On the Role of Consumption and Decreasing Absolute Risk Aversion in the Theory of Job Search

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Revised, October 1977

Working Paper #: 90

PACS File #: 3030

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January 1978

Introduction

The agents in job search theories are different than the agents in most other microeconomic theories; and it isn't just that they are unemployed. In job search theories, individuals are typically assumed to maximize the expected utility of the present value of income net of search costs.[1/] This contrasts with most other microeconomic models which assume individuals maximize the expected utility of lifetime consumption. It seems natural to ask whether or not this difference is of any consequence, but it is our impression that this question has been neither asked nor answered in the relevant literature.

Perhaps the question has been ignored because maximization of a utility of income function is not necessarily inconsistent with preferences being defined directly in terms of consumption. There are well-known conditions which, if satisfied, allow income to be viewed as a composite good and included as the sole argument of an indirect utility function. However, if the direct utility function is not linear in consumption, these conditions are generally violated in models which, like job search, involve intertemporal utility maximization under uncertainty.[2/] Thus, maximizing the expected utility of income is not necessarily (or even likely to be) equivalent to maximizing the expectation of some reasonable utility of consumption function.[3/]

Even granting this nonequivalence, though, one could still argue that the difference is inconsequential if the empirical content of the theory is not affected by the choice of objective function. But the empirical content is affected by this choice. If one assumes that individuals maximize the expected utility of consumption, then, as Sargent [13] and Hall [7] have demonstrated, consumption data can provide information on individual tastes and expectations. If, on the other hand, individuals are assumed to maximize the expected utility of income, consumption data lacks any such informational content.

These observations have prompted us to analyze a model of job search, wherein the job seeker's preferences are defined directly on random sequences of consumption. This basic approach is intrinsically more general than that found in conventional models of job search since, as argued above, it places fewer restrictions on preferences and it encompasses a wider set of economic decisions. We have, however, sacrificed some of this intrinsic generality in order to increase the number of testable hypotheses yielded by our theory. In particular, we restrict our attention to preferences that give rise to additively separable utility functions and which display decreasing absolute risk aversion (DARA). The first of these restrictions, separability, allows us to ignore past consumption levels in formulating a rule for current and future decisions.[4/] The second restriction, DARA, captures the widely held (and empirically supported[5/]) belief that individuals are willing to give up less in order to avoid a given amount of variability in consumption as their average level of consumption increases.

Plan of the Paper and Summary of Results

The paper is divided into two sections. In the first section we present a finite horizon model of job search. In the second section we consider the infinite horizon analogue of that model.

We begin the first section with a presentation of the basic assumptions of the finite horizon model. The form of the optimal job search-cum-consumption allocation strategy is then deduced. This strategy specifies current consumption, next period's asset holdings, and a set of acceptable wage offers as a function of age and current asset endowments.

Two aspects of the unemployed individual's behavior can be inferred directly from our general solution. First, his strategy cannot, in general, be decomposed into separate expected income maximization and consumption allocation problems. Second, his decisions relating to job acceptability are sensitive to his current financial endowments.

The precise nature of this sensitivity is revealed in three propositions which follow from the DARA hypothesis. The first of these propositions may be referred to as "the rich are more selective" result. It asserts that an increase in initial wealth endowments induces a reduction in the set of wage offers which would be accepted in the current period if received. The second proposition is that "the rich search longer." That is, a ceteris paribus increase in assets leads to an increase in the expected duration of search unemployment for the individual. The last of these three results states that "the rich get richer." More precisely, this proposition observes that the expected present value of the returns from job search are directly related to current asset holdings.

In the second section we present an infinite horizon analogue of the model presented in Section 1. While the propositions of Section 1 remain valid for the model presented here, the influence of career length is not present in the infinite horizon setting. Thus, the only state variable for this model is asset holdings. We show that if the rate at which future utility is discounted is as large as the market discount rate for future income, then the job seeker will find it optimal to draw down his asset holdings and reduce his acceptance wage as time passes. This proposition reveals that one need not rely on an arbitrary deadline for terminating the search activity or a nonstationary distribution of wage offers in order to generate a declining acceptance wage.

I. A Model of Job Search: The Finite Horizon Version

In this section we present a model of optimal search with the following underlying structure. At each date the individual's current situation or state is completely described by his employment status (employed at some particular wage or unemployed) and his current asset holdings. He can exercise some control over movement between states through his choice of current consumption and his decision to accept or reject specific wage offers. Transition from one state to another is also influenced by which random wage offer is observed. The individual ranks alternative feasible control laws or strategies on the basis of the expected utility of the random consumption stream resulting from their adoption.

Assumptions[6/]

We assume that the individual's satisfaction depends solely on his lifetime path of single-period consumption levels, c(1), c(2), ..., c(N). His ordering of alternative consumption sequences is given by a von Nuemann-Morgenstern utility function, V: $R_+^N \to R^1$, having the intertemporally separable form:

$$(1) \qquad V(c(1),c(2),\ldots,c(N)) = \sum_{i=1}^{N} \beta^i u(c(i)),$$

where u is assumed to be increasing, bounded, twice differentiable, and strictly concave.

Since u is strictly concave, the individual is, of course, risk averse. In the sequel we shall utilize the stronger condition on attitudes toward risk alluded to in the introduction, namely, decreasing absolute risk aversion. Mathematically we require[7/] that $r_u \equiv -u''/u'$, the absolute risk aversion measure, is a decreasing function of c(i), i=1, 2, ..., N. This assumption, the implications of which will be discussed in precise terms below, basically guarantees that in any period, the quantity of certain consumption the individual is just willing to exchange for a given random supplement to consumption increases as the nonrandom component of consumption increases.

