Price Setting 'Perfect Competitors'

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In this paper it is shown that a distribution of prices occurs in a market of numerous small identical firms with identical product and with free entry, and of identical customers having zero search costs.

For want of an alternative, many economic models assume perfect competition. In particular, they assume price taking firms and market clearing prices. However, in many markets firms are price setters and prices do not clear the market. The hope is that this departure from the assumption of perfect competition does not significantly affect behavior.

It is shown below that the distinction between price setters and price takers is not important. However, the assumption that price clears the market is an essential element in the behavior of firms. To generate market clearing we assume that output and price decisions are made after observing the demand schedule. To violate market clearing we assume that both output and price decisions are made before observing the (stochastic) demand schedule. There is some technological constraint that prohibits setting output and price after demand is observed. Under this circumstance the market is characterized by a distribution of prices rather than by a single price.

I. Certainty

We will consider first the case of output and price being set with demand known in the one-period problem.

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Assume there are N consumers with identical demand curves, $d(P)$. These consumers have costless search. The consumers are ordered and enter the market sequentially.[1/] The customer's optimal strategy is to go to the lowest price firm which is not stocked out. The identical firms must set price and output before the customers arrive.

The proposed equilibrium is the perfect competition result. Firms set price at $\overline{P}$ and output at $\overline{X}$ where $\overline{P} = C'(\overline{X})$ meeting the zero-profit requirement $P\overline{X} - C(\overline{X}) = 0$. In other words, they produce at the minimum of their average cost curve, with price equal to their average cost. Consumers buy $d(\overline{P})$ and the number of firms, K, is determined by $K\overline{X} = Nd(\overline{P})$.[2/]

This is not a Nash equilibrium in the set of price and output strategies. Given that firms 1, ..., j-1, j+1, ..., K follow the strategy $P = \overline{P}$, $X = \overline{X}$ and firms K+1, ... do not enter the market, consider the problem of firm j. For simplicity assume that $(K-1)\overline{X}$ satisfies N_0 customers, $N_0 < N$. Then firm j should act as a monopolist facing demand curve $(N-N_0)d(P)$ and set $P_j > \overline{P}$, $X_j < \overline{X}$ such that marginal revenue equals marginal cost and $X_j = (N-N_0)d(P_j)$.

Clearly there is no Nash equilibrium with all prices equal and marginal cost different from price. Consider a proposed equilibrium with a nonempty set of prices $\{P_\Omega | P_\Omega > \overline{P}\}$ and zero profits. Then a deviant could enter and act as a monopolist as in the above paragraph, but with the constraint that he set $P < \min\{P_\Omega\}$, and make positive profits thereby. It appears that there is no pure strategy equilibrium.

[1/] Their ordering and entering the market sequentially is convenient, but not important to the argument. Customers could hold a lottery at each firm visited.

[2/] We ignore the possibility that $Nd(\overline{P})$ may not be an even multiple of $\overline{X}$.

The reason that there may be no pure strategy equilibrium can be easily demonstrated. For any set of prices $(P_0, P_1, \ldots, P_K)$ define $f_j[P_0, P_1, \ldots, P_K] = P_j^*$ where P_j^* is an optimal price strategy for firm j given $P_1, P_2, \ldots, P_{j-1}, P_{j+1}, \ldots P_K$. If f_j are continuous for all j, then one can show using the Brouwer fixed-point theorem that $F(P_1, \ldots, P_K) = [f_1(P_0, \ldots, P_K), \ldots, f_K(P_0, \ldots, P_K)]$ has a fixed point. This fixed point is a Nash equilibrium. Our problem is twofold: (1) f_j is not defined and (2) if it were defined f_j would not be continuous. (1) follows from the fact that the optimal strategy may be to set price "just" below that of the competition. (2) follows from the fact that there is a set of prices such that if a competitor increases his price at all it is optimal for a firm to jump its price "just" below that of this competitor. The payoff functions to individual firms are discontinuous and, therefore, not concave. As the payoff functions are discontinuous it is even unclear that a mixed strategy equilibrium exists.

This lack of a pure strategy Nash equilibrium in the model is disconcerting. Is the role of the auctioneer so crucial? Perfect competition rules out gaming with the auctioneer by assumption. If gaming with the auctioneer is allowed, the same problem arises. Below, the structure of the market of price setters is modified so that the perfect competition solution is the solution for price setters. If this modification is added to the perfect competition model, then gaming with the auctioneer will not occur. The auctioneer is an unnecessary fiction.

