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Online Appendix for:
The Welfare Costs of Inflation*

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A Model Solution

The problem of the agent is to maximize utility (Equation (1) in the paper) by choosing c_t, n_t, M_t, B_t, and W_{t+1} subject to the cash in advance constraint (Equation (2) in the paper) and the two budget constraints (Equations (3) and (4) in the paper). Assume that the function $\theta(n_t, \nu_t)$ is differentiable.

If we let ξ_t, λ_t and δ_t be the corresponding Lagrange multipliers, the first order conditions are given by

$$\beta^t U'(c_t) = P_t\lambda_t + P_t\delta_t \tag{1}$$

$$P_t\lambda_t\theta_n(n_t, \nu_t)z_t = M_t\delta_t \tag{2}$$

$$\xi_t = \lambda_t(1 + r_t^m) + \delta_t n_t \tag{3}$$

$$\xi_t = \lambda_t(1 + r_t^b) \tag{4}$$

$$\lambda_t = E_t\xi_{t+1}. \tag{5}$$

*The views expressed in this paper do not necessarily reflect those of the Federal Reserve Bank of Minneapolis, or of the Federal Reserve System.

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Conditions (3) and (4) imply that as long as $r_t^b - r_t^m > 0$,

$$\lambda_t = \frac{\delta_t n_t}{r_t^b - r_t^m},$$

and replacing this on (2), we obtain

$$P_t \frac{\delta_t n_t}{r_t^b - r_t^m} \theta_n(n_t, \nu_t) z_t = M_t \delta_t,$$

or

$$\frac{n_t}{r_t^b - r_t^m} \theta_n(n_t, \nu_t) z_t = \frac{M_t}{P_t}. \tag{6}$$

Note also that as long as $r_t^b - r_t^m > 0$, (3) and (4) imply that $\delta_t > 0$, which means that the cash-in-advance constraint is binding, so

$$c_t = n_t \frac{M_t}{P_t},$$

which, together with feasibility

$$c_t = z_t(1 - \theta(n_t, \nu_t)),$$

implies

$$\frac{z_t(1 - \theta(n_t, \nu_t))}{n_t} = \frac{M_t}{P_t}.$$

Replacing this on (6) above, we get

$$n_t^2 \frac{\theta_n(n_t, \nu_t)}{(1 - \theta(n_t, \nu_t))} = r_t^b - r_t^m.$$

Thus, the solution for n_t depends only on the two stochastic processes $r_t^b - r_t^m$ and ν_t. Note, in particular, that it does not depend on z_t, so the theory implies a unit income elasticity.

B The Return on Money

As mentioned above, we assume that the monetary aggregate is the sum of cash and deposits, in fixed proportions. Thus, if we let D_t and C_t be the stock of deposits and cash, then

$$D_t = (1-\gamma)M_t \text{ and } C_t = \gamma M_t,$$

where $\gamma \in (0,1)$. If we let r_t^d and r_t^c be the nominal returns on deposits and cash, then

$$r_t^m = (1-\gamma)r_t^d + \gamma r_t^c.$$

As for cash, we assume that $r_t^c = -r^c$ for some non-negative constant r^c. This reflects the chances that cash is being lost or stolen, which, for simplicity, we assume independent of the state of the economy s^t.[1] In discussing the interest rate on deposits, we refer the reader to models similar to the one we discussed above but enlarged by modeling banks that create deposits backed with goverenment bonds, as the one in Freeman and Kydland (2000), for example. In those models, owing to costs of creating deposits, the interest rate on deposits is a function of the bond rate, so $r_t^d = R(r_t^b)$, with the following properties:

$$R(r_t^b) \leq r_t^b, \ 1 \geq R_r(r_t^b) \geq 0. \tag{7}$$

These qualities imply that the spread between the interest rate in bonds and the interest rate in deposits is positive, that the interest rate on deposits is a non-decreasing function of the interest rate in bonds, and that it does not change more than one to one with the interest rate in bonds.