Wage offers are the source of uncertainty in the model. We assume that the individual believes per-period wage offers to be identically and independently distributed random variables, $\mathcal{Y}_1, \mathcal{Y}_2, \ldots, \mathcal{Y}_N$. The subjective cumulative distribution function and density functions for each of these variables will be denoted by F and f, respectively.

On observing a per-period wage offer of y in period i(<N) the individual may either accept it and receive y dollars in each of periods i+1, i+2, ..., N, or reject it and sample again in period i+1, incurring a search cost of s dollars at that time.

The individual uses his labor income as well as his assets to finance consumption expenditures. We assume for convenience that each composite consumption good can be purchased at a price of one dollar per unit.

The individual is allowed to borrow and lend at a constant interest rate, r. We assume, however, that lenders do place limits on borrowing so as to ensure that the individual's outstanding debts at the end of period N do not exceed some fixed upper bound, B. The necessity of satisfying this terminal constraint then imposes limitations on the individual's consumption and savings which depend on his current employment status. In particular, if the individual has not accepted employment as of date $t-1 \leq N$, his asset holdings $A(t)$ at date t must satisfy

$$(2) \qquad A(t) = [A(t-1)-c(t-1)-s](1+r),$$

and

$$(2') \qquad A(t) \geq \sum_{i=t}^{N} s(1+r)^{t-i} - B(1+r)^{-(N+1-t)} \equiv B_{N-t+1},$$

where r is the one-period market rate of interest. Alternatively,

$$(3) \qquad A(t) = [A(t-1)-c(t-1)+y](1+r),$$

and

$$(3') \qquad A(t) \geq -\sum_{i=t}^{N} y(1+r)^{t-i} - B(1+r)^{-(N+1-t)} \equiv B^{y}_{N-t+1}$$

if the individual is employed at wage y as of date $t-1 \leq N$. Equations (2) and (3) are just recursive budget equations, while (2') and (3') are the borrowing constraints imposed on the individual when he is unemployed and employed, respectively.

Finally, we assume that if the individual has not accepted a job prior to date t=1, 2, ..., N, then c(t) is chosen before the value of y_t is observed.[8/]

The Optimal Strategy

Our task now is to characterize the highest ranked or optimal feasible strategy. The technique we shall employ in obtaining this characterization is dynamic programming. Accordingly we define $J_{N-t}(A,y)$ as the maximal utility attainable in the N-t periods between dates t+1 and N+1 when initial assets are A and the individual is employed at wage y. Then $J_0 \equiv 0$, and

$$J_n(A,y) = \max_{0 \le c \le A-B_n^y} \{u(c)+\beta J_{n-1}[(A-c+y)(1+r),y]\}. \tag{4}$$

Similarly, if the individual is unemployed at date t+1 with asset holdings equal to A, then let $S_{N-t}(A)$ be the maximal utility attainable in the N-t periods between dates t+1 and N+1. Then $S_0 \equiv 0$, and

$$S_n(A) = \max_{0 \le c \le A-B_n} \{u(c)+\beta\int_0^\infty \max\{S_{n-1}[(A-s-c)(1+r)]; \tag{5}$$

$$J_{n-1}[(A-s-c)(1+r),y]\}dF(y)\}.$$

it is clear, given the definitions of J_n and S_n, that an offer of y will be accepted at date N-n+1 if

$$J_n(A,y) > S_n(A)$$

and will be rejected if

$$J_n(A,y) < S_n(A).$$

If neither inequality is satisfied, both acceptance and rejection are optimal. We thus define the continuous function $\hat{y}_n$: $[B_n,\infty) \to R_1$ by

$$\hat{y}_n(A) = \begin{cases} 0 \text{ if } J_n(A,0) \ge S_n(A) \\ y \text{ such that } J_n(A,y) = S_n(A) \text{ if} \\ \qquad J_n(A,0) < S_n(A). \end{cases} \tag{6}$$

Therefore, (5) may be rewritten as

$$S_n(A) = \max_{0 \le c \le A-B_n} \{u(c) + \beta\int_{\hat{y}_{n-1}((A-c-s)(1+r))}^{\infty} J_{n-1}((A-c-s)(1+r),y)f(y)dy \tag{7}$$

$$+ \beta F(\hat{y}_{n-1}((A-c-s)(1+r)))S_{n-1}((A-c-s)(1+r))\}.$$ [9/]

Since $\hat{y}_n$ may not be a constant function, strict concavity of J_{n+1} and S_{n+1} need not imply concavity of (7). Indeed, one can easily construct examples where (7) is a locally convex function of A. The underlying reason nonconcavities can arise here, even though the individual's utility function is concave, is that the constraint set is nonconvex. That is, the individual can choose to accept employment or to remain unemployed but not a convex combination of the two.

The behavioral implications of this type of nonconcavity are twofold. First, there may be a multiplicity of expected utility-maximizing search strategies. Though each strategy yields the same expected utility, the consumption, savings, and job acceptance decisions may differ among them. As we shall see below, no fundamental analytical difficulties result from such nonuniqueness.

Second, if the maximum expected utility of search is a nonconcave function of assets, the unemployed individual will be a risk preferrer for some levels of wealth holdings. Thus, while the individual would never think of accepting an actuarially unfair gamble so as to alter his assets when employed, he might be anxious to do so when unemployed.

