho) \cdot G_{\beta}(\beta') \cdot H(z') \cdot G_{e}(e') \cdot \mathbf{1}_{\{a'=0\}} \cdot \mathbf{1}_{\{s'=G_{\beta}\}}.$$

The second line in equation (16) is the probability of a survivor transiting to (β', z', e', a', s') while the third line is the probability that a newborn arrives in state (β', z', e', a', s').

An invariant distribution is a fixed point $\overline{\mu}(\cdot|f) = T\,\overline{\mu}(\cdot|f)$. In Online Appendix A.3 we prove:

Lemma 2. *There exists a unique invariant distribution $\overline{\mu}(\cdot|f)$ and $\{\mu_0 T^n\}$ converges to $\bar{\mu}(\cdot|f)$ at a geometric rate for any initial distribution μ_0.*

Note that although the invariant distribution is critical for computing cross-sectional moments used to map the model to the data, none of the other equilibrium objects (i.e. the set of functions f, the value function V or the decision rule σ take μ as an argument. This simplifies the model and eases the computational burden, but is not necessary. Other specifications in which knowledge of the distribution is required are possible, but we do not consider these in the baseline model.

3.4 Existence

We can now give the definition of a stationary recursive competitive equilibrium.

Definition 3. A stationary *Recursive Competitive Equilibrium* (RCE) is a pricing function q^*, a type scoring function ψ^*, a choice probability function σ^*, and a steady state distribution $\overline{\mu}^*$ such that:

(i). Optimality: $\sigma^{(d,a')*}(\beta, z, \omega|f^*)$ satisfies (6) and (7) for all $(\beta, z, \omega) \in \mathcal{B} \times \mathcal{Z} \times \Omega$ and $(d, a') \in \mathcal{F}(z, \omega|f^*)$,

(ii). Zero Profits: $q^{(0,a')*}(\omega|f^*)$ satisfies (12) with equality for all $\omega \in \Omega$ and $(d, a') \in \mathcal{F}(z, \omega|f^*)$ with $p^{(0,a')*}(\omega|f^*)$ satisfying (14) for all $\omega \in \Omega$ and $(d, a' < 0) \in \mathcal{F}(z, \omega|f^*)$,

(iii). Bayesian Updating: $\psi_{\beta'}^{(d,a')*}(\omega|f^*)$ satisfies (13) for all $(\beta', \omega) \in \mathcal{B} \times \Omega$, and

(iv). Stationary Distribution: $\overline{\mu}^*(\beta, z, \omega|f^*)$ solves (15) for $T(\beta', z', \omega'; \beta, z, \omega|f^*)$.

The fact that there are zero profits in equilibrium implies $q^{(0,a')}(\omega|f) = \frac{\rho}{1+r}$ for $a' \geq 0$ (i.e. the price on savings is a function only of parameters). In what follows we take $F^* \subset F$ to contain only those f_1 for which $f_1(a', \omega) = \frac{\rho}{1+r}$ for $a' \geq 0$.

The key step in proving the existence of a competitive equilibrium is proving that the value function $W(\beta, z, \omega|f)$ is continuous in f. In Online Appendix A.4 we prove:

Lemma 3. *$W(\beta, z, \omega|f)$ is continuous in f and for any $(d, a') \in \mathcal{F}(z, \omega|f)$, $\sigma^{(d,a')}(\beta, z, \omega|f)$ is continuous in f.*

Using these results, we prove in Online Appendix A.5:

Theorem 2. *A stationary recursive competitive equilibrium exists.*

3.5 Equivalence to an Economy with Credit Scores

In the economy described thus far, an individual's reputation is her type score. In U.S. credit markets, one critical measure of a person's reputation is her *credit score*. A credit score is an index that is inversely related to the likelihood of serious delinquency. This index, typically ranging from around 300 to 850, provides an ordinal ranking across individuals of creditworthiness.[15] The goal of this subsection is to show that under certain conditions, the equilibrium described in the previous subsections can be implemented via an arrangement in which lenders use a model equivalent of a credit score. Lenders need only know how credit scores and loan sizes translate into default probabilities, not why.

We formalize the vague notion that a credit score depicts a consumer's creditworthiness, by defining it to be the probability of repayment on a loan of some standard size $a' = \bar{a} < 0$. According to the timing in Definition 1 part 4(a), since (e, a, s) is known at the end of $t-1$, an individual's credit score can be calculated at the end of period $t-1$ to be $m \equiv p^{(0,\bar{a})}(e, a, s)$ given in equation (14). Newborns in period $t-1$ are assigned $m \equiv p^{(0,\bar{a})}(e, 0, G_\beta)$. Intuitively speaking, an equivalence between the type-scoring and credit-scoring environments will exist if there is a one-to-one mapping between s and m, holding fixed all other factors that affect credit scores (i.e., e and a). Then, wherever s appears in the theoretical model, it can be replaced by m.

An individual in state $\hat{\omega} = (e, a, m)$ of the financial arrangement with credit scores takes as given a pricing function $q^{(0,a')}(\hat{\omega})$ and a credit-score transition function $Q^m(m'|(d, a'), e', \hat{\omega})$ which tells her the likelihood of her future credit score conditional on her actions. Intermediaries take as given the pricing function (which must satisfy the zero profit condition) and the probability of repayment function $p^{(0,a')}(\hat{\omega})$ (which must be consistent with the individuals objective likelihood of repayment). This

[15]See, for example, Investopedia.

repayment function makes use of a one-to-one and onto mapping between s and m which we denote $M(\hat{\omega})$.[16] Specifically, $M(\hat{\omega})$ is given by the inverse mapping defined by $m = p^{(0,\bar{a})}(e, a, M(e, a, m))$. These functions can be defined to exactly replicate the RCE with type scores.

In Online Appendix A.6, we restate the household and financial intermediary problems using this definition of a credit score. We then provide a definition of a recursive competitive equilibrium with credit scores (RCECS) and prove:

Theorem 3. *Suppose an RCE exists and let $m = p^{(0,\bar{a})*}(e, a, s)$. Suppose $M^*(\hat{\omega})$ given by the inverse function $m = p^{(0,\bar{a})*}(e, a, M^*(e, a, m))$ exists. Then an RCECS also exists in which the choice probabilities $\sigma^{(d,a')*}(\beta, z, e, a, m) = \sigma^{(d,a')*}(\beta, z, e, a, s)$.*

As noted above, a credit score is an *ordinal* measure of creditworthiness, not a direct estimate of the probability of repayment. To close this gap, we associate with $p^{(0,\bar{a})}(\omega)$ a number in the unit interval that gives $p^{(0,\bar{a})}(\omega)$'s position (i.e. ranking) in the overall distribution of $p^{(0,\bar{a})}(\omega)$ in the model economy.

Definition 4. An individual's *credit ranking* in state ω is given by

$$\chi^{(0,\bar{a})}(\omega) = \sum_{\tilde{\omega} \in J^{(0,\bar{a})}(\omega)} \mu(\tilde{\omega}) \tag{17}$$

where $J^{(0,\bar{a})}(\omega) = \{\tilde{\omega} : p^{(0,\bar{a})}(\tilde{\omega}) \leq p^{(0,\bar{a})}(\omega)\}$ and $\mu(\omega) = \sum_{\beta,z} \mu(\beta, z, \omega)$.

Clearly, $\chi^{(0,\bar{a})}(\omega) \in [0, 1]$.

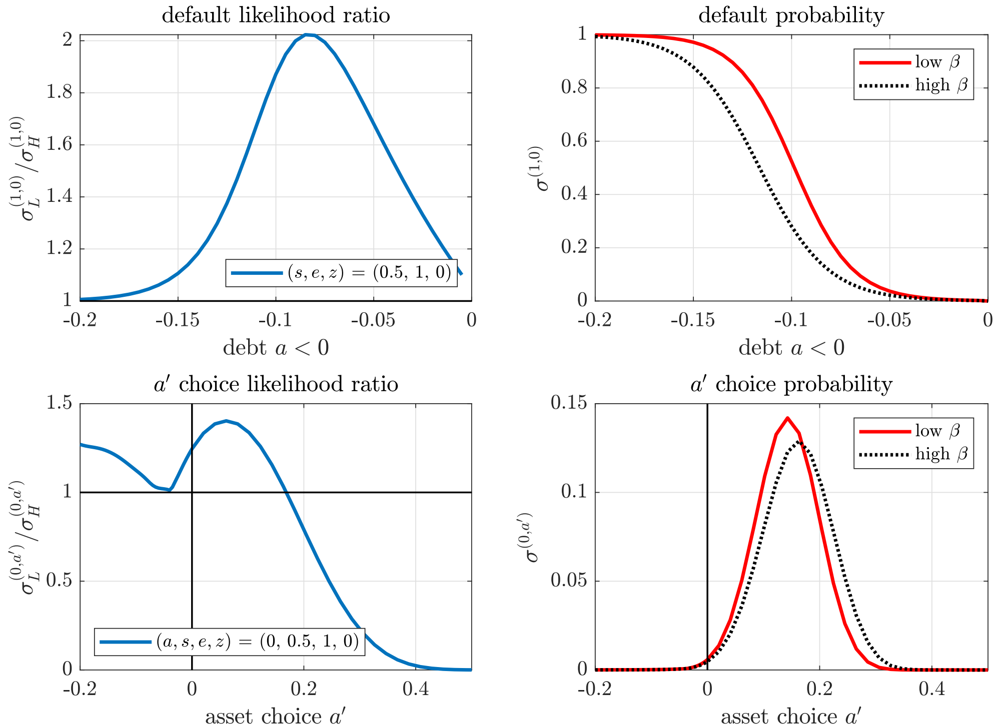
Furthermore, real-world credit scores do not mention any specific level of borrowing. One way to interpret this fact is to think that the ranking of individuals with respect to probability of repayment holds for *any* level of debt. For this to be true, we need the following property:

Definition 5. Let $\hat{a} < \bar{a} < 0$. Then, $p^{(0,a')}(\omega)$ *preserves order with respect to a'* if $p^{(0,\bar{a})}(\omega) \geq p^{(0,\bar{a})}(\tilde{\omega})$ if and only if $p^{(0,\hat{a})}(\omega) \geq p^{(0,\hat{a})}(\tilde{\omega})$.

If $p^{(0,a')}(\omega)$ preserves order, then $J^{(0,\hat{a})}(\omega) = J^{(0,\bar{a})}(\omega)$ and $\chi^{(0,\bar{a})}(\omega)$ becomes invariant to the choice of $\bar{a}$. This order preserving property holds for a wide range of debt levels for our estimated model in Section 5.

[16] If not one-to-one, then two individuals with the same e, a, and m choosing the same amount of debt, could face different prices (because their s's are different).

Figure 1: **Likelihood Ratios of Default and Borrowing/Saving Decisions**



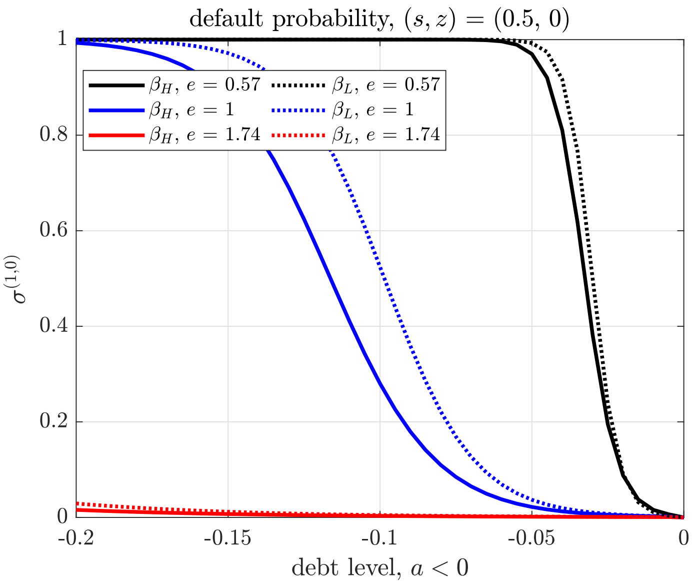
4 Model Mechanics

The workings of our model depend on differences in patience among types. The differences could be so small as to be close to irrelevant or so large as to make types reveal themselves immediately by their actions. We are interested here in the sweet spot between where the differences are large enough to affect behavior but at the same time small enough to prevent complete learning because there are incentives for less creditworthy types to partially emulate more creditworthy types. Because our estimates in Section 5 indeed yield this sweet spot, we hereafter discuss the workings of our model for the particular parameterization in which there are only two levels of impatience.

The fact that patience matters in the simple sense that agents of different types take different actions is illustrated by means of choice probabilities and likelihood ratios in Figure 1. The top panel shows those objects for default for agents with the same type score and earnings as a function of debt, indicating that it is always more likely for the low type to default (right graph) implying a likelihood ratio above one (left graph). The bottom panel shows similar objects; a particular type's choice probabilities over assets chosen (right graph) and its associated likelihood ratio (left graph). To summarize, low types tend to borrow more frequently, to borrow more, and once in debt they default more frequently.

It is also true that an individual's earnings class matters for the probability of default as depicted in

Figure 2: **Default Probability by Earnings and Type**



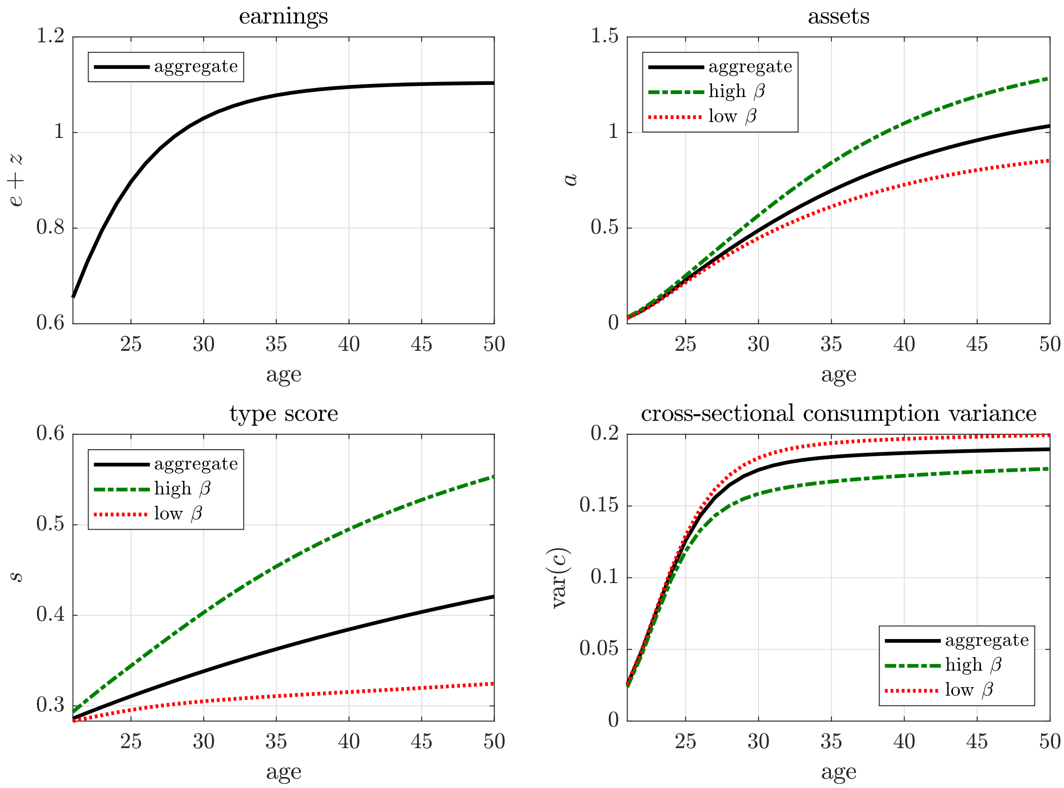
Notes: This figure plots $\sigma^{(1,0)}(\beta, e, z = 0, a, s = 0.5)$ over a range of debt levels. Solid (dashed) lines indicate high (low) beta, and black / blue / red indicate low / medium / high earnings.

Figure 2. Specifically, individuals with higher earnings are less likely to default. A crucial implication of this is that the realization of earnings levels at the start of the period changes the credit score but not the type score, which highlights the composite notion of the credit score.

While age is not a state variable in the decision problems of individuals and lenders, demographics play a role through how we model the arrival of newborns and the Markov process for hidden type. Specifically, since all newborns begin with the lowest earnings class, our Markov process for earnings implies that earnings are expected to rise through an individual's life as shown in the top left panel of Figure 3. The earnings profile induces an increasing wealth profile in the top right panel. Using our estimates from Section 5 of G_β and Q^β where the fraction of type H newborns is lower than the long run fraction of type H, there is upward drift in the average type score with age given by $\bar{s}' = \bar{s} \cdot Q^\beta(H|H) + (1 - \bar{s}) \cdot Q^\beta(H|L)$ evident in the bottom left panel of Figure 3. Finally, the bottom right panel shows that the "within-group" variance of consumption is higher for type L than type H, consistent with more precautionary saving by type H as well as greater credit access via higher type scores in our baseline model.[17] A notable feature of the bottom right panel of Figure 3 is that the cross-sectional variance of consumption is growing across the early part of the life cycle consistent with the empirical evidence in Figure 14 of Heathcote et al. (2010).

[17] Krueger and Perri (2006) term "across-group" variation owing to observable differences like education and "within-group" variation the residual which includes idiosyncratic income. Here we are grouping people on observables like age and also unobservables like type.

Figure 3: **Moments by Age in Baseline Model**



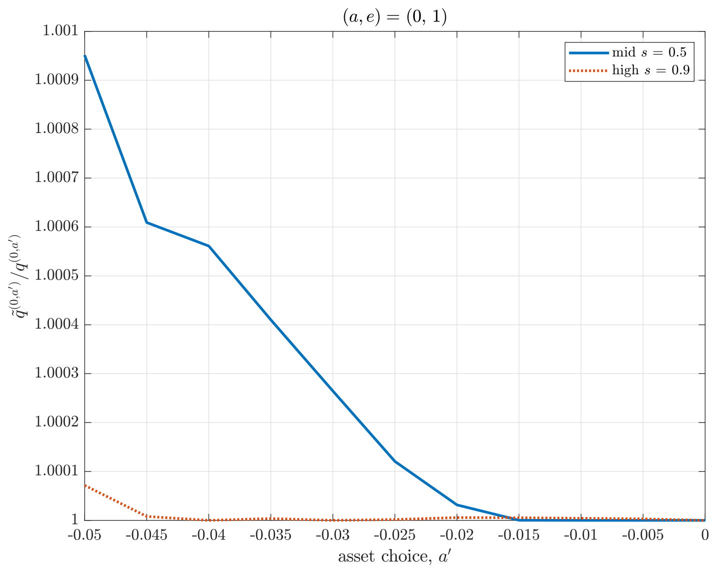
Notes: In each panel, each line corresponds to the average moment indicated at the specified age in the baseline model. For the type-specific measures, the average is computed conditional on type.

Since actions are partially revealing of type, we decompose the consequence of actions into static consequences – which only occur when borrowing as they are immediately incorporated into the price of credit (Section 4.1) – and the dynamic consequences that go beyond the current price through the dynamic update of type scores (Section 4.2). The static cost for an impatient type to imitate a patient type in order to build a better reputation for even small differences in patience is assessed in Section (4.3).

4.1 Static Selection Effects of Asset Choice

If an individual of observable type $\omega = (e, a, s)$ were to borrow a' she would be facing a price that depends on the default probabilities of her expected type tomorrow $\omega' = (e', a', s')$. Since earnings class levels evolve exogenously and are publicly observed, there is no interaction between a' and e'. The evolution of type score s', however, is affected both by the known exogenous type evolution given by Q^{β} but also by the Bayesian update of the individual's type score *conditional on the choice of* a' given by $Q^s(s'|\psi^{(d,a')})$. To isolate the contribution of the latter, we compare the baseline equilibrium price

Figure 4: **Static Effect of Borrowing Choice**

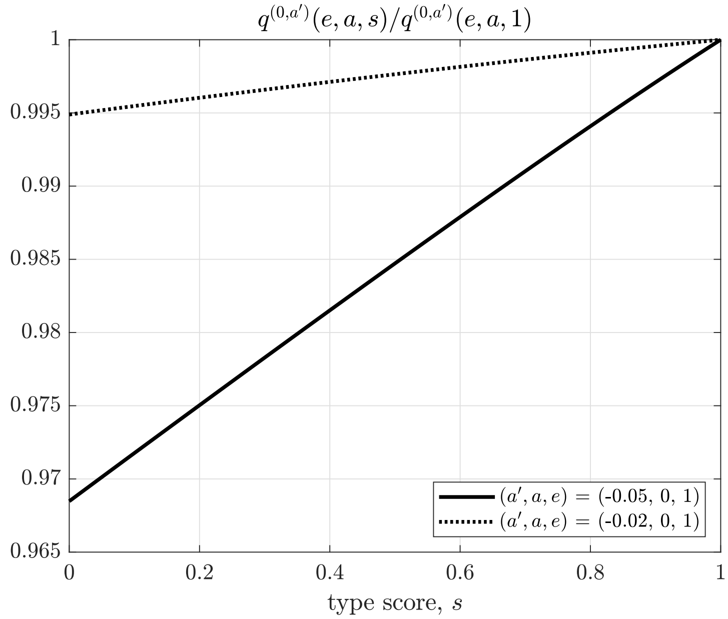


with a price schedule that results from excluding a' from the Bayesian updating formula. This price schedule is given by $\widetilde{q}^{(0,a')}(a, s, e) = \rho\widetilde{p}^{(0,a')}(a, s, e)/(1 + r)$ where

$$\widetilde{p}^{(0,a')}(a, s, e) = \sum_{\beta',e',z'} H(z')\ Q^{\beta}(\beta'|\beta)\ Q^{e}(e'|e)\ s(\beta)\ \left[1 - \sigma^{(1,0)}(\beta', e', z', a', \bar{s}')\right]. \tag{18}$$

In (18), recall $\bar{s}' = \bar{s} \cdot Q^{\beta}(H|H) + (1 - \bar{s}) \cdot Q^{\beta}(H|L)$ updates the prior using only Q^{β} and ignores the borrowing choice a'. For the estimated economy, Figure 4 shows the ratio of $\widetilde{q}$ to the baseline equilibrium price q. We can see that for a high enough probability of being patient (i.e. $s = 0.9$), the effect is small and the equilibrium price imputes the choice mostly to the noise of the extreme value shocks. For a more agnostic prior (i.e. $s = 0.5$), the effect is larger. In our baseline economy, the more borrowing an individual undertakes the more likely she is imputed to be a low type and hence faces a more adverse price. In the counterfactual world, lenders are not taking into account that as the level of debt rises, the proportion of low β (less creditworthy) types in the pool of people who borrow that amount also rises. Thus, ignoring adverse selection effects induces $\widetilde{q}$ to be higher than the equilibrium zero profit price q. If we were to assume the same $\bar{s}'$ updating but require lenders to use the information revealed by an individual's a' choice when pricing debt, that price would be lower than $\widetilde{q}$. It could potentially even be lower than q because the discipline from the dynamic reputation effects of asset choice would be absent.