With these two assumptions, the diffference between the interest rate in bonds and the

[1] See Alvarez and Lippi (2009) for some survey-based evidence on this cost.

interest rate on money is given by

$$r_t = (1-\gamma)\left[r_t^b - R(r_t^b)\right] + \gamma\left[r_t^b - r^c\right]. \tag{8}$$

The maximum problem is well defined for values of r_t such that $r_t \geq 0$. The properties specified in (7) imply that r_t is non-decreasing on r_t^b. Thus, feasibility implies that $r_t \geq r^{\min}$, where $r^{\min}$ satisfies $r_t = 0$. It immediately follows that as long as $r^c > 0$ or $R(0) < 0$, then $r^{\min} < 0$, which implies that r_t^b may indeed be negative.

Kurlat (2019) convincingly argues that the function $R(r_t^b)$ is linear. He uses micro data from the United States, for which the slope is very precisely estimated, while the constant is essentially zero.

On the other hand, Alvarez and Lippi (2009) estimate the return on cash $r^c = -0.02$, using survey data. As a consequence, we use in the paper a liner return for money of the form

$$r_t^m = -a + br_t^b.$$

Our benchmark case sets $a = b = 0$, which is the standard assumption in the literature. To account for the countries that experienced negative rates, we also consider two more cases. In both, we set $b = 0.15$, following the findings in Kurlat (2019). We then explore two alternative values for the constant $a \in \{1, 2\}$. The first value implies that the lower bound on r_t^b is given by

$$r_t^b - r_t^m = r_t^b + a - br_t^b \geq 0,$$

or

$$r_t^b \geq -1.17\%,$$

which is lower than the negative rates in the euro area and Sweden, which were always above -1%. However, rates in Switzerland went all the way down to -1.85%. That is why we also explore the case of $b = 2$, which implies a lower bound of -2.35%.

C The Data

This section presents a detailed description of the dataset. All data are from official sources—that is, either central banks or national statistical agencies.

C.1 United States

For the United States, seasonally adjusted series for nominal GDP and the standard M1 aggregate and series for the three-month Treasury bill rate and the 10-year government bond yield are all from the St. Louis Fed's internet data portal, FRED II (their acronyms are GDP, M1SL, TB3MS, and GS10, respectively). The standard M1 aggregate starts in 1959Q1. Before that, the series has been linked to the series M173Q4 in the spreadsheet m1QvMd.xlsx from the Federal Reserve Bank of Philadelphia's real-time data portal, which starts in 1947Q1. Over the period of overlapping the two M1 series are virtually identical, which justifies the linking. The series for Money Market Deposits Accounts (MMDAs), starting in 1982Q4, is from the Federal Reserve's mainframe. A series for currency is from the Federal Reserve's website.

C.2 United Kingdom

For the United Kingdom, the seasonally adjusted series for nominal GDP ("YBHA, Gross Domestic Product at market prices: Current price, Seasonally adjusted £m") is from the Office for National Statistics. The seasonally adjusted and break-adjusted stock of M1 is from "A millennium of macroeconomic data for the UK, The Bank of England's collection of historical macroeconomic and financial statistics, Version 3 - finalised 30 April 2017," which is from the Bank of England's website. Series for a 10-year bond yield and a Treasury bill rate are from the same spreadsheet.

C.3 Canada

For Canada, the seasonally adjusted series for nominal GDP ("Gross domestic product (GDP) at market prices, Seasonally adjusted at annual rates, Current prices") is from Statistics Canada. Series for the three-month Treasury bill auction average rate and the benchmark 10-year bond yield for the government of Canada, are from Statistics Canada. M1 ("v41552787, Table 176-0020: currency outside banks, chartered bank chequable deposits, less inter-bank chequable deposits, monthly average") is from Statistics Canada. Data on currency are from Statistics Canada ("Table 176-0020: Currency outside banks and chartered bank deposits, monthly average, Bank of Canada, monthly").