Of course, the job search process itself may be viewed as a type of gamble or lottery. Each outcome is a particular income stream. The lottery evolves over time as various unacceptable job offers are encountered and terminates when a specific job is accepted. Given any level of asset holdings, there is some dollar amount, denoted $Y_n(A)$, which the individual would be just willing to accept when n periods remain rather than continue the job search lottery. When $J_n(A,0) \leq S_n(A)$, this amount is simply $\sum_{i=1}^{n} \hat{y}_n(A)/(1+r)^{i-1}$, the present discounted value of an "annuity" paying the period t-1 acceptance wage in periods t through N.

When $J_n(A,0) > S_n(A)$, we solve for the $\bar{y} < 0$ such that $J_n(A,\bar{y}) = S_n(A)$ and $Y_n(A) = \sum_{i=1}^{n} \bar{y}(1+i)^{i-1}$. By virtue of the continuity of S_n and J_n and the monotonicity of J_n, Y_n is a continuous function.

Implications of DARA

As was indicated in the introduction, there exists ample empirical evidence suggesting that consumers generally place higher dollar values on risky ventures as their wealth holdings are increased. Arrow [1] and Pratt [11] have shown that when the domain of an individual's utility function is the real line, i.e., V: $R^1 \to R^1$, such a positive correlation between wealth and risk value will obtain if and only if

$$r_V(x) = -V''(x)/V'(x)$$

is a decreasing function of x. When this condition is satisfied, we say that V displays or is characterized by DARA.

In [5] we have considered conditions which imply and are implied by a positive correlation between wealth and multidimensional risk value. One of the conclusions contained therein (Theorem 1, page 58) was that if an individual's utility function is additively separable, as we have assumed in this paper, then the present dollar value of arbitrary multidimensional risks is positively correlated with wealth if and only if

$$r_u(x) = -u''(x)/u'(x)$$

is decreasing throughout, i=1, 2, ..., N. Within the context of our search model, this result implies that Y_n is an increasing function of A since we have assumed r_u to be decreasing. In words, if the one-period utility function displays DARA, then the certain dollar value of job search is an increasing function of asset holdings. Furthermore, if the one-period utility function is strictly concave but does not display DARA, then the certain dollar value of job search can be a decreasing or constant function of asset holdings.

As an immediate consequence of the monotonicity of Y_n we have:

<u>Theorem 1</u>: It $\delta > 0$ and $A \geq B_n$, then $\hat{y}_n(A+\delta) \geq \hat{y}_n(A)$ $(\hat{y}_n(A+\delta) > \hat{y}_n(A)$ if $\hat{y}_n(A) \neq 0)$.

Theorem 1 states that the acceptance wage is a monotone increasing function of asset holdings. Since the probability that an individual will no longer be unemployed in the next period is inversely related to the size of his current acceptance wage, Theorem 1 provides a link between asset holdings and state transition probabilities. One might suspect that this link could be easily extended to establish an empirically testable relationship between the expected duration of unemployment and wealth. The possible multiplicity of expected utility-maximizing strategies mentioned above, however, means that the expected duration of unemployment may depend on which expected utility-maximizing strategy is chosen. While this complicates our analysis to some extent, we are able to demonstrate that the expected duration of unemployment associated with <u>any</u> expected utility-maximizing strategy for asset holdings A is less than the expected duration of unemployment associated with <u>any</u> expected utility-maximizing strategy for asset holdings A + δ.

First let $\sum(A,t)$ represent the individual's nonempty set of expected utility-maximizing, completely specified strategies when his state as of date t=N-n+1 is "unemployed with asset holdings $A \geq B_n$." Any $\sigma \varepsilon \sum(A,t)$ specifies a particular feasible sequence of consumption, asset stocks, and acceptance wages for each possible sequence of wage offers during the n periods between date t and date N+1. Of course, once an offer is accepted, no more wage offers are observed so that subsequent realizations of the wage offer process can have no effect on decisions. Furthermore, our assumptions of nonrecall and independent wage offer distributions imply that the level of previous unacceptable wage offers are irrelevant to current opportunities and utility. Thus, if the individual decides at date t to employ strategy $\sigma\varepsilon\sum(A,t)$, there is a unique level of wealth, denoted A^{σ}_{t+i}, which will be held at date t+i if the wage offers realized in the intervening periods, y(t), y(t+1), ..., y(t+i-1), have been unacceptable.[10/]

The acceptance wage in period (t+i), given that strategy $\sigma\varepsilon\sum(A,t)$ is being employed and that previous wage offers have been unacceptable, is $\hat{y}_{n-i-1}(A^{\sigma}_{t+i+1})$. The probability that the individual will be unemployed and searching at date j may, therefore, be expressed as

(9) $$P^{\sigma}_{j} \equiv F(\hat{y}_{n-1}(A^{\sigma}_{t+1}))F(\hat{y}_{n-2}(A^{\sigma}_{t+2})) \; . \; . \; . \; F(\hat{y}_{N+1-j}(A^{\sigma}_{j})).$$

The mathematical expectation of the duration of search unemployment when strategy σ is employed, DS^{σ}, is then given by

(10) $$DS^{\sigma} \equiv \sum_{i=t}^{N} (1+i-t)[P^{\sigma}_{i}-P^{\sigma}_{i+1}].$$

Observing that $P^{\sigma}_{t} = 1$ and $P^{\sigma}_{N+1} = 0$, (10) may be rewritten in the more convenient form of

(10') $$DS^{\sigma} = 1 + \sum_{i=t+1}^{N} P^{\sigma}_{i}.$$

Our next theorem observes that a ceteris paribus increase in the unemployed individual's current asset holdings will increase the expected duration of his search unemployment.