Figure 5: **Reputation and Prices**



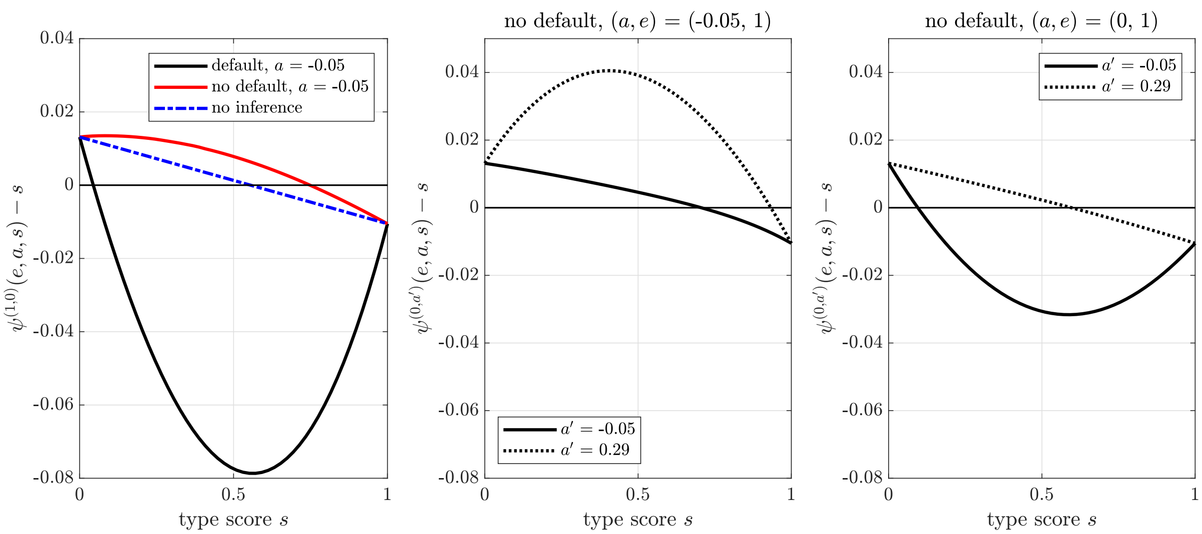
4.2 Dynamic Reputation Effects of Asset Choice

In addition to those static effects discussed in the previous section, asset market choice can also have long lasting effects. This requires two conditions to hold: (i) prices depend on an individual's current reputation (i.e. type score) independent of her current action and (ii) her choice today affects her future reputation (and hence future prices). Both requirements hold.

Figure 5 establishes the first requirement. It plots the ratio of debt prices that an individual with current type score $s \leq 1$ can obtain relative to a person with an impeccable $s = 1$ score for two different debt choices. The fact that both lines are downward sloping establishes that type scores matter; for a given debt choice, a higher s fetches a higher price. The reason is simple; since the high β type repays with higher probability than the low β type, there is information about the probability of repayment in the current type score (which, in turn, reflects the history of the individual's past actions). The fact that the line is steeper for the larger debt level arises from adverse selection (as in Figure 4).

Figure 6 establishes the second requirement. It plots the change in the public assessment of an individual's type resulting from Bayesian updating as a function of her current type score and her current actions (i.e. $\psi^{(d,a')}(e,a,s)$). The left plot shows the differential update for defaulters and non defaulters (the latter integrated over all asset choices). At the extremes, ($s = 0$ or $s = 1$), where the priors are very strong, there is no update and hence actions are not relevant, but for other values the posterior is clearly affected and shows the impact of choice. Note, however, that if Q^{β} has positive

Figure 6: **Type Score Responses**



off-diagonal elements, which we find in our estimation, an individual's type can change from one period to the next. This means that even if an individual's asset market actions are not observed, her posterior would change over time as it converges to the mean type score implied by Q^β alone. This mean reversion in type score is shown by the blue dashed line and accounts for why a person's type score falls upon repayment if the current score is sufficiently high or rises upon default if it is sufficiently low. Still, it remains true that repaying leads to a higher type score than defaulting for all interior values of s.

The center and right plots of Figure 6 show the net change in reputation that result from different actions taken by an individual either already in debt (center) or with zero assets (right). For an individual already in debt, staying in debt leaves the assessment mostly unchanged, but sizeable savings gives a boost to the probability of being a high type. For an agent with no assets the opposite is true: borrowing lowers the imputation of being a high type, while sizeable savings keeps the assessment unchanged.

4.3 Signalling Costs

In models with hidden types, the "bad" types have an incentive to imitate the "good" types in order to pool with them and obtain better terms of trade, while the "good" types have an incentive to separate themselves from the "bad" types to get even better terms. As we discussed at the beginning of this section, the hidden type difference (i.e. $\beta_H - \beta_L$) could be so small that adverse selection is virtually irrelevant or so large as to induce near perfect separation of type from observable actions. Here we assess the costliness for an impatient type L to imitate the actions of a patient type H for even small differences in patience in the "sweet spot."

Table 1: **Signalling Costs and Benefits**

	% Average Gain in:		
	Consumption ($\widehat{C}$)	**Wealth** ($\widehat{A}$)	**Reputation** ($\widehat{\psi}$)
All	-3.78	6.09	5.10
Newborns	-0.84	15.87	3.73

Notes: The first column measure is defined as

$$\hat{C} = \frac{\sum_{z,\omega} \mu(\beta_L, z, \omega) \left[\sum_{(d,a') \in F(z,\omega)} \left(\sigma^{(d,a')}(\beta_H, z, \omega) - \sigma^{(d,a')}(\beta_L, z, \omega)\right) c^{(d,a')}(z, \omega)\right]}{\sum_{z,\omega} \mu(\beta_L, z, \omega) \left[\sum_{(d,a') \in F(z,\omega)} \sigma^{(d,a')}(\beta_L, z, \omega) c^{(d,a')}(z, \omega)\right]}$$

while the second and third columns substitute $a'(z, \omega)$ and $\psi^{(d,a')}(\omega)$ for $c^{(d,a')}(z, \omega)$.

Changing one's action has three effects: (i) a change in today's consumption; (ii) a change in tomorrow's net wealth; and (iii) a change in tomorrow's reputation. To explore these three effects for a type L to imitate a type H, we conduct the following experiment. To imitate a type H individual, we assume the type L follows the choice probability function $\sigma(\beta_H, z, \omega)$ instead of $\sigma(\beta_L, z, \omega)$. One measure of the consumption cost from a type L individual mimicking a type H is the average difference in consumption between H types and L types implied by the differences in their choice probabilities relative to the average consumption of a type L individual. Similar measures can be computed for next period net wealth and next period reputation (as measured by the difference in type score posterior).

Table 1 provides these calculations for the case where there are small differences in patience between type H and type L (i.e. where $(\beta_H - \beta_L)/\beta_L = 3.3\%$). The table illustrates an important point. Type L newborns have a much lower consumption loss to mimicking a type H individual. This is because the imitation costs are increasing in earnings and assets, both of which rise on average through one's life. Since type H choose to save more, this imposes a bigger consumption loss to type L from mimicking. Alternatively, it is easier to mimic when young as the dispersion in assets and scores are lower in youth. The fact that it is less costly to mimic when young implies there is more pooling among the young and the fact that is is more costly to mimic when old implies there will be more separation among the old. Importantly, the table makes clear that even small differences in hidden type can lead to large static costs of mimicking a good type in order to gain a better dynamic reputation.

5 Mapping the Model to Data

We now examine the U.S. unsecured credit market through the lens of our model. We rely on the equivalence result between the model with type scores (RCE) and the model with credit scores

(RCECS) described in Theorem 3 of Section 3.5 when there are two β types. Specifically, it allows us to use the model with type scores in order to target the joint behavior of earnings, aggregate credit market moments, and credit rankings over the life cycle.[18] We then verify that the sufficient condition in Theorem 3 is satisfied for the estimated parameters so that the equivalence result holds.

As described briefly in the introduction, credit scores, earnings and assets all grow with age on average, and we want our model to capture those features. Unfortunately, we do not have access to a panel dataset which contains all these dimensions. So we use a version of simulated method of moments to estimate our model. Specifically, we take some non-controversial information from outside the model: the earnings process, the risk free rate of return, demographics, preferences over risk, a measurement of the costs of bankruptcy filings, and a generic value of debt ($\overline{a}$) to which the credit score is normalized.[19]

Next we obtain a set of data moments that summarize the properties of the unsecured credit market (default rates, average interest rates, dispersion of interest ratios, fraction of households in debt, debt to income ratio) and we approximate the behavior of credit scores as a function of age (specifically, affine functions of the mean and the standard deviation of credit scores and the autocorrelation of the annual change in individual scores). Establishing a credit history is, in light of our model, just code for age.

We then proceed to estimate the parameters of interest which are the values of patience for both types, the transition probabilities of types and their frequency at birth, as well as measures of noise (the variances of the extreme value shocks) by minimizing the weighted sum of squared differences between the values of the moments in the data and their model counterparts. We have tried various alternative sets of moments with minimal effects on the findings. While earnings, credit and bankruptcy statistics have been used since Chatterjee et al. (2007) and Livshits et al. (2007a), credit scores, and their evolution by age, have not. The evolution of credit scores are crucial for understanding the building of a reputation over the early part of the life-cycle.

Computation of equilibrium requires solving for two endogenous functions: the bond price function and the type-score updating function. The bond price function is standard in unsecured debt models like Chatterjee et al. (2007), except that the endogenous type score is an additional dimension. The type scoring function is new: individuals take as given how feasible actions change the market's perception of their type, and this perception has to be consistent with the actions taken by both types.

[18] An analogy would be that real business cycle models solve the simpler Planner's problem to compute consumption and investment allocations consistent with a decentralized competitive equilibrium. It is faster to compute the RCE because Bayesian updating gives a candidate input used to generate $p^{(0,a')}(\hat{\omega})$, while iterating on an initial guess of $p^{(0,a')}(\hat{\omega})$ in an RCECS does not use information on how an agent's type affects her repayment decision.

[19] We verify that the choice of $\overline{a}$ does not matter provided it is higher than the bankruptcy filing costs.

5.1 Preliminaries: Parameters Chosen Outside the Model

A model period is one year. We take the relevant life span of people to be 40 years as the bulk of borrowing is by young people, implying a survival probability of 0.975. We choose a CRRA utility function with risk aversion parameter 3. We pose a risk free rate of 1% which implies an effective interest rate of 3.59% in the presence of perfect annuity markets. We take the cost of filing for bankruptcy to be about 2% of median earnings as estimated in Albanesi and Nosal (2015). Since it is a dominant action not to default on debt less than the filing cost, we choose $\bar{a}$ (the value used to compute the probability of default in a credit score) to be 3.5% of median earnings (i.e. sizeably above those costs). Finally we take the earnings class to be the persistent AR1 process estimated by Floden and Lindé (2001) and assume agents are born with the lowest earnings level to replicate the upward earnings path in the life cycle.[20] This is all summarized in Table 2.

Table 2: **Parameters Chosen Outside the Model**

Parameter		**Value**	**Notes**
Demographics and preferences			
Survival probability	ρ	0.975	avg. life span 40 years
Risk aversion	γ	3.0	CRRA preferences
Earnings at birth	$\underline{e}$	0.57	Footnote 22
Technology			
Risk-free rate (%)	r	1.000	
Filing cost	κ	0.020	2% of median earnings
Debt level for computing credit score	$\bar{a}$	-0.035	3.5% of median earnings
Earnings			
Persistence of log(e)	ρ_e	0.9136	Floden and Lindé (2001)
Std. dev. of innovations to log(e)	ν_e	0.0426	Floden and Lindé (2001)
Std. dev. of log(z)	ν_z	0.0421	Floden and Lindé (2001)

[20]We approximate this process by a three-state Markov chain using the Adda and Cooper (2003) method, which yields support $\mathcal{E} = \{0.57, 1.00, 1.74\}$, transition matrix $Q^e(e'|e) = \begin{bmatrix} 0.818 & 0.178 & 0.004 \\ 0.178 & 0.643 & 0.178 \\ 0.004 & 0.178 & 0.818 \end{bmatrix}$, and a transitory component with a three-point uniform distribution on support $\mathcal{Z} = \{-\bar{z}, 0, \bar{z}\}$, where $\bar{z} = \sqrt{\frac{3}{2} \cdot 0.0421} = 0.18$.

5.2 Estimation

We start by describing the data moments that we target in our simulated method of moments estimation.

5.2.1 Data and Targets

As indicated above the set of statistics that we deem important to target pertain to the main aggregate characteristics of the U.S. unsecured credit market: credit usage (the fraction of households in net debt and the debt-to-income ratio), credit terms (average interest rates and their dispersion), how many net borrowers and how many defaulters. Importantly, we are also interested in the age profile of credit rankings.

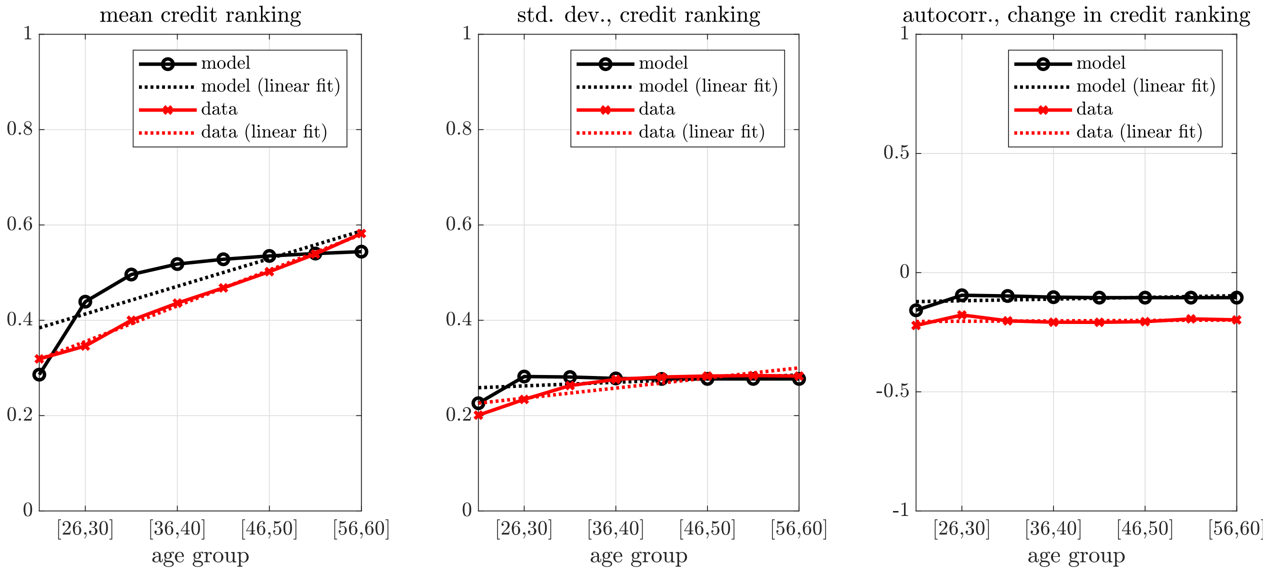
To obtain these data targets, we use three primary sources: the Survey of Consumer Finances (SCF), the administrative records of the U.S. Bankruptcy Courts, and the Federal Reserve Bank of New York Consumer Credit Panel/Equifax (FRBNY CCP/Equifax). The first provides information on individual level variation in debt and interest rates, the second provides information on aggregate bankruptcy filing rates, and the last contains individual level information on credit records from an anonymized panel which provides us with moments on the variation and evolution of credit scores. The credit score measure is the Equifax Risk Score (hereafter Risk Score), which is a proprietary credit score similar to other risk scores used in the industry.

We choose 2007 as our baseline year. In the SCF, we focus on the subset of households with heads between the ages of 20 and 60 years excluding the top 5% of the wealth distribution for whom we think our theory is not relevant. The fraction of indebted households is the fraction of such households with negative networth. The average debt-to-income ratio is the ratio of total unsecured debt of indebted households to 2007 per household U.S. GDP. For the mean and standard deviation of interest rates we used the interest rates reported on unsecured debt by all households with negative networth.

The default rate is the ratio of the total number of nonbusiness Chapter 7 filings in 2004 reported by the U.S. Bankruptcy Courts, scaled by the total number of U.S. households in 2004. We chose 2004 filing rates rather than later ones because they are unaffected by the Bankruptcy Abuse Prevention and Consumer Protection Act of 2005 (which changed the eligibility requirements for a discharge in ways we do not model in this paper).

While the previous credit market data targets have been used in numerous quantitative theory papers on bankruptcy, what is novel is our use of the age profile of Risk Score moments. For this we use a

Figure 7: **Credit Ranking Age Profile: Model vs. Data**



Notes: The credit ranking data is based on author calculations using FRBNY CCP/Equifax data in Table 11 in Appendix D. The linear approximation to the model generated credit ranking age profiles use the regression coefficients from Table 3.

2% sample (approximately 150,000 observations) of the FRBNY CCP/Equifax panel (described in detail in Appendix D). With this data, we create credit rankings (defined as a percentile ranking of individuals among the overall sample using Risk Scores) and group individuals between the ages of 20 and 60 in 5 year bins. We then compute the means and standard deviations of credit rankings within each age bin. With these age bin data values, we estimate affine age profiles for means, standard deviations, and autocorrelations of year-to-year changes in credit rankings (2004Q1, 2005Q1 and 2006Q1).[21] Figure 7 shows the data and auxiliary affine approximations to the data (in red) as well as the model generated data and approximation (in black).

Our system is overidentified (we have 7 parameters and 10 moments) so not all moments will be exactly replicated. While all moments jointly identify the parameters of the model, we find it useful to provide an informal discussion of how different moments in the data relate to the identification of different parameters. This discussion is guided by a sensitivity analysis examining how model moments change in response to changes in the parameters which can be found in Appendix B.9.

It is important to recall that the process for earnings was estimated outside the model, so all these considerations are conditional on such a process, which we assume is the same for both types. For instance, we find the aggregate debt-to-income ratio is primarily determined by that exogenous process and our assumption that newborns start with the lowest earnings class level. Further, along with the

[21]We do not use the slope of the age profile of the autocorrelation of year-to-year credit score changes as a target because it is zero, which makes matching the relative deviation between model and data a problem.

extreme value parameter λ, the exogenous earnings process is also important for the autocorrelation of changes in credit rankings given that repayment probabilities are highly dependent on the earnings process.

The affine age profile of average credit rankings helps identify the "evolution" parameters of the type transition process. The constant primarily determines the fraction of high types at birth G_{β_H}, while the slope is informative about the difference between $Q^{\beta}(L'|H)$ and $Q^{\beta}(H'|L))$ along with the upward sloping earnings profile. The constant and the slope of the age profile of the standard deviations is partially determined by the Q^{β} parameters and the extreme value parameter λ, both of which affect the speed of learning in the economy. Furthermore, the fact that earnings dispersion is rising with age contributes to the slope.

The four other aggregate credit statistics, namely, the default rate, average interest rate, interest rate dispersion, and the fraction in debt, are important to identify the remaining parameters. One can think of the difference in discount factors $\beta_H - \beta_L$ and the extreme value parameter α as also governing the speed of learning in our economy. That is, if $\beta_H - \beta_L$ is small (large), then there will be more (less) pooling and therefore quicker (slower) learning. A higher $\beta_H - \beta_L$, implying more separation, leads to higher aggregate default, interest rates, and interest rate dispersion. This occurs in the model because type L are less able to imitate type H and hence face worse terms and have less to lose from defaulting (so that the aggregate default rate helps identify this difference). While a higher β_H relative to β_L leads to a lower fraction of individuals in debt, more separation implies type H face better borrowing terms, which leads them to select higher amounts of debt (conditional on borrowing) thereby contributing to higher average rates as well as dispersion. The fraction of households in debt helps identify α because it attenuates the precautionary savings motive and along with the earnings age profile contributes to a larger fraction of households in debt. A higher fraction of households in debt contributes to higher aggregate default, average interest rates, and dispersion moments despite more pooling.