C.4 Australia

Nominal GDP ("Gross domestic product: Current prices, $ Millions, Seasonally Adjusted, A2304418T") is from the Australian Bureau of Statistics. The short rate ('3-month BABs/NCDs, Bank Accepted Bills/Negotiable Certificates of Deposit-3 months; monthly average, quarterly average, per cent, ASX, 42767, FIRMMBAB90') is from the Reserve Bank of Australia (henceforth, RBA). M1 since 1975Q2 ('M1: Seasonally adjusted, $ millions') is from the Reserve Bank of Australia, and data from FRED II (at the St. Louis Fed's website) are used for the period 1972Q1-1975Q1 (over the period of overlapping— i.e., since 1975Q2— the two series are identical, which justifies their linking). 5- and 10-year government bond yields are from the RBA. Specifically, they are from the RBA's spreadsheet "F2.1 Capital Market Yields – Government Bond," which is available at the RBA's website. A quarterly, seasonally adjusted series for the "Unemployment rate, Unemployed persons as percentage of labour force" has been computed by taking averages within the quarter of the corresponding monthly series from the Australian Bureau of Statistics (the series' code is GLFSURSA).

C.5 Switzerland

For Switzerland, both M1 and the short rate ("Monetary aggregate M1, Level" and "Switzerland - CHF - call money rate (tomorrow next)", respectively) are from the Swiss National Bank's internet data portal. The seasonally adjusted series for nominal GDP ("Gross domestic product, ESA 2010, quarterly aggregates of gross domestic product, expenditure approach, seasonally and calendar adjusted data, In Mio. Swiss Francs, at current prices") is from the State Secretariat for Economic Affairs (SECO) at https://www.seco.admin.ch/seco/en/home. The series for the 10-year government bond yield is from the St. Louis Fed's internet data portal, FRED II (the acronym is IRLTLT01CHM156N).

C.6 Sweden

For Sweden, a seasonally adjusted series for nominal GDP ("BNPM - GDP at market prices, expenditure approach (ESA2010) by type of use, seasonally adjusted current prices, SEK million") is from Statistics Sweden. Series for M1 and the three-month Treasury bill rate ("Money supply, notes and coins held by Swedish non-bank public, M1 (SEK millions)" and "Treasury Bills, SE 3M," respectively) are from Statistics Sweden. A series for the 10-year government bond yield is from the St. Louis Fed's internet data portal, FRED II (the acronym is IRLTLT01SEM156N).

C.7 Euro area

For the euro area, all of the data are from the European Central Bank.

C.8 Denmark

For Denmark, M1 ("Money stock M1, end of period, units: DKK bn") is from Denmark's central bank. Nominal GDP ("B.1GF gross domestic product at factor cost, seasonally adjusted, current prices, 1-2.1.1 production, GDP and generation of income (summary table) by seasonal adjustment, price unit, transaction and time, units: DKK mio") and real GDP ("B.1*g gross domestic product, real, seasonally adjusted, 2010-prices, real value, Units: DKK mio") are from Statistics Denmark. The central bank's discount rate is from the central bank's website.

C.9 South Korea

For South Korea, all of the data are from the central bank: nominal and real GDP ("10.2.1.1 GDP and GNI by economic activities (seasonally adjusted, current prices, quarterly), gross domestic product at market prices (GDP), bil.Won" and "10.2.2.2 expenditures on GDP (seasonally adjusted, chained 2010 year prices, quarterly), expenditure on GDP, bil.Won," respectively); M1 ("1.1. money & banking (monetary aggregates, deposits, loans & discounts etc.), seasonally adjusted M1(end of), bil.Won since 1969Q4; before that: 1.1. money & banking (monetary aggregates, deposits, loans & discounts etc.), M1 (narrow money, end of), bil.Won, adjusted via ARIMA X-12"); and the central bank's discount rate.

C.10 Japan

A series for the discount rate is from the Bank of Japan. A seasonally adjusted series for nominal GDP is from the Economic and Social Research Institute, Cabinet Office, Government of Japan. A seasonally adjusted series for M1 has been constructed based on MA'MAM1NAM3M1MO ("M1/Average amount outstanding/money stock") and MA'MAM1YAM3M1MO ("M1/Percent changes from the previous year in average amounts outstanding/Money Stock"), both from the Bank of Japan.