Let us change the structure of the market slightly. The new game can be divided into two simple games. In the first game economic agents decide whether to be firms in the market or potential entrants. In the second game firms are Stackelberg versus potential entrants.

Firms announce price and set output. Then potential entrants sequentially announce prices and set output (if any). Then consumers are released on the market.

Let our proposed equilibrium be the perfect competition result as before: $\overline{P} = C'(\overline{X})$, $\overline{PX} - C(\overline{X}) = 0$, $K\overline{X} = Nd(\overline{P})$, with K firms in the market and no potential entrants in the market. Consider the second subgame first. Any one of the firms could act as a monopolist and raise his price. But if he did so, he would be undercut by a new entrant and have negative profit.[3/] At $\overline{P}$ the firm will sell all it wants, $\overline{X}$. A potential entrant could sell only by setting $P \leq \overline{P}$ and taking a loss. He will not enter. Consider now the first subgame. It is in a Nash equilibrium.

In the second description of the market the set of admissible strategies has been changed. Potential entrants use contingent strategies. The reader must decide which solution concept corresponds to his view of the role of potential entrants. Does the market consist of an undifferentiated group of economic agents facing an opportunity, or does it consist of participants and potential entrants that differ in an essential way?

A real issue of how the market behaves is involved here. In the real world, is there an opportunity for firms to unexpectedly or randomly raise their prices and capture positive profits before their competitors can react? Is this behavior important enough to model? Perfect competition rules this out.

[3/] We assume here a lexicographic return function to potential entrants. The first dimension is expected profit and the second is whether they enter--they like to enter.

The reader may object that the model is not perfect competition. Firms are not small and numerous and therefore do not face infinitely elastic demand. Moreover, the use of the Stackelberg equilibrium in the second description is just a trick to keep the firms from using the inelasticity to their advantage.

One can argue that to strictly get the results of perfect competition we should assume a continuum of firms. With a continuum of firms a deviant has zero mass. Therefore, if the deviant raises his price above $\overline{P}$, the other producers will fully meet demand and the deviant will face no demand. His demand curve is infinitely elastic. The Nash equilibrium is $\overline{P} = C'(\overline{X})$, $\overline{PX} - C(\overline{X}) = 0$, $K\overline{X} = Nd(\overline{P})$, where K is the measure of the firms.

If one wants to take this route, there is an issue which must be faced. In what sense can the behavior of a continuum of firms be considered the limiting behavior of many firms? One needs to show that as the number of firms grows the behavior which we have difficulty predicting is reduced to a narrow band. Prices may not equal $\overline{P}$ and output $\overline{X}$, but $\overline{P} - \varepsilon < P_i < \overline{P} + \varepsilon$, $\overline{X} - \varepsilon < X_i < \overline{X} + \varepsilon$ for all i and ever smaller ε. Why does the infinitesimal deviant firm face zero demand but have an infinitesimal amount to sell? Is the Nash equilibrium a reasonable solution concept for infinitesimal firms? It is worthwhile for firms to form a coalition of positive mass and deviate together. Why does ruling out such coalitions provide a more reasonable description of reality? Suppose firms are in the process of learning the structure of the game, and some firms try deviating from $P = \overline{P}$, $X = \overline{X}$. If the deviants have a positive mass, they will not observe an infinitely elastic demand.

Because of the above problems with a continuum of firms, we will continue to consider firms to be finite in number and Stackelberg with respect to potential entrants. It is shown in the next section that under demand uncertainty such firms do not set equal prices. The firms being a continuum does not affect this result.

II. Aggregate Demand Uncertainty

Let us turn now to the case where there is uncertain aggregate demand so that the market does not always clear. Consumers still have zero search costs, so the only source of randomness to the firm is the aggregate demand variation. Aggregate demand is $Nd(P,U)$ where U is a random variable and $\partial d/\partial U > 0$. Firms set X and P before observing U.