Theorem 2: If $\sigma^1 \varepsilon \sum(A^1,t)$, $\sigma^2 \varepsilon \sum(A^2,t)$ and $A^2 > A^1$, then

$DS^{\sigma^1} \leq DS^{\sigma^2}$; the inequality is strict if $\hat{y}_{n-1}(A_{t+1}^{\sigma^1}) > 0$ and $A_{t+1}^{\sigma^1} > B_{n-1}$.

Proof: A change in A affects expected duration of unemployment through the impact of such a change on future asset holdings and, consequently, on future acceptance wages. In particular, if increasing A increases asset holdings in future periods, then future acceptance wages will increase, thus increasing the probability of being unemployed in any future period.

Our proof will, therefore, be very nearly complete if it can be shown that asset holdings at dates t+1 through N are "normal goods" in the sense that increasing current asset holdings will lead to an increase in asset holdings in each subsequent period. Given that u is strictly concave, it is not surprising that in our model asset holdings are normal goods in this sense. The proof of this unsurprising fact is, however, rather lengthy due to the possible nonconcavities in and non-differentiability of the S_{t+i}'s. We, therefore, refer the interested reader to [4] (Lemma 9, pages 46-49) wherein we establish that if

$$\sigma^1 \varepsilon \sum(A^1,t),\ \sigma^2 \varepsilon \sum(A^2,t), \text{ and } A^2 > A^1,$$

then either

$$A_{j+1}^{\sigma^2} > A_{j+1}^{\sigma^1}$$

or

$$A_{j+1}^{\sigma^2} = A_{j+1}^{\sigma^1} = B_{j+1},\ j=t,\ \ldots,\ N-1.$$

This result together with Theorem 1 ensures that each element of the product defining P_j is an increasing function of A for j=t+1, ..., N. Hence, $P_j^{\sigma^1} \leq (<) P_j^{\sigma^2}$, j=t+1, ..., N, and, therefore, by (10'), $DS^{\sigma^1} \leq (<) DS^{\sigma^2}$ (if $\hat{y}_{n-1}(A_{t+1}^{\sigma^1}) > 0$ and $A_{t+1}^{\sigma^1} > B_{n-1}$) and our proof is complete. Q.E.D.

Job search is often described as a type of investment in human capital with the duration of search measuring the size of this investment. Theorem 2 establishes that if an individual is characterized by DARA, his demand for search, like his demand for other investments having risky returns, increases as his asset holdings increase. If, as we believe, the analogy between increased expected duration of search and increased participation in other risky investments is a good one, then the expected return to this increased search should be positive. The remainder of this section will be devoted to investigating the sign of the change in expected returns accompanying an increase in the expected duration of search unemployment.

Since the returns to search take the form of increases in lifetime earnings (net of search costs), the expected return of a particular search strategy is the expected present value of lifetime labor income less search costs resulting from the adoption of that strategy. It will prove convenient to have a recursive expression for this expected present value. Therefore, we define the expected present value of income less search costs as of date t+i (where again t=N-n+1) if the individual is employing strategy $\sigma \varepsilon \sum(A,t)$ and has not accepted employment by

$$\text{(11a)} \qquad PVY^{\sigma}_{n-i} = -s + \int_{\hat{y}_{n-i-1}(A^{\sigma}_{t+i+1})}^{\infty} \sum_{j=t+i+1}^{N} y(1+r)^{i-j} f(y)dy$$

$$+ (1+r)^{-1} PVY^{\sigma}_{n-i-1} F(\hat{y}_{n-i-1}(A^{\sigma}_{t+i+1})).$$

and

$$\text{(11b)} \qquad PVY^{\sigma}_{1} = -s.$$

We now have two dollar figures to place on the individual's optimal search strategy, the certain dollar value of the strategy, $Y_n(A)$, and the expected present value of income less search costs resulting from the adoption of the strategy, PVY^{σ}_n. Since we have assumed the labor force participant to be risk averse, i.e., u strictly concave, one can easily verify that the difference,

$$PVY^{\sigma}_n - Y_n(A),$$

is nonnegative for $\sigma \varepsilon \sum(A,t)$. This difference can be interpreted as a risk premium.

In our model this risk premium is closely related to the marginal change in expected net income associated with an increase in the acceptance wage. To see this, observe that if a wage offer equal to the acceptance wage is received, $Y_n(A)$ is the present value of income if it is accepted and PVY^{σ}_n is the expected present value of income less search costs if it is rejected. Therefore, since an increase in asset holdings increases the expected duration of search by raising acceptance wages, it will also increase expected income less search costs if the risk premium is positive. This is the basis for our proof of

<u>Theorem 3</u>: If $\sigma^1 \varepsilon \sum(A^1,t)$, $\sigma^2 \varepsilon \sum(A^2,t)$ and $A^1 < A^2$, then

$$PVY_n^{\sigma^1} \leq (<) PVY_n^{\sigma^2} \quad (\text{if } \hat{y}_{n-1}(A_{t+1}^{\sigma^1}) > 0 \text{ and } A_{t+1}^{\sigma^1} > B_{n-1}).$$