5.2.2 Estimated Parameters

To estimate the parameters, we proceed by a mixture of simulated method of moments for the aggregate statistics and indirect inference for the credit score auxiliary affine model of means, standard deviations, and autocorrelations across the age profile. Our model does not provide for an explicit source of errors in the cross section (given the large sample sizes, measurement error is not really a problem), so we cannot use standard procedures to choose the weighting matrix in the sum of squared differences between model and data moments. Consequently, we choose to weight the two sets of moments, aggregate credit market statistics and the credit ranking age profile moments, equally.[22, 23] The values of the data moments and their model counterparts, as well as the average mean square errors (both total and for each of the two blocks of moments), are reported in Table 3 and graphed for the auxiliary model in Figure 7.

The estimated parameter values are in Table 4. The values for patience are $\beta \in \{0.886, 0.915\}$, so that low types discount the future 3.6% more than high types. This differential is capable of allowing reputation acquisition to play a role in equilibrium: types are far enough apart to want to behave differently, but close enough that imitation is not too costly.

The estimates for the transition matrix Q^{β} display a slight secular increase of the high types but with large turnover of nearly 20% (i.e. $Q^{\beta}(H|L)/Q^{\beta}(L|H) = 0.013/0.011$). This implies an ergodic distribution featuring 55% high types, but the life-cycle demographic structure implies a lower stationary fraction equal to 41%. [24] Note that the demographic improvement in the average assessment of an individual's type (and hence creditworthiness) given by $\overline{s}' = \overline{s} \cdot Q^{\beta}(H|H) + (1 - \overline{s}) \cdot Q^{\beta}(H|L)$ over one's life with estimated initial fraction of high types $G_{\beta} = 0.28$, is independent of alternative credit arrangements we consider. The fact that G_{β} is estimated to be below the stationary fraction of high types is a robust consequence of the rising average credit score over the age profile.

We view our estimates of the parameters α and λ of the extreme value distributions as consistent with a modest amount of noise in behavior, but not so much that fundamentals are over-ridden. This is especially true for the binary default/no-default action where the share of modal defaulters is 87%, and somewhat less so for no-default actions. For debt, where there are (potentially large) pricing and reputational differences between nearby actions, 71% of choices fall within 1 grid point of the mode. For assets, where pricing penalties are non-existent and reputational differences are smaller, the analogous figure is 47%.[25]

5.3 Model Fit

We next assess how the model does against certain non-targeted properties in the data.

[22]We have explored alternative weighting schemes without finding relevant differences.

[23]Specifically, we estimate the parameters of the model by minimizing the sum of squared residuals normalized by the size of the moments) with equal weighting of the two sets of targets. Because the slope of the age-profile of the autocorrelation of year-to-year changes of credit scores is almost zero, we do not use it in the estimation, instead we use it as an over identifying restriction. We use the derivative-free, least squares minimization routine developed in Zhang et al. (2010) to implement the minimization routine. For more details on computation and estimation see Appendix B.4.

[24]The fraction of type H in the stationary distribution (call it μ_H) solves $\mu_H = \rho \cdot \left[(1 - Q^{\beta}(L'|H))\mu_H + Q^{\beta}(H'L)(1 - \mu_H)\right] + (1 - \rho) \cdot G_{\beta_H}$ or $\mu_H = \frac{\rho Q^{\beta}(H'|L) + (1-\rho) G_{\beta_H}}{1 - \rho(1 - Q^{\beta}(L'|H)) + \rho Q^{\beta}(H'|L)}$. For our estimated parameter values in Table 4, $\mu_H = 0.41$. The fraction of type H asymptotes (when $\rho = 1$) to 0.55.

[25]See Appendix B.5 for a description of these calculations.

Table 3: **Estimation Targets**

Moment (%)	**Data**	**Model**
Aggregate credit market moments		
default rate	0.99	0.98
average interest rate	12.87	13.96
interest rate dispersion	6.56	7.28
fraction of HH in debt	10.43	10.58
debt to income ratio	0.35	0.25
Credit ranking age profile moments		
intercept, mean credit ranking	0.281	0.355
slope, mean credit ranking	0.037	0.029
intercept, std. dev. credit ranking	0.216	0.255
slope, std. dev. credit ranking	0.011	0.004
average autocorrelation of change in credit ranking	-0.202	-0.109
Mean squared errors		
Aggregate credit market moments		0.020
Credit ranking age profile moments		0.152
Total		0.085

Notes: The credit ranking data is based on author calculations using FRBNY CCP/Equifax data. MSEs are computed in percentage deviation terms to control for relative magnitudes of moments. That is, for data moment x with model analog $\widehat{x}$, the contribution of x to the MSE is $(\widehat{x}/x - 1)^2$. Within each group of moments, each moment receives equal weight (1/5 on each moment), and each group of moments receives weight 1/2 of the total MSE (hence 0.085 = (0.020 + 0.152)/2).

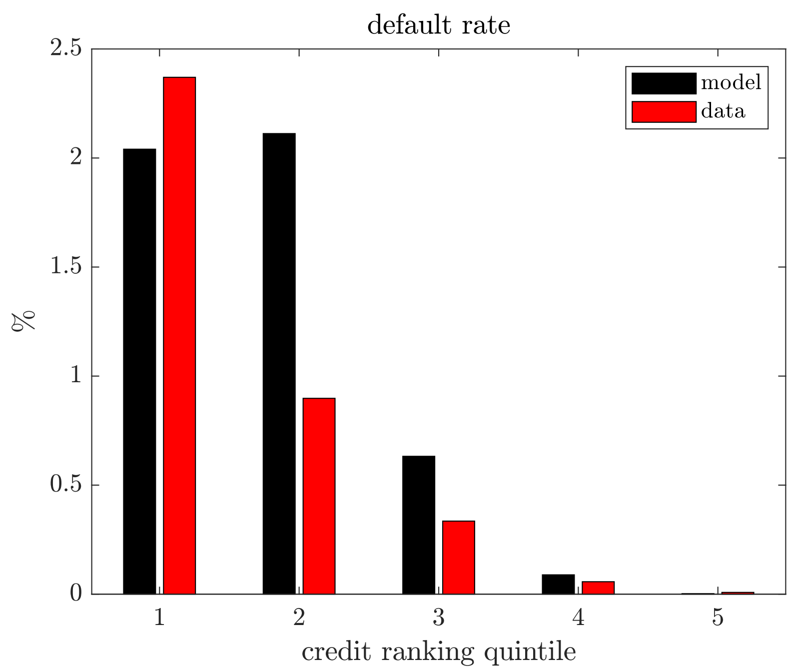
5.3.1 Credit Ranking of Defaulters

Figure 8 shows the non-targeted default rate *by* credit ranking quintiles in the data and in the model. As in the data, the model generates high default rates among individuals with low credit rankings and low default rates among individuals with high credit rankings. The model replicates the decreasing pattern in the data, except for the second quintile. This arises due to the coarseness of the earnings class grid which generates a high proportion of low earners in credit ranking quintiles 1 and 2.

Table 4: **Parameters Chosen Within the Model**

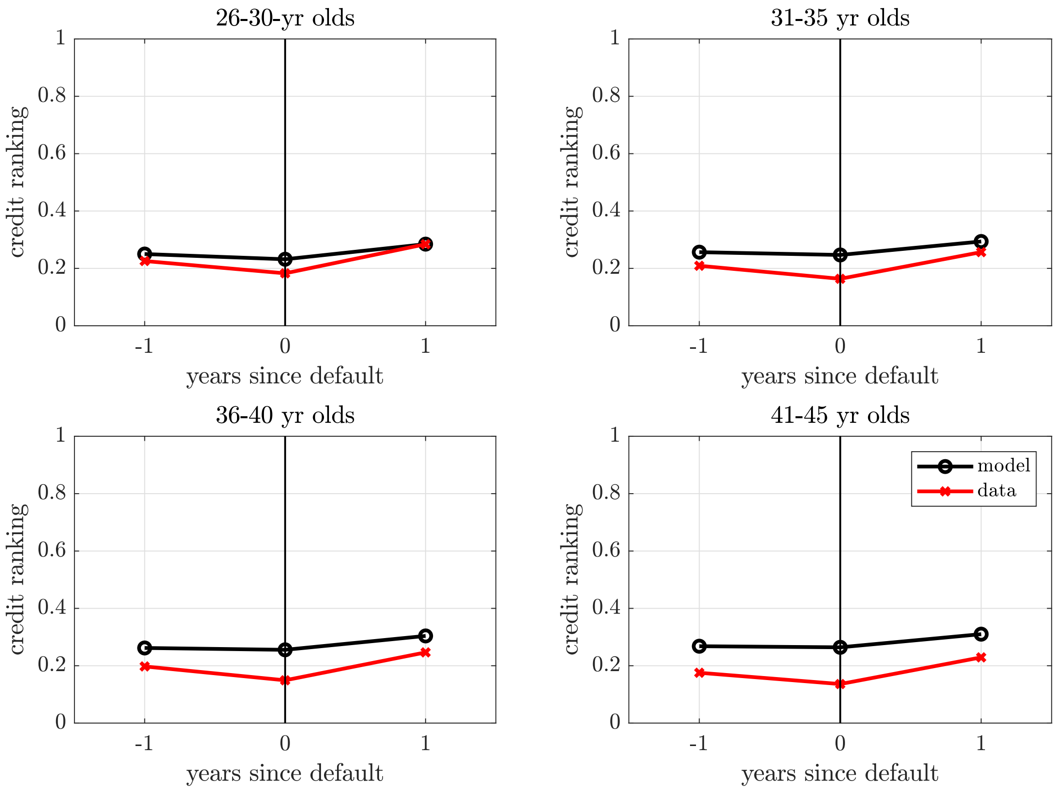
Parameter		**Value**
Evolution of types		
high discount factor	β_H	0.915
low discount factor	β_L	0.886
high to low β transition	$Q^{\beta}(L'\|H)$	0.011
low to high β transition	$Q^{\beta}(H'\|L)$	0.013
fraction high β at birth	G_{β_H}	0.280
Extreme value shocks		
scale parameter ($\times 10^{-3}$)	α	3.387
correlation parameter	λ	0.991

Figure 8: **Default Rate by Credit Ranking Quintiles**



Notes: : The credit ranking data is based on author calculations using FRBNY CCP/Equifax data.

Figure 9: **Default Event Study**



Notes: The credit ranking data is based on author calculations using FRBNY CCP/Equifax data. Model results are obtained by simulating a panel of 10,000 individuals for 1,000 periods and dropping the first 100 periods. Default events are then isolated, and each data point reported represents the mean of credit rankings across all default events for the given lead or lag from the date of default (normalized to 0).

5.3.2 Credit Rankings Around Default

A key property of real world Risk Scores is that they fall upon bankruptcy and mean revert. We illustrate this property for credit rankings in both the data and the model in Figure 9. Specifically, we pose the non-targeted average change in credit rankings during a default episode for various age bins, a form of event study. There are three things to emphasize here. First, the average ranking of defaulters is around 25% for all ages in the data and a bit higher in the model, but not much. Second, more importantly, default does not trigger a tremendous fall in credit rankings and is followed by a mild improvement both in the data and in the model, confirming the mean reversion of scores following bankruptcy documented in Musto (2004).

Why is there so little change in credit rankings? The reason is that bankruptcy discharges the debt burden, which ceteris paribus lowers the likelihood of subsequent default. Third, there is no reversion to the mean (see the bottom right panel of Figure 13) just before bankruptcy indicating that both the model and the data are picking up some bad news that are likely being confirmed subsequently via default.

5.3.3 Credit Access Following Default

Musto (2004) documents that credit access upon default is limited and slow to grow especially for those with initially low credit scores (see Figures 2 through 4 in his paper). Is our model consistent with these properties? This is an important question, since in standard models of unsecured debt, bankruptcy is modeled as an event that exogenously punishes the defaulter by exclusion from borrowing for a random number of periods. Punishment in our model is endogenous and works through the effect of dynamic reputation on future borrowing costs. While the model is in principle clearly capable of delivering these properties of the data cited by Musto (2004), we have just seen that an individual's credit ranking barely drops upon default, so how can default trigger these difficulties? The reason is that there is regression to the mean, and the lack of improvement of the ranking of the defaulters suffices to give them more adverse terms than those faced by the non- defaulters.

To see this we plot in Figure 10 the average percent difference in the prices faced two agents who had the same state in the previous period (earnings class, wealth and credit ranking). One defaulted (and hence has zero assets today), while the other did not, but this latter agent chose to have zero assets today (our extreme value shocks implies it is always possible to find two such individuals). As the figure shows for all except the tiniest of loan sizes (for which approximation error blurs the effect), the agent who defaulted got a worse update on their score, leading her to face lower debt prices (higher interest rates). The drop in spread at higher borrowing levels is due to more confusion about type.

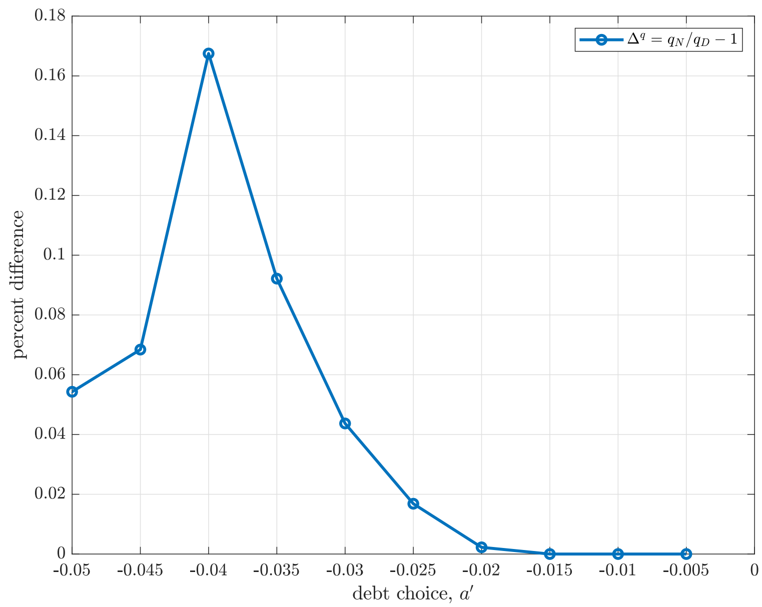
5.4 Implication of the Estimates for Credit Rankings

5.4.1 Cross-Section

Figure 11 shows the tight relation between credit rankings and an individual's earnings class for selected asset levels. The figure also verifies a form of the sufficient condition (one-to-one mapping between type scores and credit scores) in Theorem 3 that establishes the equivalance between the fundamental type score equilibrium (RCE) and the credit score equilibrium (RCECS). Since there is little overlap in credit rankings across earnings classes in the figure, earnings are a primary predictor of credit rankings. However, within an earnings class, a credit ranking is a very discerning market assessment of an individual's type.

Because of the tight connection of credit scores to the earnings class, credit rankings may not always

Figure 10: **Average Price Effects, Default vs. No Default**



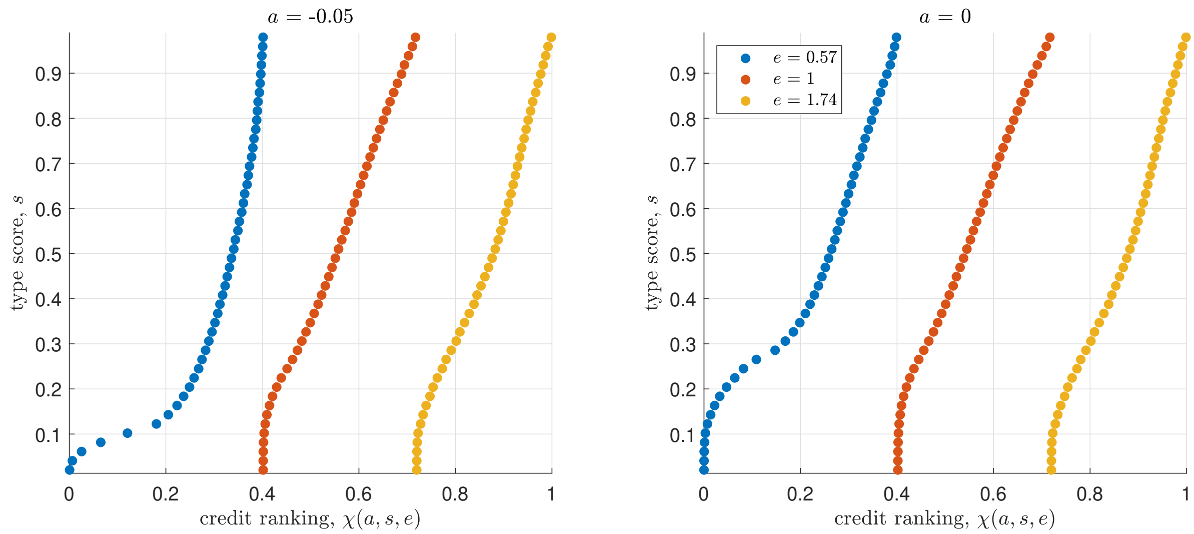
be a good predictor of actual default rates. This can be seen in the top left panel of Figure 12. While perhaps counterintuitive, it is due to that fact that higher earners do borrow more when they borrow (which generates the positive relation between borrowing and ranking in the top right panel) even if fewer of them do borrow (top center panel). The debt to income ratio is, however, monotonic (bottom right panel), lower score individuals have much higher debt *relative* to their own income, which is related to the reversion to the mean in income, low income individuals expect it to go up so they will borrow a bigger fraction than high income indiviudals that expect it to go down. These relations about debt, income, and credit status are consistent with the empirical findings in Diaz-Gimenez et al. (2011) [Table 17, p. 19]. The nonmonotonicity in default rates similarly show up in the relation between the credit ranking and the average and standard deviation of the interest rate.

5.4.2 Age Profile

Our parameter estimates in Table 4 imply there is demographic drift; specifically that the passage of time increases the fraction of type H. Our economy also displays demographic drift in earnings class as all individuals are born as low earners and over time an increasing fraction of them switch to mid and high earners. These two features are what capture the tight relation in the data between ages and the main variables that we are interested in even though in the model age per se does not affect behavior.

These properties are evident in Figure 13. Households earnings on average improve with age and the standard life cycle wealth accumulation can be seen in the top two panels of the figure as a function of

Figure 11: **Credit Scoring Function**



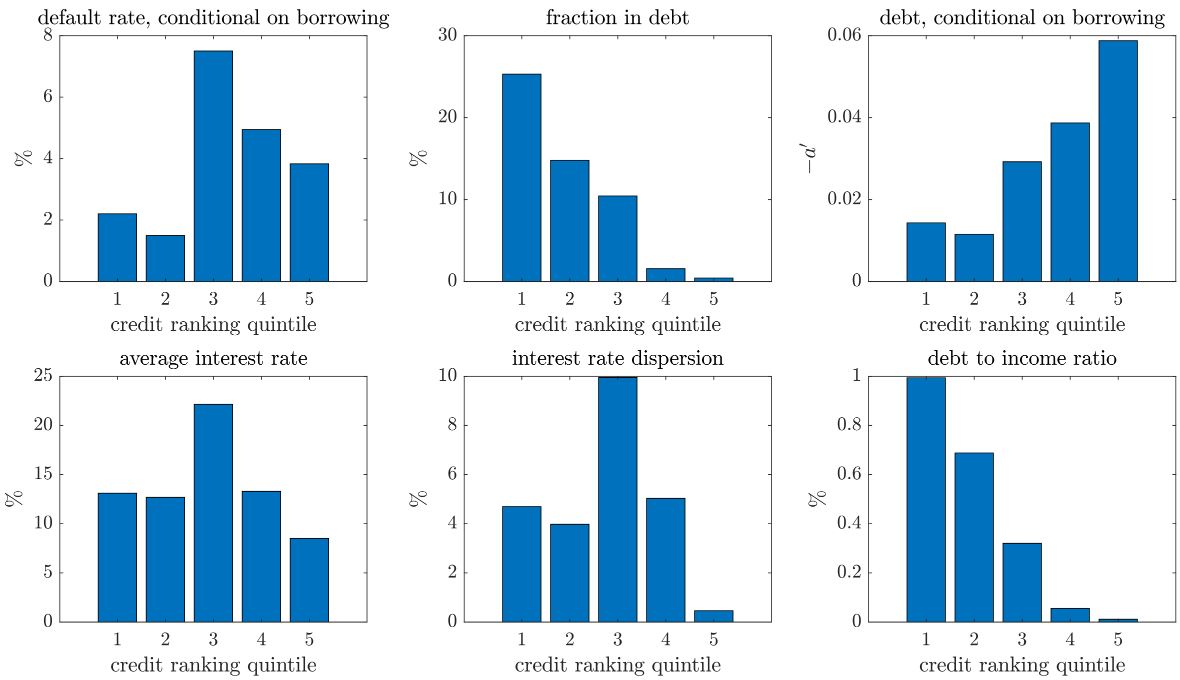
age. The bottom left shows how the economy's assessments of type is aware of the demographic drift of type starting from 28% to an asymptotic fraction of 54%. Note that the drift itself is *independent of* behavior as we are including all individuals of any given age, but the dispersion is increasing indicating how there is learning over time: the 5th and the 25th percentiles are drifting down. These are individuals whose actions have revealed that they are more likely to be type L. The right panel shows the age evolution of credit rankings where the effect of age is sharper indicating its joint determination by type and earnings both of which are moving in the same direction.