This problem differs in an important way from the preceding one. In both problems the payoff function is discontinuous. If all firms set the same price, $\overline{P}$, a deviant can capture the whole market by setting $P < \overline{P}$. In the previous problem we could rule this out by having firms sell all they want, $\overline{X}$, at $P = \overline{P}$. In the uncertainty case firms will not necessarily be able to sell all they want. By decreasing price an arbitrarily small amount the deviant can hugely increase his probability of selling. Consider the perfect competition solution with $Nd(\overline{P},U') = K\overline{X}$, where $Prob(U<U') > 0$. If realized $U \geq U'$ profits are zero, and if $U < U'$ profits are negative. Therefore, expected profits are negative and this is not an equilibrium.

In the certainty case an upward deviant or potential entrant setting $P > \overline{P}$ could be cut out of the market by K firms setting $P = \overline{P}$, $C'(\overline{X}) = \overline{P}$, $\overline{PX} - C(X) = 0$, $K\overline{X} = d(\overline{P})$. In the uncertainty case the upward deviant cannot be cut out of the market with certainty. Suppose K, $\overline{P}$, and $\overline{X}$ satisfy $Prob\{Nd(\overline{P},U)<K\overline{X}\} = 0$ so that the downward deviant can be

ruled out. Then $\text{Prob}\{Nd(\overline{P},U)<K\overline{X}\} > 0$ except for the degenerate (certainty) case. There is motivation for a potential entrant to enter as there is positive probability of facing an upward sloping demand curve. Indeed, even if there are a continuum of firms, the entrant has some monopoly power.

These results suggest our proposed equilibrium. For simplicity let us assume that if a consumer can buy some of his demand at a given price, but not all, his Fairy God Mother will sell to him the rest of what he wants at that price. Moreover, if firms set the same price, customers will spread themselves evenly between firms.[4/]

Let U^* satisfy the property $\text{Prob}\{U<U^*\} = 0$ and if $\text{Prob}\{U<U'\} = 0$ then $U' \leq U^*$. K_0 firms set P_0, X_0 where $P_0 = C'(X_0)$, $P_0X_0 - C(X_0) = 0$, $K_0X_0 = Nd(P_0,U^*)$. These K_0 firms live in a certainty world and they will not deviate downward. Next, there is a firm with price equals $P_1 > P_0$, which sets output to maximize expected profits given price equals P_1. If P_1 is too close to P_0, the firm will have negative expected profits. P_1 is set at the smallest value such that the firm gets non-negative expected profits. Next there is a firm at $P_2 > P_1$ setting the expected profit maximizing output given price equals $P_2 > P_1$, and P_2 is the smallest nonnegative expected profit point above P_1. The "stacking" continues until the number of firms equals $K = K_0 + K_1$ where $P_{K_1} > P_{K_1-1}$. P_{K_1} is the smallest nonnegative expected profits price above P_{K_1-1}, and price equals P_{K_1+1} is a negative expected profit position for all prices $P_{K_1+1} > P_{K_1}$ given price and output of the K firms. If there is no U^* satisfying the above property, then all firms are of this second

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4/ The result of a lexicographic ordering--customers are not gregarious.

type. If there is no P_1 satisfying the above property, then all firms are of the first type.

Consider the second Stackelberg subgame. A firm has no motive to change its price or output. If a firm raises its price, it can be undercut by a potential entrant and will necessarily face negative expected profits. If the firm decreases price, it gets negative expected profits. For the same reason there is no motivation for a potential entrant to produce. Therefore, in the first subgame there is no reason for a potential entrant to become a firm. As expected profits equal zero, there is no reason for a firm to leave the market.

Our solution consists of a group of firms in a world of certainty acting like perfect competitors. Stacked above them are firms acting as monopolists, but constrained to prices that yield zero expected profits. This is the only equilibrium.[5/] These firms are price setters, but only in a narrow sense. There is only one price that a firm can set that will yield nonnegative expected profits, given other economic agents' decisions.

In perfect competition the auctioneer is instructed to equate supply and demand. In this setting such an instruction is impossible to follow. Therefore, there is no exact perfect competition analog to this problem. Let us change the instructions to the auctioneer. The auctioneer designates prices and firms and these designated firms are price takers. The auctioneer is to set prices so that no firm drops out of the market, and no (price setting) potential entrant enters the market. The prices

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[5/] The reader can convince himself of this by considering any other proposed pure or mixed strategy solution. The potential entrants will reduce all firms to nonpositive expected profits and some firms to negative expected profits.

the auctioneer sets are the solution described above. Once again, the distinction between price setter and price taker is unimportant. The auctioneer is an unncessary fiction.