<u>Proof</u>: Take as an induction hypothesis for date $t+i \leq N$

$$PVY_{n-i}^{\sigma^1} \leq PVY_{n-i}^{\sigma^2}.$$

Now recall from our proof of Theorem 2 that

$$\hat{y}_{n-i-1}(A_{t+i+1}^{\sigma^1}) \leq (<) \hat{y}_{n-i-1}(A_{t+i+1}^{\sigma^2})$$

and hence

$$(12) \qquad \int_{\hat{y}_{n-i-1}(A_{t+i+1}^{\sigma^1})}^{\hat{y}_{n-i-1}(A_{t+i+1}^{\sigma^2})} [Y_{n-i-1}(A_{t+i+1}^{\sigma^2}) - \sum_{j=t+i+1}^{N} y(1+r)^{i-j}] f(y)dy \geq (>) 0$$

$$(\text{if } \hat{y}_{n-i-1}(A_{t+i+1}^{\sigma^1}) > 0 \text{ and } A_{t+i+1}^{\sigma^1} > B_{n-i-1}).$$

Also, since the "risk premium" is nonnegative,

$$(13) \qquad \int_{\hat{y}_{n-i-1}(A_{t+i+1}^{\sigma^1})}^{\hat{y}_{n-i-1}(A_{t+i+1}^{\sigma^2})} [PVY_{n-i-1}^{\sigma^2} - Y_{n-i-1}(A_{t+i+1}^{\sigma^2})] f(y)dy \geq 0.$$

Our induction hypothesis implies

$$(14) \qquad F(\hat{y}_{n-i-1}(A_{t+i+1}^{\sigma^1}))[PVY_{n-i-1}^{\sigma^2} - PVY_{n-i-1}^{\sigma^1}] \geq 0.$$

Summing the LHS's of (12), (13), and (14) and referring to (11a), we conclude that

$$PVY^{\sigma^2}_{n-i} - PVY^{\sigma^1}_{n-i} \geq (>) 0$$

$$(\text{if } \hat{y}_{n-i-1}(A^{\sigma^1}_{t+i+1}) > 0 \text{ and } A^{\sigma^1}_{t+i+1} > B_{n-i-1}).$$

Since our induction hypothesis is clearly valid for t+i+1=N, the proof is complete//

Theorem 3 confirms the claim that there is a positive return to increased investment in job search. It is also worthwhile noting that this result provides a formal justification for Milton Friedman's statements (see reference [6]) regarding the existence of a "link between differences in natural endowments or inherited wealth and the realized distribution of wealth or income" which results from a systematic relationship between individual propensities to take risks and his initial endowments. Thus, if labor force participants are characterized by DARA and differ only as to their initial asset holdings, our model predicts that, on average, the rich will get richer.

II. A Model of Job Search: The Infinite Horizon Version

The model of this section differs in only one respect from the one considered in the previous section. The horizon is infinite rather than finite. The primary reason for this switch in horizons is that it allows us to isolate the effects of changes in asset holdings from changes in the length of the future as time passes.

The assumptions of the preceding section are easily altered to give rise to the infinite horizon analogue to our finite horizon model.

First, the objective function given in (1) is changed to

$$(1') \qquad V(c(1),c(2),\ldots) = \sum_{i=1}^{\infty} \beta^i u(c(i)).$$

Second, jobs are assumed to last forever. Thus, when a wage offer of y is accepted, the individual will receive a "perpetuity" of y dollars per period. Finally, the borrowing constraints given in (2') and (3') become

$$(2'') \qquad A(t) \geq s(1+r)/r \equiv B$$

and

$$(3'') \qquad A(t) \geq -y(1+r)/r \equiv B^y,$$

respectively. Of course, $B=\lim_{n\to\infty} B_n$ and $B^y=\lim_{n\to\infty} B_n^y$.

The procedure employed in deriving an optimal strategy within this altered framework is a variant of Bellman's "method of successive approximations" discussed in [2]. First, a sequence of objective functions is specified which converges uniformly to V. For each element of this sequence, a strategy which maximizes its expected value is determined. Finally, the resulting sequence of maximizing strategies and expected values is used to characterize a strategy which maximizes the expected value of V.

Since the objective function and constraints of this model are the limits of those posited in the first section, it is not surprising to find that the sequence of functions which shall be used is $\{V_n\}$, where

$$V_n(c(1),c(2),\ldots) = \sum_{i=1}^{n} \beta^i u(c(i)).$$

Of course, we know a great deal about strategies which maximize the expected value of any particular V_n, since this is the utility function dealt with in the previous section.

Thus, if the labor force participant's state as of date 1 is "employed at wage y with asset holdings A," then the maximum attainable value of V_n from date 1 forward is $J_n'(A,y)$ where J_n' is defined as in (4), except that B_n^y is replaced by B^y. Similarly, if the individual's state as of date t is "unemployed with asset holdings A," then the maximum attainable expected value of V_n from date 1 forward is S_n' where S_n' is defined as in (5), except that B_n is replaced by B. Replacing J_n and S_n in (6) with J_n' and S_n' we obtain the definition of the acceptance wage function, $\hat{y}_n'$, for this truncated problem. J_n', S_n', and $\hat{y}_n'$ are easily shown to possess the same continuity, differentiability, monotonicity, and concavity properties as J_n, S_n, and $\hat{y}_n$, respectively.

Since u is bounded, there is no loss of generality in assuming that $u(0) \geq 0$, so that J_n' and S_n' are nondecreasing functions of n for any value of A. Also, since $0 < \beta < 1$ and u is bounded from above by some number $\bar{u}$, both J_n' and S_n' are uniformly bounded from above by $\bar{u}/(1-\beta)$ for all values of n. J_n' and S_n', therefore, converge uniformly to continuous and increasing functions, J and S, as n tends to infinity.