6 Impact of Alternative Information Structures

How important is the information structure for allocations? Is the presence of private information and hence of adverse selection quantitatively significant? What would happen if society outlawed tracking of individual credit histories and with it the incentives to build a good reputation? Would the credit market shrink dramatically as the usefulness of maintaining a good reputation disappears? These are natural questions that we can provide quantitative answers to in the context of our model by comparing the outcomes of economies that only differ from our baseline in their information structures.

Before we get into the details of our answers to these questions it is important to keep in mind certain features of private information that are present in our model. First, because of imperfect separation, low types are being subsidized by high types. Second, there are additional incentives to repay one's debt and to save more so as to imitate a high type. Third, there can be important interactions of private information across the age profile. Specific to our model, all newborns are low earners and face an

Figure 12: **Outcomes by Credit Ranking Quintiles**



Notes: Average moments are computed as the average conditional on credit ranking quintile. Default rate is total default across all borrowers as in the calibration, not default rate conditional on borrowing, and debt to income ratio is computed likewise. The size of debt conditional on borrowing averages across all choices made by each agent in a given state.

(expected) upward sloping age-earnings profile. Thus, newborns and young have *a life-cycle reason to borrow* and so are more impacted by by private information. Finally, all individuals face idiosyncratic earnings shocks against which direct insurance is unavailable. Since borrowing to smooth consumption is costly in all the economies that we explore, all individuals have *a precautionary savings reason to accumulate assets*.

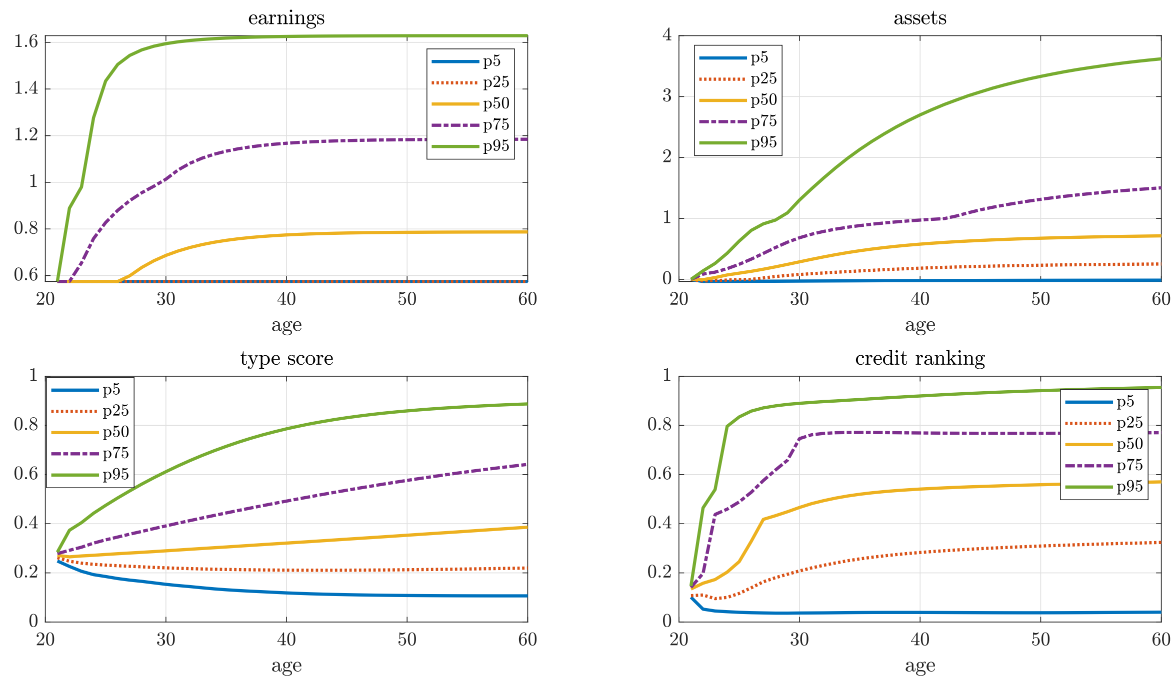
Differences between the economies imply not only that individuals behave differently on account of the incentives that they face, but also that the equilibrium prices reflect these changes and accordingly we have to look at both aspects simultaneously.

6.1 Description of alternative economies

We now consider two stark alternative information structures in which reputation plays a limited role: one where past actions cannot be used to price discriminate but demographic drift can be used to infer type and another where type is public information.[26]

[26] For the formal specification of these alternative economies, see Appendix B.8.

Figure 13: **Life Cycle Properties**

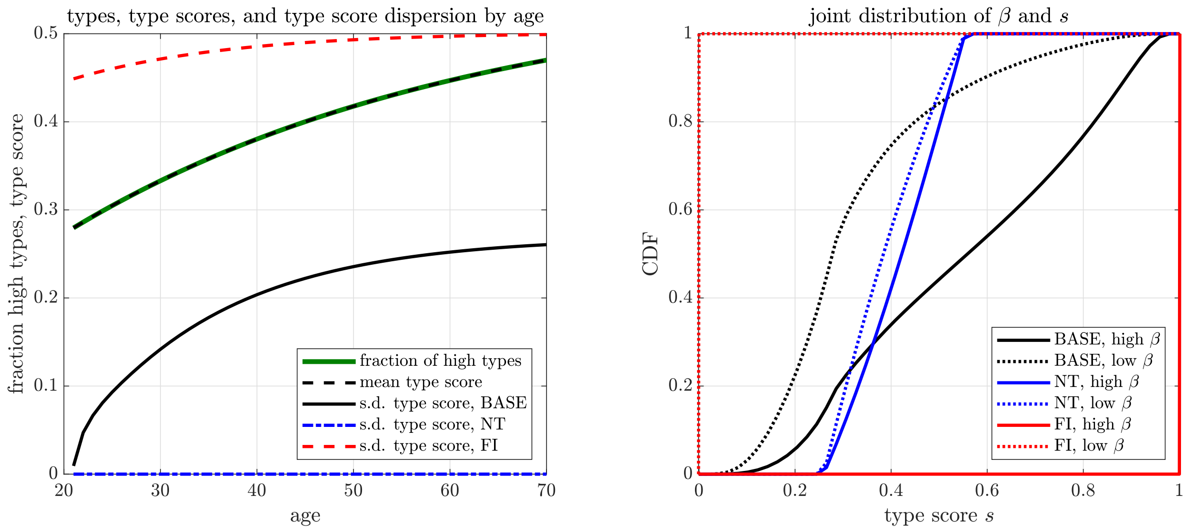


Notes: Each figure plots the percentiles of the variable specified in the title, conditional on the age specified on the horizontal axis. For example, a credit ranking of 0.2 puts an individual in the 95th percentile of 21 year olds, but only the 25th percentile of 30 year olds. Earnings in the figure include only the persistent component, e, since the transitory shocks z do not impact one's credit ranking and are i.i.d. and mean zero across individuals regardless of age.

Our first alternative economy poses hidden information as in the baseline model (hereafter termed BASE) but creditors are prohibited from using a person's past to price loans. We assume that age (or the length of one's credit history) is both publicly observable and legally used to price debt. We make this assumption to isolate the role of the incentives from the role of the demographic drift in the credit market. In this economy some information about the individual's type is inferred by her current asset choice but this information cannot be carried across periods. We refer to this economy as the *no-tracking* (NT) economy. To be concrete, in this alternative economy individuals start their adult life with a type score equal to the fraction of high types among newborns and it evolves thereafter according to the demographic drift slowly improving over time.[27] Note that this implies a one-to-one mapping between an individual's age and the prior that she is a high type (her type score in this alternative economy). Consistent with no-tracking, lenders are also not allowed to use information about an individual's beginning-of-period asset holdings when pricing loans since this also contains information about their past actions. However, lenders are able to use the current action and the cross-sectional distribution of agents in the NT economy when forming a posterior about the likelihood of repayment

[27]As stated previously, the evolution is simply given by $\bar{s}' = \bar{s} \cdot Q^{\beta}(H|H) + (1-\bar{s}) \cdot Q^{\beta}(H|L)$ with initial condition $\bar{s} = 0.28$.

Figure 14: **Evolution of Types and Type scores in Alternative Economies**



Notes: The green and black dashed lines of the left panel correspond to the fraction of high types at the indicated age and the mean type score at each age in each economy. These lines coincide by construction. The three model variants considered have different type score standard deviations. The right panel plots the type-specific CDF of type scores in each model economy. Black / blue / red refer to BASE / NT / FI model economies, and solid (dashed) lines refer to high (low) types.

necessary to price loans. In this NT economy there is private information and indeed, there is cross-subsidization, but there are no incentives to look like a high type, as actions cannot be used to impute type like they can in the BASE economy.

Our second alternative features *full information* (hereafter termed FI) where the type is directly observed by lenders that use it to price discriminate. Except for our extreme value shocks, this alternative economy is similar in many ways to Chatterjee et al. (2007). There is no need to infer a person's type and the price for a loan of size a' depends directly on β and all other relevant observables. Importantly, prices do not depend on s nor a because they are not directly payoff relevant. Comparing the FI economy to the BASE reveals the full impact of hidden information: in the FI there is no subsidizing nor incentives to look like a high type.

Some of the main distinguishing features of the three economies can be seen already in Figure 14. The right panel shows the CDF of the type scores for each type indicating the degree to which information about type is revealed to creditors. In the BASE economy, the CDF of type L individuals rises steeply and fast, indicating that most type L individuals have low scores. In contrast, the CDF of type H individuals rises more gradually, indicating that type H individuals have more dispersed types scores. In the NT economy, type scores (priors) are trapped between 0.28 (score at birth) and 0.54 (score at age infinity). Although people of all types share the same age-specific type score at each age, there are more type L individuals at each age (in the figure) than type H and, consequently, the CDF of low types rises somewhat faster. Most importantly, the CDFs are closest to each other for the NT economy, indicating that less is being learned about an individual's type as she ages, compared to the BASE and FI economies.

The left panel of Figure 14 plots the mean and standard deviation of type scores for each age across the alternative economies. Importantly, the mean type score at each age is correctly assessed in all economies to be equal to the fraction of high types. In the NT economy the standard deviation of type scores at all ages is zero as nothing is learned.[28] For the FI economy, the dispersion in type scores at any age is the dispersion of types themselves in the economy that has not yet reached its maximum by age 70. In the BASE economy it is increasing at a faster rate than in the FI economy, consistent with learning.

6.2 No-Tracking Economy

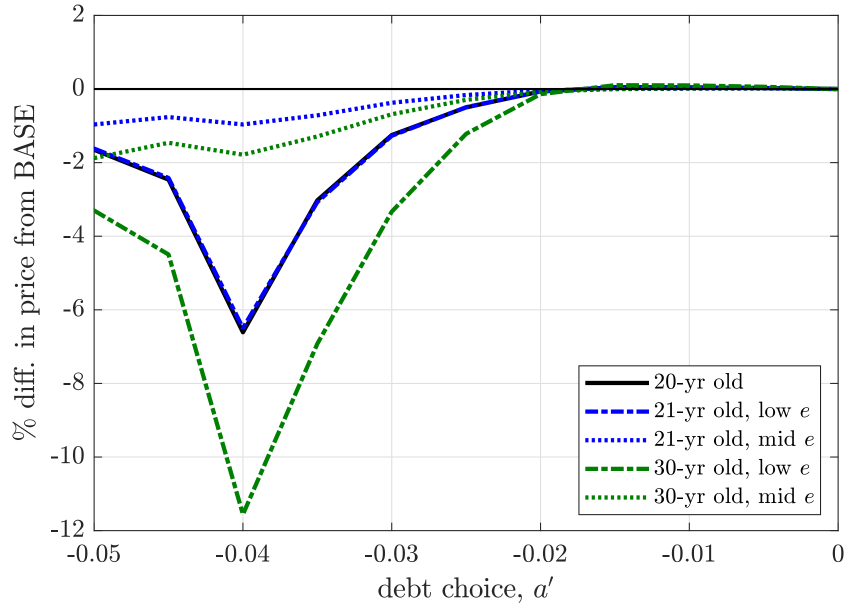
Recall that while creditors are prohibited from using a person's past to price loans in the NT economy, some information about the individual's type is inferred by her current asset choice and can be used to price each debt level accurately. Unlike the BASE economy, the assessment of default probabilities is not permitted to depend on beginning-of-period assets (which reflect past actions prohibited by no-tracking) but includes their age (since that determines the composition of types in a given borrowing pool). The key feature of NT is that the only information that can be used by lenders in the future (i.e. earnings class and the type score implied by their age) is exogenous so incentives to behave in asset markets are absent.

We start by looking at loan prices in Figure 15. The figure shows the percentage differences between the prices faced by some agents in the NT economy relative to the BASE economy in comparable states. For newborns (i.e. 20 year olds in our mapping to the data) the comparison is easy, as all newborns begin life in the same observable state in both economies (i.e. they have zero assets (and it is common knowledge they do), low earnings class, and they are high types with probability 0.28). The price difference comes only from the different probabilities of repayment across the two economies owing to differences in dynamic incentives. Prices are equal up to a loan size of 0.02 and for larger loans the prices are lower in NT, reflecting higher default probabilities at each loan size. This is due to the lower incentives to repay in the NT economy.

As individuals age, besides incentive differences between the economies, selection effects begin to

[28] Strictly speaking, because the evolution of s implied by the Markov type transition function for the NT economy typically does not yield scores which fall on the grid points in $\mathcal{S}$, there is some negligible dispersion in type scores.

Figure 15: **Loan Price Comparison Between BASE and NT Economies**

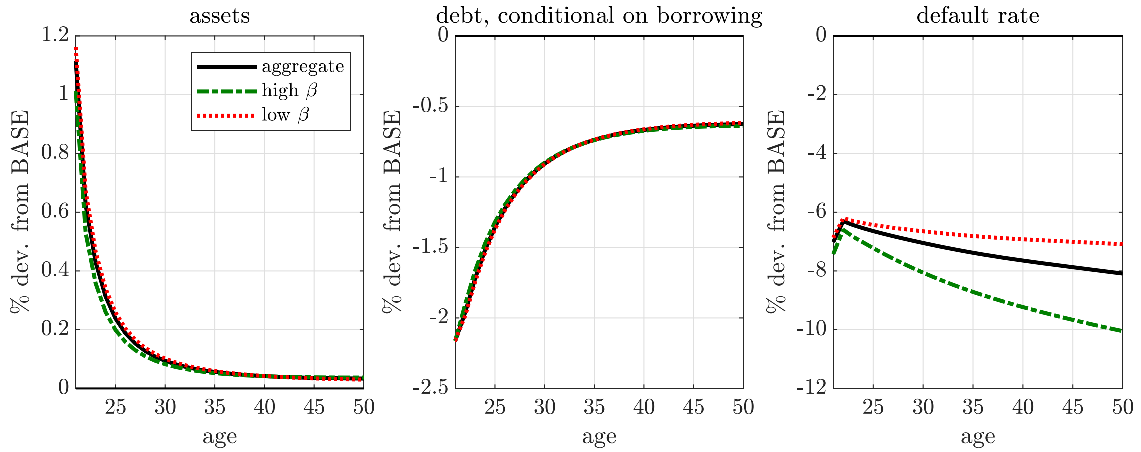


Notes: Let s_j denote the average type score for an agent of age j, and let a_j^{NT} be the average wealth of an agent of age j in the NT economy. Each line in the figure represents $100 \cdot (q_{NT}^{(0,a')}(j,e)/q_{BASE}^{(0,a')}(a_j^{NT}, s_j, e) - 1)$.

play a role. For 21 year olds, there are low and middle earning class individuals. The 21 year old low earners in the NT economy are of high type with probability 0.29 and have a certain amount of wealth $\widetilde{a}$, but they cannot be differentiated by asset holdings since the NT economy does not allow lenders to price using an individual's $\widetilde{a}$. Consequently, we compare the prices that they face with those faced by an agent with $s = 0.29$ and the same amount of wealth $\widetilde{a}$ in the BASE economy. We plot this for the case where $\widetilde{a}$ is the average asset holdings of 21 year olds in the NT economy. For low earners, the price difference from the baseline is slightly different while for mid earners the differences from the baseline are smaller. By age 30, most individuals have accumulated assets and so the need to borrow is much lower. Consequently, if an individual seeks to borrow, lenders assign a higher probability to the person having low assets and, so, of being the low type. In the baseline economy, assets are observable, so this selection effect on price is absent. This adverse selection effect is present for both 21 year olds and 30 year olds but is weaker at younger ages because individuals have not had enough time to accumulate assets.

In summary, an important takeaway from a legal restriction which prohibits tracking an individual's previous credit market actions is an equilibrium rise in borrowing rates individuals face on sizeable levels of debt. How do agents respond to these higher borrowing rates in the NT economy relative to BASE? A way to see this is via Figure 16 that compares allocations in the NT and the BASE by age. In the

Figure 16: **Percentage Differences by Age in No-Tracking Relative to Baseline Economy**



Notes: In each panel, each line corresponds to the percent difference in the SI economy relative to the Base in the average moment indicated at the specified age. For the type-specific measures, the average is computed conditional on type in both economies.

left panel agents save more in the NT economy, especially the young who have the lowest wealth and hence the most affected by a bad shock in the near future. Whatever incentives exist in BASE to save more than in NT to signal type are dwarfed by the actual price differences themselves. Conditional on borrowing, both types borrow less in the NT economy. The right panel shows there is a sizeable decrease in default, a feature that may look counterintuitive given the fact that default is not tracked into next period. However, lower default is a result of the lower individual borrowing in the NT economy, shifting from relatively large individual loans in BASE to smaller amounts of debt in NT. This is especially true for type H individuals who can shift to lower amounts of debt more easily since they are less impatient.

An important implication of these differences across economies is that the stronger incentives to pay back loans in BASE can accommodate a more active loan market than the NT economy. Indeed, Table 5 shows how much less debt there is in the NT economy, especially for the high types, both in absolute terms and in terms of the fractions of debt-to-income. As a result of this smaller debt volume there is less default in the NT economy.

Finally we turn to welfare. While it is immediate to talk about how newborns fare in both economies, there is more than one way to think of how the switch from the BASE to NT is implemented for people already alive. One possibility is to immediately outlaw the use of personal asset market history beyond the length of one's credit history (i.e. age). Alternatively, one could treat older individuals just like newborns, using information on their asset holdings and type score for the period of the policy switch but then knowledge about subsequent savings or defaults cannot be used. Rather than make a choice on implementation, we focus on newborns.[29]

The findings are sharp. Eliminating incentives based on the use of credit histories reduces the average welfare of newborns (measured using consumption equivalents) by 0.5% in Table 5.[30] The losses are larger for impatient agents mostly because they weigh the present more which is when they would be more likely using the credit market. As we might expect, the losses are higher – almost 1.0% – for those with the lowest transitory earnings. That all newborns are made worse off in the NT economy implies that incentive effects which affect all types outweigh the cross-subsidization of type L by type H.

Table 5: **Comparison of No-Tracking vs Baseline**

		Ratio to BASE (in %)		
		High Types	Low Types	All
Volume of Debt		-4.20	-3.16	-3.50
Debt to Income Ratio		-4.02	-2.97	-3.32
Default Rate		-9.03	-6.85	-7.52
		Consumption equivalent gain (in %)		
Transitory Earnings (z):	**Low**	**Median**	**High**	**Mean**
All Newborns	-0.88	-0.41	-0.25	**-0.513**
High type Newborns	-0.69	-0.33	-0.21	**-0.414**
Low type Newborns	-0.96	-0.44	-0.26	**-0.551**

6.3 Full Information

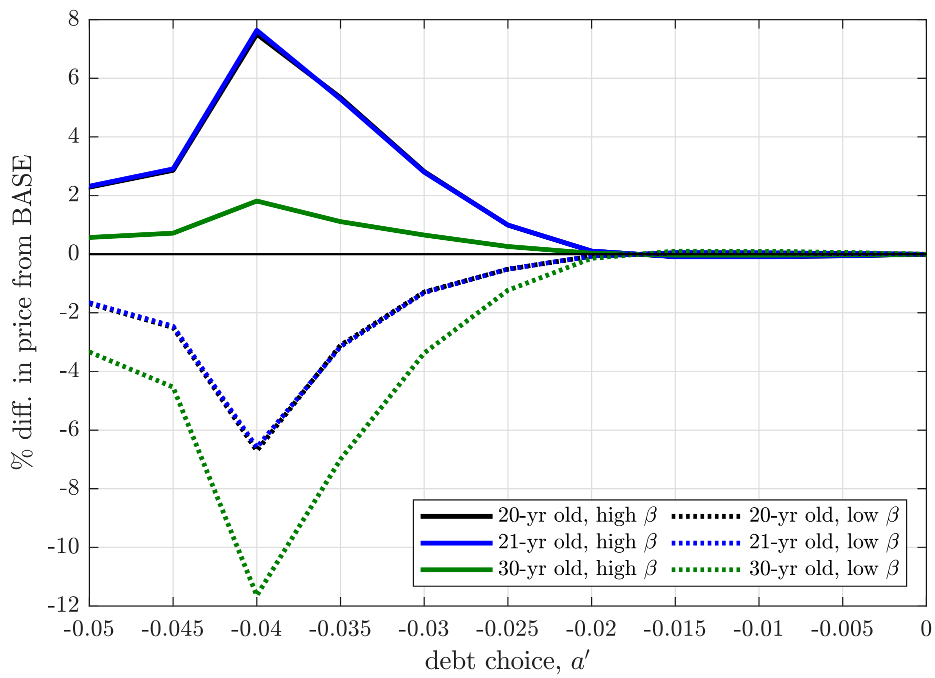
Under full information (FI), types are observed, cross-subsidization ends, and so do incentives for a type L to imitate a type H. In Figure 17, we compare the prices that individuals of a given age face in the FI economy with their counterpart in the BASE economy who has the average type score for that age and the FI economy's average asset holdings for that age. Since we use the average type score for a given age, the price comparison does take into account the learning that naturally occurs in the Base economy (i.e. type H (L) have a higher (lower) type score than the average type score for their given age).