Let us look at the problem of the "monopolist" firms, say firm $j-1+K_0$, setting price P_j. Let v_0 be the number of customers satisfied by the first K_0 firms, v_1 the number of customers satisfied by firm $K_0 + 1$, and so on, given these outputs and prices and U. Let g_j be the value of U such that the firms with prices below P_j just satisfy all their customers. g_j is a function of these lower prices and associated outputs. Let h_j be the value of U such that the firms setting prices below P_j and firm $j-1+K_0$ setting price P_j together just satisfy all customers. h_j is a function of the arguments of g_j and of P_j and X_j as well. Assume U has a probability density function f(U).

The firm's problem can be decomposed as

(A)
$$\phi(P_j) = \max_{X_j} P_i \int_{g_j}^{h_j} (N-\sum_{m=1}^{j-1} v_m) d(P,U) f(U) dU$$
$$+ P_j X_j \int_{h_j}^{\infty} f(U) dU - C(X_j)$$

(B)
$$\phi(P_j) = 0.$$

In part (A) of the problem the first-order condition on X_j is

$$P_j \int_{h_j}^{\infty} f(U) dU = C'(X_j)$$

as the effects of X_j on h_j cancel. The zero profit condition is:

$$P_j \int_{g_j}^{h_j} (N-\sum v_m) d(P_j,U) f(U) dU + P_j X_j \int_{h_j}^{\infty} f(U) dU - C(X_j) = 0$$

where X_j is at its optimal value. Dividing by X_j and substituting in the first-order condition yields:

$$\frac{P_j}{X_j}\int_{g_j}^{h_j}(N-\sum v_m)d(P_j,U)f(U)dU + C'(X_j) = \frac{C(X_j)}{X_j} .$$

The firm will produce to the "left" of the minimum of its average cost curve.

Let us assume that firms are numerous and small. If this holds

$$\int_{g_j}^{h_j} f(U)dU \approx 0$$

and

$$C'(X_j) \approx \frac{C(X_j)}{X_j} .$$

If the probability of selling some but not all output is near zero, the market will consist of firms producing near the minimum of their average cost curves. However, these firms will be selling at different prices with different probabilities of selling all or nothing. The perfect competition result of equal prices does not appear in a market with uncertain demand even if there is free entry and firms are small and numerous. Rather the products of price and probability of selling are equal. Nonstochastic aggregate demand is crucial to the equality of prices.

In such a market the range of prices can be taken as a measure of the uncertainty of demand over the interval for which prices are set. A dispersion of prices in a market cannot be taken as proof of lack of competition or of costly search or of different information sets. In our market temporary fluctuations in aggregate demand will result in fluctuations in the index of output price, if correctly measured, but this occurs with constant prices of individual firms. Changes in aggregate demand will be reflected in inventories.

III. The Multiperiod Problem

If one wants to solve the multiperiod problem for goods producing firms, inventories should be included in the model. The presence of inventories clouds the picture. With inventories firms will be solving different problems and thus will differ in an essential way. To determine what will happen the distribution of inventories must be derived.

The problem of inventories can be swept under the rug if some strong assumptions are made. Let us assume that firms all have the same cost function of holding inventories, and the cost function is linear. Further, suppose that firms are small so that the probability of selling some output but not all can be treated as zero. Under these circumstances the firm's value will be linear in inventories and its decisions independent of its inventory stock.[6/] The two parameters (intercept, slope) determining the firm's cost of inventory holding will influence its decision. However, these parameters are assumed the same for each firm, and will just enter in determining the zero profit condition.

If these strong assumptions are not made, more structure must be put on the model to determine a solution. If costs are convex in inventories, one anticipates that the market may be characterized by churning. Speculators, who set high prices in hopes of high demand, after a series of weak or normal demand cut price in order to liquidate inventories. Those firms with low inventories due to low price become higher priced speculators.

[6/] Bryant, John, "Demand Anticipation and Speculation on Inventories: The Decisions of a Price Setting Firm" (mimeographed).

In the multiperiod problem with inventories, a new lower bound on a firm's price is introduced. No firm sets a price below its expected discounted marginal worth of inventories next period. This is a result similar to the submartingale property of efficient markets.