J(A,y) is thus the maximum attainable value of V given the individual's state is "employed at wage y with asset holding A." S(A) is the maximum attainable value of V given the individual's state is "unemployed with asset holdings A." The acceptance wage, $\hat{y}$, is, therefore, the solution to

$$J(A,\hat{y}) - S(A) = 0,$$

which depends on A but not on t.

Since the conditions under which employment will be accepted or rejected are completely summarized in the acceptance wage, all that remains to be said about the individual's strategy is how he chooses current consumption. Of course, this is no great mystery. If the individual's state is "unemployed with asset holdings A," he should choose c, $0 \leq c \leq A-B$, to maximize

$$(17) \qquad u(c) + \int_{\hat{y}((A-c-s)(1+r))}^{\infty} \beta J((A-c-s)(1+r),y)f(y)dy$$

$$+ \beta F(\hat{y}((A-c-s)(1+r)))S((A-c-s)(1+r)).$$

The maximized value of this sum is S(A). Similarly, if the individual's state is "employed at wage y with asset holdings A," he should choose c, $0 \leq c \leq A-B^y$, so as to maximize

$$(18) \qquad u(c) + \beta J((A-c+y)(1+r),y).$$

The maximized value of (18) is then J(A,y).

Our model shares with standard models of consumption allocation over time the property that the intertemporal pattern of consumption and asset accumulation or deaccumulation is sensitive to the relationship between β, the psychological rate of time discount, and the interest rate r. For example, if $\beta(1+r) > 1$, then the individual, having sold his resource, will increase his level of consumption and his financial asset holdings in each successive period without bound. Conversely, if $\beta(1+r) < 1$, then the individual who has sold his resource will plan to consume less in future periods than in the present and will draw down his financial asset stock over time. If $\beta(1+r)=1$, the individual will neither increase nor decrease his asset holdings as time passes.

In this paper we are primarily interested in pre-job-acceptance asset dynamics so that the relationships cited in the preceding paragraph, while suggestive, are not directly applicable. In particular, the rate of change of financial asset holdings for an individual engaged in price search depends not only upon β and r but also upon the distribution F of wage offers. Although the addition of the wage distribution to our calculations makes it exceedingly difficult to determine conditions under which a job seeker will accumulate assets, restrictions which imply that financial asset holdings will be used up over time are straightforward. As the following theorem reveals, $\beta(1+r) \leq 1$ is a sufficient, though not necessary, condition to ensure that the job seeker's financial assets will decline over time.

Theorem 4: If $A(t) > B$, $\beta(1+r) \leq 1$, and c^* is the constrained maximizer for (17), then $A(t) > A(t+1) = (A(t)-c^*-s)(1+r)$.

Proof: This theorem can be proven in three steps. First, one can establish by induction that

$$dS_n(A)/dA \leq \partial J_n(x,0)/\partial x|x = A-B$$

n=1, 2, ..., and therefore the derivatives of the limit functions satisfy

$$dS(A)/dA \leq \partial J(x,0)/\partial x|x = A-B.$$

Second, the relationship between the slopes of S and J together with the strict concavity of J (established in Appendix B) can be shown to imply that the value of c which maximizes

$$u(c) + \beta J((A-s-c)(1+r),0),$$

c°, is less than the value of c which maximizes (17), c^*. Third, one may easily verify that $\beta(1+r) \leq 1$ implies

$$A(t) \geq (A(t)-s-c^\circ)(1+r)$$

and thus

$$A(t) > (A(t)-s-c^*)(1+r) = A(t+1) //$$

Our attention will be confined to search strategies which specify declining asset balances in our subsequent discussion. This emphasis can be justified on at least two grounds. First, conditions which are consistent with alternative savings behavior are illusive. Second, we believe asset deaccumulation by agents involved in price search is more plausible than asset accumulation. We, therefore, assume throughout the remainder of the paper that $\beta(1+r) \leq 1$.

Since utility, prices, and labor demand are all stationary in this infinite horizon model, the unemployed individual's optimal choice of controls, i.e., consumption and acceptance wage, will change over time solely in response to his declining asset holdings. We are, therefore, obliged to investigate the relationship which exists between the acceptance wage and asset holdings in this stationary setting if we are to infer anything about the dynamics of the first of these two variables. The nature of this relationship is partially revealed in the following lemma.

Lemma 1: The acceptance wage $\hat{y}$ is a nondecreasing function of the asset level A.

This result is a consequence of Theorem 1 of Section 1, which asserts that the acceptance wage in a finite horizon optimal search strategy, $\hat{y}_n$, is an increasing function of A if $r'_u(c) < 0$. Since $J_n \to J$ and $S_n \to S$ uniformly as $n \to \infty$, $\hat{y}_n \to \hat{y}$ uniformly as $n \to \infty$ and the limit of a uniformly convergent sequence of increasing functions is nondecreasing.

Theorem 4 and Lemma 1 together imply that the acceptance wage will be nonincreasing as time passes. This, however, is not all that was promised in the introduction. We asserted that the acceptance wage would be declining over time. This distinction rules out the constant acceptance wage which is observed in an infinite horizon model of expected income maximizing job search.

In order to prove our stronger result, we must establish that $\hat{y}$ is a strictly increasing function. The main tool used in proving this is verifying that the indirect utility function U displays DARA, where U is given by

Lemma 2: $U(x) = J(x,0)$. The indirect utility function has strictly decreasing risk aversion, that is, r_U is strictly decreasing.