C.11 Hong Kong

For Hong Kong, the HIBOR (Hong Kong Inter-Bank Offered Rate) is from the Hong Kong Monetary Authority (HKMA). M1 ("M1, Total, (HK$ million)") is from HKMA, and it has been seasonally adjusted via ARIMA X-12. Nominal GDP ("GDP, HK$ million, from: Table 031: GDP and its main expenditure components at current market prices, national income section (1)1,") is from Hong Kong's Census and Statistics Department. It has been seasonally adjusted via ARIMA X-12.

D Why We Do Not Use Divisia Aggregates

Throughout the entire paper, we work with "simple-sum" M1 aggregates. In this appendix, we briefly discuss why we have chosen to ignore Divisia indices. A first problem is that to the very best of our knowledge, such indices are available only for the U.S. (from the Center for Financial Stability, henceforth CFS) and for the U.K(from the Bank of England). A second problem is that for the U.S., the Divisia M1 series constructed by the CFS does not feature MMDAs (which are instead included in Divisia M2). This means that although the resulting index of monetary services has been constructed by optimally weighting the underlying individual assets, it suffers from the crucial shortcoming that it does not include a key component of the transaction technology. As a result, although Divisia M1 is in principle superior to the standard simple-sum M1 aggregate, it ultimately suffers from the same shortcoming of not including MMDAs.

Tables for the online appendix

Table A.1 Bootstrapped *p*-values for Elliot, Rothenberg, and Stock unit root tests[a]

		M1 velocity				short rate			
		p=2	p=4	p=6	p=8	p=2	p=4	p=6	p=8
United States	1959Q1-2019Q4	0.8633	0.8362	0.9048	0.8764	0.4382	0.2861	0.1903	0.4334
United Kingdom	1955Q1-2019Q4	0.9129	0.8806	0.8170	0.8609	0.4010	0.4436	0.5495	0.5951
Canada	1947Q3-2006Q4	0.4641	0.6307	0.3987	0.5405	0.2298	0.2466	0.2224	0.3600
	1967Q1-2019Q4	0.9780	0.9725	0.9636	0.9641	0.5020	0.5018	0.4775	0.7151
Australia	1969Q3-2019Q4	0.9766	0.9766	0.9695	0.9418	0.4745	0.3996	0.6217	0.7796
Switzerland	1980Q1-2019Q4	0.8918	0.7756	0.8488	0.8296	0.1631	0.2756	0.1845	0.2488
Sweden	1998Q1-2019Q4	0.7086	0.6588	0.7850	0.9379	0.3931	0.5558	0.6600	0.5850
Euro area	1999Q1-2019Q4	0.2303	0.1188	0.0399	0.0074	0.6275	0.3936	0.3316	0.3453
Denmark	1991Q1-2019Q4	0.3557	0.6095	0.4426	0.5297	0.1394	0.0431	0.0288	0.0102
South Korea	1964Q1-2019Q4	0.0205	0.0001	0.0346	0.0289	0.5296	0.5266	0.4200	0.1089
Japan	1960Q1-2019Q4	0.7583	0.7323	0.8419	0.6102	0.2549	0.4236	0.4468	0.4413
Hong Kong	1985Q1-2019Q4	0.6960	0.7432	0.6730	0.6678	0.3566	0.2372	0.1851	0.3773

[a] Based on 10,000 bootstrap replications of estimated ARIMA processes. Tests are with an intercept and no time trend. [b] The short rate has a few negative observations at the end of the sample.

Table A.1 (continued) Bootstrapped p-values for Elliot, Rothenberg, and Stock unit root tests[a]