[29] One consequence of focusing on newborns is that we do not need to compute a transition. At the moment of the policy switch, the average asset holdings of older cohorts are potentially different from those of the same age group in the steady state of the NT economy. Hence, even in a small open economy, all except the newborns face a transition of prices as the cross-sectional distribution used to infer future default probabilities evolves to the invariant distribution.

[30] See Appendix B.7 for the derivation of our consumption equivalent measures.

As one might expect, Figure 17 shows that for each age, type L in the FI economy face lower loan prices (higher interest rates) in the default territory (loans larger than .02) and type H face higher prices (lower interest rates) than the BASE economy where there is some cross-subsidization. The figure also shows that these price differences change with age. As individuals age, $1 - s$ falls for type H while $s - 0$ rises for type L so the elimination of cross-subsidization leads to even lower loan prices for type L and less of an improvement in loan prices for type H.

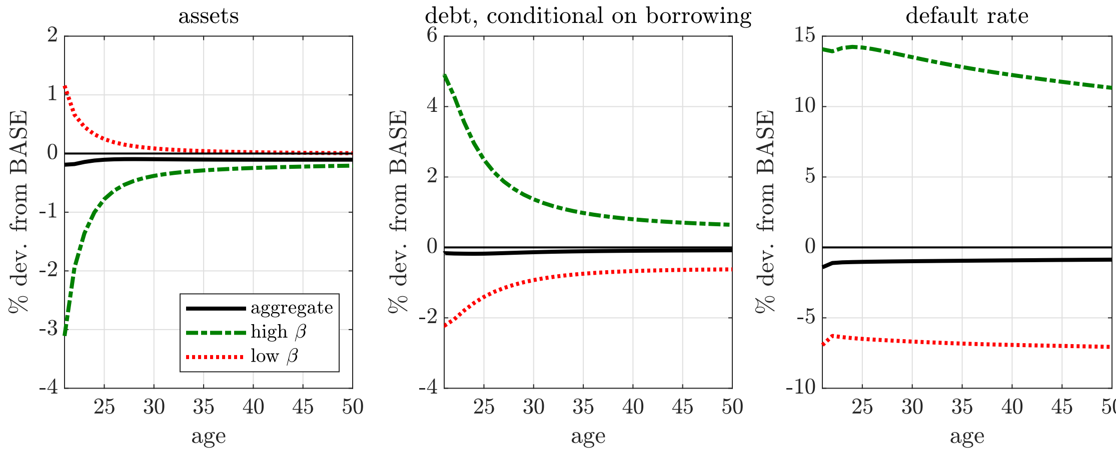
Figure 17: **Loan Price Comparison Between Full Information and Baseline Economies**



Notes: Let s_j denote the average type score for an agent of age j, and let a_j^{FI} be the average wealth of an agent of age j in the FI economy. Each line in the figure represents $100 \cdot (q_{FI}^{(0,a')}(\beta, e)/q_{BASE}^{(0,a')}(a_j^{FI}, s_j, e) - 1)$.

As a result of the price differences, agents take different actions which can be seen in Figure 18 that plots the differences between the FI and BASE economies in important observables by type and age, expressed again as percentage deviation relative to the BASE. The higher loan prices faced by the high types allows them to borrow more effectively and to fear less low future earnings which induce them to save less, while the opposite is true for the low types as the plot on assets holdings show. As time passes the differences shrink. Interestingly, there is a small aggregate effect with the FI economy showing lower assets overall despite the high types being a slight minority, which is due to the incentives built to save in the BASE. Due to the better prices faced by the high types and their lack of need to show that they are indeed high types, these agents default a lot more. The opposite occurs for the low types, they borrow less and default less.

Figure 18: **Percentage Differences by Age Between Full Information and Baseline**



Notes: In each panel, each line corresponds to the percent difference in the FI model relative to the baseline in the average moment indicated at the specified age. For the type-specific measures, the average is computed conditional on type in both economies.

Perhaps the most important takeaway from Figure 18 is that while the type-specific age profiles are very different from the base model, the *average* profiles are not. This explains why the aggregate credit market moments in Table 6 are quite similar between the BASE and FI economies, while the type-specific averages are quite different. High types both borrow and default more in the FI economy than BASE. The opposite happens to low types. As expected, due to the knowledge of types, we find that interest rate dispersion "within" types diminishes dramatically (over 90% lower for each type) while the overall dispersion rate (including "across" types) drops only about 2%.

The welfare implications for newborns are clear: high types rather live with full information where they do not subsidize the low types while the opposite is true for low types. On average, partly because of higher frequency of low types, the BASE is preferred.

7 Conclusion and Directions for Future Research

In this paper, we present a hidden information model of unsecured consumer credit with risk of default. People are subject to unobserved persistent and transitory shocks and the history of people's asset market actions help forecast future defaults. The setup is possibly the simplest environment to quantitatively study the role of credit scores in regulating consumer credit. We showed how this can be done using shocks drawn from an extreme value distribution and recursive updating of beliefs.

Our quantitative model with persistent hidden heterogeneity in types is capable of not only accounting for aggregate credit market moments but also the joint behavior of the age profile of credit rankings. Furthermore, the model generates an age profile of credit ranking patterns like those in U.S. data. In

Table 6: **Comparison of Full Information vs. Baseline**

	Ratio to BASE (in %)		
	High	Low	All
Volume of Debt	6.28	-3.14	-0.08
Debt to Income Ratio	5.77	-2.95	-0.09
Default Rate	12.21	-6.84	-1.00

	Consumption equivalent gain (in %)			
	Transitory Earnings (z)			
	Low	**Median**	**High**	**Mean**
All Newborns	-0.169	-0.018	0.033	**-0.051**
High type Newborns	1.684	0.846	0.579	**1.036**
Low type Newborns	-0.890	-0.354	-0.179	**-0.474**

Notes: All numbers are reported in percentages.

this sense, our model provides a quantitative theory of the credit score.

Two implications of our theory are worth highlighting. First, we found that restricting lenders' access to an individual's history of asset market actions (no tracking) leads to a welfare loss for all young adults. Since the young tend to borrow against their future income, the negative incentive effects from not having to maintain a good reputation raises interest rates faced by the young which offsets any benefits from pooling afforded to the low types. Our "big data" baseline model affords all of the young with better intertemporal insurance at the expense of worse intratemporal insurance for a subset of the population in a "small data" economy.

Second, even though our model allowed lenders unrestricted access to the history of all actions relevant for inferring an individual's type, the equilibrium allocations at an individual level remain far removed from those of a full information economy. This stems from the fact that individuals *select* actions that only partially reveal their type while in the full information economy they get that revelation for free. Despite big differences at the micro level, the macro (aggregate) differences are small.

Where next? First, type does not have to correspond to an individual's hidden time preference. Alternatively, it could correspond to hidden ability differences that exogenously affect earnings. Furthermore, hidden time preference could affect a hidden human capital decision (i.e. moral hazard) to endogenously affect earnings or to a variety of other personal traits. Second, while the theory is written for many types, the quantitative work assumed two types since it was adequate for the task at hand. Extending the analysis to higher dimensional type differences is left for future research. Third, to keep the state space reasonable, we restricted attention to a model with one asset so that we consider simply positions of net debt. A model with separate debt and assets (e.g. housing) can generate stronger reputation effects. In Online Appendix C we provide a simple example where reputation in the unsecured credit market spills over to other asset markets, reinforcing reputation effects. More generally, a person's reputation (or type score) in the unsecured credit market may have implications for other markets (e.g. insurance, labor) and other interactions (marriage) that are worth exploring.

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Online Appendices

A Model Appendix

A.1 Construction of $Q^s(s'|\psi)$ and Proof of Lemma 1

Let $G \equiv \{0, 1/K, 2/K, \dots 1\}$ be a uniform discrete approximation of $[0,1]$. Let $\mathcal{S} = \{(s_1, s_2, \dots s_B) | s_i \in G$ and $\sum_{i=1}^{B} s_i = 1\}$ be the associated probability simplex. Let $D = 1/K$ denote the distance between adjacent (grid) points of G.

Lemma 4. *Let $s_i \in G$ for $i = 1, 2, \dots B-1$. If $\sum_{i=1}^{B-1} s_i < 1$, then $1 - \sum_{i=1}^{B-1} s_i \in G$*

Proof. $\sum_{i=1}^{B-1} s_i < 1 \Rightarrow \sum_{i=1}^{B-1} (\ell_i/K) < 1 \Rightarrow \sum_{i=1}^{B-1} \ell_i < K$ where the ℓ_i's are integers between 0 and K. Since a sum of integers is an integer and a difference of two integers is also an integer, $K - \sum_{i=1}^{B-1} \ell_i$ is a positive integer and it is less than K. Therefore, by the definition of G, $1 - \sum_{i=1}^{B-1} \ell_i/K \in G$. $\square$

Assumption 2. *All elements of the matrix Q^β are strictly positive.*

Lemma 5. *Let $\psi = (\psi_1, \psi_2, \dots, \psi_B)$ be any vector of type scores resulting from the Bayesian update. Then, $\psi_i \geq \underline{Q} > 0$.*

Proof. Let $\underline{Q}$ be the smallest element of Q^β. By Assumption 1, $\underline{Q} > 0$.

$$\begin{aligned} \psi_i &= \sum_j Q^\beta(i|j) \times \text{ posterior probability of } j|\text{actions} \\ &\geq \sum_j \underline{Q} \times \text{ posterior probability of } j|\text{actions} \\ &= \underline{Q}. \end{aligned}$$

The first equality follows from the definition of ψ_i, the inequality follows from Assumption 1 and the last line follows from the fact that the sum of posterior probabilities is 1. $\square$

We now identify the elements of $\mathcal{S}$ that approximate any given type-score vector ψ resulting from the Bayesian update. Let $s_{i,L} = \max_{s \in G} s \leq \psi_i$ and $s_{i,H} = s_{i,L} + D$. Consider the collection of 2^{B-1} vectors:

$$S_\psi = \left\{ (s_{1,I(1)}, s_{2,I(2)}, \dots, 1 - \sum_{i=1}^{B-1} s_{i,I(i)}) \right\} \text{ where for each } i,\ I(i) \in \{L, H\}$$

Lemma 6. *If $D < \underline{Q}/(B-1)$ then $S_\psi \subset \mathcal{S}$.*

Proof. By construction, $s_{i,L} \in G$. Next, observe that $s_{i,L}$ cannot be 1 since that would imply that $\psi_i = 1$ and, therefore, $\psi_{j \neq i} = 0$ in contradiction to Lemma 5. Therefore, $s_{i,H} = s_{i,L} + D$ must belong in G for all i. To show that $(s_{1,I(1)}, s_{2,I(2)}, \dots, 1 - \sum_{i=1}^{B-1} s_{i,I(i)})$ belongs in $\mathcal{S}$ it is sufficient to show, by virtue of Lemma 4, that $\sum_{i=1}^{B-1} s_{i,I(i)} < 1$.

$$\begin{aligned}\sum_{i=1}^{B-1} s_{i,I(i)} &\leq \sum_{i=1}^{B-1} s_{i,H} \\ &\leq \sum_{i=1}^{B-1}(\psi_i + D) \\ &= (1-\psi_B) + (B-1)D \\ &\leq 1 - \underline{Q} + (B-1)D \\ &< 1\end{aligned}$$

The first inequality follows because $s_{i,I(i)} \leq s_{i,H}$. The second inequality follows because $s_{i,L} = s_{i,H} + D$ and $\psi_i \geq s_{i,L}$. The third equality follows because $\sum_{i=1}^{B} \psi_i = 1$. The fourth inequality follows from Lemma 5 and the final inequality follows from the hypothesis of the Lemma. □

By virture of lemma 6 we can take S_ψ to be the collection of approximating vectors. Note that for each member of this set, the first $B-1$ components are within $\psi_i \pm D$ so, in this sense, the vectors are close to ψ.

We now determine the probability assigned to each of these vectors. To this end, let

$$p(s_{i,L}) = \frac{s_{i,H} - \psi_i}{D} \text{ and } p(s_{i,H}) = \frac{\psi_i - s_{i,L}}{D} \text{ for } i = 1, 2, 3, \dots B-1 \tag{19}$$

Since $s_{i,L} \leq \psi_i < s_{i,H}$ and $s_{i,H} - s_{i,L} = D$ both $p(s_{i,L})$ and $p(s_{i,H})$ are nonnegative and their sum is 1. We set

$$\Pr\left[\left(s_{1,I(1)}, s_{2,I(2)}, s_{3,I(3)}, \dots, 1 - \sum_{i=1}^{B-1} s_{i,I(i)}\right)\right] = \prod_{i=1}^{B-1} p(s_{i,I(i)}), \quad I(i) \in \{L, H\}, i = 1, 2, \dots B-1.$$

Then our assignment rule $Q^s(s'|\psi) : \mathcal{S} \to [0,1]$ is given by:

$$Q^s(s'|\psi) = \begin{cases} \prod_{i=1}^{B-1} p(s'_{i,I(i)}) & \text{if } s' \in S_\psi \\ 0 & \text{otherwise} \end{cases} \tag{20}$$

For this assignment rule, we can prove:

Lemma 1. *(i)* $\sum_{s' \in \mathcal{S}} s'_i Q^s(s'|\psi) = \psi_i, \forall i$, *(ii)* $\sum_{s' \in \mathcal{S}} (s'_i - \psi_i)^2 Q^s(s'|\psi) \leq 2(B-1)D^2, \forall i$, *and (iii)* $Q^s(s'|\psi)$ *is continuous in* ψ.

Proof. (i) First, note that

$$\sum_{s' \in \mathcal{S}} s'_i Q^s(s'|\psi) = \sum_{s' \in S_\psi} s'_i Q^s(s'|\psi)$$

since (20) assigns positive probability only to vectors that are in S_ψ.

Let $i \in \{1, 2, \dots, B-1\}$. Now, group the collection of vectors in S_ψ into two: In the first group are all vectors for which $s'_i = s_{i,L}$ and in the second group are all vectors for which $s'_i = s_{i,H}$. Denote these groups as S^L_ψ and S^H_ψ. Then,

$$\begin{aligned}
\sum_{s' \in S_\psi} s'_i Q^s(s'|\psi) &= \sum_{s' \in S^L_\psi} s'_i Q^s(s'|\psi) + \sum_{s' \in S^H_\psi} s'_i Q^s(s'|\psi) \\
&= s_{i,L} \sum_{s' \in S^L_\psi} Q^s(s'|\psi) + s_{i,H} \sum_{s' \in S^H_\psi} Q^s(s'|\psi) \\
&= s_{i,L} p(s_{i,L}) + s_{i,H} p(s_{i,H}) \\
&= \psi_i
\end{aligned}$$

The third equality follows from the fact that the first and second summation terms in the second line are the probabilities of selecting a vector that belongs to group L and group H, respectively. Since the assignment of $s_{i,L}$ or $s_{i,H}$ for s'_i is done independently of the assignments to the other $B-2$ components, the probability of selecting a vector in group L is $p(s_{i,L})$ and in group H is $p(s_{i,H})$. The last equality follows from (19).

Next, let $i = B$. Then,

$$\begin{aligned}
\sum_{s' \in S_\psi} s'_B Q^s(s'|\psi) &= \sum_{s' \in S_\psi} [1 - s'_1 - s'_2 - \dots - s'_{B-1}] Q^s(s'|\psi) \\
&= \sum_{s' \in S_\psi} Q^s(s'|\psi) - \sum_{i=1}^{B-1} \sum_{s' \in S_\psi} s'_i Q^s(s'|\psi) \\
&= 1 - \sum_{i=1}^{B-1} \psi_i \\
&= \psi_B
\end{aligned}$$

(ii) Let $i \in \{1, 2, \dots, B-1\}$.

$$\begin{aligned}
\sum_{s' \in S_\psi} (s'_i - \psi_i)^2 Q^s(s'|\psi) &= \sum_{s' \in S^L_\psi} (s'_i - \psi_i)^2 Q^s(s'|\psi) + \sum_{s' \in S^H_\psi} (s'_i - \psi_i)^2 Q^s(s'|\psi) \\
&= \sum_{s' \in S^L_\psi} (s_{i,L} - \psi_i)^2 Q^s(s'|\psi) + \sum_{s' \in S^H_\psi} (s_{i,H} - \psi_i)^2 Q^s(s'|\psi) \\
&\le D^2 \sum_{s' \in S^L_\psi} Q^s(s'|\psi) + D^2 \sum_{s' \in S^H_\psi} Q^s(s'|\psi) \\
&= D^2 (p(s_{i,L}) + p(s_{i,H})) \\
&= D^2.
\end{aligned}$$

Let $i = B$. Then,

$$\sum_{s' \in S_\psi} (s'_B - \psi_B)^2 Q^s(s'|\psi) = \sum_{s' \in S_\psi} \left(1 - \sum_{i=1}^{B-1} s'_i - 1 + \sum_{i=1}^{B-1} \psi_i\right)^2 Q^s(s'|\psi)$$

$$= \sum_{s' \in S_\psi} \left(\sum_{i=1}^{B-1} (s'_i - \psi_i) \right)^2 Q^s(s'|\psi)$$

$$= \sum_{i=1}^{B-1} \sum_{s' \in S_\psi} \left(s'_i - \psi_i\right)^2 Q^s(s'|\psi) + \text{ expectations of cross product terms}$$

$$\leq (B-1)D^2.$$

The inequality in the final line follows from the bound on each of the variances and from the fact that the assignments of s'_i for $i \in \{1, 2, \ldots, B-1\}$ are independent of each other so that the expectation of all the cross product terms are zero.

(iii) Let ψ_n be a sequence converging to ψ^*. Consider first the case where $\psi_i^* \notin G$. Then, for $n > N$, N sufficiently large, $\psi_i^n \in (s^*_{i,L}, s^*_{i,H})$ and, so,

$$p^n(s_{i,L}) = \frac{s^*_{i,H} - \psi_i^n}{D} \text{ and } p^n(s_{i,H}) = \frac{\psi_i^n - s^*_{i,L}}{D}.$$

It follows that $\lim_{n\to\infty} p^n(s_{i,L}) = p^*(s_{i,L})$ and $\lim_{n\to\infty} p^n(s_{i,H}) = p^*(s_{i,H})$. Next consider the case where $\psi_i^* \in G$. Then, by construction

$$s^*_{i,L} = \psi_i^*, \;\; s^*_{i,H} = s^*_{i,L} + D \text{ and } p^*(s^*_{i,L}) = 1.$$

Then, for $n > N$, N sufficiently large, either $\psi_i^n \in (s^*_{i,L} - D, s^*_{i,L})$ or $\psi_i^n \in (s^*_{i,L}, s^*_{i,L} + D)$. Therefore, $p^n(s^*_{i,L})$ converges to $1 = p^*(s^*_{i,L})$ as ψ_i^n converges to ψ_i^*. $\square$

Note that by reducing the distance between adjacent points of G, or, equivalently, increasing the number of (uniformly-placed) grid points approximating the unit interval, the dispersion of s' around ψ can be made arbitrarily small.

A.2 Proof of Theorem 1 (Existence of the Value Function)

Theorem 1. *Given f, there exists a unique solution $W(\beta, z, \omega|f)$ to the individual's decision problem in* (3) - (5).