The applicability of this proposition is in no way limited to models of job search. It says something quite general about the preservation of a property of direct utility functions in the indirect utility function resulting from constrained maximization. Specifically the lemma observes that the indirect utility function associated with an infinite dimensional stationary direct utility function displays DARA if the single-period utility indicators $\beta^i u$ display DARA. The proof of Lemma 2 is technical and lengthy; accordingly, it is given in Appendix B.

With the following theorem we conclude our analysis of the infinite horizon search problem. The theorem asserts that, so long as an individual's asset holdings have not been completely exhausted, his acceptance wage will decline with the passage of time. Thus, even though the labor force participant's decision rule $\hat{y}$ is stationary, the optimal decision $\hat{y}(A(t))$ at time t is not.

Theorem 5: If $A(t) > B$, then $\hat{y}(A(t)) > \hat{y}(A(t+1))$.

Proof: As before, we shall denote by $\sum(A,t)$ the expected utility maximizing strategies available to the individual at date t when his state is "unemployed with asset holdings A." The acceptance wage, $\hat{y}(A)$, will then be a solution to

$$J(A,\hat{y}(A)) = u(c_t^\sigma) + \int_{\hat{y}(A_{t+1}^\sigma)}^{\infty} J(A_{t+1}^\sigma, y)f(y)dy + F(\hat{y}(A_{t+1}^\sigma))\beta J(A_{t+1}^\sigma, \hat{y}(A_{t+1}^\sigma))$$

for any $\sigma\varepsilon\sum(A,t)$.

It is easily demonstrated that

$$J(A,y) = J(A+y(1+r)/r,0)$$

so that

$$\max_{0\leq c\leq A^i+\hat{y}(A^i)(1+r)/r} u(c) + \beta J((A^i+\hat{y}(A^i)(1+r)/r-c)(1+r),0)$$

$$= u(c_t^{\sigma^i}) + \int_{\hat{y}(A_{t+1}^{\sigma^i})}^{\infty} J(A_{t+1}^{\sigma^i}+y(1+r)/r,0)f(y)dy + F(\hat{y}(A_{t+1}^{\sigma^i}))\beta J(A_{t+1}^{\sigma^i}+\hat{y}(A_{t+1}^{\sigma^i})(1+r)/r,0)$$

for i=1, 2, and $\sigma^i\varepsilon\sum(A^i,t)$. Since r_u and r_J are decreasing, the conditions of Theorem 2 of [5], discussed in connection with our Theorem 1, are satisfied. We thus conclude that if $A^2 > A^1$, then

$$\hat{y}(A^2) > \hat{y}(A^1).$$

By Theorem 4, $A(t) > B$ implies $A(t) > A(t+1)$, completing the proof. Q.E.D.

Summary and Conclusions

The optimal labor market strategy of an unemployed expected utility of consumption-maximizing individual was investigated in this paper. In the first section we considered a finitely long-lived individual. We demonstrated that the individual has at least one optimal job search-cum-consumption allocation strategy, and that any such strategy consists of a decision rule which specifies current consumption, savings, and acceptable wage offers as a function of current asset holdings and age.

Several testable hypotheses were generated under the assumption of decreasing absolute risk aversion. In particular, we proved that asset holdings are positively correlated with the acceptance wage, the expected duration of unemployment, and the expected present value of noninterest income.

These predicted correlations, especially the last one, are very much in the spirit of Friedman's comments cited in the text. If individuals are indeed characterized by decreasing absolute risk aversion, then the job search process has been shown to translate unequal nonhuman wealth endowments into unequal lifetime labor income. Thus, with the inclusion of bequests in the utility function, our analysis suggests that the apparently typical negative wealth-risk aversion relationship characterizing individual's acts as an inequality preserving factor in the dynamic determination of the personal distribution of income.

In the second section the job search-consumption allocation strategy of an infinitely long-lived individual acting in a stationary environment was derived. Here the strategy was shown to consist of a rule which specified current consumption, savings, and an acceptance wage for each level of financial asset holdings. Due to stationarity assumptions on wage expectations and preferences, this rule was also stationary, i.e., independent of date.

The stationarity of the rule did not, of course, imply that consumption, savings, and acceptable wage offers would be the same in each period. In fact, it was shown that plausible restrictions on the relationship between the psychological discount rate and the market interest rate implied declining financial asset holdings (savings) before and after the culmination of the price search process.

The time path followed by financial asset holdings, the only quantitative state variable in the model, could be translated into a time path for the acceptance wage and consumption, the two control variables of the model. We found that if the individual satisfied the DARA hypothesis, then current financial asset holdings and current reservation price would be positively related to one another. When combined with our earlier result on the time path of financial asset holdings, we obtained a theorem which stated straightforward conditions under which the individual's acceptance wage would decline over time.

Footnotes

[1] See Kohn and Shavell [8] and Lippman and McCall [9] for comprehensive discussion of this literature.

[2] See Spence and Zeckhauser [14] or Danforth [5] for a discussion of why these conditions are generally violated.

[3] By "reasonable utility of consumption function" we mean one which is increasing in each period's consumption.

[4] Expected utility of income maximization implies separability.

[5] See Cohn et al., [3] and Projector and Weiss [12] for such empirical support.

[6] In Appendix A we indicate how these assumptions can be generalized without significantly altering the main conclusions of the section.

[7] The sense in which this mathematical condition is equivalent to DARA is made precise in [5].

[8] This ordering is somewhat arbitrary in that reversing it leaves all of the implications of the analysis intact.

[9] If one assumes that u and F each belongs to a particular finite parameter family of functions, equation (7) can be used in conjunction with data on consumption and acceptance wages to obtain maximum likelihood estimates of the parameters of those functions (see Sargent [13] and Hall [7] for worked examples of this type of procedure).