		Logarithm of:							
		M1 velocity				short rate[b]			
		p=2	p=4	p=6	p=8	p=2	p=4	p=6	p=8
United States	1959Q1-2019Q4	0.9749	0.9550	0.9784	0.9611	0.4835	0.3839	0.4577	0.2198
United Kingdom	1955Q1-2019Q4	0.9728	0.9709	0.9468	0.9628	0.7598	0.7917	0.8321	0.8835
Canada	1947Q3-2006Q4	0.1103	0.2339	0.1159	0.2931	0.0590	0.0474	0.0229	0.0275
	1967Q1-2019Q4	0.9986	0.9979	0.9960	0.9974	0.4761	0.4924	0.6513	0.7351
Australia	1969Q3-2019Q4	0.9967	0.9988	0.9957	0.9933	0.9297	0.8868	0.9786	0.9822
Switzerland	1980Q1-2019Q4	0.9391	0.8697	0.9167	0.9019	[b]	[b]	[b]	[b]
Sweden	1998Q1-2019Q4	0.9332	0.8848	0.9314	0.9712	0.7487	0.7071	0.8188	0.8947
Euro area	1999Q1-2019Q4	0.6377	0.4587	0.2992	0.0952	0.8988	0.9366	0.9539	0.9768
Denmark	1991Q1-2019Q4	0.5492	0.7791	0.6813	0.6186	[b]	[b]	[b]	[b]
South Korea	1964Q1-2019Q4	0.3406	0.2072	0.4245	0.3604	0.9125	0.8993	0.8952	0.7023
Japan	1960Q1-2019Q4	0.9718	0.9639	0.9767	0.9090	0.6177	0.6604	0.6732	0.6732
Hong Kong	1985Q1-2019Q4	0.8316	0.8502	0.8396	0.8061	0.2699	0.2891	0.2719	0.3173

[a] Based on 10,000 bootstrap replications of estimated ARIMA processes. Tests are with an intercept and no time trend. [b] The short rate has a few negative observations at the end of the sample.

Table A.2*a* Model comparison exercise, semi-log *versus* log-log: mode of the log-likelihood in regressions of log velocity on lags of itself and either the short rate or its logarithm (p=2)

Country	Period	Semi-log	Log-log a=0 b=0	Log-log a=-1 b=0.15	Log-log a=-2 b=0.15
United States	1959Q1-2019Q4	766.1394	756.6280	763.3872	764.4667
United Kingdom	1955Q1-2019Q4	879.6821	877.9350	878.9924	879.3578
Canada	1947Q3-2006Q4	820.2401	807.8379	835.9188	836.6934
	1967Q1-2019Q4	775.0890	767.0845	772.1874	773.4180
Australia	1969Q3-2019Q4	650.7331	656.0624	655.5800	654.8957
Switzerland	1972Q1-2019Q4	547.7844	–[a]	–[a]	542.5159
Sweden	1998Q1-2019Q4	317.3933	–[a]	316.9838	317.3565
Euro area	1999Q1-2019Q4	333.1620	–[a]	333.0533	333.1005
Denmark	1991Q1-2019Q4	404.1328	–[a]	404.0745	404.0309
South Korea	1964Q1-2019Q4	630.9515	633.8825	633.7839	633.4732
Japan	1960Q1-2019Q4	845.3632	850.7677	850.8201	849.3694
Hong Kong	1985Q1-2019Q4	328.0148	325.5701	334.2308	332.9143

[a] The last observations for the short rate are negative.

Table A.2*b* Model comparison exercise, semi-log *versus* log-log: mode of the log-likelihood in regressions of log velocity on lags of itself and either the short rate or its logarithm (p=4)

Country	Period	Semi-log	Log-log a=0 b=0	Log-log a=-1 b=0.15	Log-log a=-2 b=0.15
United States	1959Q1-2019Q4	763.2818	751.3266	758.5811	759.4505
United Kingdom	1955Q1-2019Q4	898.6224	893.7504	895.9872	897.0579
Canada	1947Q3-2006Q4	813.8001	804.9218	832.4689	833.5199
	1967Q1-2019Q4	775.9595	766.4531	772.1997	773.9634
Australia	1969Q3-2019Q4	649.9510	655.1057	654.3638	653.9727
Switzerland	1972Q1-2019Q4	544.4609	–[a]	–[a]	538.9411
Sweden	1998Q1-2019Q4	311.4090	–[a]	312.9552	312.7534
Euro area	1999Q1-2019Q4	326.4400	–[a]	326.5178	326.6725
Denmark	1991Q1-2019Q4	405.6384	–[a]	406.5285	406.1349
South Korea	1964Q1-2019Q4	628.2222	634.6372	635.1622	634.5937
Japan	1960Q1-2019Q4	841.5156	848.6520	846.0627	844.8858
Hong Kong	1985Q1-2019Q4	326.1339	324.9236	335.0398	333.2458

[a] The last observations for the short rate are negative.