Proof. The proof relies on the Contraction Mapping Theorem. However, since $\epsilon^{(d,a')}$ can take any value on the real line, it is mathematically more convenient to seek a solution to (2), (4), (9), and (10) since the extreme value errors do not appear in these. Define the operator $(T_f)(W) : \mathbb{R}^{B+Z+|\Omega|} \to \mathbb{R}^{B+Z+|\Omega|}$ as the map that takes a vector W in $\mathbb{R}^{B+Z+|\Omega|}$ and returns a vector $(T_f)(W)$ via (4), (9), (10) using (2). We may easily verify that T_f satisfies Blackwell's sufficiency condition for a contraction map (with modulus $\beta\rho$). Since $\mathbb{R}^{B+Z+|\Omega|}$ is a complete metric space (with, say, the uniform metric $\rho(W, W') = \max_{1\leq i \leq B+Z+|\Omega|} \|W_i - W'_i\|$), by Theorem 3.2 of Stokey and Lucas Jr. (1989), there exists a unique $W(\beta, z, \omega|f)$ satisfying $(T_f)(W) = W$. $\square$

A.3 Proof of Lemma 2 (Existence of the Invariant Distribution)

Lemma 2. *There exists a unique invariant distribution $\bar{\mu}(\cdot|f)$ and $\{\mu_0 T^n\}$ converges to $\bar{\mu}(\cdot|f)$ at a geometric rate for any initial distribution μ_0.*

Proof. We will use Theorem 11.4 in Stokey and Lucas Jr. (1989) to establish this result. To connect to that theorem, let i be a typical element of the finite state space $\mathcal{B} \times \mathcal{Z} \times \Omega$. Let the transition matrix Π in their theorem corresponds to T in (16) and let π_{ij} denote the probability of transiting to j from i. Further, let $\epsilon_j = \min_i \pi_{ij}$ and $\epsilon = \sum_j \epsilon_j$. Then it is sufficient to establish that $\epsilon > 0$. To this end, consider the state $\hat{j} = (\hat{\beta}, \hat{z}, \hat{e}, 0, G_s)$ with the property that $g(\hat{\beta})H(\hat{z})G_e(\hat{e}) > 0$. Then, (16) implies $\pi_{i\hat{j}} \geq (1-\rho)g(\hat{\beta})H(\hat{z})G_e(\hat{e}) > 0$ for all i. Hence $\epsilon_{\hat{j}} \geq (1-\rho)g(\hat{\beta})H(\hat{z})G_e(\hat{e}) > 0$. Since $\epsilon_j \geq 0$ for all other j, it follows that $\epsilon > 0$.

□

A.4 Proof of Lemma 3 (Continuity of the Value Function)

Lemma 3. *$W(\beta, z, \omega|f)$ is continuous in f and for any $(d, a') \in \mathcal{F}(z, \omega|f)$, $\sigma^{(d,a')}(\beta, z, \omega|f)$ is continuous in f.*

Proof. To prove continuity of $W(\beta, z, \omega|f)$ in f we first show that the operator T_f defined in Theorem 1 is continuous in f (meaning that for any given W, small changes in f lead to small changes in $T_f(W)$). Inspection of (9) and (10) shows that this will be true if the conditional value functions $v^{(d,a')}(\beta, z, e, a, s|f)$ in (4) are continuous in f. Let $\bar{f} \in F^*$ and let $(\hat{d}, \hat{a}') \in \mathcal{F}(z, \omega|\bar{f})$. Let $f^n \in F^*$ be a sequence converging to $\bar{f}$. By Assumption 1, $(0, 0)$ and $(1, 0)$ are feasible choices regardless of the value of any inherited debt (i.e. $a < 0$), so all debt choices ($a' < 0$) and the default choice belong in $\mathcal{F}(z, \omega|f^n)$. Furthermore, if an asset choice (i.e. $a' \geq 0$) is feasible for $\bar{f}$, that asset choice remains feasible for f^n since the price of any asset is the same in $\bar{f}$ and f^n (namely, $\rho/(1 + r)$). Therefore, $(\hat{d}, \hat{a}') \in \mathcal{F}(z, \omega|f^n)$ and so $v^{(\hat{d},\hat{a}')}(\beta, z, e, a, s|f^n)$ is well-defined for all n. Observe that f^n affects $v^{(d,a')}(\beta, z, e, a, s|f^n)$ in (4) via how q^n affects the feasible set given in (2) and how ψ^n affects $Q^s(s'|\psi^n)$ in (4). Since $\lim_{n\to\infty} c^{(\hat{d},\hat{a}')}(z, \omega|f^n) = c^{(\hat{d},\hat{a}')}(z, \omega|\bar{f})$, the continuity of u gives $\lim_{n\to\infty} u(c^{(\hat{d},\hat{a}')}(z, \omega|f^n)) = u(\lim_{n\to\infty} c^{(\hat{d},\hat{a}')}(z, \omega|f^n)) = u(c^{(\hat{d},\hat{a}')})z, \omega|\bar{f}))$. From Lemma 1, $\lim_{n\to\infty} Q^s(s'|\psi_{\beta'}^{(d,a')}(\omega|f^n)) = Q^s(s'|\psi_{\beta'}^{(d,a')}(\omega|\bar{f}))$. It follows that $v^{(d,a')}(\beta, z, e, a, s|f)$ is continuous in f and hence $\lim_{n\to\infty} T_{f^n} = T_{\bar{f}}$. Since F is a Banach space and T_f is a contraction map, we may apply Theorem 4.3.6 in Hutson and Pym (1980) to conclude that W is continuous in f. The continuity of $\sigma^{(d,a')}(\beta, z, \omega|f)$ in f follows directly by continuity of σ in W.

A.5 Proof of Theorem 2 (Existence of Equilibrium)

Theorem 2. *There exists a stationary recursive competitive equilibrium.*

Proof. The proof of existence uses Brouwer's Fixed Point Theorem (Theorem 17.3 in Stokey and Lucas Jr. (1989)). To connect to that theorem, we reinterpret the function f as a point in a unit (hyper)cube in high-dimensional Euclidean space. To this end, let $\mathcal{G} = \{((d, a'), \beta, z, \omega) : (d, a') \in \mathcal{Y}, \beta \in \mathcal{B}, z \in \mathcal{Z}, \omega \in \Omega\} \subset \mathcal{Y} \times \mathcal{B} \times \mathcal{Z} \times \Omega$ where $\mathcal{Y} = \{(d, a') : (d, a') \in \{0\} \times \mathcal{A}$ or $(d, a') = (1, 0)\}$. Let M and K be the cardinalities of $\mathcal{G}$ and $\mathcal{Y} \setminus \{(1, 0)\}$. Then, $f \in F^*$ can be thought of as a vector composed by stacking $q \in [0,1]^K$ and $\psi \in [0,1]^{B\cdot M}$. Then $f \in [0,1]^{K+B\cdot M}$ and $F^* \subset [0,1]^{K+B\cdot M}$. Next, use (12) (with equality) to construct the vector $q_{\text{new}}^{(0,a')}(\omega|f)$ and use (13) to construct the vector $\psi_{\text{new}}^{(d,a')}(\omega|f)$. Then, let J be the mapping

$$f_{\text{new}} \equiv (q_{\text{new}}^{(0,a')}, \psi_{\text{new}}^{(0,a')}, \psi_{\text{new}}^{(1,0)}) = J(f) : F^* \to F^*.$$

Since $\sigma^{(d,a')}(\beta, z, \omega|f)$ is a continuous function of f (Lemma 3), J is a continuous self-map as (12) and (13) are continuous functions of $\sigma^{(d,a')}(\beta, z, \omega|f)$. And since F^* is a nonempty, closed, bounded and convex subset of a finite-dimensional normed vector space, by the Brouwer's FPT there exists $f^* \in F^*$ such that $f^* = J(f^*)$.

□

A.6 Equivalence

Let $\hat{\Omega} = \{(e, a, m) : m \in \mathcal{P}(e, a), \forall (e, a) \in \mathcal{E} \times \mathcal{A}\}$ with typical element $\hat{\omega} \in \hat{\Omega}$. An individual who considers whether to default d or choose asset a' in state $(\epsilon, \beta, z, \hat{\omega})$ takes as given

- a price function $q^{(0,a')}(\hat{\omega}) : \mathcal{A} \times \hat{\Omega} \to [0,1]$,
- a credit-score transition function $Q^m(m'|(d,a'), e', \hat{\omega}) : \mathcal{P}(e', a') \times \mathcal{A} \times \mathcal{E} \times \hat{\Omega} \to [0,1]$,

when optimizing. As in (2), this implies that an individual of type β in state $(z, \hat{\omega})$ chooses $(d, a') \in \mathcal{F}(z, \hat{\omega})$ inducing consumption $c^{(d,a')}(z, \hat{\omega})$ satisfying:

$$c^{(d,a')}(z, \hat{\omega}) = \begin{cases} e + z + a - q^{(0,a')}(\hat{\omega}) \cdot a' & \text{if } (d, a') = (0, a') \\ e + z - \kappa & \text{if } a < 0 \text{ and } (d, a') = (1, 0) \end{cases} \tag{21}$$

For all $(d, a') \in \mathcal{F}(z, \hat{\omega})$, the value functions given by equations (3), (5), (9), (10) and choice probabilities given by equations (6), (7), (8) associated with the individual's problem are unchanged in form after substituting $\hat{\omega}$ for ω except for equation (4) now given by:

$$\begin{aligned} v^{(d,a')}(\beta, z, \hat{\omega}) \;=\; & (1-\beta\rho)u\left(c^{(d,a')}(z, \hat{\omega})\right) \\ & +\beta\rho \cdot \sum_{\beta', z', e', m'} Q^\beta(\beta'|\beta) Q^e(e'|e) H(z') Q^m(m'|(d,a'), e', \hat{\omega}) W(\beta', z', \hat{\omega}) \end{aligned} \tag{22}$$

Intermediaries issue a positive measure of contracts taking the price function $q^{(0,a')}(\hat{\omega})$ and probability of repayment function $p^{(0,a')}(\hat{\omega})$ as given to maximize profits:

$$\pi^{(0,a')}(\hat{\omega}) = \begin{cases} \rho \cdot \frac{p^{(0,a')}(\hat{\omega})\cdot(-a')}{1+r} - q^{(0,a')}(\hat{\omega}) \cdot (-a') & \text{if } a' < 0 \\ q^{(0,a')} \cdot a' - \rho \cdot \frac{a'}{1+r} & \text{if } a' \geq 0 \end{cases}. \tag{23}$$

If the intermediary issues a strictly positive measure of credit contracts, then zero profits requires:

$$q^{(0,a')}(\hat{\omega}) = \begin{cases} \frac{\rho \cdot p^{(0,a')}(\hat{\omega})}{1+r} & \text{if } a' < 0 \\ \frac{\rho}{1+r} & \text{if } a' \geq 0 \end{cases} \tag{24}$$

which is the analogue of (12).

Consistency requires that the probability of repayment satisfy the analog of (14), namely,

$$p^{(0,a')}(\hat{\omega}) = \sum_{\beta',z',e',m'} H(z') \cdot Q^e(e'|e) \cdot Q^m(m'|(d,a'),e',\hat{\omega}) \cdot M_{\beta'}(\hat{\omega}') \cdot \left(1 - \sigma^{(1,0)}(\beta',z',\hat{\omega}')\right). \tag{25}$$

Here, $M_\beta(\hat{\omega}) : \hat{\Omega} \to \mathcal{S}$ is a function mapping m to the probability an individual is of a given type β. We will denote $M(\hat{\omega}) = (M_{\beta_1}(\hat{\omega}), ..., M_{\beta_B}(\hat{\omega}))$.

The transition function in equation (16) which tracks the probability that an individual in state $(\beta, z, \hat{\omega})$ transits to state $(\beta', z', \hat{\omega}')$ is now given by:

$$\begin{aligned} T(\beta', z', \hat{\omega}'; \beta, z, \hat{\omega}) = \\ &\rho \cdot Q^\beta(\beta'|\beta) \cdot H(z') \cdot Q^e(e'|e) \cdot \sigma^{(d,a')}(\beta, z, m) \cdot Q^m(m'|(d,a'),e',\hat{\omega}) \\ &\quad + (1-\rho) \cdot G_\beta(\beta') \cdot H(z') \cdot G_e(e') \cdot \mathbf{1}_{\{a'=0\}} \cdot \mathbf{1}_{\{s'=G_\beta\}}. \end{aligned} \tag{26}$$

We can now give the definition of a stationary recursive competitive equilibrium with credit scores.

Definition 6. Stationary Recursive Competitive Equilibrium with Credit Scores A stationary Recursive Competitive Equilibrium with Credit Scores (RCECS) is a pricing function $q^{(0,a')*}(\hat{\omega})$, a credit-scoring function $Q^{m*}(m'|(d,a'),e',\hat{\omega})$, a choice probability function $\sigma^{(d,a')*}(\beta,z,\hat{\omega})$, a probability of repayment function $p^{(0,a')*}(\hat{\omega})$, a credit-score-to-type-probability function $M^*(\hat{\omega})$, and a steady state distribution $\overline{\mu}^*(\hat{\omega})$ such that:

(i). Given $q^{(0,a')*}(\hat{\omega})$ and $Q^{m*}(m'|(d,a'),e',\hat{\omega})$, $\sigma^{(d,a')*}(\beta,z,\hat{\omega})$ satisfies (6) and (7) for all $(\beta,z,\hat{\omega}) \in \mathcal{B} \times \mathcal{Z} \times \hat{\Omega}$ and $(d,a') \in \mathcal{F}(z,\hat{\omega})$,

(ii). Given $Q^{m*}(m'|(d,a'),e',\hat{\omega})$, $M^*(\hat{\omega})$, and $\sigma^{(1,0)*}(\beta,z,\hat{\omega})$, $p^{(0,a')*}(\hat{\omega})$ satisfies (25) for all $\hat{\omega} \in \hat{\Omega}$ and $(d, a' < 0) \in \mathcal{F}(z,\hat{\omega})$,

(iii). Given $p^{(0,a')*}(\hat{\omega})$, $q^{(0,a')*}(\hat{\omega})$ satisfies (24) with equality for all $\hat{\omega} \in \hat{\Omega}$ and $(d,a') \in \mathcal{F}(z,\hat{\omega})$,

(iv). Given $Q^{m*}(m'|(d,a'),e',\hat{\omega})$ and $\sigma^{(1,0)*}(\beta,z,\hat{\omega})$, $\overline{\mu}^*(\beta,z,\hat{\omega})$ is a fixed point of $\mu'(\beta',z',\hat{\omega}') = \sum_{\beta,z,\hat{\omega}} T^*(\beta',z',\hat{\omega}'|\beta,z,\hat{\omega}) \cdot \mu(\beta,z,\hat{\omega})$ for T^* in (26).

Theorem 3. Equivalence: *Suppose an RCE exists and let $m = p^{(0,\bar{a})*}(e,a,s)$ using (14). Suppose $M^*(\hat{\omega})$ given by the inverse function $m = p^{(0,\bar{a})*}(e,a,M^*(e,a,m))$ exists. Let $Q^{m*}(m' = \tilde{m}|(d,a'),e',\hat{\omega})) \equiv Q^s(s'(\beta') = M^*_{\beta'}(e',a',\tilde{m})|\psi^{(d,a')*}(e,a,s))$ using (13) $\forall \tilde{m} \in \mathcal{P}(e',a')$. Then an RCECS also exists in which the choice probabilities $\sigma^{(d,a')*}(\beta,z,e,a,m) = \sigma^{(d,a')*}(\beta,z,e,a,s)$.*

Proof. Provided $q^{(0,a')*}(\hat{\omega}) = q^{(0,a')*}(\omega)$, then $\mathcal{F}(z,\omega) = \mathcal{F}(z,\hat{\omega})$ in (2) and (21). Since $Q^{m*}(m' = \tilde{m}|(d,a'),e',\hat{\omega})) = Q^s(s'(\beta') = M^*_{\beta'}(e',a',\tilde{m})|\psi^{(d,a')*}(e,a,s))$, then $v^{(d,a')}(\beta,z,\omega)$ in (4) is identical to $v^{(d,a')}(\beta,z,\hat{\omega})$ in (22) as well as all other value functions. Hence $\sigma^{(d,a')}(\beta,z,\omega) = \sigma^{(d,a')}(\beta,z,\hat{\omega})$ satisfying (i). If the choice probabilities are the same, then repayment probabilities in (14) and (25) are the same since $s'(\beta') = M^*_{\beta'}(\hat{\omega}')$ and $Q^{m*} = Q^s$, thereby satisfying (ii). If the repayment probabilities in (14) and (25) are the same, then prices in (12) and (24) are the same, thus satisfying (iii) and our initial conjecture $q^{(0,a')*}(\hat{\omega}) = q^{(0,a')*}(\omega)$. Since $\sigma^{(d,a')}(\beta, z, \omega) = \sigma^{(d,a')}(\beta, z, \hat{\omega})$ and $Q^{m*} = Q^{s}$, then (16) is the same as (26) so that (iv) holds. □

B Quantitative Appendix

B.1 Computational algorithm

In this section, we describe the algorithm used to compute the baseline model presented in this paper. Note that the model is calibrated by using the procedure below to solve the model for a given set of parameters, and then updating these parameters to minimize the distance between the model moments and the data moments.

1. Set parameters and tolerances for convergence and create grids for (β, e, z, a, s).
2. Initialize the following equilibrium objects with sensible initial conditions:[31]
   (a) value function, $W(\beta, e, z, a, s)$ from (10)
   (b) type scoring function, $\psi^{(d,a')}(e, a, s)$ from (13)
   (c) repayment probability, $p^{(0,a')}(e, a, s)$ from (14). Note that this implies a loan price schedule given condition (12).
   (d) stationary distribution, $\mu(\beta, e, z, a, s)$ from (15)
3. Taking the current guess of the equilibrium functions $f_0 = \{q_0, \psi_0\}$ as given, enter the equilibrium computation loop:
   (a) Solve for the expected value function $W_1(\cdot|f_0)$ taking as given $W_0(\cdot|f_0)$:
      i. Assess budget feasibility, finding the set of feasible actions $\mathcal{F}(e, z, a, s|f_0)$.
      ii. Compute the conditional value associated with each action $(d, a') \in \mathcal{F}(e, z, a, s|f_0)$, $v_1^{(d,a')}(\beta, e, z, a, s|f_0)$, according to (4), with $W(\cdot) = W_0(\cdot)$
      iii. Having looped over all feasible actions, aggregate the conditional values $v_1^{(d,a')}(\cdot|f_0)$ into the new expected value function $W_1(\cdot|f_0)$ according to (3).
      iv. Assess value function convergence in terms of the sup norm metric, $\sup|W_1(\cdot|f_0) - W_0(\cdot|f_0)|$. If less than tolerance, go to 3.b; otherwise, set $W_0(\cdot|f_0) = W_1(\cdot|f_0)$ and go back to 3.a.ii.
   (b) Compute the decision probabilities $\sigma^{(d,a')}(\beta, e, z, a, s|f_0)$ implied by $W_1(\cdot|f_0)$ according to (6), (7), and (9).
   (c) Given the decision probabilities $\sigma(\cdot|f_0)$, compute the new set of equilibrium functions, $f_1 = \{q_1, \psi_1\}$:
      i. Compute $\psi_1^{(d,a')}(e, a, s)$ according to (13).
      ii. Compute $q_1^{(0,a')}(e, a, s)$ according to (12).

[31] Since the full information version of the model solves very quickly, the value functions, loan price schedules, and stationary distributions from this case are good initial conditions. For type scores, a consistent initial guess is $\psi^{(d,a')}(e, a, s) = sQ^{\beta}(\beta_H|\beta_H) + (1-s)Q^{\beta}(\beta_H|\beta_L)$.

(d) Assess equilibrium function convergence in terms of the sup norm metric

$$\max \{\sup |\psi_1 - \psi_0|, \sup |q_1 - q_0|\}$$

If less than tolerance, proceed to step 4; otherwise, set $f_0 = f_1$ and go back to the beginning of step 3.

4. Compute the stationary distribution associated with the equilibrium behavior and the equilibrium functions computed in step 3.

   (a) For each state, compute $\mu_1(\beta, e, z, a, s)$ according to (15), with $\mu(\cdot) = \mu_0(\cdot)$ and given the set of equilibrium functions solved for above.

   (b) Assess convergence based on the sup norm metric $\sup |\mu_1(\cdot) - \mu_0(\cdot)|$. If less than tolerance, go to step 5; otherwise, set $\mu_0 = \mu_1$ and go back to 4.a.

B.2 The Role of Extreme Value Preference Shocks

One of the key modifications in our model relative to standard consumer bankruptcy models in macroeconomics is the inclusion of the additive, action-specific preference shocks.[32] The mean of these extreme value shocks is irrelevant, but we calibrate the scale parameter α, which governs the overall variance of the shocks, as well as the correlation parameter λ, which governs the correlation between the shocks associated with non-default actions. How does behavior in the model change with respect to these parameters? In this section we address this question in two ways. First, we use an analytical approach to show how these parameters directly affect choice probabilities, taking the value of specific actions (i.e. all equilibrium functions) and earnings processes as given. Then, in the context of the full information version of the model, we compute actual decision rules under different parameter combinations and describe the differences graphically.[33]

B.2.1 Deriving the impact of EV parameters

To ease notation in this section, let an agent's entire state be denoted by $x = (\beta, e, z, a, s)$, and the set of feasible actions for that agent be denoted by $\mathcal{F}(x)$. The goal of this section is to show how the choice probability function σ varies with the extreme value scale and correlation parameters, α and λ. We first cover the non-default actions, and then default.

Non-default actions Equation (7) describes the probability of choosing a feasible action $(0, a')$ conditional on not defaulting. To ease computations in this section rather than compute derivatives with respect to α or λ directly, we will compute them with respect to $1/\alpha$ and $1/\lambda$.[34] Considering first α,

$$\begin{aligned}\frac{\partial \tilde{\sigma}^{(0,a')}(x)}{\partial(1/\alpha)} = & \left\{ \exp\left\{v^{(0,a')}(x)/\lambda\alpha\right\} \frac{v^{(0,a')}(x)}{\lambda} \sum_{(0,\tilde{a})\in\mathcal{F}(x)} \exp\left\{v^{(0,\tilde{a})}(x)/\lambda\alpha\right\}\right. \\ & \left. - \exp\left\{v^{(0,a')}(x)/\lambda\alpha\right\} \sum_{(0,\tilde{a})\in\mathcal{F}(x)} \exp\left\{v^{(0,\tilde{a})}(x)/\lambda\alpha\right\} \frac{v^{(0,\tilde{a})}(x)}{\lambda}\right\}\end{aligned}$$

[32] Dvorkin et al. (????) have employed extreme value shocks to smooth out decision rules in models of sovereign default.