[10] Notice that since $c(t)$ is determined prior to the observation of $y(t)$, the level of assets held at date $t+1$ for strategy σ does not depend on wage offers observed subsequent to period $t-1$.

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Appendix A

Some Generalizations of the Finite Horizon Model

In a footnote to our discussion of the finite horizon model of job search, we suggested that several assumptions of that model could be relaxed without affecting our main results. We suggest five possible generalizations in this appendix. The reader wishing to see a rigorous analysis of the model when these more general assumptions are adopted should refer to [4].

First, we can assume that the individual believes wage offers to be generated as a fairly general stochastic process. This allows for the possibility of learning through sampling.

Second, the individual can be given the opportunity to recall previous offers for some specified number of periods. Thus, offers which were received in period t would be held open through period t+s.

Third, consumption good prices, interest rates, and search costs can be made period specific. So long as these prices and costs are known with certainty, they can have any intertemporal pattern one might care to assume and our analysis will be essentially unaffected.

Fourth, we can allow the individual a period of retirement.

Fifth, the utility function can have the more general form: $V(c(1),\ldots,c(N)) = \sum_{i=1}^{N} u_i(c(i))$, where each of the u_i satisfy $u_i' > 0$, $u_i'' < 0$, and $\frac{-u_i''}{u_i'}$ decreasing.

Aside from complicating our notation, the third through fifth of these generalizations have a negligible effect on our analysis and conclusions. The first and second generalizations listed here, however, add more than notational complexity to our analysis. Each of these weakened assumptions has the effect of adding another state variable to our problem, namely, previously observed wage offers. Some difficult technical problems relating to the measurability of strategies arise when previous realizations of the stochastic process are state variables. Nevertheless, Theorems 2 and 3 are correct as stated even if each of these four generalizations is adopted.

On the other hand, Theorem 1 must be slightly modified since the set of acceptable wage offers at any particular date need not consist of all wage offers above some acceptance wage. That is, the set of acceptable wage offers may be disconnected. Our modified Theorem 1 asserts that the set of acceptable wage offers will shrink in response to an increase in asset holdings.

Appendix B

Proof of Lemma 2

The requirements of our proof differ depending on whether

$$\lim_{c\to 0} u'(c) < \infty \text{ or } \lim_{c\to 0} u'(c) = \infty,$$

so we shall consider these cases separately.

Case I: $\lim_{c\to 0} u'(c) < \infty$.

It is easy to verify that

$$J(x,0) = \max_{c_0, c_1, \ldots} \sum_{i=0}^{\infty} \beta^i u(c_i)$$

subject to $c_i \geq 0$ and $\sum_{i=0}^{\infty} c_i/(1+r)^i = x$. Necessary conditions for this maximum are

$$(1) \qquad u'(c_0) = (1+r)^i \beta^i u'(c_i) \qquad \text{if } c_i > 0$$

and

$$(2) \qquad u'(c_0) \geq (1+r)^i \beta^i u'(c_i) \qquad \text{if } c_i = 0.$$

For any $x < \infty$ there exists an $n(x) < \infty$ such that the maximizing value of c_i is 0 for all $i > n(x)$. Since $u' < 0$, $n(x)$ is nondecreasing in x. Therefore, for any $M < \infty$ and $x < M$,

$$J(x,0) = \max_{c_0, \ldots, c_{n(M)}} \sum_{i=0}^{n(M)} \beta^i u(c_i)$$

subject to $c_i \geq 0$, $i=0, \ldots, n(M)$, and $\sum_{i=1}^{n(M)} c_i/(1+r)^i = x$.

Neave ([8] Lemma 1, page 46) has shown that if $-u''/u'$ is decreasing, then the assertion of Lemma 2 is valid for $x \varepsilon [0,M]$. Since M is an arbitrary real number, this establishes the lemma for Case I.

Case II: $\lim_{c \to 0} u'(c) = \infty$.

The first-order conditions for this case are

$$u'(c_0) = (1+r)^i \beta^i u'(c_i), \quad i=0, 1, \ldots . \tag{3}$$

Each of these conditions implicitly defines a twice differentiable function, g_i, with

$$u'(c_0) = (1+r)^i \beta^i u'(g_i(c_0)).$$

Since $\beta(1+r) < 1$ and $-u''/u'$ is decreasing, $g_i(c) < c$ and $g_i'(c) < 1$ for all i=1, 2, ... The budget constraint,

$$x = \sum_{i=0}^{\infty} g_i(c_i)/(1+r)^i, \tag{4}$$

therefore, gives rise to a twice differentiable function, h, such that (3) and (4) are satisfied when $c_i = g_i(h(x))$, i=0, 1, ... Given our assumptions on u, the necessary conditions are also sufficient, so that

$$J(x,0) = \sum_{i=0}^{\infty} \beta^i u(g_i(h(x))).$$

Differentiating this sum and making use of the first-order conditions we obtain,

$$r_J(x) = -u''(g_i(h(x)))g_i'(h(x))h'(x)/u'(g_i(h(x)))$$

for i=0, 1, ... This expression is decreasing in x since $g_i' > 0$ and $h'(x) > 0$ implies

$$-u''(g_i(h(x)))/u'(g_i(h(x)))$$

is decreasing in x for all i, and

$$g_i'(h(x))h'(x)$$

is nonincreasing in x for at least one i since differentiation of (4) implies

$$1 = \sum_{i=0}^{\infty} g_i'(h(x))h'(x)/(1+r)^i.$$

This establishes the lemma for Case II//