[33] We choose the full information model purely for convenience, since the reduction in the state space reduces computation time and the number of states we need to account for in showing results. The same insights hold for other model variants.

[34] This keeps the analysis clean by avoiding repeated applications of the quotient rule for derivatives to the extent possible.

$$
\Bigg/ \left[\sum_{(0,\tilde{a})\in\mathcal{F}(x)} \exp\left\{v^{(0,\tilde{a})}(x)/\lambda\alpha\right\} \right]^2
$$

$$
\begin{aligned}
&= \frac{\exp\left\{v^{(0,a')}(x)/\lambda\alpha\right\}}{\sum_{(0,\tilde{a})\in\mathcal{F}(x)} \exp\left\{v^{(0,\tilde{a})}(x)/\lambda\alpha\right\}} \frac{\sum_{(0,\tilde{a})\in\mathcal{F}(x)} \exp\left\{v^{(0,\tilde{a})}(x)/\lambda\alpha\right\} \left[\frac{v^{(0,a')}(x)}{\lambda} - \frac{v^{(0,\tilde{a})}(x)}{\lambda}\right]}{\sum_{(0,\tilde{a})\in\mathcal{F}(x)} \exp\left\{v^{(0,\tilde{a})}(x)/\lambda\alpha\right\}} \\
&= \frac{1}{\lambda}\tilde{\sigma}^{(0,a')}(x) \sum_{(0,\tilde{a})\in\mathcal{F}(x)} \tilde{\sigma}^{(0,\tilde{a})}(x) \left[v^{(0,a')}(x) - v^{(0,\tilde{a})}(x)\right].
\end{aligned}
$$

We can sign this derivative according to

$$
\begin{aligned}
\frac{\partial\tilde{\sigma}^{(0,a')}(x)}{\partial(1/\alpha)} > 0 \quad &\Longleftrightarrow \quad \sum_{(0,\tilde{a})\in\mathcal{F}(x)} \tilde{\sigma}^{(0,\tilde{a})}(x) \left[v^{(0,a')}(x) - v^{(0,\tilde{a})}(x)\right] > 0 \\
&\Longleftrightarrow \quad v^{(0,a')}(x) > \sum_{(0,\tilde{a})\in\mathcal{F}(x)} \tilde{\sigma}^{(0,\tilde{a})}(x) v^{(0,\tilde{a})}(x)
\end{aligned}
$$

where the second line uses the fact that $\sum_{(0,\tilde{a})\in F(x)} \tilde{\sigma}^{(0,\tilde{a})}(x) = 1$ by construction. Therefore, the probability of choosing $(0, a')$ conditional on not defaulting increases in $1/\alpha$ (decreases in α) if and only if the conditional value of choosing $(0, a')$, $v^{(0,a')}(x)$, exceeds the expected value of choosing from the set of alternative actions $(0, \tilde{a})$ at the current decision rule. A symmetric calculation reveals that

$$
\begin{aligned}
\frac{\partial\tilde{\sigma}^{(0,a')}(x)}{\partial(1/\lambda)} \quad &= \quad \frac{1}{\alpha}\tilde{\sigma}^{(0,a')}(x) \sum_{(0,\tilde{a})\in\mathcal{F}(x)} \tilde{\sigma}^{(0,\tilde{a})}(x) \left[v^{(0,a')}(x) - v^{(0,\tilde{a})}(x)\right] \\
\Longrightarrow \quad \frac{\partial\tilde{\sigma}^{(0,a')}(x)}{\partial(1/\lambda)} > 0 \quad &\Longleftrightarrow \quad v^{(0,a')}(x) > \sum_{(0,\tilde{a})\in F(x)} \tilde{\sigma}^{(0,\tilde{a})}(x) v^{(0,\tilde{a})}(x)
\end{aligned}
$$

Therefore, with respect to choice probabilities conditional on not defaulting, the effects of both extreme value parameters are symmetric.

Default actions Equation (6) defines the probability of default as a function of the conditional value of the default action and the inclusive value of not defaulting, $W_{ND}(x)$, which takes the familiar log-sum form of (9). Since it will be useful in computing how $\sigma^{(1,0)}(x)$ varies with α and λ, we can begin by taking derivatives of W_{ND}:

$$
\begin{aligned}
\frac{\partial W_{ND}(x)}{\partial(1/\alpha)} &= \alpha \frac{\sum_{(0,a')\in\mathcal{F}(x)} \exp\left\{v^{(0,a')}(x)/\lambda\alpha\right\} \frac{v^{(0,a')}(x)}{\lambda}}{\sum_{(0,a')\in\mathcal{F}(x)} \exp\left\{v^{(0,a')}(x)/\lambda\alpha\right\}} \\
&\quad -\alpha^2 \log\left(\sum_{(0,a')\in\mathcal{F}(x)} \exp\left\{v^{(0,a')}(x)/\lambda\alpha\right\} \right) \\
&= \frac{\alpha}{\lambda} \sum_{(0,a')\in\mathcal{F}(x)} \tilde{\sigma}^{(0,a')}(x) v^{(0,a')}(x) - \alpha W_{ND}(x) \\
\frac{\partial W_{ND}(x)}{\partial(1/\lambda)} &= \alpha \frac{\sum_{(0,a')\in\mathcal{F}(x)} \exp\left\{v^{(0,a')}(x)/\lambda\alpha\right\} \frac{v^{(0,a')}(x)}{\alpha}}{\sum_{(0,a')\in\mathcal{F}(x)} \exp\left\{v^{(0,a')}(x)/\lambda\alpha\right\}}
\end{aligned}
$$

$$= \sum_{(0,a')\in F(x)} \tilde{\sigma}^{(0,a')}(x) v^{(0,a')}(x).$$

Returning to the default decision,

$$\begin{aligned}
\frac{\partial \sigma^{(1,0)}(x)}{\partial(1/\alpha)} &= \left\{ \exp\left\{v^{(1,0)}(x)/\alpha\right\} v^{(1,0)}(x) \left(\exp\left\{v^{(1,0)}(x)/\alpha\right\} + \exp\left\{\lambda W_{ND}(x)/\alpha\right\} \right) \right. \\
&\quad \left. - \exp\left\{v^{(1,0)}(x)/\alpha\right\} \left(\exp\left\{v^{(1,0)}(x)/\alpha\right\} v^{(1,0)}(x) + \exp\left\{\lambda W_{ND}(x)/\alpha\right\} \lambda \frac{\partial W_{ND}(x)}{\partial(1/\alpha)} \right) \right\} \\
&\quad \Big/ \left[\left(\exp\left\{v^{(1,0)}(x)/\alpha\right\} + \exp\left\{\lambda W_{ND}(x)/\alpha\right\} \right) \right]^2 \\
&= \frac{\exp\left\{v^{(1,0)}(x)/\alpha\right\} \exp\left\{\lambda W_{ND}(x)/\alpha\right\}}{\left[\left(\exp\left\{v^{(1,0)}(x)/\alpha\right\} + \exp\left\{\lambda W_{ND}(x)/\alpha\right\} \right) \right]^2} \left[v^{(1,0)}(x) - \lambda \frac{\partial W_{ND}(x)}{\partial(1/\alpha)} \right] \\
&= \sigma^{(1,0)}(x) \left(1 - \sigma^{(1,0)}(x)\right) \left[v^{(1,0)}(x) - \alpha \sum_{(0,a')\in \mathcal{F}(x)} \tilde{\sigma}^{(0,a')}(x) v^{(0,a')}(x) + \alpha\lambda W_{ND}(x) \right]
\end{aligned}$$

which is positive if an only if

$$\frac{v^{(1,0)}(x)}{\alpha} > \sum_{(0,a')\in \mathcal{F}(x)} \tilde{\sigma}^{(0,a')}(x) v^{(0,a')}(x) - \lambda W_{ND}(x).$$

Therefore, as α decreases, the probability of default increases if and only if the conditional value of defaulting, scaled by $1/\alpha$, exceeds the expected value of all non-default actions less the inclusive value of not defaulting. In the case when $\lambda = 1$ (i.e. when our nested logit structure collapses to standard logit), this condition reduces to evaluating the default action in the same way as the non-default actions above.

Likewise, turning to the correlation parameter λ,

$$\begin{aligned}
\frac{\partial \sigma^{(1,0)}(x)}{\partial(1/\lambda)} &= -\exp\left\{v^{(1,0)}(x)/\alpha\right\} \exp\left\{\lambda W_{ND}(x)/\alpha\right\} \left[\frac{\lambda}{\alpha} \frac{\partial W_{ND}(x)}{\partial(1/\lambda)} - \frac{\lambda^2}{\alpha} W_{ND}(x) \right] \\
&\quad \Big/ \left[\left(\exp\left\{v^{(1,0)}(x)/\alpha\right\} + \exp\left\{\lambda W_{ND}(x)/\alpha\right\} \right) \right]^2 \\
&= \sigma^{(1,0)}(x) \left(1 - \sigma^{(1,0)}(x)\right) \left[\lambda W_{ND}(x) - \sum_{(0,a')\in \mathcal{F}(x)} \tilde{\sigma}^{(0,a')}(x) v^{(0,a')}(x) \right]
\end{aligned}$$

This implies that

$$\frac{\partial \sigma^{(1,0)}(x)}{\partial(1/\lambda)} > 0 \iff \lambda W_{ND}(x) > \sum_{(0,a')\in \mathcal{F}(x)} \tilde{\sigma}^{(0,a')}(x) v^{(0,a')}(x),$$

i.e. if a scaling of the inclusive value of not defaulting exceeds the expected value of not defaulting.

Finally,

$$\arg \max_{(d,a')\in \mathcal{F}(x)} \sigma^{(d,a')}(x) = \arg \max_{(d,a')\in \mathcal{F}(x)} v^{(d,a')}(x),$$

so that the action which delivers the highest total utility *before* the extreme value shock is chosen with the highest probability. Combining these two pieces of information, we see that as α increases, the probability of choosing the action with the highest conditional value increases relative to all other feasible actions. Put differently, as α increases, (i) more and more weight is placed on the modal action, and (ii) the mean action converges to the modal action. For actions that are "suboptimal" in the sense that they deliver lower conditional value than the modal action, the change in weight depends on the difference in conditional value, weighted by the total value of these actions. This can have a meaningful impact on the mean action taken, if not the mode, which can effect prices and type scores significantly.

B.2.2 Changing EV shocks in the full information model

Figure 19 demonstrates the impact of changing α and λ on decisions in the full information model, discussed in Section B.8.1 below. Each figure contains three lines, corresponding to: (i) the baseline parameterization of Table 4 (black solid); (ii) a "low variance" parameterization (blue dotted) where α is reduced by 75% and λ is held fixed; and (iii) a "high correlation" parameterization (red dashed) in which λ is reduced by 75% and α is held fixed. All figures are presented for an agents with $(\beta, e, z) = (\beta_H, 1, 0)$. In each parameterization, the equilibrium pricing function, and therefore the conditional action values, are held fixed, and so the changes in response shown here can be thought of as partial equilibrium in order to highlight the direct effects on decisions.

Consider first the default decision. The top left panel shows how the default decision varies over a range of levels of debt. By lowering the variance, the slope of increase in the probability of default as the level of indebtedness increases is much sharper than in the baseline parameterization. This is because there is less chance for a high value shock to be realized for an action with lower fundamental value, so the decision rule becomes more centered at the mode for each level of a. By raising the correlation, the expected value of not defaulting is reduced, and so the default decision rule "shifts to the right," i.e. agents default more frequently for lower levels of debt.

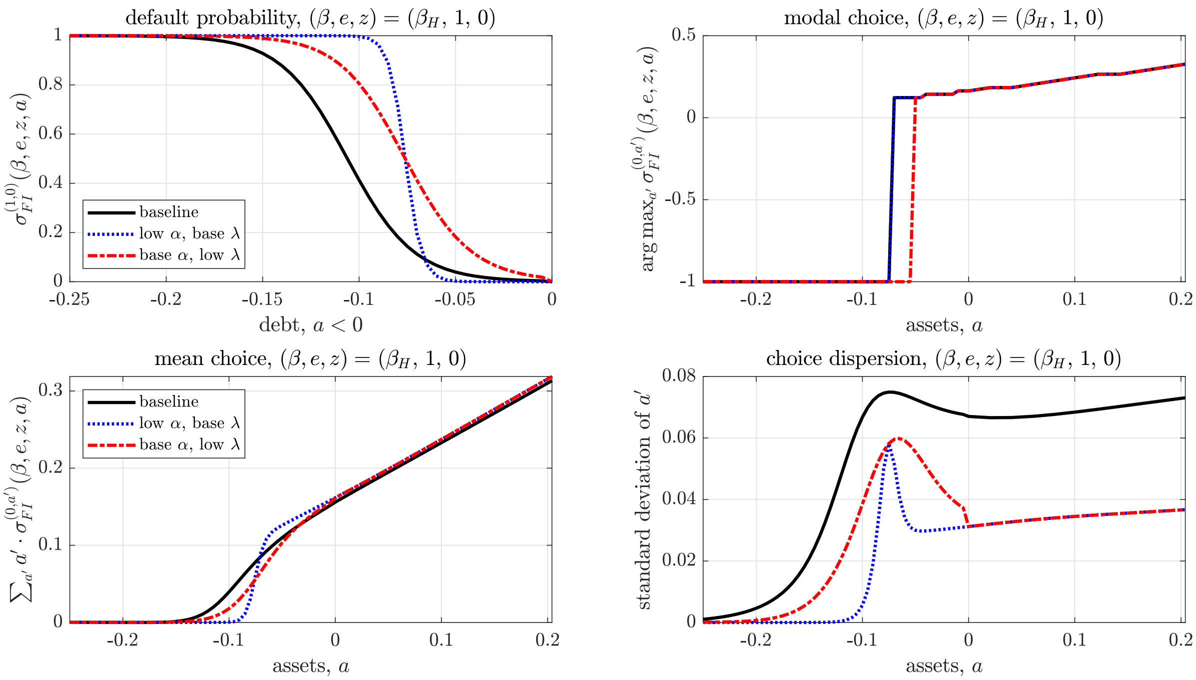
The remaining three figures show how non-default decisions are affected by changes in the extreme value parameters. The top right panel depicts the modal decision across each case (with default depicted as choosing $a' = -1$ for simplicity). Since the action values are held fixed, according to the analysis above the only parameter which can induce a change in the modal decision is the correlation parameter λ, and even this change can only apply to the default / no default decision. The top right panel confirms this.

The bottom left and bottom right panels show the mean and standard deviation of the decision rule, respectively. The actions of zero to the left of the figure reflect default. The major changes across parameterizations occur when default is an option, i.e. when $a < 0$. In this case, we see a steeper shift in the mean decision for low variance, consistent with the analysis above. Additionally, we see that the actions are biased towards default with a higher correlation. Turning to the standard deviation in choices, both alternatives feature lower dispersion in decision rules than our baseline economy. In the low variance economy, that dispersion is the lowest of the three cases, and reaches its peak at the level of assets under which the default decision is "interior." In the high correlation economy, there is similarly a smaller range of default decisions and more centering around non-default choices.

B.3 Grids

Table 7 presents the key grids used in the computational analysis. Note in particular that the asset and type score grids are quite dense in order to insure convergence, while in contrast the earnings and

Figure 19: **Impact of extreme value preference shocks**



Notes: Baseline refers to the parameterization of the extreme value shock process from Table 4. Low α (λ) is a quarter of the baseline value: $\alpha' = \alpha/4$ ($\lambda' = \lambda/4$). All panels are constructed from the full information model, and fix the state of an agent at $(\beta, e, z) = (\beta_H, 1, 0)$.

type grids are coarse in order to ease the computational burden and simplify the analysis.

B.4 Model moment definitions

Default rate The default rate is computed as the total fraction of the population who defaults within a given period. The probability of a given state is given by $\mu(\cdot)$, and the probability of default given a state is $\sigma^{(1,0)}(\cdot)$, and so the *aggregate* default rate is

$$\text{aggregate default rate} = \sum_{\beta,z,\omega} \sigma^{(1,0)}(\beta, z, \omega) \cdot \mu(\beta, z, \omega). \tag{27}$$

By type, we have $\sum_{\omega} \sigma^{(1,0)}(\beta, z, \omega) \cdot \mu(\beta, z, \omega) / \sum_{\hat{\omega}} \mu(\beta, z, \hat{\omega})$

Fraction in debt This is the fraction of the population with $a < 0$ in a given period, so

$$\text{fraction in debt} = \sum_{\beta,z,\omega|a<0} \mu(\beta, z, \omega). \tag{28}$$

By type, the analogous figure is $\sum_{\omega|a<0} \mu(\beta, z, \omega) / \sum_{\hat{\omega}} \mu(\beta, z, \hat{\omega})$.

Debt to income Income in the model is given by the sum of earnings (persistent and transitory) and net interest on assets. That is, income $= e + z + (1/q(a', p) - 1) \cdot a$. Therefore, debt to income is

Table 7: **Grids used in computational analysis**

Variable	x	$N(x)$	**Range / Values**	**Notes**
Discount factor	β	2	$\{0.915, 0.886\}$	2-point support makes Bayesian functions scalar-valued.
Earnings (per)	e	3	$\{0.57, 1.00, 1.74\}$	See Table 2
Earnings (trans)	z	3	$\{-0.18, 0.00, 0.18\}$	See Table 2
Assets	a	150	$[-0.25, 15.00]$	50 points in neg. region, 100 in pos. Density close to 0 is critical.
Type score	s	50	$[0.0, 1.0]$	bounded below by low β to high β transition, above by high to low.

computed as the weighted average of the ratio of assets, a, to income conditional on a being negative:

$$\text{debt to income} = \sum_{\beta,z,\omega} \frac{a}{e + z + (1/q - 1) \cdot a} \cdot \frac{\mu(\beta, z, \omega)}{\sum_{\hat{\beta},\hat{z},\hat{\omega}} \mu(\hat{\beta}, \hat{z}, \hat{\omega})}, \tag{29}$$

Average interest rate The average interest paid (or received) by the agents in the economy is the weighted average of the interest rates paid, $1/q - 1$, over the stationary distribution and decision probabilities.

$$\text{average interest rate} = \sum_{\omega} \overline{\mu}(\omega) \cdot \sum_{\beta,z} \frac{\mu(\beta, z, \omega)}{\sum_{\hat{\beta},\hat{z}} \overline{\mu}(\omega)} \sum_{a'} \frac{\sigma^{(0,a')}(\beta, z, \omega)}{\sum_{\tilde{a}} \sigma^{(0,\tilde{a})}(\beta, z, \omega)} \left(\frac{1}{q^{(0,a')}(\omega)} - 1 \right) \tag{30}$$

where $\overline{\mu}(\omega) = \sum_{\beta,z} \mu(\beta, z, \omega)$.

B.5 Modal choice metrics

In this section, we describe a series of metrics which quantify the dispersion in decision implied by the extreme value shocks in our framework. These results are summarized in Table 8, but we first describe the construction of the metrics.

Let $x = (\beta, z, \omega)$ be the state variable of an agent, let $\sigma^{(d,a')}(x)$ denote her decision rule, and let $\mu(x)$ be the stationary distribution over individual states in the baseline economy. We want to get a sense of dispersion around the highest value (or modal) choice, which may be defined as

$$y^*(x) \equiv \arg \max_{(d,a') \in \mathcal{F}(x)} \sigma^{(d,a')}(x)$$

Let $\mathcal{Y} \subseteq \{(1, 0), \{(0, a')\} | a' \in \mathcal{A}\}$ denote a set of possible actions. The share of agents for whom an action in set $\mathcal{Y}$ is modal is

$$m(\mathcal{Y}) = \sum_{x} \mu(x) 1 \left[y^*(x) \in \mathcal{Y} \right]$$

Action type	share for whom action type is modal (%)	share of total action from modal agents (%)	share of decisions w/in k grid pts. of mode (%) $k=0$	$k=1$	$k=2$
Default	1.84	87.14	-	-	-
Non-Default	98.16	99.13	18.96	49.00	65.77
Borrowing	7.28	66.20	30.32	70.36	86.92
Saving	90.88	99.36	18.05	47.29	65.41

Table 8: **Modal choice metrics**

Notes: For the right 3 columns, the share is computed over the total population of agents for whom the indicated action type is the modal action.

The total mass of agents choosing an action in this set includes those for whom the action is not modal, and so we can compute the share of the actions in this set accounted for by "modal agents," those for whom an action in this set is the mode, via

$$\frac{\sum_x \mu(x)\sigma^{(d,a')}(x)1\left[y^*(x) \in \mathcal{Y}\right]}{\sum_x \mu(x)\sigma^{(d,a')}(x)}$$

Lastly, for agents whose modal action is not default, we can compute the share of decisions within k grid points of the modal action. For a given individual (whose modal action is not default), let $i^*(x)$ denote the grid index of the modal choice $y^*(x)$. Then let a k-band of actions around the mode be defined by

$$\mathcal{Y}_k(x) = \{i^*(x) - k, ..., i^*(x), ..., i^*(x) + k\}.$$

Finally, define the total weight on decisions in the k-band of the mode for agent x via

$$\eta(x) = \frac{\sum_{(0,a')\in\mathcal{Y}_k(x)} \sigma^{(0,a')}(x)}{1 - \sigma^{(1,0)}(x)},$$

where the denominator renormalizes the weights on actions to exclude default. We can aggregate this metric over any group of actions $\mathcal{Y}$ via

$$\bar{\eta}(\mathcal{Y}) = \frac{\sum_{\{x|y^*(x)\in\mathcal{Y}\}} \eta(x)\mu(x)}{m(\mathcal{Y})}$$

B.6 Credit Access Following Default

For all $a' < 0$, define the two price schedules

$$\begin{aligned} q_D^{(0,a')}(e,a,s) &\equiv q^{(0,a')}\left(e, 0, \psi^{(1,0)}(e,a,s)\right), \\ q_N^{(0,a')}(e,a,s) &\equiv q^{(0,a')}\left(e, 0, \psi^{(0,0)}(e,a,s)\right), \end{aligned}$$

where the former corresponds to default (D) and the latter corresponds to no default (N). In order to compute an "average" effect of defaulting, we can weight the price differences for each action by the stationary distribution of agents who have the option to default. Specifically, define

$$\overline{\mu}(e,a,s) = \frac{\sum_{\beta,z}\mu(\beta,e,z,a,s)}{\sum_{\beta,z,\tilde{a}<0}\mu(\beta,e,z,\tilde{a},s)} \text{ for all } a<0.$$

Then, we can compute the aggregate metrics for each debt choice $a'<0$

$$\Delta^q(a') = \sum_{e,a<0,s}\overline{\mu}(e,a,s)\left[\frac{q_N^{(0,a')}(e,a,s)}{q_D^{(0,a')}(e,a,s)} - 1\right].$$

B.7 Welfare Metric

First, recognize that for a given state (β_t, x), the value of an agent is

$$V(\beta_t,x) = \mathbb{E}\left[\sum_{t=0}^{\infty}\rho^t B_t \frac{c_t^{1-\gamma}-1}{1-\gamma}\right],$$

where $B_0 = 1$, and for all $t \geq 1$

$$B_t = \prod_{i=0}^{t-1}\beta_i,$$

so that $B_1 = \beta_0$, $B_2 = \beta_0\beta_1$, etc.

Let an agent's current β be givem, and let W be an agent's value under an alternative market arrangement. Let x capture all other state variables besides β. Then, the consumption equivalent measure is defined by

$$\begin{aligned}
W(\beta,x) &= \mathbb{E}\left[\sum_{t=0}^{\infty}\rho^t B_t \frac{[(1+\lambda(\beta,x))c_t]^{1-\gamma}-1}{1-\gamma}\right] \\
&= (1+\lambda(\beta,x))^{1-\gamma}\mathbb{E}\left[\sum_{t=0}^{\infty}\rho^t B_t \frac{c_t^{1-\gamma}}{1-\gamma}\right] - \underbrace{\mathbb{E}\left[\sum_{t=0}^{\infty}\frac{\rho^t B_t}{1-\gamma}\right]}_{\equiv A(\beta)} \\
\implies W(\beta,x) + A(\beta) &= (1+\lambda(\beta,x))^{1-\gamma}\left(\mathbb{E}\left[\sum_{t=0}^{\infty}\rho^t B_t \frac{c_t^{1-\gamma}-1}{1-\gamma}\right] + A(\beta)\right) \\
\implies 1+\lambda(\beta_t,x) &= \left[\frac{W(\beta,x)+A(\beta)}{V(\beta,x)+A(\beta)}\right]^{\frac{1}{1-\gamma}}
\end{aligned}$$

We approximate $A(\beta)$ by simulating N paths of length T of β starting from a given β_0, $\{\beta_0(n), ..., \beta_T(n)\}_{n=1}^N$ and pick T sufficiently large that ρ^t becomes vanishingly small. For each path n, we compute

$\{B_0(n), ..., B_T(n)\}$ from which we compute the approximation

$$\hat{A}(\beta) = \frac{1}{N}\sum_{n=1}^{N}\left(\sum_{t=0}^{T}\rho^t B_t(n)/(1-\gamma)\right).$$

B.8 Details of Alternative Economies

B.8.1 Full information (FI)

Since there is no incentive to infer one's type, there is no type score in this model. Therefore, an agent's full state is (β, e, z, a), and the set of equilibrium functions does not include ψ. For comparability, and since it is purely i.i.d. and contains no information for inference, we maintain the assumption that z is unobservable. Therefore, the lender can observe $\omega_{FI} = (\beta, e, a)$ for each individual.

The household problem and equilibrium stationary distribution are exactly the same as in the main text, with the state variable s removed. The only substantial change is in the pricing and repayment probability equations. The repayment probability function in this case is $p_{FI}^{(0,a')}(\omega_{FI}) = \Pr(\text{repay } a'|\omega_{FI})$. Since ω_{FI} directly includes β and z is i.i.d., there is no further inference to be done. Therefore, a has no impact on pricing, and we obtain

$$p_{FI}^{(0,a')}(\beta, e) = \sum_{\beta',e',z'}\left[1-\sigma_{FI}^{(1,0)}(\beta', e', z', a')\right] Q^{\beta}(\beta'|\beta)Q^{e}(e'|e)H(z'). \tag{31}$$

The loan pricing function, $q_{FI}^{(0,a')}(\beta, e)$, simply adjusts the expression for the interest rate as in the baseline model.

B.8.2 No tracking (NT)

The key departure in the formal specification of this economy from the baseline comes from the separation of the type score updating which follows individuals and the static assessment of types that is relevant for pricing.

Evolution of type score Here, an individual's type score updates only based on the exogenous transition probabilities, and so there is no dynamic incentive to acquire reputation. As a result, s' evolves from s according to

$$\psi^1_{NT,\beta'}(s) = \sum_{\beta} Q^{\beta}(\beta'|\beta)s(\beta).$$

In the two-type case we employ in our quantitative model, we have

$$\psi^1_{NT}(s) = sQ^{\beta}(\beta_H|\beta_H) + (1-s)Q^{\beta}(\beta_H|\beta_L). \tag{32}$$

Static assessment of type In this version of the model, lenders to perform **intraperiod** updating of type assessments based on the a' chosen by the borrower. That is, the lenders compute

$$\psi^2_{NT,\beta'}(a', s, e) \equiv \Pr(\beta'|a', s, e) = \sum_{\beta} Q^{\beta}(\beta'|\beta)\Pr(\beta|a', s, e).$$

All of the action is in the last term of the expression above, and so we analyze it here:

$$\begin{aligned}
\Pr(\beta|a',s,e) &= \frac{\Pr(\beta,a',s,e)}{\Pr(a',s,e)} \\
&= \frac{\sum_{z,a}\Pr(\beta,a',s,e,z,a)}{\sum_{\tilde{\beta},z,a}\Pr(\tilde{\beta},a',s,e,z,a)} \\
&= \frac{\sum_{z,a}\sigma^{(0,a')}(\beta,e,z,a,s)\mu(\beta,e,z,a,s)}{\sum_{\tilde{\beta},z,a}\sigma^{(0,a')}(\tilde{\beta},e,z,a,s)\mu(\tilde{\beta},e,z,a,s)}
\end{aligned}$$

where the first line uses Bayes' Rule, the second sums over unobserved idiosyncratic states, and the third once more applies Bayes' Rule via

$$\Pr(a',\beta,e,z,a,s) = \Pr(a'|\beta,e,z,a,s)\Pr(\beta,e,z,a,s) = \sigma^{(0,a')}(\beta,e,z,a,s)\mu(\beta,e,z,a,s).$$

Therefore, we obtain

$$\psi^2_{NT,\beta'}(a',s,e) = \sum_{\beta} Q^{\beta}(\beta'|\beta)\frac{\sum_{z,a}\sigma^{(0,a')}(\beta,e,z,a,s)\mu(\beta,e,z,a,s)}{\sum_{\tilde{\beta},z,a}\sigma^{(0,a')}(\tilde{\beta},e,z,a,s)\mu(\tilde{\beta},e,z,a,s)} \tag{33}$$

Repayment probability function What the lender must compute is the probability that a' is repaid tomorrow given s, e observed today. For each choice of a', the lender revises the **borrower's assessed type today** via (33). At the same time, though, due to the implicit "anonymity" assumption in this economy, they recognize that the **borrower's type score tomorrow** (which is relevant for tomorrow's default decision) will be determined via (32). Therefore, the $p(\cdot)$ function in this economy is

$$\begin{aligned}
p(a',s,e) &= \Pr(\text{repay } a'|s,e) \\
&= \frac{\Pr(\text{repay } a',s,e)}{\Pr(s,e)} \\
&= \frac{\sum_{\beta',e',z',s'}\Pr(\text{repay } a'|\beta',e',z',s',a',s,e)\Pr(\beta',e',z',s'|a',s,e)}{\sum_{\beta,a,z}\Pr(\beta,e,z,a,s)} \\
&= \frac{\sum_{\beta',e',z'}\left[1-\sigma^{(1,0)}(\beta',e',z',a',\psi_1(s))\right]\Pr(\beta',e',z'|a',s,e)}{\sum_{\beta,a,z}\mu(\beta,e,z,a,s)} \\
&= \frac{\sum_{\beta,\beta',e',z'}\left[1-\sigma^{(1,0)}(\beta',e',z',a',\psi_1(s))\right]Q^e(e'|e)H(z')Q^{\beta}(\beta'|\beta)\Pr(\beta|a',s,e)}{\sum_{\beta,a,z}\mu(\beta,e,z,a,s)} \\
&= \psi^2_{NT}(a',s,e)\sum_{e',z'}\left[1-\sigma^{(1,0)}(\beta_H,e',z',a',\psi^1_{NT}(s))\right]Q^e(e'|e)H(z') \\
&\quad + \left(1-\psi^2_{NT}(a',s,e)\right)\sum_{e',z'}\left[1-\sigma^{(1,0)}(\beta_L,e',z',a',\psi^1_{NT}(s))\right]Q^e(e'|e)H(z')
\end{aligned}$$

where the last line once more applies the two-type implementation from our quantitative model.

B.9 Sensitivity Analysis

We further explore the effect of our parameters on our target moments by solving the model changing each parameter by a small amount individually. The aggregate credit market statistics resulting from increasing each parameter by 5% in turn are reported in Table 9.

Increasing β_H increases the "distance" between types. High types default slightly less, while low types default 21% more because it is more costly for them to imitate high types, and so the aggregate default rate rises. The average interest rate paid in aggregate rises sharply because of the inference problem: even though high types default less, they face higher prices on average because the low types behave so much worse and the types cannot be perfectly separated. Correspondingly, the fraction of high types in debt decreases, and despite an increase in indebtedness by low types, the mass of debtors in the economy decreases overall.

In contrast, increasing β_L lowers this between-type distance. High types default more relative to the baseline, while low types default less. Both outcomes hinge on reputation; high types have less incentive to separate themselves from low types through good behavior, while low types find it more attractive to imitate high types and enhance their reputations. Notably, this economy features a large incentive to avoid debt and the reputational hit associated with it, as evidenced by the large declines in the debt to income ratio and fraction of households in debt.

Changes to the type transition process Q^{β} have similar, but smaller effects. Increasing $Q^{\beta}_{HL'}$ lowers the stationary fraction of high types, which is analogous to increasing β_L or decreasing β_H. This is because high types in this economy have a lower expected future β, and therefore effectively discount the future more steeply. The reverse holds for increasing $Q^{\beta}_{LH'}$.

Lastly, increasing α increases the overall variance of the extreme value shocks. As actions become noisier, default rates increase for both types. Interest rates follow the same pattern, but critically agents do not react to these increased interest rates by borrowing less: both the fraction of households in debt and the debt to income ratio increase. Fixing the scale parameter and varying λ instead has a markedly different effect. While the default vs. no default decision remains as noisy as in the baseline case, the choice of a' effectively becomes less noisy: since the shocks for specific a' are now less correlated, it is more likely that an action with low conditional value will receive a high enough ϵ shock to merit choosing it over an action with high conditional value. Therefore, agents choose actions with a lower "bang-for-buck" and the average interest rate rises due to this selection effect.

C Credit Score Interactions Across Markets (AM)

In our baseline model, one's type score (or reputation) only plays a role by affecting unsecured debt prices. In reality, though, credit scores impact more than credit card interest rates. Financially, they impact mortgage rates, auto loan rates, and many other forms of credit; in other areas, they can affect employment outcomes. In this section, we consider in a simplified way how these interactions affect the acquisition of reputation and behavior in the unsecured credit market.

To capture the role that credit scores play across markets (AM), we assume that each period an individual must finance an expenditure weakly calibrated to proxy mortgage payments on a home loan. This expenditure is decreasing in type score, which proxies the empirical fact that individuals with higher credit scores tend to pay lower interest rates on standard mortgage contracts. All of the equilibrium type scoring and pricing functions are specified in exactly the same way as in the baseline economy; the only thing that differs is the household's budget constraint. Specifically, we model the financial burden of a bad reputation as an additional expense

$$m(s) = m_0 - m_1 s. \tag{34}$$

Table 9: **Sensitivity Analysis for Credit Market Moments**

Moment (%)		**Base.**	**Sensitivity to 5% increase in**						
			β_H	β_L	$Q^{\beta}(L'\|H)$	$Q^{\beta}(H'\|L)$	λ	α	G_{β_H}
Aggregate credit statistics									
default rate	agg.	0.98	1.12	0.93	0.98	0.98	0.99	1.02	0.98
	high β	0.73	0.70	0.82	0.73	0.73	0.73	0.77	0.74
	low β	1.16	1.42	1.01	1.15	1.16	1.16	1.20	1.16
average int. rate	agg.	13.96	15.88	14.30	13.95	13.96	14.10	14.98	13.96
	high β	14.75	19.67	14.10	14.76	14.73	14.92	15.98	14.75
	low β	13.40	13.22	14.45	13.40	13.41	13.53	14.26	13.39
int. rate dispersion	agg.	7.28	10.67	6.32	7.28	7.28	7.31	7.51	7.27
	high β	6.13	12.05	6.12	6.13	6.12	6.18	6.47	6.14
	low β	7.95	8.64	6.45	7.94	7.96	7.97	8.10	7.94
fraction HH in debt	agg.	10.58	9.73	9.65	10.59	10.57	10.63	10.79	10.57
	high β	8.47	6.31	8.51	8.49	8.44	8.51	8.67	8.54
	low β	12.06	12.13	10.44	12.03	12.12	12.11	12.27	12.03
debt / income ratio	agg.	0.25	0.26	0.24	0.25	0.25	0.25	0.27	0.25
	high β	0.20	0.18	0.21	0.20	0.20	0.20	0.21	0.20
	low β	0.29	0.32	0.26	0.29	0.30	0.30	0.31	0.29
Age Profile of Credit Rankings									
intercept, mean		0.35	0.35	0.24	0.32	0.32	0.32	0.36	0.33
slope, mean		0.03	0.03	0.05	0.03	0.03	0.03	0.03	0.03
intercept, std. deviation		0.25	0.24	0.24	0.24	0.24	0.19	0.25	0.24
slope, std. deviation		0.00	0.01	0.00	0.01	0.01	0.04	0.00	0.01
average, autocorrelation		-0.11	-0.10	-0.13	-0.11	-0.11	-0.13	-0.11	-0.11

Notes: For parameters for which raising by 5% would violate core assumptions of the model, we raise them by a smaller amount. For example, a 5% increase in β_L would make $\beta_L > \beta_H$, and so instead we set $\beta_L = 0.999 \times \beta_H$. Likewise, raising λ by 5% would yield $\lambda > 1$, and so we raise λ only to 1.

The household's flow income becomes $e + z - m(s)$, as opposed to simply $e + z$.[35]

[35] The parameters m_0 and m_1 in equation (34) are chosen to be consistent with a scaled down version of the cost of holding a 30 year fixed-rate mortgage on a home of median price as of 2007.

The baseline results are collected in the last column (AM) of Table 10. We see that the effects of modeling reputation in this way are large. Relative to the baseline economy, the default rate drops by 12%. While average interest rates remain close to their baseline levels, dispersion in interest rates rises, and quantities of debt decline sharply on both the extensive and intensive margins. There are three sharp changes in the credit ranking life cycle moments. First, the intercept of the lifetime path of credit rankings is much higher in the AM economy: given the additional incentives to acquire a good reputation, young agents are in general much less risky in the AM economy than in the baseline. Second, the average improvement in credit ranking over one's lifetime is much lower, dropping by more than 1/3 relative to the baseline. Third, the mean reversion in credit rankings is much sharper in the AM economy. Given the additional costs of having a bad reputation, agents tend to more sharply "correct" past actions which were destructive of their credit scores.

D Data Appendix

This appendix describes the construction of the data underlying the life cycle credit ranking moments reported in Table 3 and Figures 7, 8, and 9.

We begin with a 2 percent random sample of the FRBNY CCP/Equifax anonymized panel containing each individual's birth year and the individual's credit score in the first quarters of 2004, 2005 and 2006. The credit score measure is the Equifax Risk Score (hereafter Risk Score), which is a proprietary credit score similar to other risk scores used in the industry. We drop individuals who are not within the ages of 21 and 60 years in 2004Q1 and any individual whose Risk Score is missing for any of the quarters. This yields our *base sample*.

For this sample, we construct empirical cumulative distribution functions of Risk Scores for each of the three years. And, for each individual, we computed the percentile ranking of his or her Risk Score for each year. We call this the individual's *credit ranking* — it is a number that gives the fraction of people who had Risk Scores not exceeding the individual's score in that year. We then placed individuals in 5-year age bins according to their age in 2004Q1.

Table 11 reports the mean and standard deviations of the credit rankings in each bin. These moments were used in the regressions that determine the coefficients in the first 4 rows of the middle panel of Table 3. For the final row of the middle panel of Table 3, we computed the change in an individual's credit ranking from 2004Q1 to 2005Q1 and, again, from 2005Q1 to 2006Q1. For each age bin, we computed the correlation between these pair of changes across individuals in that bin. These correlations are reported in the final column of Table 11. The final row of the middle panel of Table 3 reports the average of these correlations.

Turning next to the construction of the data for the default event study in Figure 9, we first isolated about 50,000 individuals who filed for Chapter 7 bankruptcy in sometime in 2004 and obtained a discharge sometime in 2005. As above, we eliminated all individuals younger than 21 years and older than 60 years. This yielded our *base sample of bankrupts*. For each individual in this sample, we recorded their birth year and Risk Score in the filing quarter and in the four quarters preceding and following the filing quarter. We placed each individual in the appropriate 5-year age bin based on their age in 2004. We computed the average Risk Score in each age bin for each of the nine quarterly observations and then computed the fraction of individuals in the *base sample* who had scores not exceeding this average. This gave us the average credit ranking by age bin of people in the *base sample of bankrupts* reported in Table 12.

Table 10: **Comparison between Baseline and Across Market (AM) Economies**

Moment (%)	**Subgroup**	**Model**	
		BASE	AM
Aggregate credit market statistics			
default rate	aggregate	0.981	0.915
	high β	0.731	0.662
	low β	1.156	1.099
average interest rate	aggregate	13.96	13.84
	high β	14.75	14.71
	low β	13.40	13.21
interest rate dispersion	aggregate	7.282	8.595
	high β	6.128	9.058
	low β	7.946	8.184
fraction of HH in debt	aggregate	10.58	9.12
	high β	8.472	6.988
	low β	12.06	10.68
debt to income ratio	aggregate	0.252	0.230
	high β	0.196	0.174
	low β	0.293	0.271
Credit ranking age profile moments			
intercept, mean credit ranking		0.355	0.424
slope, mean credit ranking		0.029	0.018
intercept, std. dev. credit ranking		0.255	0.240
slope, std. dev. credit ranking		0.004	0.008
average autocorr of change in credit ranking		-0.109	-0.143

Table 11: **Age Profile of Credit Rankings**

Age Bins	**Mean, Score Pctl**	**SD, Score Pctl**	**Corr(ΔPctl$^{05}_{04}$, ΔPctl$^{06}_{05}$)**
21-25 years	0.32	0.20	-0.22
26-30 years	0.35	0.23	-0.18
31-35 years	0.40	0.26	-0.20
36-40 years	0.44	0.28	-0.21
41-45 years	0.47	0.28	-0.21
46-50 years	0.50	0.28	-0.21
51-55 years	0.54	0.28	-0.19
56-60 years	0.58	0.28	-0.20

Notes: The credit ranking data is based on author calculations using FRBNY CCP/Equifax data.

Table 12: **Default Event Study Data**

Quarters	**26-30 years**	**31-35 years**	**36-40 years**	**41-45 years**
-4	0.23	0.21	0.20	0.18
-3	0.19	0.18	0.17	0.15
-2	0.16	0.15	0.14	0.12
-1	0.13	0.12	0.10	0.10
0	0.18	0.16	0.15	0.14
1	0.25	0.22	0.21	0.19
2	0.28	0.25	0.24	0.21
3	0.28	0.26	0.25	0.23
4	0.28	0.26	0.25	0.23

Notes: The credit ranking data is based on author calculations using FRBNY CCP/Equifax data. The data presented in this table corresponds to the black lines in Figure 9.